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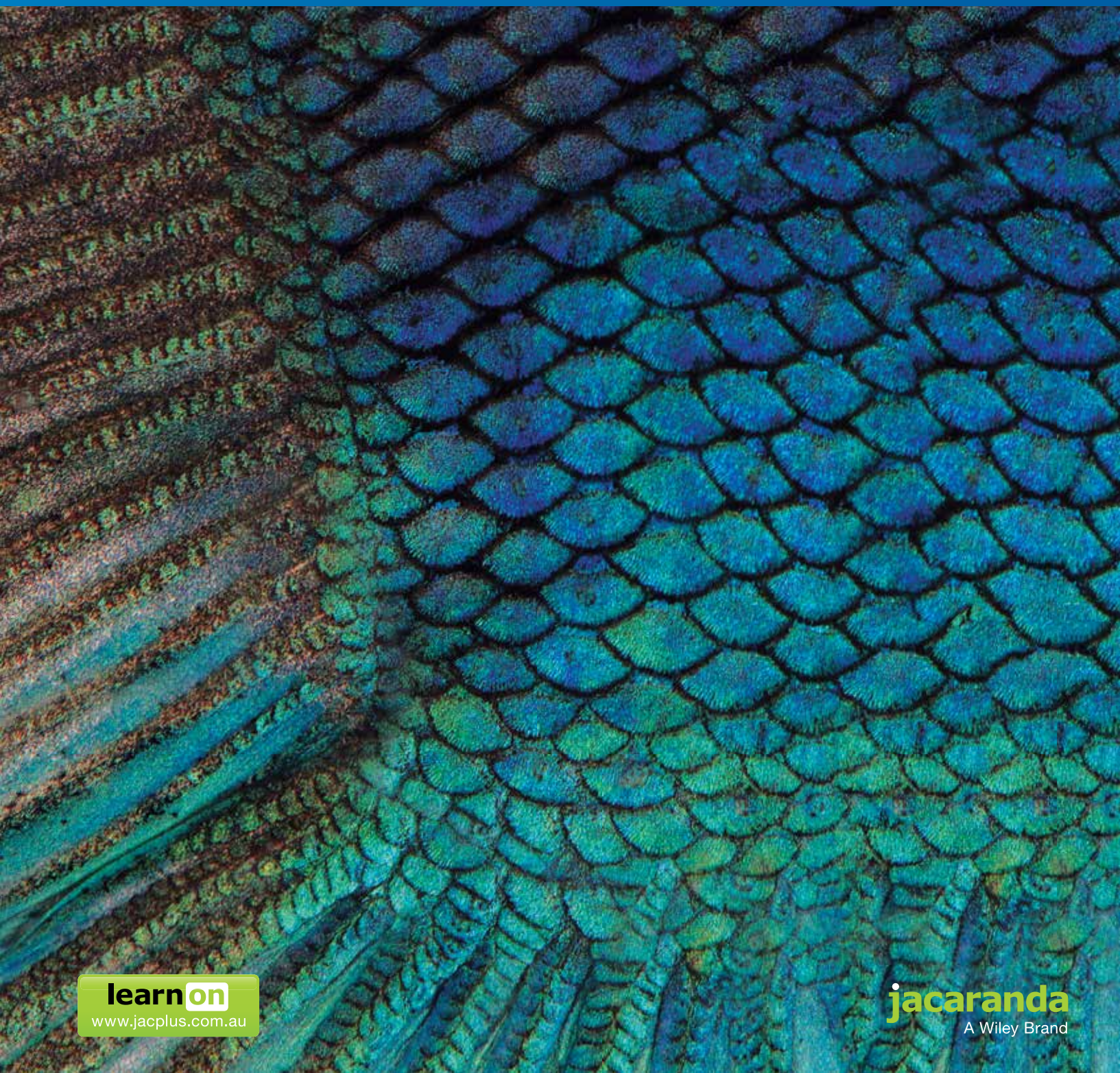
MATHS QUEST 7

AUSTRALIAN CURRICULUM | THIRD EDITION



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AUSTRALIAN CURRICULUM | THIRD EDITION

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ABOUT THIS RESOURCE

Jacaranda Maths Quest 7 Australian Curriculum Third Edition has been completely revised to help teachers and students navigate the Australian Curriculum Mathematics syllabus. The suite of resources in the Maths Quest series is designed to enrich the learning experience and improve learning outcomes for all students.

Maths Quest is designed to cater for students of all abilities: no student is left behind and none is held back. Maths Quest is written with the specific purpose of helping students deeply understand mathematical concepts. The content is organised around a number of features, in both print and online through Jacaranda's *learnON* platform, to allow for seamless sequencing through material to scaffold every student's learning.

NUMBER AND ALGEBRA

TOPIC 4
Rational numbers

4.1 Overview

Numerous **videos** and **interactivities** are embedded just where you need them, at the point of learning, in your learnON title at www.jacplus.com.au. They will help you to learn the content and concepts covered in this topic.

4.1.1 Why learn this?

There are many everyday situations where we do not work with whole numbers. Fractions are a way of expressing parts of whole things, amounts or quantities. They are important in everyday situations such as measuring, cooking, shopping, telling the time and sport. Did you know that approximately $\frac{1}{7}$ of the population of Australia lives in Victoria?

4.1.2 What do you know?

assessON

1. **THINK** List what you know about fractions. Use a thinking tool such as a concept map to show your list.
2. **PAIR** Share what you know with a partner and then with a small group.
3. **SHARE** As a class, create a thinking tool such as a large concept map that shows your class's knowledge of fractions.

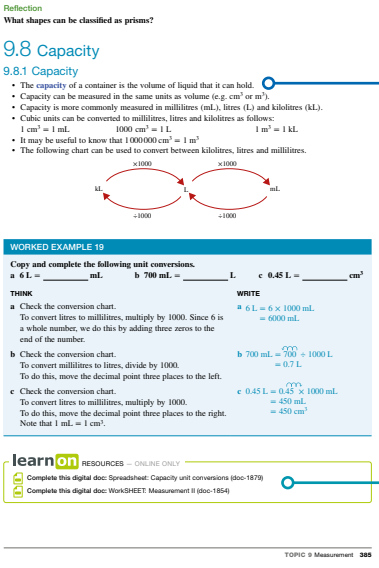
LEARNING SEQUENCE

4.1 Overview
4.2 What are rational numbers?
4.3 Simplifying rational numbers
4.4 Mixed numbers and improper fractions
4.5 Adding and subtracting rational numbers
4.6 Multiplying rational numbers
4.7 Dividing rational numbers
4.8 Fractions as ratios
4.9 Working with mixed numbers
4.10 Review

TOPIC 4 Rational numbers 91

Topic introductions put the topic into a real-world context.

The learning sequence at a glance


Reflection

What shapes can be classified as prisms?

9.8 Capacity

9.8.1 Capacity

- The **capacity** of a container is the volume of liquid that it can hold.
- Capacity can be measured in the same units as volume (e.g. cm^3 or m^3).
- Capacity is more commonly measured in millilitres (mL), litres (L) and kilolitres (kL).
- Cubic units can be converted to millilitres, litres and kilolitres as follows:
  $1\ cm^3 = 1\ mL$ $1000\ cm^3 = 1\ L$ $1\ m^3 = 1\ kL$
- It may be useful to know that $1\,000\,000\ cm^3 = 1\ m^3$
- The following chart can be used to convert between kilolitres, litres and millilitres.

×1000 ×1000
kL L mL
÷1000 ÷1000

WORKED EXAMPLE 19

Copy and complete the following unit conversions.
a 6 L = _____ mL **b** 700 mL = _____ L **c** 0.45 L = _____ cm^3

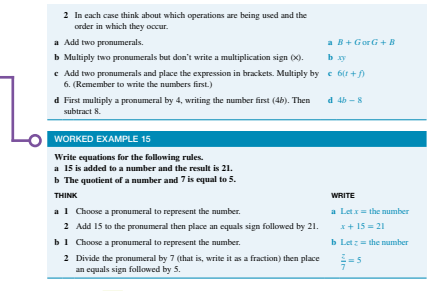
THINK	WRITE
a Check the conversion chart. To convert litres to millilitres, multiply by 1000. Since 6 is a whole number, we do this by adding three zeros to the end of the number.	**a** 6 L = 6 × 1000 mL = 6000 mL
b Check the conversion chart. To convert millilitres to litres, divide by 1000. To do this, move the decimal point three places to the left.	**b** 700 mL = 700 ÷ 1000 L = 0.7 L
c Check the conversion chart. To convert litres to millilitres, multiply by 1000. To do this, move the decimal point three places to the right. Note that 1 mL = 1 cm^3.	**c** 0.45 L = 0.45 × 1000 mL = 450 mL = 450 cm^3

learnON RESOURCES — ONLINE ONLY
Complete this digital doc: Spreadsheet: Capacity unit conversions (doc-1879)
Complete this digital doc: WorkSHEET: Measurement II (doc-1854)

TOPIC 9 Measurement 385

An extensive glossary of mathematical terms in print, and as a hover-over feature in your *learnON* title

Visit your *learnON* title to watch videos which tell the story of mathematics.


2 In each case think about which operations are being used and the order in which they occur.

a Add two pronumerals. **a** $B + G$ or $G + B$
b Multiply two pronumerals but don't write a multiplication sign (x). **b** xy
c Add two pronumerals and place the expression in brackets. Multiply by 6. (Remember to write the numbers first.) **c** $6(t + f)$
d First multiply a pronumeral by 4, writing the number first ($4b$). Then subtract 8. **d** $4b - 8$

WORKED EXAMPLE 15

Write equations for the following rules.
a 15 is added to a number and the result is 21.
b The quotient of a number and 7 is equal to 5.

THINK	WRITE
a 1 Choose a pronumeral to represent the number.	**a** Let x = the number
2 Add 15 to the pronumeral then place an equals sign followed by 21.	$x + 15 = 21$
b 1 Choose a pronumeral to represent the number.	**b** Let z = the number
2 Divide the pronumeral by 7 (that is, write it as a fraction) then place an equals sign followed by 5.	$\frac{z}{7} = 5$

learnON RESOURCES — ONLINE ONLY
Complete this digital doc: SkillSHEET: Alternative expressions used to describe the four operations (doc-6502)

Exercise 8.5 Terms, expressions and equations assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1, 2, 3, 4, 5a–h, 6, 7, 8, 9, 10, 14, 16	Questions: 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 16	Questions: 5, 6, 7, 8, 9, 10, 12, 13, 15, 16, 17

Individual pathway interactivity: int-4352 learnON ONLINE ONLY

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Fluency

1. WRITE Write an expression for the total number of coins in each of the following, using c to represent the number of coins in a moneybox.
a. b. c.
d. e.

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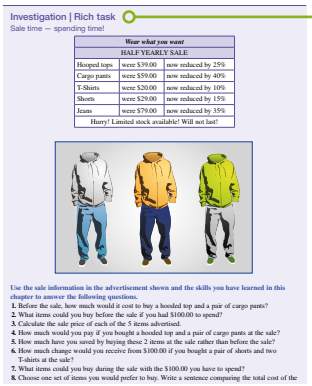
Fully worked examples throughout the text demonstrate key concepts.

Your FREE online *learnON* resources contain hundreds of videos, interactivities and traditional WorkSHEETs and SkillSHEETs to support and enhance learning.

Carefully graded questions cater for all abilities. Question types are classified according to strands of the Australian Curriculum.

Individual pathway interactivities in each sub-topic ensure consolidation of learning for every skill level.

Fully worked solutions to every question are provided online, and answers are provided at the end of each print topic.


Investigation | Rich task

Sale time — spending time!

Wear what you want	
HALF YEARLY SALE	
Hooped tops	were $39.00	now reduced by 25%
Cargo pants	were $59.00	now reduced by 40%
T-Shirts	were $20.00	now reduced by 10%
Shorts	were $29.00	now reduced by 15%
Jeans	were $79.00	now reduced by 35%
Hurry! Limited stock available! Will not last!		

Use the sale information in the advertisement shown and the skills you have learned in this chapter to answer the following questions.
1. Before the sale, how much would it cost to buy a hooded top and a pair of cargo pants?
2. What items could you buy before the sale if you had $100.00 to spend?
3. Calculate the sale price of each of the 5 items advertised.
4. How much would you pay if you bought a hooded top and a pair of cargo pants at the sale?
5. How much have you saved by buying these 2 items at the sale rather than before the sale?
6. How much change would you receive from $100.00 if you bought a pair of shorts and two T-shirts at the sale?
7. What items could you buy during the sale with the $100.00 you have to spend?
8. Choose one set of items you would prefer to buy. Write a sentence comparing the total cost of the

4.10 Review

4.10.1 Review questions

Fluency

1. What fraction of the eggs in this carton are brown?
2. Write the sequence of the first three equivalent fractions for the following.
a. $\frac{1}{3}$ b. $\frac{2}{5}$ c. $\frac{3}{10}$ d. $\frac{5}{8}$
3. Fill the gaps to make equivalent fractions.
a. $\frac{1}{2} = \frac{\square}{4} = \frac{4}{\square}$ b. $\frac{1}{5} = \frac{\square}{10} = \frac{\square}{15}$ c. $\frac{1}{7} = \frac{2}{\square} = \frac{\square}{21}$
d. $\frac{2}{3} = \frac{\square}{12} = \frac{16}{\square}$ e. $\frac{1}{4} = \frac{\square}{20} = \frac{\square}{100}$ f. $\frac{1}{10} = \frac{\square}{100} = \frac{200}{\square}$
4. Ricky, Leo and Gabriel were the highest goal scorers for their local football team throughout the season. Ricky scored $\frac{1}{2}$ of the team's goals, Leo scored $\frac{1}{4}$ of the goals and Gabriel scored $\frac{1}{8}$ of the goals.
a. Who scored most of the goals?
b. Who scored more goals; Ricky or Gabriel?
5. Simplify these fractions.
a. $\frac{6}{10}$ b. $\frac{10}{12}$ c. $\frac{16}{20}$ d. $\frac{10}{15}$
e. $\frac{36}{48}$ f. $\frac{200}{250}$ g. $\frac{70}{100}$ h. $\frac{2}{10}$
6. Simplify the following mixed numbers.
a. $6\frac{4}{10}$ b. $3\frac{6}{8}$ c. $4\frac{12}{16}$ d. $1\frac{60}{100}$
7. What fraction of the months of the year begin with the letter J? Express your answer in simplest form.
8. Year 7 students at Camberwell Secondary College raised a total of $1600 for the Royal Children's Hospital Good Friday appeal. The table below shows how much money each class raised.

Class	7.1	$200	7.2	$400	7.3	$250
Amount raised ($)	7.4	$250	7.5	$280	7.6	$220

Express as a simple fraction how much of the total each class raised.
9. Convert these improper fractions to mixed numbers.
a. $\frac{10}{3}$ b. $\frac{30}{8}$ c. $\frac{50}{9}$ d. $\frac{11}{7}$
e. $\frac{7}{2}$ f. $\frac{60}{8}$ g. $\frac{60}{9}$ h. $\frac{72}{10}$

Engaging Investigations at the end of each topic to deepen conceptual understanding

Each topic concludes with comprehensive Review questions, in both print and online.

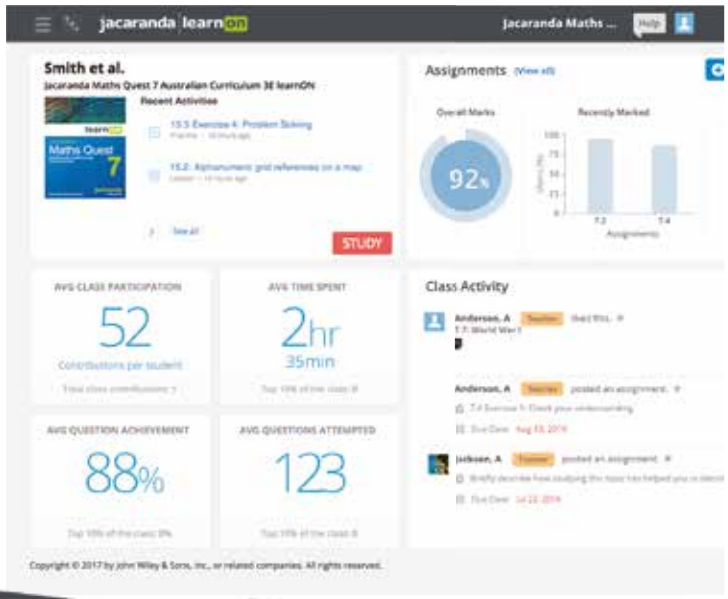
learnON

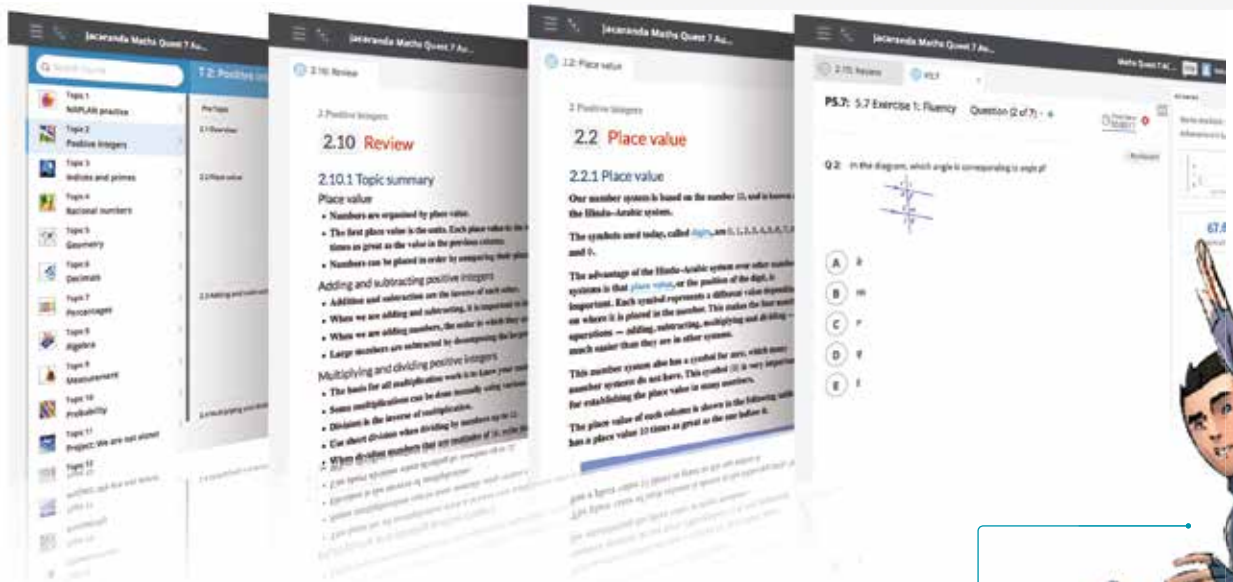
LearnON is Jacaranda's immersive and flexible digital learning platform that transforms trusted Jacaranda content to make learning more visible, personalised and social. Hundreds of engaging videos and interactivities are embedded just where you need them — at the point of learning. At Jacaranda, our 'learning made visible' framework ensures immediate feedback for students and teachers, with customisation and collaboration to drive engagement with learning.

Maths Quest contains a free activation code for *learnON* (please see instructions on the inside front cover), so students and teachers can take advantage of the benefits of both print and digital, and see how *learnON* enhances their digital learning and teaching journey.

learnON includes:

- Students and teachers connected in a class group
- Hundreds of videos and interactivities to bring concepts to life
- Fully worked solutions to every question
- Immediate feedback for students
- Immediate insight into student progress and performance for teachers
- Dashboards to track progress
- Collaboration in real time through class discussions
- Comprehensive summaries for each topic
- Code puzzles and dynamic interactivities help students engage with and work through challenging concepts
- Formative and summative assessments
- And much more …





Also available for purchase is **SpyClass**. An online game combining comic book-style art with problem based learning

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Text

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TOPIC 1
Numeracy 1

1.1 Overview

Numerous **videos** and **interactivities** are embedded just where you need them, at the point of learning, in your learnON title at www.jacplus.com.au. They will help you to learn the content and concepts covered in this topic.

1.1.1 Why learn this?

Understanding numeracy helps us make sense of mathematical ideas and how they relate to everyday life.

Numeracy includes counting, addition, subtraction, multiplication and division, fractions, decimals, measuring, understanding shapes and graphs.

1.1.2 What do you know?

assessON

1. **THINK** List what you know about numeracy. Use a thinking tool such as a concept map to show your list.
2. **PAIR** Share what you know with a partner and then with a small group.
3. **SHARE** As a class, create a thinking tool such as a large concept map that shows your class's knowledge of numeracy.

LEARNING SEQUENCE

1.1 Overview
1.2 Set A
1.3 Set B

learnON RESOURCES — ONLINE ONLY

Watch this eLesson: NAPLAN: strategies and tips (eles-1688)

1.2 Set A

1.2.1 Calculator allowed

1. Our feet produce approximately 100 litres of sweat a year. Approximately how many millilitres of sweat do your feet produce each day?

A. 270 mL **B.** 27 mL
C. 37 mL **D.** 2.7 mL

2. The brain grows at an amazing rate during development. At times during development 250 000 neurons (nerve cells) are added every minute. If neurons multiplied at the given rate for an hour, how many neurons would be added?

A. 1 500 000 **B.** 15 000 000 **C.** 6 000 000 **D.** 600 000

3. Nerve impulses to and from the brain travel as fast as 274 metres every second. Which is the closest estimate of the number of metres a nerve impulse travels in an hour?

A. 970 000 **B.** 980 000 **C.** 990 000 **D.** 100 000

4. On average, human beings blink their eyes 6 205 000 times each year. How many times do you blink in a day?

5. How much timber would be left if eleven 1.75 m lengths are cut from a 22 m piece of timber?

A. 3.25 **B.** 2.25
C. 2.35 **D.** 2.75

6. On three tests, a student scored a total of 255 marks. If the student improved her performance on each successive test by 4 marks, what was the mark for each test?

A. 80, 84, 88
B. 81, 85, 89
C. 78, 86, 91
D. 79, 83, 87

7. A student scored 42 out of 50 for a Maths test. What percentage did she obtain for the Maths test?

A. 82% **B.** 84% **C.** 75% **D.** 16%

8. In a notebook, an author writes a short story of 4000 words. He can type 125 words in three minutes. Keeping the same rate, how long would it take him to type the story?

A. 32 minutes **B.** $41\frac{2}{3}$ minutes **C.** 96 minutes **D.** $10\frac{2}{3}$ minutes

9. In a soccer team, $\frac{1}{3}$ of the players ride a bike to practice, 25% walk to practice and the remaining 15 players are driven by their parents. How many players are there in the team?

A. 42 **B.** 36 **C.** 30 **D.** 21

10. Jonathan is saving his money for a two-week surfing trip to Hawaii. He has estimated the costs of his adventure to be:

- return airfare — $2000
- accommodation — $1400
- food — $500
- extra costs — $400.

If Jonathan can earn $80 per day laying bathroom tiles, how many full days of work does he need to complete in order to save the money for the trip?

11. A bottle containing 1 litre of lemonade was poured into big glasses of volume 120 mL and small glasses of volume 80 mL. How much lemonade was left in the bottle after 5 big glasses and 4 small glasses of lemonade had been poured out?

A. 620 mL **B.** 378 mL **C.** 280 mL **D.** 80 mL

12. There are 184 students who need to be seated for a dinner in a hall. If all the tables need to be full, select whether the school should use:

A. 18 tables **B.** 19 tables **C.** 23 tables **D.** 31 tables

13. The following table shows the cost of road tax for motor cars.

Engine capacity	Road tax
1001 to 1600 cc	75 cents per cc
1601 to 2000 cc	85 cents per cc
2001 to 2600 cc	95 cents per cc

The road tax on a motor car of engine capacity 2400 cc is:

A. 95 cents **B.** $950 **C.** $228 **D.** $2280

14. A waiter works 40 hours in order to earn $640. Assuming the same rate of pay per hour, calculate how much she will earn in 15 hours.

15. There is space in a multistorey car park for 15 rows of 20 cars on each of the 8 floors. How many spaces are left if 2124 cars have already parked?

16. Your friend is training for a triathlon. On one training day she swam 1500 m, rode a bike 47 000 m and ran 14 km. How many kilometres did she complete?

17. $5320 was raised by a group of students to give to 38 refugees. How much does each refugee receive?

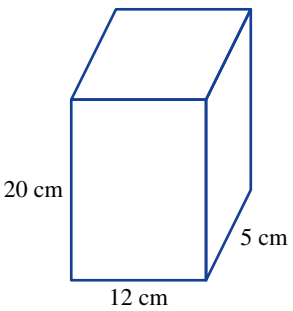
18. A plane leaves Melbourne at 0935 and arrives in Osaka at 2007 (Melbourne time). How long did the trip take?

A. 9 hours 42 minutes
B. 10 hours 72 minutes
C. 9 hours 28 minutes
D. 10 hours 32 minutes

19. During the day, a patient had her temperature measured as 37.3 °C, 38 °C, 37.8 °C and 38.5 °C. Her average temperature was:

A. 37.8 °C **B.** 37.9 °C **C.** 38 °C **D.** 38.2 °C

20. Boxes for packaging are in the shape of a cuboid, as shown. A company offers a promotion of an extra 20% volume. What is the new volume?



A. 1200 cm^3
B. 240 cm^3
C. 1440 cm^3
D. 81.4 cm^3

21. A couple bought a laptop with an interest-free loan of $2400. They paid a deposit of $800, and paid off the remainder over the next 5 months. How much were the monthly repayments?

A. $480
B. $320
C. $160
D. $200

22. A restaurant served 315 meals. One-third of the people ate a two-course meal while the remainder ate a three-course meal. How many courses did the chef prepare?

A. 840 **B.** 210 **C.** 630 **D.** 1575

23. The following is the exchange rate as given by Travelex.

	A$1
US$	0.92
€	0.58

You have $1500 in Australian currency and you want to change $500 to US currency and the remainder to Euros. How much will you receive of each currency?

24. At the Healesville supermarket, shopping trolleys are 1 metre long. You work collecting them and stacking them into an area. Each shopping trolley that is added to the stack adds on 25 cm. How long would the line be if you stacked 20 trolleys?

A. 6 m **B.** 5.75 m **C.** 60 m **D.** 57.5 m

25. At another supermarket, shopping trolleys are also 1 metre long. Your friend works collecting them and stacking them into an area. Each shopping trolley in the stack adds on 0.25 m. How many trolleys could be stacked in a single line of 10 metres?

A. 37 **B.** 36 **C.** 35 **D.** 34

26. There are 128 Year 7 students. Three-quarters of them own a pet. A survey shows that 37 students have cats, $\frac{1}{3}$ of the students who own a pet have dogs and the remaining students have horses. How many have horses?

27. How many 55 cent stamps can you buy for $10?

A. 2 **B.** 18 **C.** 11 **D.** 19

28. In 1969 the first 12-sided 50 cent coin (as shown) came into existence. It weighs 15.55 g. Coins are usually stored in bags of 400 coins for transport. How much would the bag weigh (in kilograms)?

29. Clare buys a fashion magazine for $10.25 and chocolates for $3.99. She pays with a $20 note. How much change does she receive? (Assume rounding to the nearest 5c.)

A. $6.75 **B.** $5.80 **C.** $5.76 **D.** $5.75

30. What number would you insert into the following expression to get an answer of 19?

$5 \times 3 + _ \div 6$

1.3 Set B

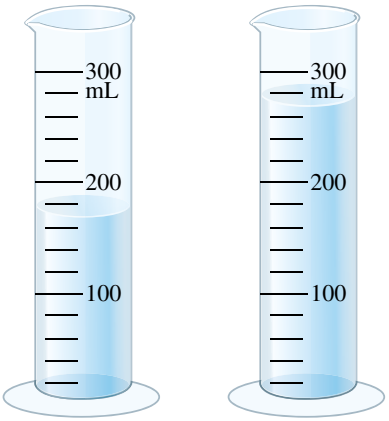
1.3.1 Non-calculator

1. The figure 2 010 415 is closest to:

A. 2 000 000 **B.** 2 010 000 **C.** 2 020 000 **D.** 2 100 000

2. Jennifer's softball match will take 90 minutes from start to finish, including breaks. Her team must finish by 12.20 pm. What is the latest time the match can start?

3. Jug A and Jug B contain different amounts of water. How much more water is in Jug B?



Jug A Jug B

4. What is the probability of choosing a number from the following table that is divisible by 7?

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25

A. $\frac{1}{25}$ **B.** $\frac{2}{25}$ **C.** $\frac{3}{25}$ **D.** $\frac{4}{25}$

5. Human blood travels 675 780 km per week on its journey through the body. How far does the blood travel, to the nearest thousand km, in one week?

A. 670 000 **B.** 700 000 **C.** 675 000 **D.** 676 000

6. The average human heart will beat three thousand million times in its lifetime. This can be represented as:

A. 3×10^6 **B.** 3×10^7

C. 3×10^8 **D.** 3×10^9

7. A chef made ten apple pies. For one apple pie he used $\frac{7}{12}$ of a tablespoon of cinnamon and $\frac{5}{12}$ of a tablespoon of nutmeg. How much more cinnamon than nutmeg was used for all the apple pies?

A. $\frac{1}{6}$ **B.** $\frac{5}{6}$

C. $\frac{5}{3}$ **D.** $1\frac{1}{3}$

8. Hue leaves home and rides on a bike 4 km west, then 2 km north, followed by 4 km east and finally 5 km south. Which direction and how far must Hue ride to get back home?

A. South 3 km **B.** South 1 km

C. North 3 km **D.** North 1 km

9. Peta has 36 small buttons and n large buttons. Which expression shows how many buttons Peta has altogether?

A. $n - 36$ **B.** $36 - n$
C. 36 **D.** $n + 36$

10. Of the 48 children in the park, $\frac{3}{4}$ are wearing a hat. How many children are not wearing a hat?

A. 12 **B.** 36
C. 1 **D.** 48

11. Three-quarters of the students on the athletics team compete in the high jump. Of the students who do the high jump, one-half also do the long jump. What fraction of the athletics team does the long jump?

A. $\frac{1}{2}$ **B.** $\frac{3}{8}$ **C.** $\frac{2}{3}$ **D.** $\frac{3}{4}$

12. The Ambelfield aquifer pumps 15 million litres per day from its bore field. If the water is to be shared among the 200 000 residents, how many litres would each person receive per day?

A. 750 L
B. 75 L
C. 30 L
D. 7.5 L

13. The Ambelfield aquifer is estimated to hold 50 billion litres. Assuming no addition or depletion of water, for approximately how many days could water be pumped if 15 million litres were extracted each day?

A. 34 **B.** 340
C. 3400 **D.** 34 000

14. A boy borrowed two-fifths of his friend's CD collection. If he borrowed 20 CDs, how many were in the collection?

A. 50 **B.** 70
C. 8 **D.** 24

15. On a map, the distance between Port Fairy and Warrnambool is represented by 10.3 cm. What is the actual distance if 1 cm represents 7 km?

A. 17.3 km **B.** 103 km
C. 72.1 km **D.** 70 km

16. Seven cubic metres can be written as:

A. $(7\text{ m})^3$ **B.** 7 m^3 **C.** 21 m **D.** 343 m

17. At the local Sunday market, you have sold $\frac{2}{3}$ of your mangoes. You started with 24 boxes, each containing 20 mangoes. How many mangoes are left?

A. 320 **B.** 160 **C.** 16 **D.** 32

18. Six thousand people attended a tennis match. If $\frac{3}{10}$ of them entered with a free pass, how many people paid for a ticket?

A. 1800 **B.** 600 **C.** 2000 **D.** 4200

19. The difference between the volumes of water in two tanks is 436 L. If the smaller tank has a volume of 7844 L, what is the volume of water in the larger tank?

20. A nurse takes your temperature at 9 am and then again at 4 pm. What is the difference in the temperature?

A. 0.3 °C **B.** 0.1 °C
C. 0.4 °C **D.** 1.1 °C

21. It takes Lyng 15 minutes to walk to school. If school begins at 9.03 am, what time must she leave home so that she can get to school just as school begins?

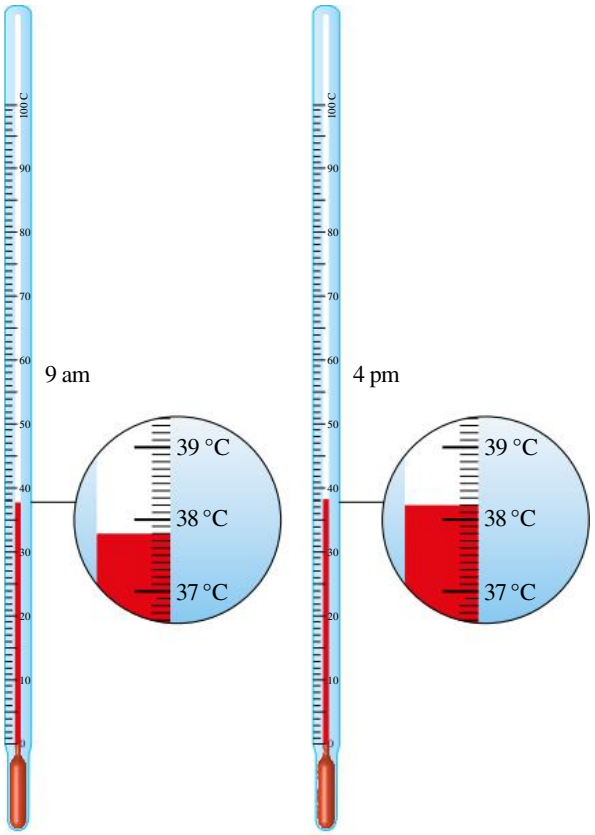
22. There are 42 apples in a box. If you picked up 28 apples, what fraction of apples would be left in the box?

A. One-quarter **B.** One-third
C. One-half **D.** Three-quarters

23. It takes Shannon exactly three hours to walk 18 km. How long would it take Shannon to walk (assuming the same speed) 30 km?

24. A piece of rope had 160 cm cut from the end. The rope that remained measured 75 metres. How long was the original piece of rope?

25. Jarrod goes to sleep at 2130 and wakes at 6:00 am. His brother goes to sleep at 2215 and sleeps for the same length of time that Jarrod did. What time does he wake up?



26. A student works in a shop each day from 14 January to 24 January, inclusive. The student is paid $40 for each day. How much does he earn in total?

27. You can walk 100 m per minute and can run 200 m per minute. How far can you travel if you walk for 20 minutes and run for 15 minutes?

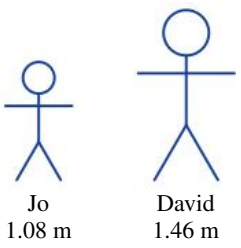
A. $20 \times 100 + 15 \times 200$ **B.** $20 \times 200 - 15 \times 200$
C. $15 \times 100 + 20 \times 200$ **D.** $20 \times 200 + 15 \times 100$

28. A DVD recorder was set to begin taping a show at 1835. It ended at 2015. How long was the show?

A. 110 min **B.** 100 min **C.** 85 min **D.** 80 min

29. What is the difference between Jo's height and David's height?

A. 0.32 m **B.** 0.38 m
C. 0.42 m **D.** 0.48 m



30. A sketchbook contains 100 pieces of paper and is 14 mm thick. What is the thickness of one sheet of paper?

A. 1.4 mm **B.** 10 mm
C. 0.14 mm **D.** 7.14 mm

Answers

Topic 1 Numeracy 1

1.2 Set A

1.2.1 Calculator allowed

1. A
2. B
3. C
4. 17 000
5. D
6. B
7. B
8. C
9. B
10. 54 days
11. D
12. C
13. D
14. $240
15. 276
16. 62.5 km
17. $140
18. D
19. B
20. C
21. B
22. A
23. US$460, €580
24. B
25. A
26. 27
27. B
28. 6.22 kg
29. D
30. 24

1.3 Set B

1.3.1 Non-calculator

1. B
2. 10.50 am
3. 100 mL
4. C
5. D
6. D
7. C
8. C
9. D
10. A
11. B
12. B
13. C
14. A
15. C
16. B
17. B
18. D
19. 8280 L
20. C
21. 8:48 am
22. B
23. 5 hours
24. 76.6 m
25. 6:45 am
26. $440
27. A
28. B
29. B
30. C

TOPIC 2
Positive integers

2.1 Overview

Numerous **videos** and **interactivities** are embedded just where you need them, at the point of learning, in your learnON title at www.jacplus.com.au. They will help you to learn the content and concepts covered in this topic.

2.1.1 Why learn this?

Addition, subtraction, multiplication and division are the building blocks of our understanding of Mathematics. Many of our daily activities depend on us knowing how to answer questions like 'how much?', 'how far?' and 'how many?' We use addition, subtraction, multiplication and division to answer such questions.

2.1.2 What do you know?

assessON

1. **THINK** List what you know about positive integers. Use a thinking tool such as a concept map to show your list.
2. **PAIR** Share what you know with a partner and then with a small group.
3. **SHARE** As a class, create a thinking tool such as a large concept map that shows your class's knowledge of positive integers.

LEARNING SEQUENCE

2.1 Overview
2.2 Place value
2.3 Adding and subtracting positive integers
2.4 Multiplying and dividing positive integers
2.5 Long division
2.6 Order of operations
2.7 Factors and multiples
2.8 Lowest common multiple and highest common factor
2.9 Estimation
2.10 Review

2.2 Place value

2.2.1 Place value

- Our number system is based on the number 10, and is known as the Hindu–Arabic system.
- The symbols used today, called **digits**, are 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9.
- The advantage of the Hindu–Arabic system over other number systems is that **place value**, or the position of the digit, is important. Each symbol represents a different value depending on where it is placed in the number. This makes the four number operations — adding, subtracting, multiplying and dividing — much easier than they are in other systems.
- This number system also has a symbol for zero, which many number systems do not have. This symbol (0) is very important for establishing the place value in many numbers.
- The place value of each column is shown in the following table. Working from the right, each column has a place value 10 times as great as the one before it.

Millions	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Units
1 000 000	100 000	10 000	1000	100	10	1

WORKED EXAMPLE 1

Write the following numbers in expanded notation.
a 59 176 **b 108 009**

THINK	WRITE
a 1 Read the number to yourself, stating the value of each digit.	
2 Write the number as the sum of each value.	**a** $59\,176 = 50\,000 + 9000 + 100 + 70 + 6$
b 1 Read the number to yourself, stating the value of each digit.	
2 Write the number as the sum of each value.	**b** $108\,009 = 100\,000 + 8000 + 9$

- Numbers are ordered according to their place values. For whole numbers, the number with the most digits is the greatest in value because the first digit will have the highest place value.
- If two numbers have the same number of digits, then the digits with the highest place value are compared. If they are equal, the next higher place values are compared, and so on.

WORKED EXAMPLE 2

Write the following numbers in descending order.
858 58 85 8588 5888 855

THINK	WRITE
1 Look for the numbers with the most digits.	8588 and 5888
2 There are two numbers with 4 digits. The number with the higher digit in the thousands column is larger; the other is placed second.	8588, 5888
3 Compare the two numbers with 3 digits. Both have the same hundreds and tens values, so compare the values of the units.	858, 855
4 Compare the two 2-digit numbers.	85, 58
5 Write the answer.	8588, 5888, 858, 855, 85, 58

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Complete this digital doc: SkillSHEET: Place value (doc-6409)

Complete this digital doc: SkillSHEET: Ascending and descending order (doc-6410)

Exercise 2.2 Place value

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1, 2, 3, 4, 5, 6, 7a, b, c, 8, 9a, b, 10, 13	Questions: 1, 2, 3, 4, 5, 6, 7, 8, 9a, b, 10, 11, 13	Questions: 1a, d, f, g, h, 2, 3a, d, 4, 5, 6, 7d, 8c, 9, 10, 11, 12, 13

Individual pathway interactivity: int-4311

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Fluency

1. WE1 Write the following numbers in expanded notation.

a. 925	**b.** 1062	**c.** 28 469	**d.** 43
e. 502 039	**f.** 800 002	**g.** 1 080 100	**h.** 22 222

2. Write the following numbers in words.

a. 765	**b.** 9105	**c.** 90 450	**d.** 100 236

3. Write the numeral for each of the following.

a. Four hundred and ninety-five
b. Two thousand, six hundred and seventy
c. Twenty-four thousand
d. One hundred and nine thousand, six hundred and five

4. Give the value of the digits (in brackets) in the distances to the following destinations in these road signs. Use words to state these values.

a.

i. Broken Hill (2)
ii. Yancannia (4)
iii. White Cliffs (8)

b.

i. Geelong (2)
ii. Ballarat (1)
iii. Bendigo (3)

c.

i. Katherine (0)
ii. Alice Springs (2)

d.

i. Coober Pedy (3)
ii. Alice Springs (5)
iii. Marla (7)

Understanding

5. a. MC Which of the following numbers is the largest?

A. 4884 **B.** 4488 **C.** 4848 **D.** 4844 **E.** 4888

b. Justify your answer to part **a** using mathematical reasoning.

6. MC Which of the following numbers is the smallest?

A. 4884 **B.** 4488 **C.** 4848 **D.** 4844 **E.** 4888

7. WE2 In each of the following, write the numbers in descending order.

a. 8569, 742, 48 987, 28, 647
b. 47 890, 58 625, 72 167, 12 947, 32 320
c. 6477, 7647, 7476, 4776, 6747
d. 8088, 8800, 8080, 8808, 8008, 8880

8. In each of the following, write the numbers in ascending order.

a. 58, 9, 743, 68 247, 1 258 647
b. 78 645, 58 610, 60 000, 34 108, 84 364
c. 9201, 2910, 1902, 9021, 2019, 1290
d. 211, 221, 212, 1112, 222, 111

9. Did you know that we can use the abbreviation K to represent 1000? For example, $50 000 can be written as $50K.

a. What amounts do each of the following represent?

i. $6K **ii.** $340K **iii.** $58K

b. Write the following using K as an abbreviation.

i. $430 000 **ii.** $7000 **iii.** $800 000

c. Find a job or real estate advertisement that uses this notation.

Reasoning

10. WE1 A new mobile phone company is investigating the habits of telephone users. From the following sample of telephone calls, find out the most popular length of phone call. First, round each call time to the nearest half minute.

132 s	10 s	43 s	71 s	243 s	52 s
142 s	332 s	6 s	38 s	63 s	32 s
132 s	32 s	43 s	52 s	352 s	101 s
124 s	28 s	153 s	10 s	243 s	34 s

11. What is the largest five-digit number you can write if each digit must be different and no digit may be prime? Justify your answer.

Problem solving

12. a. How many two-digit numbers can you make using the digits 1 and 2 if:
- **i.** you are not allowed to repeat any digits
- **ii.** you can repeat the digits?

b. How many three-digit numbers can you make using the digits 1, 2 and 3 if:
- **i.** you are not allowed to repeat any digits
- **ii.** you can repeat the digits?

c. Is there a relationship between the initial number of digits and the number of arrangements when:
- **i.** repetition is not allowed? Explain your answer.
- **ii.** repetition is allowed? Explain you answer.

13. How long would it take you to count out loud from 0 to 1 000 000? Find a way of estimating the time without actually counting to one million.

Reflection

What strategies will you use to place numbers in order?

2.3 Adding and subtracting positive integers

- **Integers** are positive and negative whole numbers, including zero. Integers are denoted by the letter Z.

2.3.1 Addition of positive integers

- To add larger numbers, write them in columns according to place value and then add them.
- The **sum** is the answer obtained when numbers are added together.

WORKED EXAMPLE 3

Arrange these numbers in columns, then add them.

1462 + 78 + 316

THINK

1 Set out the sum in columns.

2 Add the digits in the units column in your head (2 + 8 + 6 = 16). Write the 6 in the units column of your answer and carry the 1 to the tens column, as shown in red.

3 Now add the digits in the tens column (1 + 6 + 7 + 1 = 15). Write the 5 in the tens column of your answer and carry the 1 to the hundreds column, as shown in orange.

WRITE

$$\begin{array}{r} 1\,4\,6\,2 \\ \quad\;\; 7\,8 \\ +\;\; 3_{1}1_{1}6 \\ \hline 1\,8\,5\,6 \end{array}$$

4 Add the digits in the hundreds column (1 + 4 + 3 = 8). Write 8 in the hundreds column of your answer, as shown in green. There is nothing to carry.

5 There is only a 1 in the thousands column. Write 1 in the thousands column of your answer.

WORKED EXAMPLE 4

Mentally perform the addition 27 + 19 + 141 + 73 by finding suitable pairs of numbers.

THINK	WRITE
1 Write the question.	27 + 19 + 141 + 73
2 Look for pairs of numbers that can be added to make a multiple of 10. Reorder the sum, pairing these numbers.	= (27 + 73) + (141 + 19)
3 Add the number pairs.	= 100 + 160
4 Complete the addition.	= 260

2.3.2 Commutative and associative properties for addition

- The **commutative law for addition** means that you can add in any order.
  For example, 3 + 4 gives the same result as 4 + 3.
- The **associative law for addition**, simply stated, is that the order in which additions are calculated is not important and does not change the result.
  For example, 2 + 3 + 7 could be calculated as 5 + 7 = 12 or 2 + 10 = 12.
  This is a very useful property when performing mental calculations.

2.3.3 Subtraction of positive integers

- Subtraction is the inverse of addition. To evaluate 800 − 360 you could ask the question: 'What do I need to add to 360 to give 800?' The answer is 440.
- To subtract numbers in a more formal manner, the digits are lined up vertically according to place value, as we saw for addition.
- The most commonly used method of subtraction is called the **decomposition method**. This is because the larger number is decomposed, or taken apart.
  The 10 which is added to the top number is taken from the previous column of the same number. So 32 − 14 is written as

$$\begin{array}{r} 32 \\ -\,14 \\ \hline \end{array} \quad \text{or} \quad \begin{array}{r} 30 + 2 \\ -\,(10 + 4) \\ \hline \end{array}$$

and becomes

$$\begin{array}{r} {}^{2}\not{3}^{1}2 \\ -1\ 4 \\ \hline \end{array} \qquad \begin{array}{r} 20 + 12 \\ -\,(10 + \ 4) \\ \hline \end{array}$$

Now 4 can be taken from 12 and 10 from 20 to give 18.

$$\begin{array}{r} {}^{2}\not{3}^{1}2 \\ -1\ 4 \\ \hline 1\ 8 \end{array}$$

- Subtraction is neither *commutative* nor *associative* since the order in which the subtraction is performed is very important.

For example, $6 - 3$ does not give the same result as $3 - 6$.

- The **difference** is the answer obtained when 2 numbers are subtracted.

WORKED EXAMPLE 5

Evaluate: a 6892 − 467 b 3000 − 467.

THINK	WRITE
a 1 Since 7 cannot be subtracted from 2, take one ten from the tens column of the larger number and add it to the units column of the same number. So the 2 becomes 12, and the 9 tens become 8 tens.	**a** $6\,8^{8}\not{9}^{1}2$ $-\ \ 4\,6\,7$ $6\,4\,2\,5$
2 Subtract the 7 units from the 12 units$(12 - 7 = 5)$.	
3 Subtract 6 tens from the 8 remaining tens $(8 - 6 = 2)$.	
4 Subtract 4 hundreds from the 8 hundreds $(8 - 4 = 4)$.	
5 Subtract 0 thousands from the 6 thousands $(6 - 0 = 6)$.	
b 1 Since 7 cannot be taken from 0, 0 needs to become 10.	**b** $^{2}\not{3}^{9}\not{0}^{9}\not{0}^{1}0$ $-\ \ 4\,6\,7$ $2\,5\,3\,3$
2 We cannot take 10 from the tens column, as it is also 0. The first column that we can take anything from is the thousands, so 3000 is decomposed to 2 thousands, 9 hundreds, 9 tens and 10 units.	
3 Subtract the units $(10 - 7 = 3)$.	
4 Subtract the tens $(9 - 6 = 3)$.	
5 Subtract the hundreds $(9 - 4 = 5)$.	
6 Subtract the thousands $(2 - 0 = 2)$.	

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- **Complete this digital doc:** SkillSHEET: Adding and subtracting whole numbers less than 20 (doc-6411)
- **Complete this digital doc:** Spreadsheet: Adding numbers (doc-6419)
- **Complete this digital doc:** Spreadsheet: Subtracting numbers (doc-2086)
- **Complete this digital doc:** Positive integers I (doc-0273)

Exercise 2.3 Adding and subtracting positive integers

assessON

Individual pathways

PRACTISE
Questions:
1a, c, e, g, i, k, m, 2–8, 9, 10, 11, 13, 15, 16, 18, 19, 21, 26

CONSOLIDATE
Questions:
1b, d, f, h, j, l, n, 2–8, 9, 11, 13, 15, 16, 17, 18, 19, 21–23, 26, 27

MASTER
Questions:
2–8, 9, 12, 14, 16, 17, 18, 19, 20, 21–24, 25, 27, 28

Individual pathway interactivity: int-4312

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Fluency

1. Answer these questions, doing the working in your head.

a. $7 + 8 =$
b. $18 + 6 =$
c. $20 + 17 =$
d. $80 + 41 =$
e. $195 + 15 =$
f. $227 + 13 =$
g. $420 + 52 =$
h. $1000 + 730 =$
i. $7300 + 158 =$
j. $17000 + 1220 =$
k. $125000 + 50000 =$
l. $2 + 8 + 1 + 9 =$
m. $6 + 8 + 9 + 3 + 2 + 4 + 1 + 7 =$
n. $12 + 5 + 3 + 7 + 15 + 8 =$

2. Add these numbers, setting them out in columns as shown. Check your answers using a calculator.

a.
$$\begin{array}{r} 34 \\ +\ 65 \\ \hline \\ \hline \end{array}$$

b.
$$\begin{array}{r} 482 \\ +\ 517 \\ \hline \\ \hline \end{array}$$

c.
$$\begin{array}{r} 1418 \\ +\ 2765 \\ \hline \\ \hline \end{array}$$

d.
$$\begin{array}{r} 68\,069 \\ 317 \\ 8 \\ +\ 4\,254 \\ \hline \\ \hline \end{array}$$

e.
$$\begin{array}{r} 696 \\ 3\,421\,811 \\ +\ 63\,044 \\ \hline \\ \hline \end{array}$$

f.
$$\begin{array}{r} 399 \\ 1489 \\ 2798 \\ +\ 8943 \\ \hline \\ \hline \end{array}$$

3. WE3 Arrange these numbers in columns, then add them.

a. $137 + 841$
b. $723 + 432$
c. $149 + 562 + 55$
d. $47 + 198 + 12$
e. $376 + 948 + 11$
f. $8312 + 742 + 2693$
g. $8 + 12\,972 + 59 + 1423$
h. $465 + 287\,390 + 45\,012 + 72 + 2$
i. $1\,700\,245 + 378 + 930$
j. $978\,036 + 67\,825 + 7272 + 811$

Check your answers with a calculator.

4. WE4 Mentally perform each of the following additions by pairing numbers together.

a. $56 + 87 + 24 + 13$
b. $69 + 45 + 55 + 31$
c. $74 + 189 + 6 + 11$
d. $254 + 187 + 6 + 13$
e. $98 + 247 + 305 + 3 + 95 + 42$
f. $180 + 364 + 59 + 141 + 47 + 20 + 16$

5. Answer these questions without using a calculator.

a. $11 - 5$
b. $20 - 12$
c. $53 - 30$
d. $100 - 95$
e. $87 - 27$
f. $150 - 25$
g. $820 - 6$
h. $1100 - 200$
i. $1700 - 1000$
j. $22000 - 11500$
k. $100 - 20 - 10$
l. $75 - 25 - 15$
m. $1000 - 50 - 300 - 150$
n. $80 - 8 - 4 - 5$
o. $24 - 3 - 16$
p. $54 - 28$
q. $78 - 39$

6. Answer these questions which involve adding and subtracting whole numbers.

a. $10 + 8 - 5 + 2 - 11$
b. $40 + 15 - 35$
c. $16 - 13 + 23$
d. $120 - 40 - 25$
e. $53 - 23 + 10$
f. $15 + 45 + 25 - 85$
g. $100 - 70 + 43$
h. $1000 - 400 + 250 + 150 + 150$

7. WE5 Evaluate:

a. $98 - 54$
b. $167 - 132$
c. $47\,836 - 12\,713$
d. $149 - 63$
e. $642\,803 - 58\,204$
f. $3642 - 1811$

g. 664 − 397 **h.** 12 900 − 8487
i. 69 000 − 3561

Check your answers using a calculator.

8. Evaluate:

a. 406 564 − 365 892 **b.** 2683 − 49
c. 70 400 − 1003 **d.** 64 973 − 8797
e. 27 321 − 25 768 **f.** 518 362 − 836
g. 812 741 − 462 923 **h.** 23 718 482 − 4 629 738

Check your answers using a calculator.

Understanding

9. Julia was performing in a ballet and needed to buy a tutu, pointe shoes and white tights. How much did she spend in total on her costume?

10. The Sydney telephone directory has 1544 pages in the A–K book and 1488 pages in the L–Z book. How many pages does it have in total?

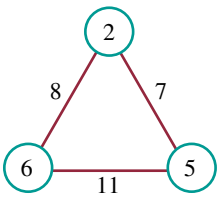
11. Nathan has taken his sister and her friends to lunch for her birthday. What will be the total cost of the items shown?

12. Hussein's family drove from Melbourne to Perth. In the first 2 days they drove from Melbourne to Adelaide, a distance of 738 kilometres. After a couple of days' sightseeing in Adelaide, Hussein's family took a day to drive 321 kilometres to Port Augusta, and another to drive the 468 kilometres to Ceduna. They drove 476 kilometres to Norseman the following day, then took 3 more days to travel the remaining 1489 kilometres to Perth.

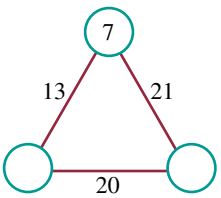
a. How many days did it take for Hussein's family to drive from Melbourne to Perth?
b. How far is Norseman from Melbourne?
c. How many kilometres is Perth from Melbourne?
d. How far is Adelaide from Perth?

13. Of all the world's rivers, the Amazon in South America and the Nile in Africa are the two longest. The Amazon is 6437 kilometres in length and the Nile is 233 kilometres longer than the Amazon. How long is the Nile River?

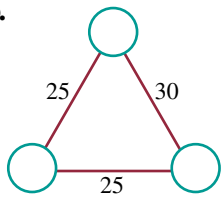
14. An **arithmagon** is a triangular figure in which the two numbers at the end of each line add to the number along the line. An example is shown at right. Work in pairs to try and solve each of these arithmagons.



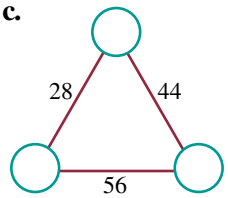
a.



b.



c.



15. Hayden received a box of 36 chocolates. He ate 3 on Monday, 11 on Tuesday and gave 7 away on Wednesday. How many did he have left?

16. A crowd of 29 641 attended the 2012 NRL match between Canterbury–Bankstown Bulldogs and St George Illawarra Dragons. If 17 492 people supported the Bulldogs and the rest supported the Dragons, how many supporters did the Dragons have?

17. A school bus left Laurel High School with 31 students aboard. Thirteen of these passengers alighted at Hardy Railway Station. The bus collected 24 more students at Hardy High School and a further 11 students disembarked at Laurel swimming pool. How many students were still on the bus?

18. The most commonly spoken language in the world is Mandarin, spoken by approximately 575 000 000 people (in north and east central China). Approximately 360 000 000 people speak English and 140 000 000 Spanish.

a. How many more people speak Mandarin than English?

b. How many more people speak English than Spanish?

19. The following photographs show three of the highest waterfalls in the world. How much higher are the:

a. Victoria Falls than the Iguazu Falls?

b. Iguazu Falls than the Niagara Falls?

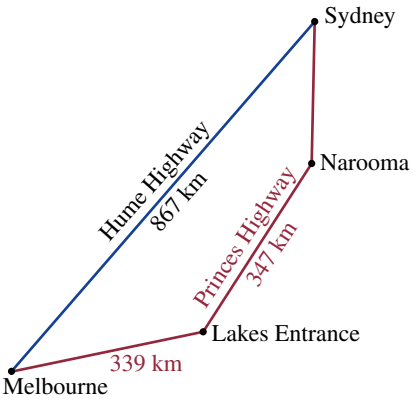
c. Victoria Falls than the Niagara Falls?

d. Explain how you obtained your answers.

20. Lucy and Ty were driving from Melbourne to Sydney for a holiday. The distance via the Hume Highway is 867 kilometres, but they chose the more scenic Princes Highway even though the distance is 1039 kilometres. They drove to Lakes Entrance the first day (339 kilometres), a further 347 kilometres to Narooma on the second day and arrived in Sydney on the third day.

a. How much further is Melbourne to Sydney via the Princes Highway than via the Hume Highway?

b. How far did Lucy and Ty travel on the third day?



Reasoning

21. Can you fill in the missing numbers? The * can represent any digit.

a.
$$\begin{array}{r} 6*8\,*2* \\ -488\,417 \\ \hline *49\,9*4 \end{array}$$

b.
$$\begin{array}{r} 3*9* \\ -*6*5 \\ \hline 1*07 \end{array}$$

22. Without using a calculator, and in less than 10 seconds, find the answer to 6 849 317 − 999 999. Explain how you reached your answer.

23. A beetle has fallen into a hole that is 15 metres deep. It is able to climb a distance of 3 metres during the day but at night the beetle is tired and must rest. However, during the night it slides back 1 metre. How many days will it take the beetle to reach the top of the hole to freedom? Explain your answer.

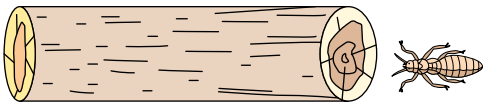
24. Each letter in the following equation stands for a digit from 1 to 9. Find the value of each letter so that the addition statement is true.

$$\text{BIG} + \text{JOKE} = \text{HAHA}$$

Show how you reached your answer.

Problem solving

25. Five termites start munching into a log. After an hour:
Pixie is 19 mm ahead of Bitsie.
Trixie is 5 mm ahead of Mixie.
Mixie is twice as far as Itsie.
Itsie is 10 mm behind Bitsie.
Mixie has eaten through 52 mm.
Which termite is the fastest eater?



26. In a class of 22 students, each attends swimming practice, or lifesaving, or both. If 16 attend swimming practice and 9 attend lifesaving, find the number of students who attend both.

27. Vernon works in a restaurant where people choose to work Monday to Friday, or weekends, or Monday to Sunday. There are 15 employees altogether. If 13 work Monday to Friday and 14 work weekends, find the number of employees who do not work Monday to Sunday.

28. There are 20 chocolate bars on the table all containing fruit, nuts, or fruit and nuts. If 12 contain nuts and 14 contain fruit, find the numbers of chocolate bars that contain both fruit and nuts.

Reflection

In what situations do you use addition and subtraction of positive integers in everyday life?

2.4 Multiplying and dividing positive integers

2.4.1 Multiplying positive integers

- Short multiplication can be used when multiplying a large number by a single digit number.
- The **product** is the answer obtained when numbers are multiplied.

WORKED EXAMPLE 6

Calculate 1456 × 5.

THINK **WRITE**

1 Multiply the units (5 × 6 = 30). Write the 0 and carry the 3 to the tens column.

2 Multiply the tens digit of the question by 5 and add the carried number ($5 \times 5 + 3 = 28$). Write the 8 in the tens column and carry the 2 to the hundreds column.

3 Multiply the hundreds digit by 5 and add the carried number ($5 \times 4 + 2 = 22$). Write the last 2 in the hundreds column and carry the other 2 to the thousands column.

4 Multiply the thousands digit by 5 and add the carried number ($5 \times 1 + 2 = 7$). Write 7 in the thousands column of the answer.

$$\begin{array}{r} {}^{2}1{}^{2}4{}^{3}5\,6 \\ \times \quad 5 \\ \hline 7\,2\,8\,0 \end{array}$$

WORKED EXAMPLE 7

Calculate 1456×132 using long multiplication.

THINK	WRITE
1 Multiply the larger number by the units digit in the smaller number using short multiplication ($1456 \times 2 = 2912$). Write the answer directly below the problem, as shown in blue.	$\begin{array}{r} 1\,4\,5\,6 \\ \times \quad 1\,3\,2 \\ \hline 2\,9\,1\,2 \end{array}$
2 Place a zero in the units column, as shown in red, when multiplying the larger number by the tens digit of the smaller number ($1456 \times 3 = 4368$); this is because you are really working out $1456 \times 30 = 43\,680$. Write the answer directly below the previous answer, as shown in green.	$\begin{array}{r} 1\,4\,5\,6 \\ \times \quad 1\,3\,2 \\ \hline 2\,9\,1\,2 \\ 4\,3\,6\,8\,0 \end{array}$
3 Place zeros in the units and tens columns, as shown in pink, when multiplying the larger number by the hundreds digit of the smaller number ($1456 \times 1 = 1456$); this is because you are really working out $1456 \times 100 = 145\,600$. Write the answer directly below the previous answer, as shown in purple.	$\begin{array}{r} 1\,4\,5\,6 \\ \times \quad 1\,3\,2 \\ \hline 2\,9\,1\,2 \\ 4\,3\,6\,8\,0 \\ 1\,4\,5\,6\,0\,0 \end{array}$
4 Add the numbers in each column of the three rows and write the answer, as shown in orange.	$\begin{array}{r} 1\,9\,2\,1\,9\,2 \end{array}$

- The **distributive law** applies only to multiplication. It is defined as $a(b + c) = ab + ac$. Put simply, it helps with mental calculation because it means that 8×13 can be thought of as:

$$\begin{aligned} 8 \times 13 &= 8 \times (3 + 10) \\ &= 8 \times 3 + 8 \times 10 \\ &= 24 + 80 \\ &= 104 \end{aligned}$$

2.4.2 Mental strategies for multiplication

- In many cases it is not practical to use pen and paper or even a calculator to perform multiplication.
- Some mental strategies can make multiplication easier.
- Multiplication is commutative because the order in which numbers are multiplied is not important. For example, $3 \times 8 \times 6 = 6 \times 3 \times 8$.
- Multiplication is associative because larger numbers can be paired up for multiplication.

  For example, $\begin{aligned} 2 \times 17 \times 5 &= 17 \times (2 \times 5) \\ &= 17 \times 10 \\ &= 170 \end{aligned}$

  By finding the multiplication pair that made 10, this question has been made simpler.

WORKED EXAMPLE 8

Use mental strategies to calculate $4 \times 23 \times 25$.

THINK	WRITE
1 Write the question.	$4 \times 23 \times 25$
2 Rearrange it looking for a number pair that makes a simpler multiplication.	$= 23 \times (4 \times 25)$
3 Mentally calculate 4×25.	$= 23 \times 100$
4 Mentally calculate the final answer.	$= 2300$

WORKED EXAMPLE 9

Use a mental strategy to calculate 34×200.

THINK	WRITE
1 Write the question.	34×200
2 Break 200 into 2×100.	$= 34 \times 2 \times 100$
3 Calculate 34×2.	$= 68 \times 100$
4 Calculate 68×100.	$= 6800$

- If both numbers are multiples of 10, 100 and so on, ignore the zeros, multiply the remaining numbers then add the total number of zeros to the answer. For example, $900 \times 6000 = 5\,400\,000$.
- Consider the multiplication 9×58. This multiplication can be regarded as: $10 \times 58 - 1 \times 58$. Using this, the answer can be mentally calculated by multiplying 58 by 10 then subtracting 58 from the answer.

WORKED EXAMPLE 10

Use mental strategies to calculate 77×9.

THINK	WRITE
1 Write the question.	77×9
2 Use the strategy of 'multiply by 10'.	$= 77 \times 10 - 77$
3 Calculate 77×10 and subtract 77.	$= 770 - 77$ $= 693$

2.4.3 Dividing positive integers

- Division is the inverse of multiplication. To evaluate $56 \div 8$, you could ask the question: 'By which number do I multiply 8 to get 56?' The answer is 7.
- Short division can be used when dividing by numbers up to 10.

WORKED EXAMPLE 11

Calculate 89 656 ÷ 8.

THINK	WRITE
1 Divide 8 into the first digit and carry the remainder to the next digit; 8 goes into 8 once. Write 1 above the 8 as shown. There is no remainder.	$\begin{array}{r} 1\ 1\ 2\ 0\ 7 \\ 8\overline{)8\ 9^{1}6\ 5^{5}6} \end{array}$
2 Divide 8 into the second digit and carry the remainder to the next digit; 8 goes into 9 once with 1 left over. Write 1 above the 9 and carry 1 to the hundreds column.	
3 Divide 8 into the third digit and carry the remainder to the next digit; 8 goes into 16 twice with no remainder. Write 2 above the 6 as shown.	
4 Divide 8 into the fourth digit and carry the remainder to the next digit; 8 doesn't go into 5. Write the 0 above the 5. Carry 5 to the next digit.	
5 Divide 8 into 56; 8 goes into 56 seven times.	
6 Write the answer.	$89\,656 \div 8 = 11\,207$

2.4.4 Dividing numbers that are multiples of 10

- When we are dividing numbers that are multiples of 10, an equivalent number of zeros can be cancelled from both the **dividend** and the **divisor**.
- The dividend is the quantity to be divided.
- The divisor is the number by which the dividend is to be divided.
- The **quotient** is the answer obtained when one number is divided by another number.

WORKED EXAMPLE 12

Calculate 48 000 ÷ 600.

THINK	WRITE
1 Write the question.	$48\,000 \div 600$
2 Write the question as a fraction.	$= \frac{48\,0\not{0}\not{0}}{6\not{0}\not{0}}$
3 Cancel as many zeros as possible, crossing off the same number in both numerator and denominator.	$= \frac{480}{6}$
4 Perform the division.	$= 6\overline{)\begin{array}{r}080\\480\end{array}}$
5 Write your answer.	$48\,000 \div 600 = 80$

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Complete this digital doc: Spreadsheet: Dividing numbers (doc-2089)

Exercise 2.4 Multiplying and dividing positive integers

assessON

Individual pathways

PRACTISE
Questions:
1–10, 11, 12a, d, g, h, 6, 15–18, 19, 21, 25, 32, 40

CONSOLIDATE
Questions:
1–10, 11, 12a, b, e, h, 15–18, 20, 22, 25, 26, 29, 32, 35, 37, 40

MASTER
Questions:
1–10, 11–14, 15–18, 23, 24, 27, 28, 30–39

Individual pathway interactivity: int-4313

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Fluency

1. Write the answer to each of the following without using a calculator.

a. 4×3	**b.** 9×5	**c.** 2×11
d. 8×7	**e.** 12×8	**f.** 10×11
g. 6×9	**h.** 12×11	**i.** 9×8

2. Find the product of each of the following without using a calculator.

a. 13×2	**b.** 15×3	**c.** 25×2	**d.** 3×13
e. 25×4	**f.** 45×2	**g.** 16×2	**h.** 35×2
i. 14×3	**j.** 21×3	**k.** 54×2	**l.** 25×3
m. $3 \times 4 \times 6$	**n.** $2 \times 5 \times 9$	**o.** $3 \times 3 \times 3$	**p.** $5 \times 6 \times 3$
q. $5 \times 4 \times 5$	**r.** $8 \times 5 \times 2$		

3. WE6 Calculate these using short multiplication.

a. 16×8	**b.** 29×4	**c.** 137×9	**d.** 857×3
e. 4920×5	**f.** $15\,984 \times 7$	**g.** 7888×8	**h.** 472×4
i. 2015×8	**j.** $10\,597 \times 6$	**k.** $34\,005 \times 11$	**l.** $41\,060 \times 12$

Check your answers using a calculator.

4. WE7 Calculate these using long multiplication.

a. 52×44	**b.** 97×31	**c.** 59×28	**d.** 16×57
e. 173×41	**f.** 850×76	**g.** 407×53	**h.** 47×2074
i. $80\,055 \times 27$	**j.** $19 \times 256\,340$	**k.** $57\,835 \times 476$	**l.** 8027×215

Check your answers using a calculator.

5. WE8 Use mental strategies to calculate each of the following.

a. $2 \times 8 \times 5$ **b.** $2 \times 4 \times 5 \times 6$ **c.** $4 \times 19 \times 25$

d. $50 \times 45 \times 2$ **e.** $2 \times 9 \times 50$ **f.** $4 \times 67 \times 250$

6. WE9 Use mental strategies to calculate each of the following.

a. 45×20 **b.** 61×30 **c.** 62×50

d. 84×200 **e.** 500×19 **f.** 86×2000

7. Find each of the following.

a. 200×40 **b.** 30×700 **c.** 600×800

d. 90×80 **e.** 120×400 **f.** 1100×5000

g. $900\,000 \times 7000$ **h.** $120\,000 \times 1200$ **i.** 800×7000

j. 9000×6000 **k.** 4000×110 **l.** $12\,000 \times 1100$

8. WE10 Use mental strategies to calculate each of the following.

a. 34×9 **b.** 83×9 **c.** 628×9 **d.** 75×99 **e.** 24×19 **f.** 26×8

9. a. Calculate 56×100.

b. Calculate 56×10.

c. Use your answers to parts **a** and **b** to calculate the answer to 56×90.

10. Use the method demonstrated in question **9** to calculate each of the following.

a. 48×90 **b.** 74×90 **c.** 125×90

d. 32×900 **e.** 45×80 **f.** 72×800

11. a. Calculate 25×6.

b. Multiply your answer to part **a** by 2.

c. Now calculate 25×12.

d. Use the answers to parts **a**, **b** and **c** to describe a method for mentally multiplying by 12.

12. Use the method that you discovered in question **11** to mentally calculate the value of each of the following.

a. 15×12 **b.** 70×12 **c.** 18×12 **d.** 105×12

e. 25×14 **f.** 40×16 **g.** 11×18 **h.** 34×20

13. a. What is the value of 9×10?

b. What is the value of 9×3?

c. Calculate the value of 9×13.

d. Use the answers to parts **a**, **b** and **c** above to describe a method for mentally multiplying by 13.

14. Use the method that you discovered in question **13** to calculate the value of each of the following.

a. 25×13 **b.** 30×13 **c.** 24×13 **d.** 102×13

15. Evaluate these divisions without using a calculator. There should be no remainder.

a. $24 \div 6$ **b.** $24 \div 8$ **c.** $36 \div 9$

d. $72 \div 8$ **e.** $49 \div 7$ **f.** $96 \div 12$

g. $108 \div 9$ **h.** $56 \div 7$ **i.** $16 \div 4$

j. $28 \div 7$ **k.** $40 \div 2$ **l.** $26 \div 2$

m. $45 \div 15$ **n.** $32 \div 16$ **o.** $27 \div 3 \div 3$

p. $96 \div 8 \div 6$ **q.** $48 \div 12 \div 2$ **r.** $72 \div 2 \div 9$

s. $56 \div 7 \div 4$ **t.** $100 \div 2 \div 10$ **u.** $90 \div 3 \div 2$

Check your answers using multiplication.

16. Perform these calculations which involve a combination of multiplication and division. Always work from left to right.

a. $4 \times 5 \div 2$ **b.** $9 \times 8 \div 12$ **c.** $80 \div 10 \times 7$

d. $45 \div 9 \times 7$ **e.** $144 \div 12 \times 7$ **f.** $120 \div 10 \times 5$

g. $4 \times 9 \div 12$ **h.** $121 \div 11 \times 4$ **i.** $81 \div 9 \times 6$

17. WE11 Calculate each of the following using short division.

a. $3\overline{)1455}$	**b.** $4\overline{)27\,768}$	**c.** $7\overline{)43\,456}$
d. $9\overline{)515\,871}$	**e.** $11\overline{)30\,371}$	**f.** $8\overline{)640\,360}$
g. $3\overline{)255\,194}$	**h.** $6\overline{)516\,285}$	**i.** $12\overline{)103\,717}$
j. $7\overline{)6\,628\,530}$	**k.** $5\overline{)465\,777}$	**l.** $8\overline{)480\,594}$

Check your answers using a calculator.

18. WE12 Divide these numbers, which are multiples of 10.

a. 4200 ÷ 6	**b.** 700 ÷ 70	**c.** 210 ÷ 30
d. 720 000 ÷ 800	**e.** 8100 ÷ 900	**f.** 4 000 000 ÷ 8000
g. 600 000 ÷ 120	**h.** 560 ÷ 80	**i.** 880 000 ÷ 1100

Understanding

19. John wants to make a telephone call to his friend Rachel who lives in San Francisco. The call will cost him $3 per minute. If John speaks to Rachel for 24 minutes:
- **a.** what will the call cost?
- **b.** what would John pay if he made this call every month for 2 years?

20. Chris is buying some generators. The generators cost $12 000 each and she needs 11 of them. How much will they cost her?

21. Julie was saving money to buy a digital camera. She was able to save $75 each month.
- **a.** How much did she save after 9 months?
- **b.** How much did she save over 16 months?
- **c.** If Julie continued to save at the same rate, how much will she save over a period of 3 years?

22. A car can travel 14 kilometres using 1 litre of fuel. How far could it travel with 35 litres?

23. As Todd was soaking in the bath, he was contemplating how much water was in the bath. If Todd used 85 litres of water each time he bathed and had a bath every week:
- **a.** how much bath water would Todd use in 1 year?
- **b.** how much would he use over a period of 5 years?

24. In 1995, a team of British soldiers at Hameln, Germany, constructed a bridge in the fastest time ever. The bridge spanned an 8-metre gap and it took the soldiers 8 minutes and 44 seconds to build it. How many seconds did it take them to build it?

25. You are helping your Dad build a fence around your new swimming pool. He estimates that each metre of fence will take 2 hours and cost $65 to build.
- **a.** How long will it take you and your Dad to build a 17-metre fence?
- **b.** How much will it cost to build a 17-metre fence?
- **c.** How much would it cost for a 29-metre fence?

26. Narissa does a paper round each morning before school. She travels 2 kilometres each morning on her bicycle, delivers 80 papers and is paid $35. She does her round each weekday.
- **a.** How far does she travel in 1 week?
- **b.** How much does she get paid in 1 week?
- **c.** How far does she travel in 12 weeks?
- **d.** How much would she be paid over 52 weeks?
- **e.** How many papers would she deliver in 1 week?
- **f.** How many papers would she deliver in 52 weeks?

27. **a.** What is the value of 580 ÷ 10?
b. Halve your answer to part **a**.
c. Now find the value of 580 ÷ 20.
d. Use the answers to parts **a**, **b** and **c** to describe a method for mentally dividing by 20.

28. Use the method that you discovered in question **27** to evaluate:
a. 4280 ÷ 20 **b.** 1840 ÷ 20 **c.** 790 ÷ 20 **d.** 960 ÷ 30

29. Spiro travels 140 kilometres per week travelling to and from work. If Spiro works 5 days per week:
a. how far does he travel each day?
b. what distance is his work from home?

30. Kelly works part time at the local pet shop. Last year she earned $2496.
a. How much did she earn each month?
b. How much did she earn each week?

31. At the milk processing plant, the engineer asked Farid how many cows he had to milk each day. Farid said he milked 192 cows because he obtained 1674 litres of milk each day and each cow produced 9 litres. Does Farid really milk 192 cows each day? If not, calculate how many cows he does milk.

32. When Juan caters for a celebration such as a wedding he fills out a form for the client to confirm the arrangements. Juan has been called to answer the telephone so it has been left to you to fill in the missing details. Copy and complete this planning form.

Celebration type	Wedding
Number of guests	152
Number of people per table	8
Number of tables required	
Number of courses for each guest	4
Total number of courses to be served	
Number of courses each waiter can serve	80
Number of waiters required	
Charge per guest	$55
Total charge for catering	

33. Janet is a land developer and has bought 10 450 square metres of land. She intends to subdivide the land into 11 separate blocks.
a. How many square metres will each block be?
b. If she sells each block for $72 250, how much will she receive for the subdivided land?

34. Shea has booked a beach house for a week over the summer period for a group of 12 friends. The house costs $1344 for the week. If all 12 people stayed for 7 nights, how much will the house cost each person per night?

Reasoning

35. Some people like to keep mice as pets. Female mice give birth to about six litters of babies per year, with about six babies in each litter.
a. If you own a female mouse, how many pet mice will you have at the end of 1 year if none of the female's babies have any litters of their own? Explain how you reached this answer.
b. How many female mice did you have at the start if, at the end of the year, you had 111 mice and none of your females' babies had any litters of their own? Show your working.

36. The photograph at right shows a mobile phone and its associated costs.

Voice calls:
47c per 30 seconds
Text messages:
15c per message
Picture messages:
50c per message

a. How much does it cost for a 1-minute call on this phone?
b. How much does it cost for a 35-minute call on this phone?
c. If you make five 3-minute calls each day, how much will calls cost you for a year using this phone? (There are 365 days in a standard year.)
d. How much does it cost to send 20 text messages per day from this phone?
e. What is the difference in cost between sending 100 text messages and sending 50 picture messages using this phone?

Problem solving

37. Imagine a simplified form of car numberplate where the plate consists of a letter followed by a number; that is, A0 up to Z9. How many numberplates could be issued under this system? Remember that all numberplates issued have to be different.

38. Marcus eats one of the strawberries in a basket, then he and his two friends split them evenly. If his friends get 15 strawberries each, how many were in the basket to begin with?

39. Peter is unaware that his computer has a virus. Every time he shuts down, the virus deletes half of his hard drive. Peter has also decided that he will have a 'clean-up' to free up some of his computer's memory. Each time he starts up his computer, he transfers 2 GB of work onto a USB drive (hence deleting this from his hard drive) then shuts down. Soon after, Peter gets a message: 'Hard drive wiped'. He realises his computer must have a virus because he hasn't saved or deleted any work since he started his clean-up, and he is certain he had much more work than the 10 GB that has been transferred to the USB drive in front of him. How much work (in GB) has Peter lost?

40. The product of two consecutive numbers is 1190. What are the two numbers and what is their sum?

Reflection

How are multiplication and division related?

CHALLENGE 2.1

Consider numbers with two identical digits multiplied by 99. Work out:

11×99 22×99 33×99.

Can you see a pattern? Explain it.

Without using a calculator or long multiplication, write down the answers to:

a. 44×99 **b.** 55×99 **c.** 66×99
d. 77×99 **e.** 88×99 **f.** 99×99.

2.5 Long division

2.5.1 Long division

- Long division uses the same process as short division, but all the calculations are recorded.
- As a general rule, when dividing by numbers greater than 12, use long division.

WORKED EXAMPLE 13

Calculate 2685 ÷ 15.

THINK	WRITE
1 15 > 12, so long division is needed.	$15\overline{)2685}$
2 As 15 > 2, divide 15 into the first 2 digits, as shown in red.	$\begin{array}{r} 1 \\ 15\overline{)2685} \end{array}$
3 To find the remainder, multiply 1 by 15, write the result under the digits 26 and subtract.	$\begin{array}{r} 1 \\ 15\overline{)\ 2685} \\ -15 \\ \hline 11 \end{array}$
4 Bring the remaining digits down next to the 11.	$\begin{array}{r} 1 \\ 15\overline{)\ 2685} \\ -15\downarrow\downarrow \\ \hline 1185 \end{array}$
5 As 15 > 11, divide 15 into the first 3 digits.	$\begin{array}{r} 17 \\ 15\overline{)\ 2685} \\ -15 \\ \hline 1185 \end{array}$
6 Multiply 7 by 15 and write the result under the digits 118. Subtract the numbers and bring the 5 down next to the 3.	$\begin{array}{r} 17 \\ 15\overline{)\ 2685} \\ -15 \\ \hline 1185 \\ -105\downarrow \\ \hline 135 \end{array}$
7 Divide 15 into 135 and repeat.	$\begin{array}{r} 179 \\ 15\overline{)\ 2685} \\ -15 \\ \hline 1185 \\ -105 \\ \hline 135 \\ -135 \\ \hline 0 \end{array}$
8 Write the answer.	2685 ÷ 15 = 179

WORKED EXAMPLE 14

Calculate 2297 ÷ 17.

THINK	WRITE
1 17 > 12, so long division is needed.	$17\overline{)2297}$
2 Divide 17 into the first 2 digits. Multiply this result by 17 and subtract.	$\begin{array}{r} 1 \\ 17\overline{)\ 2297} \\ -17 \\ \hline 5 \end{array}$
3 Bring the remaining digits down next to the 5.	$\begin{array}{r} 1 \\ 15\overline{)\ 2297} \\ -17\downarrow\downarrow \\ \hline 5\,97 \end{array}$
4 Divide 17 into the first 2 digits. Multiply this result by 17 and subtract.	$\begin{array}{r} 13 \\ 17\overline{)\ 2297} \\ -17 \\ \hline 597 \\ -51 \\ \hline 8 \end{array}$
5 Bring the remaining digit 7 down next to the 8.	$\begin{array}{r} 13 \\ 17\overline{)\ 2297} \\ -17 \\ \hline 597 \\ -51\downarrow \\ \hline 87 \end{array}$
6 Divide 17 into 87 and repeat.	$\begin{array}{r} 135 \\ 17\overline{)\ 2297} \\ -17 \\ \hline 597 \\ -51 \\ \hline 87 \\ -85 \\ \hline 2 \end{array}$
7 Write the answer.	$2297 \div 17 = 135$ remainder 2 or $2297 \div 17 = 135\frac{2}{17}$

Exercise 2.5 Long division

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–3, 6, 7	Questions: 1–4, 6, 7	Questions: 1–3, 5–7

Individual pathway interactivity: int-4314

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Fluency

1. WE13 Using long division, evaluate the following.

a. 195 ÷ 13 **b.** 308 ÷ 22 **c.** 544 ÷ 17 **d.** 589 ÷ 31 **e.** 9683 ÷ 23

f. 8554 ÷ 13 **g.** 5525 ÷ 17 **h.** 9050 ÷ 25 **i.** 1302 ÷ 21 **j.** 11 004 ÷ 2

2. WE14 Using long division, evaluate the following.

a. 847 ÷ 13 **b.** 951 ÷ 15 **c.** 1210 ÷ 17 **d.** 1600 ÷ 19 **e.** 5050 ÷ 41

f. 8289 ÷ 33 **g.** 6831 ÷ 21 **h.** 7721 ÷ 31 **i.** 14 997 ÷ 23 **j.** 20 011 ÷ 27

Understanding

3. Complete the following long division problems.

a. $19\overline{)448\square}$ with quotient $2\square6$

b. $17\overline{)82\square9}$ with quotient $48\square$

c. $21\overline{)512\square}$ with quotient $2\square3$ r 17

Reasoning

4. A bakery is using 435 eggs per week to bake their cakes.

a. How many cartons do they need if the eggs are delivered in 12-egg cartons?

b. How many cartons do they need if the eggs are delivered in 15-egg cartons?

c. Do you need an exact number of cartons in both **a** and **b**?

d. Explain why one type of packaging has a remainder while the other one doesn't.

5. Choose a four-digit odd number, for example, 2371. Divide this number by 2, 4 and 8. Record the remainder of each division in the table given. Then continue with the next consecutive odd number and repeat the divisions by 2, 4 and 8. Repeat these steps for 10 consecutive odd numbers.

Dividend	**Divisor**		
	2	**4**	**8**
	Remainder		
2371	1	3	3
2373			
2375			

a. Can you see a pattern? Explain your answer.

b. Why do you think that this pattern occurs?

Problem solving

6. a. Choose a digit from 2 to 9. Write it six times to form a six-digit number. For example, if you choose 7, your six-digit number will be 777 777.

b. Divide the six-digit number by 33.

c. Divide the number from part **b** by 37.
d. Divide the number from part **c** by 91.
e. What do you notice about your result from part **d**?

7. Repeat question **6** with a different six-digit number, and then use mathematics to explain how the 'magic division' works.

Reflection

Why would you use long division instead of short division?

2.6 Order of operations

2.6.1 Order of operations

- In mathematics, conventions are followed so that we all have a common understanding of mathematical operations.
- Consider the question $6 + 6 \div 3$. If you perform the addition first the answer is 4. If you perform the division first the answer is 8. The correct answer is 8.
- There is a set order in which mathematicians calculate problems. The order is:
  1. **B**rackets
  2. **I**ndices or square roots
  3. **D**ivision and **M**ultiplication (from left to right)
  4. **A**ddition and **S**ubtraction (from left to right).
- The acronym **BIDMAS** can be used to remember the correct order of operations.

WORKED EXAMPLE 15

Calculate $6 + 12 \div 4$.

THINK	WRITE
1 Write the question.	$6 + 12 \div 4$
2 Perform the division before the addition.	$= 6 + 3$
3 Calculate the answer.	$= 9$

WORKED EXAMPLE 16

Calculate:
a $12 \div 2 + 4 \times (4 + 6)$ **b** $80 \div \{ [(11 - 2) \times 2] + 2 \}$.

THINK	WRITE
a 1 Write the question.	**a** $12 \div 2 + 4 \times (4 + 6)$
2 Remove the brackets by working out the addition inside.	$= 12 \div 2 + 4 \times 10$
3 Perform the division and multiplication next, working from left to right.	$= 6 + 40$
4 Complete the addition last and calculate the answer.	$= 46$

b 1	Write the question.	**b** $80 \div \{[(11 - 2) \times 2] + 2\}$
2	Remove the innermost brackets by working out the difference of 11 and 2.	$= 80 \div \{[9 \times 2] + 2\}$
3	Remove the next pair of brackets by working out the multiplication inside them.	$= 80 \div \{18 + 2\}$
4	Remove the final pair of brackets by working out the addition inside them.	$= 80 \div 20$
5	Perform the division last and calculate the answer.	$= 4$

WORKED EXAMPLE 17

Insert one set of brackets in the appropriate place to make the following statement true.
$3 \times 10 - 8 \div 2 + 4 = 7$

THINK		WRITE
1	Write the left-hand side of the equation.	$3 \times 10 - 8 \div 2 + 4$
2	Place one set of brackets around the first two values.	$= (3 \times 10) - 8 \div 2 + 4$
3	Perform the multiplication inside the bracket.	$= 30 - 8 \div 2 + 4$
4	Perform the division.	$= 30 - 4 + 4$
5	Perform the subtraction and addition working from left to right. *Note*: Since this is not the answer, the above process must be repeated.	$= 26 + 4$ $= 30$ Since this is not equal to 7 we must place the brackets in a different position.
6	Place one set of brackets around the second and third values.	$3 \times 10 - 8 \div 2 + 4$ $= 3 \times (10 - 8) \div 2 + 4$
7	Perform the subtraction inside the bracket.	$= 3 \times 2 \div 2 + 4$
8	Perform the multiplication and division working from left to right.	$= 6 \div 2 + 4$ $= 3 + 4$
9	Perform the addition last and calculate the answer. This is correct, so the brackets are in the correct place.	$= 7$

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Exercise 2.6 Order of operations

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1a–n, 2, 3, 4, 5, 6, 10	Questions: 1e–r, 2–8, 10, 11	Questions: 1i–v, 2, 3–7, 9–11

Individual pathway interactivity: int-4315

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Fluency

1. WE 15, 16 Calculate each of these, following the order of operations rules.

a. $3 + 4 \div 2$
b. $8 + 1 \times 1$
c. $24 \div (12 - 4)$
d. $15 \times (17 - 15)$
e. $11 + 6 \times 8$
f. $30 - 45 \div 9$
g. $56 \div (7 + 1)$
h. $12 \times (20 - 12)$
i. $3 \times 4 + 23 - 10 - 5 \times 2$
j. $42 \div 7 \times 8 - 8 \times 3$
k. $10 + 40 \div 5 + 14$
l. $81 \div 9 + 108 \div 12$
m. $16 + 12 \div 2 \times 10$
n. $(18 - 15) \div 3 \times 27$
o. $4 + (6 + 3 \times 9) - 11$
p. $52 \div 13 + 75 \div 25$
q. $(12 - 3) \times 8 \div 6$
r. $88 \div (24 - 13) \times 12$
s. $(4 + 5) \times (20 - 14) \div 2$
t. $(7 + 5) - (10 + 2)$
u. $\{[(16 + 4) \div 4] - 2\} \times 6$
v. $60 \div \{[(12 - 3) \times 2] + 2\}$

Understanding

2. WE 17 Insert one set of brackets in the appropriate place to make these statements true.

a. $12 - 8 \div 4 = 1$
b. $4 + 8 \times 5 - 4 \times 5 = 40$
c. $3 + 4 \times 9 - 3 = 27$
d. $3 \times 10 - 2 \div 4 + 4 = 10$
e. $12 \times 4 + 2 - 12 = 60$
f. $17 - 8 \times 2 + 6 \times 11 - 5 = 37$
g. $10 \div 5 + 5 \times 9 \times 9 = 81$
h. $18 - 3 \times 3 \div 5 = 9$

3. MC $20 - 6 \times 3 + 28 \div 7$ is equal to:

A. 46
B. 10
C. 6
D. 4
E. 2

4. MC The two signs marked with * in the equation $7 * 2 * 4 - 3 = 12$ are:

A. $-$, $+$
B. $+$, $\times$
C. $=$, $\div$
D. $\div$, $\times$
E. $\times$, $\div$

5. Insert brackets if necessary to make each statement true.

a. $6 + 2 \times 4 - 3 \times 2 = 10$
b. $6 + 2 \times 4 - 3 \times 2 = 26$
c. $6 + 2 \times 4 - 3 \times 2 = 16$
d. $6 + 2 \times 4 - 3 \times 2 = 8$

Reasoning

6. James and Liz discovered that they had different answers to the same question, which was to calculate $6 + 6 \div 3$. James thought the answer was 8. Liz thought the answer was 4. Who was correct, James or Liz? Provide an explanation.

7. What is the smallest number of pebbles greater than 10 for which grouping them in heaps of 7 leaves 1 extra and grouping them in heaps of 5 leaves 3 extra? Show your working.

Problem solving

8. The student lockers at Jacaranda College are to be numbered consecutively from 1 to 800 using plastic digits. Each digit costs 10c. What will be the total cost of all the digits?

9. Minh was asked to buy enough ice-creams for everyone in his class. There are 31 people in the class. At the supermarket, identical ice-creams come in packets of 3 or packets of 5. Minh is told that the cost of one ice-cream is the same from either packet. What packets or combination of packets should Minh buy?

10. Use the digits 1, 2, 3 and 4 and the operators +, −, ×, and ÷ to construct equations that result in the numbers 1 to 5 (the numbers 2 and 4 are already done for you). You must use each digit in each expression, and you may not use the digits more than once. You may combine the digits to form larger numbers (like 21). You may also use brackets to make sure the operations are done in the correct order.

$$\begin{aligned} 1 &= \\ 2 &= 4 - 3 + 2 - 1 \\ 3 &= \\ 4 &= 4 - 2 + 3 - 1 \\ 5 &= \end{aligned}$$

11. Use the digits 1, 2, 3 and 4 and the operators +, −, ×, and ÷ to construct equations that result in the numbers 5 to 10 (the numbers 7 and 10 have already been done for you). You must use each digit in each expression and you may not use the digits more than once. You may combine the digits to form larger numbers (like 21). You may also use brackets to make sure the operations are done in the correct order.

$$\begin{aligned} 5 &= \\ 6 &= \\ 7 &= 24 \div 3 - 1 \\ 8 &= \\ 9 &= \\ 10 &= 1 + 2 + 3 + 4 \end{aligned}$$

Reflection

Can you think of a real-life situation where performing calculations in the wrong order could cause problems?

2.7 Factors and multiples

2.7.1 Multiples

- A **multiple** of a number is the answer obtained when that number is multiplied by another whole number. For example, all numbers in the 5 times table are multiples of 5: so 5, 10, 15, 20, 25, … are all multiples of 5.

WORKED EXAMPLE 18

List the first five multiples of 7.

THINK	WRITE
1 First multiple is the number × 1, that is, 7 × 1.	
2 Second multiple is the number × 2, that is, 7 × 2.	
3 Third multiple is the number × 3, that is, 7 × 3.	
4 Fourth multiple is the number × 4, that is, 7 × 4.	
5 Fifth multiple is the number × 5, that is, 7 × 5.	
6 Answer the question.	The first five multiples of 7 are 7, 14, 21, 28, 35.

WORKED EXAMPLE 19

Write the numbers in the list that are multiples of 8.
18, 8, 80, 100, 24, 60, 9, 40

THINK	WRITE
1 The biggest number in the list is 100. List multiples of 8 using the 8 times table just past 100; that is, $8 \times 1 = 8, 8 \times 2 = 16, 8 \times 3 = 24, 8 \times 4 = 32$ $8 \times 5 = 40$, and so on.	8, 16, 24, 32, 40, 48, 56, 64, 72, 80, 88, 96, 104
2 Write any multiples that appear in the list.	Numbers in the list that are multiples of 8 are 8, 24, 40, 80.

2.7.2 Factors

- A **factor** is a whole number that divides exactly into another whole number, with no remainder. For example, the number 4 is a factor of 8 because 4 divides into 8 twice with no remainder, or $8 \div 4 = 2$.
- If one number is divisible by another number, the second number divides exactly into the first number.
- Factors of a number can be written as factor pairs. These are pairs of numbers that multiply to equal a certain number.

WORKED EXAMPLE 20

Find all the factors of 14.

THINK	WRITE
1 1 is a factor of every number and the number itself is a factor; that is, $1 \times 14 = 14$.	1, 14
2 14 is an even number so 14 is divisible by 2; therefore 2 is a factor. Divide the number by 2 to find the other factor ($14 \div 2 = 7$).	2, 7
3 Write a sentence placing the factors in order from smallest to largest.	The factors of 14 are 1, 2, 7 and 14.

WORKED EXAMPLE 21

List the factor pairs of 30.

THINK	WRITE
1 1 and the number itself are factors; that is, $1 \times 30 = 30$.	1, 30
2 30 is an even number so 2 and 15 are factors; that is, $2 \times 15 = 30$.	2, 15
3 Divide the next smallest number into 30. Therefore, 3 and 10 are factors; that is, $3 \times 10 = 30$.	3, 10
4 30 ends in 0 so 5 divides evenly into 30, that is, $5 \times 6 = 30$.	5, 6
5 List the factor pairs.	The factor pairs of 30 are 1, 30; 2, 15; 3, 10 and 5, 6.

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Complete this digital doc: SkillSHEET: Multiples (doc-6416)

Complete this digital doc: SkillSHEET: Factor pairs (doc-6417)

Exercise 2.7 Factors and multiples

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1a–h, 2–8, 9a, b, 10–15, 17, 19, 21, 22, 26	Questions: 1e–l, 2–8, 9a, b, 10–17, 18, 19–22, 26, 27	Questions: 1l–p, 2–9, 12, 16–27

Individual pathway interactivity: int-4316

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Fluency

1. WE 18 List the first five multiples of the following numbers.

a. 3	**b.** 6	**c.** 100	**d.** 11	**e.** 15	**f.** 4
g. 21	**h.** 25	**i.** 14	**j.** 12	**k.** 50	**l.** 30
m. 33	**n.** 120	**o.** 45	**p.** 72		

2. WE 19 Write the numbers in the following list that are multiples of 10.
10, 15, 20, 100, 38, 62, 70

3. Write the numbers in the following list that are multiples of 7.
17, 21, 7, 70, 47, 27, 35

4. Write the numbers in the following list that are multiples of 16.
16, 8, 24, 64, 160, 42, 4, 32, 1, 2, 80

5. Write the numbers in the following list that are multiples of 35.
7, 70, 95, 35, 140, 5, 165, 105, 700

6. The numbers 16, 40 and 64 are all multiples of 8. Find three more multiples of 8 that are less than 100.

7. List the multiples of 9 that are less than 100.

8. List the multiples of 6 between 100 and 160.

9. a. MC The first three multiples of 9 are:

A. 1, 3, 9 **B.** 3, 6, 9 **C.** 9, 18, 27 **D.** 9, 18, 81 **E.** 18, 27, 36

b. The first three multiples of 15 are:

A. 15, 30, 45 **B.** 30, 45, 60 **C.** 1, 15, 30 **D.** 45 **E.** 3, 5, 15

c. Justify your answer to part **b** using mathematical reasoning.

10. WE 20 Find all the factors of each of the following numbers.

a. 12 **b.** 8 **c.** 40 **d.** 35
e. 28 **f.** 60 **g.** 100 **h.** 72
i. 39 **j.** 85 **k.** 76 **l.** 69
m. 99 **n.** 250 **o.** 51 **p.** 105

11. WE 21 List the factor pairs of:

a. 20 **b.** 18 **c.** 36 **d.** 132

12. If 3 is a factor of 12, state the smallest number greater than 12 which has 3 as one of its factors.

13. a. MC A factor pair of 24 is:

A. 2, 4 **B.** 4, 6 **C.** 6, 2 **D.** 2, 8 **E.** 3, 9

b. A factor pair of 42 is:

A. 6, 7 **B.** 20, 2 **C.** 21, 1 **D.** 16, 2 **E.** 6, 8

14. Which of the numbers 3, 4, 5 and 11 are factors of 2004?

Understanding

15. Alex and Nadia were having races running down a flight of stairs. Nadia took the stairs two at a time while Alex took the stairs three at a time. In each case, they reached the bottom with no steps left over.

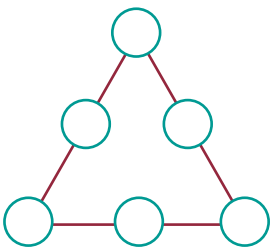
a. How many steps are there in the flight of stairs? List three possible answers.

b. What is the smallest number of steps there could be?

c. If Alex can also take the stairs five at a time with no steps left over, what is the smallest number of steps in the flight of stairs?

16. Connie Pythagoras is trying to organise her Year 4 class into rows for their class photograph. If Ms Pythagoras wishes to organise the 20 students into rows containing equal numbers of students, what possible arrangements can she have?

Note: Ms Pythagoras will not be in the photograph.



17. Place the first 6 multiples of 3 into the following triangle so the numbers along each side of the triangle add up to 27. Use each number once only.

18. a. MC Tilly Tyler has 24 green bathroom tiles left over. If she wants to use them all on the wall behind the kitchen sink (without breaking any) which of the following arrangements would be suitable?

I 4 rows of 8 tiles II 2 rows of 12 tiles III 4 rows of 6 tiles
IV 6 rows of 5 tiles V 3 rows of 8 tiles

A. I and II **B.** I, II and III **C.** II, IV and V
D. II, III and V **E.** V

b. Justify your answer to part **a** using mathematical reasoning.

Reasoning

19. I am a 2-digit number that can be divided by 3 with no remainder. The sum of my digits is a multiple of 4 and 6. My first digit is double my second digit. What number am I? Show your working.

20. Find a 2-digit number such that if you subtract 3 from it, the result is a multiple of 3; if you subtract 4 from it, the result is a multiple of 4 and if you subtract 5 from it, the result is a multiple of 5. Explain how you reached the answer.

21. In a class election with 3 candidates, the winner beat the other 2 candidates by 3 and 6 votes respectively. If 27 votes were cast, how many votes did the winner receive? Show how you reached your answer.

22. What number am I? I am a multiple of 5 with factors of 6, 4 and 3. The sum of my digits is 6. Show your working.

23. My age is a multiple of 3 and a factor of 60. The sum of my digits is 3. How old am I? (There are two possible answers.)

24. A room measures 550 centimetres by 325 centimetres. What would be the side length of the largest square tile that can be used to tile the floor without any cutting?

- **a.** Try this question now. (*Hint:* Find the factors of 550 and 325.)
- **b.** How many tiles would fit on the floor along the wall 550 centimetres long?
- **c.** How many tiles would fit on the floor along the wall 325 centimetres long?
- **d.** How many floor tiles would be needed for this room? Explain your answer.

Problem solving

25. Use the following clues to find the values of A, B, C and D.

- A is a factor of B.
- C is the second multiple of B and the fourth multiple of A.
- D is a prime number between 10 and 20.
- All numbers are different, and add up to 41.

26. A man and his grandson share the same birthday. For exactly six consecutive years the grandson's age is a factor of his grandfather's age. How old is each of them at the sixth of these birthdays, given that the grandson is no more than 10?

27. A book has pages numbered 1 to 140. This means that the numbers on each page have either one, two or three of the digits 0 to 9. How many digits would be used in total to number the pages?

Reflection

Does a number have more factors than multiples? Explain.

2.8 Lowest common multiple and highest common factor

2.8.1 Lowest common multiple and highest common factor

- The **lowest common multiple (LCM)** of two or more numbers is the smallest number that has those numbers as factors. For example, 20 is a common multiple of 5 and 10, but 10 is the lowest common multiple of 5 and 10.

- A **common factor** is a number that is a factor of two or more given numbers. For example, for the numbers 6 and 10, the number 2 is a common factor because 2 is a factor of 6, and 2 is a factor of 10.
- The **highest common factor (HCF)** is the largest of the common factors.

WORKED EXAMPLE 22

a Find the common factors of 8 and 24 by:
i listing the factors of 8
ii listing the factors of 24
iii listing the factors common to both 8 and 24.
b State the highest common factor of 8 and 24.

THINK	WRITE
a i 1 Find the pairs of factors of 8.	**a i** 1, 8 2, 4
2 Write them in order.	Factors of 8 are 1, 2, 4, 8.
ii 1 Find the pairs of factors of 24.	**ii** 1, 24 2,12 3, 8 4, 6
2 Write them in order.	Factors of 24 are 1, 2, 3, 4, 6, 8, 12, 24.
iii Write the common factors.	**iii** Common factors are 1, 2, 4, 8.
b Find the highest common factor.	**b** HCF is 8.

- Factors can make it easier to multiply numbers mentally. The following examples show the thought processes required.

WORKED EXAMPLE 23

Find 12 × 15 using factors.

THINK	WRITE
1 Write the question.	12×15
2 Write one of the numbers as a factor pair.	$= 12 \times 5 \times 3$
3 Multiply the first two numbers.	$= 60 \times 3$
4 Find the answer.	$= 180$

WORKED EXAMPLE 24

Use factors to evaluate 32 × 25.

THINK	WRITE
1 Write the question.	32×25
2 Rewrite the question so that one pair of factors is easy to multiply. The number 4 is a factor of 32 and $4 \times 25 = 100$.	$= 8 \times 4 \times 25$
3 Find the answer.	$= 8 \times 100$ $= 800$

Exercise 2.8 Lowest common multiple and highest common factor

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–7, 9, 11, 13, 14, 15, 16, 18, 22, 24	Questions: 1–7, 8, 10, 13, 14, 15, 16, 18, 20, 22, 24, 26	Questions: 1–7, 13, 14, 17–23, 25, 26

Individual pathway interactivity: int-4317

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Fluency

1. a. List the first ten multiples of 4.
b. List the first ten multiples of 6.
c. In your lists, circle the multiples that 4 and 6 have in common (that is, circle the numbers that appear in both lists).
d. What is the lowest multiple that 4 and 6 have in common? This is the lowest common multiple of 4 and 6, known as the LCM.

2. a. List the first six multiples of 3.
b. List the first six multiples of 9.
c. Circle the multiples that 3 and 9 have in common.
d. What is the lowest common multiple of 3 and 9?

3. Find the LCM of each of the following pairs of numbers.
a. 3 and 6 **b.** 6 and 15 **c.** 7 and 10
d. 12 and 16 **e.** 4 and 15

4. Answer True (T) or False (F) to each of the following statements.
a. 20 is a multiple of 10 and 2 only.
b. 15 and 36 are both multiples of 3.
c. 60 is a multiple of 2, 3, 6, 10 and 12.
d. 100 is a multiple of 2, 4, 5, 10, 12 and 25.

5. a. WE 22 List the factors of 21.
b. List the factors of 56.
c. Find the highest common factor of 21 and 56.

6. a. List the factors of 27.
b. List the factors of 15.
c. Find the highest common factor of 27 and 15.

7. a. List the factors of 7.
b. List the factors of 28.
c. Find the highest common factor of 7 and 28.

8. a. List the factors of 48.
b. List the factors of 30.
c. Find the HCF of 48 and 30.

9. Find the highest common factor of 9 and 36.
10. Find the highest common factor of 42 and 77.
11. Find the highest common factor of 36 and 64.
12. Find the highest common factor of 45, 72 and 108.

13. WE 23 Find the following using factors.

a. 12×25	**b.** 12×35	**c.** 12×55
d. 11×16	**e.** 11×14	**f.** 11×15
g. 20×15	**h.** 20×18	**i.** 30×21

14. WE 24 Use factors to evaluate the following.

a. 36×25	**b.** 44×25	**c.** 24×25
d. 72×25	**e.** 124×25	**f.** 132×25
g. 56×50	**h.** 48×125	**i.** 52×250

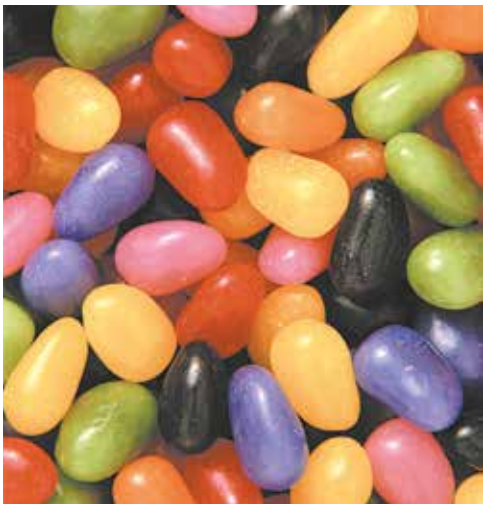
Understanding

15. Kate goes to the gym every second evening, while Ian goes every third evening.
- **a.** How many days will it be before both attend the gym again on the same evening?
- **b.** Explain how this answer relates to the multiples of 2 and 3.

16. Vinod and Elena are riding around a mountain bike trail. Each person completes one lap in the time shown on the stopwatches.
- **a.** If they both begin cycling from the starting point at the same time, how long will it be before they pass this starting point again at exactly the same time?
- **b.** Relate your answer to the multiples of 5 and 7.

17. A warehouse owner employs Bob and Charlotte as security guards. Each security guard checks the building at midnight. Bob then checks the building every 4 hours, and Charlotte every 6 hours.
- **a.** How long will it be until both Bob and Charlotte are next at the warehouse at the same time?
- **b.** Relate your answer to the multiples of 4 and 6.

18. Two smugglers, Bill Bogus and Sally Seadog, have set up signal lights that flash continuously across the ocean. Bill's light flashes every 5 seconds and Sally's light flashes every 4 seconds. If they both start together, how long will it take for both lights to flash again at the same time?



19. Twenty students in Year 7 were each given a different number from 1 to 20 and then asked to sit in numerical order in a circle. Three older girls — Milly, Molly and Mandy — came to distribute jelly beans to the class. Milly gave red jelly beans to every second student, Molly gave green jelly beans to every third student and Mandy gave yellow jelly beans to every fourth student.

a. Which student had jelly beans of all 3 colours?

b. How many students received exactly 2 jelly beans?

c. How many students did not receive any jelly beans?

20. Wah needs to cut tubing into the largest pieces of equal length that he can, without having any offcuts left over. He has three sections of tubing; one 6 metres long, another 9 metres long and the third 15 metres long.

a. How long should each piece of tubing be?

b. How many pieces of tubing will Wah end up with?

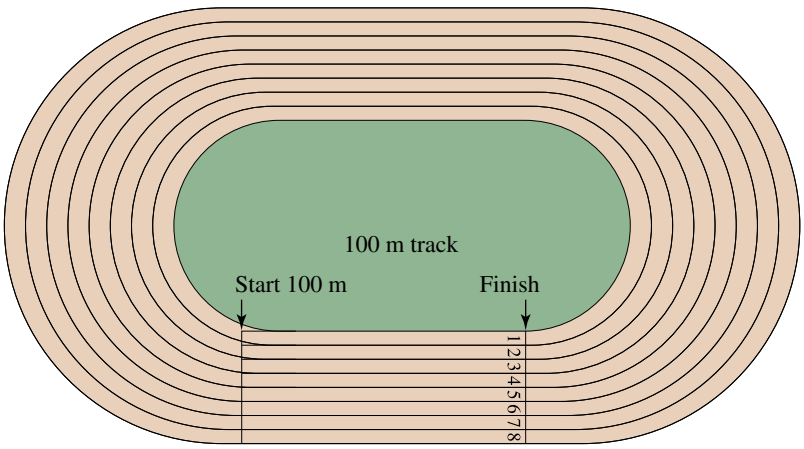
21. Mario, Luigi, Zoe and Daniella are playing on a Playstation. Mario takes 2 minutes to play one level of the game, Luigi takes 3 minutes, Zoe takes 4 minutes and Daniella takes 5 minutes. They have 12 minutes to play.

a. If they play continuously, which of the players would be in the middle of a game as time ran out?

b. After how many minutes did this player begin the last game?

Reasoning

22. The runners in a 100 m race are lined up next to each other while those in a 400 m race are staggered around the track. Look at the following diagram of a standard 400 m athletic track and take note of both the starting position and finish line for a 100 m race.

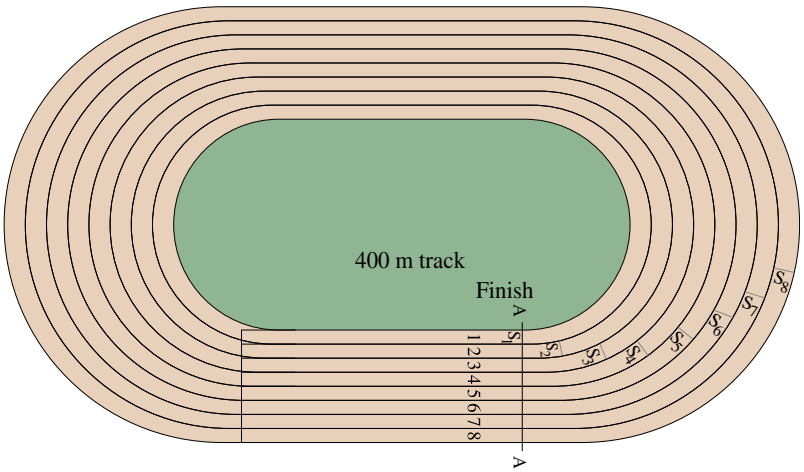


a. What shapes are the two curved ends of the track?

b. Explain why the starting line for a 100 m race has been positioned in the straight section.

c. Explain why the finish line for a 100 m race has been positioned where it is.

Track events such as the 200 m, 400 m and 800 m races have staggered starting positions. The diagram illustrates the position of the starting blocks S_1–S_8 for each lane in the 400 m race.



d. Why is there a need to stagger the starting blocks in the 200 m, 400 m and 800 m track events?

e. A runner completing one lap of the inside lane (lane 1) runs a distance of 400 m, while a runner completing one lap in the next lane (lane 2) runs a distance of 408. If this pattern continues, determine how far runners in lanes 3–8 run. Enter these results in the appropriate column in the table.

Lane number	Distance travelled (m)	Difference
1	400 m	
2	408 m	
3		
4		
5		
6		
7		
8	456 m	

f. Calculate the difference between the distances travelled by the runners in each of the lanes compared to the distance travelled by the runner in lane 1. Enter these results in the appropriate column in the table above.

g. What do you notice about the values obtained in part **f**?

Problem solving

23. Two candles of equal length are lit at the same time. The first candle takes 6 hours to burn out and the second takes 3 hours to burn out. How many hours will the slower burning candle take to be twice as long as the faster burning candle?

24. Carmen types 30 words per minute while Evan types 40 words per minute. They have to type the same number of words and they both end up typing for a whole number of minutes.

a. How long has Carmen been typing for?

b. How long has Evan been typing for?

25. Alex enjoys running after school around the oval. It takes him 10 minutes to complete one lap. His mother Claire walks the family dog at the same time. It takes her 15 minutes to complete one lap. They decide to go home when they meet next. How long will it take them to meet again?

26. The HCF of two numbers is 12 and the LCM is 840. One of the numbers is 60.

a. Calculate the value of the second number.

b. Explain your problem solving method.

Reflection

What strategies will you use to help you remember the difference between highest common factors and lowest common multiples?

2.9 Estimation

2.9.1 Rounding

- An estimate is not the same as a guess, because it is based on information.
- Estimation is useful when we are working with calculators. By mentally estimating an approximate answer, we are more likely to notice a data entry error.
- To estimate the answer to a mathematical problem, round the numbers to the first digit and find an approximate answer.
- Estimations can be made when multiplying, dividing, adding or subtracting. They can also be used when there is more than one operation in the same question.
- If the second digit is 0, 1, 2, 3 or 4, the first digit stays the same.
- If the second digit is 5, 6, 7, 8 or 9, the first digit is rounded up.
- If rounding to the first digit:
  6512 would be rounded to 7000 as it is closer to 7000
  6397 would be rounded to 6000 as it is closer to 6000
  6500 would be rounded to 7000. It is exactly halfway between 6000 and 7000. So to avoid confusion, if it is halfway the number is rounded up.

WORKED EXAMPLE 25

Estimate 48 921 × 823 by rounding to the first digit.

THINK	WRITE
1 Write the question.	48 921 × 823
2 Round each part of the question to the first digit.	≈ 50 000 × 800
3 Multiply.	≈ 40 000 000

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Complete this digital doc: SkillSHEET: Rounding to the first (leading) digit (doc-6418)

Complete this digital doc: Positive integers II (doc-0274)

Exercise 2.9 Estimation

assesson

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1, 2, 3, 5, 6, 8, 10	Questions: 1, 2, 4, 5, 6, 8, 10	Questions: 1, 2, 5–10

Individual pathway interactivity: int-4318

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Fluency

1. WE25 Estimate 67 451 × 432 by rounding to the first digit.

2. Copy and complete the following table by rounding the numbers to the first digit. The first row has been completed as an example.

- In the column headed 'Estimate', round each number to the leading digit.
- In the column headed 'Estimated answer', calculate the answer.
- In the column headed 'Prediction', guess whether the actual answer will be higher or lower than your estimate.
- Use a calculator to work out the actual answer and record it in the column headed 'Calculation' to determine whether it is higher or lower than your estimate.

		Estimate	Estimated answer	Prediction	Calculation*
Example	4129 ÷ 246	4000 ÷ 200	20	Lower	16.784 553 so lower
a	487 + 962				
b	33 041 + 82 629				
c	184 029 + 723 419				
d	93 261 − 37 381				
e	321 − 194				
f	468 011 − 171 962				
g	36 × 198				
h	623 × 12 671				
i	29 486 × 39				
j	31 690 ÷ 963				
k	63 003 ÷ 2590				
l	69 241 ÷ 1297				

*Is the actual answer higher or lower than the estimate?

3. a. MC The best estimate of 4372 + 2587 is:

A. 1000 **B.** 5527 **C.** 6000 **D.** 7000 **E.** 7459

b. The best estimate of 672 × 54 is:

A. 30 000 **B.** 35 000 **C.** 36 000 **D.** 40 000 **E.** 42 000

c. The best estimate of 67 843 ÷ 365 is:

A. 150 **B.** 175 **C.** 200 **D.** 230 **E.** 250

4. Estimate the answers to each of these.

a. 5961 + 1768 **b.** 432 − 192 **c.** 48 022 ÷ 538 **d.** 9701 × 37

e. 98 631 + 608 897 **f.** 6501 + 3790 **g.** 11 890 − 3642 **h.** 83 481 ÷ 1751

i. 112 000 × 83 **j.** 66 501 ÷ 738 **k.** 392 × 113 486 **l.** 12 476 ÷ 24

Understanding

5. MC Su-Lin was using her calculator to answer some mathematical questions, but found she obtained a different answer each time she performed the same calculation. Using your estimation skills, predict which of Su-Lin's answers is most likely to be correct.

a. 217 × 489

A. 706 **B.** 106 113 **C.** 13 203 **D.** 19 313 **E.** 105 203

b. 89 344 ÷ 256

A. 39 **B.** 1595 **C.** 89 088 **D.** 349 **E.** 485

c. 78×6703

A. 522 834 **B.** 52 260 **C.** 6781 **D.** 56 732 501 **E.** 51 624

d. $53\,669 \div 451$

A. 10 **B.** 1076 **C.** 53 218 **D.** 119 **E.** 183

6. Julian is selling tickets for his school's theatre production of *South Pacific*. So far he has sold 439 tickets for Thursday night's performance, 529 for Friday's and 587 for Saturday's. The costs of the tickets are $9.80 for adults and $4.90 for students.

a. Round the figures to the first digit to estimate the number of tickets Julian has sold so far.

b. If approximately half the tickets sold were adult tickets and the other half were student tickets, estimate how much money has been received so far by rounding the cost of the tickets to the first digit.

7. During the show's intermission, Jia is planning to run a stall selling hamburgers to raise money for the school. She has priced the items she needs and made a list in order to estimate her expenses.

a. By rounding the item price to the first digit, use the following table to estimate how much each item will cost Jia for the quantity she requires.

b. Estimate what Jia's total shopping bill will be.

c. If Jia sells 300 hamburgers over the 3 nights for $2 each, how much money will she receive for the hamburgers?

d. Approximately how much money will Jia raise through selling hamburgers over the 3 nights?

Item	Item price	Quantity required	Estimated cost
Bread rolls	$2.90/dozen	25 packets of 12	
Hamburgers	$2.40/dozen	25 packets of 12	
Tomato sauce	$1.80/litre	2 litres	
Margarine	$2.20/tub	2 tubs	
Onions	$1.85/kilogram	2 kilograms	
Tomatoes	$3.50/kilogram	2 kilograms	
Lettuce	$1.10 each	5 lettuces	

Reasoning

8. Estimate the number of graduating students shown in the following photograph. (Do not count all of the students in the photograph.) If the hall holds 12 times this number, estimate the total capacity of the hall. Show all working and write a sentence explaining how you solved this problem.

Problem solving

9. At Kooboora Secondary College there are 127 students in Year 7, 152 students in Year 8 and 319 students in Years 9 and 10. Estimate the total number of students in Years 7–10.

10. Brent bought a car 5 years ago for $21 798. The car's value has decreased to one-third of its initial value. What is the approximate value of the car today?

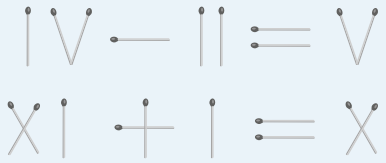
Reflection

When is it important that you do not estimate calculations by rounding numbers?

CHALLENGE 2.2

a. Matchsticks can be used to create Roman numeral sums. Move one matchstick to make a true statement.

b. How can a true statement be made without moving any of the matchsticks?



2.10 Review

2.10.1 Review questions

Fluency

1. State the place value of the digit shown in red in each of the following.
a. 74037 **b.** 541910 **c.** 1904000 **d.** 290

2. Write each of the following numbers in expanded notation.
a. 392 **b.** 4109 **c.** 42001 **d.** 120000

3. List the numbers 394, 349, 943, 934, 3994, 3499 in ascending order.

4. List the numbers 1011, 101, 110, 1100, 1101 in descending order.

5. Find the sum of these numbers.
a. 43 + 84 **b.** 139 + 3048
c. 3488 + 91 + 4062 **d.** 3486208 + 38645 + 692803

6. Calculate each of the following.
a. 20 − 12 + 8 − 14 **b.** 35 + 15 + 5 − 20 **c.** 300 − 170 + 20 **d.** 18 + 10 − 3 − 11

7. Find the difference between these numbers.
a. 688 − 273 **b.** 400 − 183 **c.** 68348 − 8026
d. 46234 − 8476 **e.** 286005 − 193048 **f.** 1370000 − 1274455

8. Use mental strategies to multiply each of the following.
a. 2 × 15 × 5 **b.** 4 × 84 × 25 **c.** 62 × 20
d. 56 × 300 **e.** 67 × 9 **f.** 31 × 19

9. Calculate each of these using short division.
a. 4172 ÷ 7 **b.** 101040 ÷ 12 **c.** 15063 ÷ 3

10. Calculate each of these.
a. 6 × 4 ÷ 3 **b.** 4 × 9 ÷ 12 **c.** 49 ÷ 7 × 12
d. 81 ÷ 9 × 5 **e.** 6 × 3 ÷ 9 ÷ 2 **f.** 12 ÷ 2 × 11 ÷ 3

11. Divide these multiples of 10.
a. 84000 ÷ 120 **b.** 4900 ÷ 700 **c.** 12300 ÷ 30

12. Evaluate the following using long division:
a. 4706 ÷ 13 **b.** 6867 ÷ 21 **c.** 10744 ÷ 17 **d.** 3762 ÷ 19

13. Evaluate the following using long division:
a. 5702 ÷ 17 **b.** 6932 ÷ 23 **c.** 11852 ÷ 13 **d.** 13187 ÷ 29

14. Write the rules for the order of operations.

15. Follow the rules for the order of operations to calculate each of the following.

a. $35 \div (12 - 5)$ **b.** $11 \times 3 + 5$ **c.** $8 \times 3 \div 4$

d. $5 \times 12 - 11 \times 5$ **e.** $(6 + 4) \times 7$ **f.** $6 + 4 \times 7$

g. $3 \times (4 + 5) \times 2$ **h.** $5 + [21 - (5 \times 3)] \times 4$

16. By rounding each number to its first digit, estimate the answer to each of the calculations.

a. $6802 + 7486$ **b.** $8914 - 3571$ **c.** $5304 \div 143$ **d.** 5706×68

e. $49581 + 73258$ **f.** $17564 - 10689$ **g.** $9480 \div 2559$ **h.** 289×671

17. List the first 5 multiples of each number.

a. 11 **b.** 100 **c.** 5 **d.** 20 **e.** 13 **f.** 35

18. Find the lowest common multiple (LCM) of the following pairs of numbers.

a. 3 and 12 **b.** 6 and 15 **c.** 4 and 7 **d.** 5 and 8

19. Find all the factors of each of the following numbers.

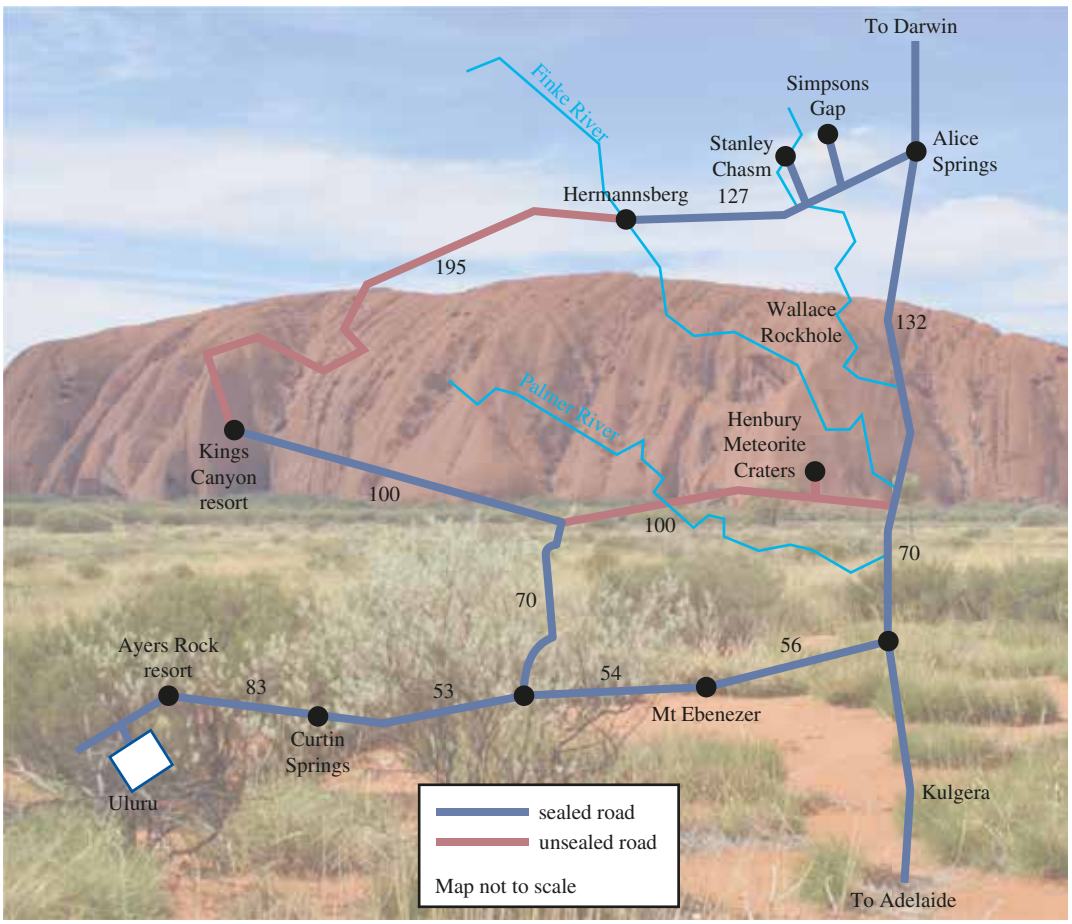
a. 16 **b.** 27 **c.** 50 **d.** 42 **e.** 36 **f.** 72

20. List the factor pairs of the following numbers.

a. 24 **b.** 40 **c.** 48 **d.** 21 **e.** 99 **f.** 100

Problem solving

21. Uluru is a sacred Aboriginal site. The map below shows some roads between Uluru and Alice Springs. The distances (in kilometres) along particular sections of road are indicated.



a. How far is Kings Canyon resort from Ayers Rock resort near Uluru?
b. What is the shortest distance by road if you are travelling from Kings Canyon resort to Alice Springs?
c. If you are in a hire car, you must travel only on sealed roads. Calculate the distance you need to travel if driving from Kings Canyon resort to Alice Springs.

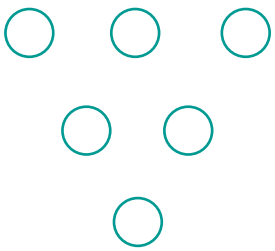
22. In summer, an ice-cream factory operates 16 hours a day and makes 28 ice-creams each hour.
a. How many ice-creams are produced each day?
b. If the factory operates 7 days a week, how many ice-creams are produced in one week?
c. If there are 32 staff who run the machines over a week, how many ice-creams would each person produce?

23. Hung and Frank are cyclists who are riding around a track. They ride past the finish line together, and from then Hung takes 25 seconds to complete a lap and Frank takes 40 seconds.
a. How long will it be until they next pass the finish line together?
b. How many laps will each have ridden when this occurs?

24. When you add two even positive integers, the answer is even; the sum of an even and an odd positive integer is an odd integer; two odd positive integers add to give an even integer. What happens if you perform multiplication on these types of integers? Give examples to support your statements. Write a general statement to summarise what would result when you multiplied more than two positive integers.

25. Joe, Claire and Daniela were having races up and down a flight of stairs. Joe took the stairs three at a time, Daniela took the stairs two at a time while Claire took the stairs four at a time. In each case, they reached the bottom with no steps left over.
a. How many steps are there in the flight of stairs? List three possible answers.
b. What is the smallest number of steps there could be?
c. If Joe can also take the stairs five at a time with no steps left over, what is the smallest number of steps in the flight of stairs?

26. Place the numbers 1 to 6 in these circles so that the number in every circle below two higher circles shows the difference between the two numbers in the higher circles.



27. A perfect number is one whose factors (all except the number itself) add to the number. For example, 6 is a perfect number because $1 + 2 + 3$ (the sum of its factors, excluding 6) is equal to 6. Show why 496 is a perfect number.

28. Complete this subtraction by filling in the missing numbers in the boxes.

$$\begin{array}{r} 8\ 2\ \square\ 0 \\ -3\ \square\ 8\ \square \\ \hline \square\ 5\ 5\ 9 \end{array}$$

29. At a confectionary factory, a machine dispenses 760 lollies into 8 jars. Assuming that an equal number of lollies is dispensed into each jar, find the number in each jar.

30. Arrange the digits 0, 2, 4, 5 and 8 to form the smallest possible 5-digit number. You must use each digit once and only once.

31. In a race, one dirt bike rider completes each lap in 40 seconds while another completes it in 60 seconds. How long after the start of the race will the two bikes pass the starting point together?

32. There are sets of numbers known as *Hailstone numbers*. Here's an example. Pick any integer and enter it into your calculator. If it is an even number, divide by 2. If it is an odd number, multiply by 3, then add 1. Repeat the process with the new number over and over. Explain what happens in the long run, and why these numbers would be called *Hailstone numbers*.

33. Julie sells 8 bottles of soft drink for $3 each and 12 bottles of water for $2 each.
   a. Write a calculation that will find the total value of Julie's sales.
   b. Find the total value of Julie's sales.

34. At a football match, Richard estimates the crowd to be 20000 people. If Richard's estimate is correct to the nearest 1000 people, what is the greatest number of people that could possibly be at the football match.

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- **Complete this digital doc:** Concept map: Topic 2 (doc-10721)

Language

It is important to learn and be able to use correct mathematical language in order to communicate effectively. Create a summary of the topic using the key terms below. You can present your summary in writing or using a concept map, a poster or technology.

approximations
arithmagon
ascending
associative law
BIDMAS
common factor
commutative law
decomposition method
descending
difference
digits
distributive law
dividend
divisor
estimate
factor
highest common factor
integer
lowest common multiple
multiples
place value
products
quotients
rounding
sum

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Investigation | Rich task

Ancient number systems

All ancient civilisations developed methods to count and use numbers. The methods they used to represent numbers did not involve the ten digits 1, 2, 3, 4, 5, 6, 7, 8, 9 and 0 that we use today, but included different systems such as letters, other symbols, pebbles, tying knots in ropes or cutting notches in sticks. As the need to use and represent larger numbers became more and more important, many old systems disappeared.

The ancient Egyptians were one of the oldest known civilisations to have a recorded number system. About 5000 BC, the Egyptians used hieroglyphics, a language system of symbols called *hieroglyphs*. An example of hieroglyphs can be seen in this picture. The hieroglyphs used for numbers are shown below.

Number	1	10	100	1000	10 000	1 00 000	1 000 000	10 000 000
Symbol								

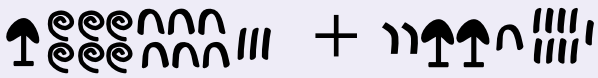
The following example shows how the symbols are used to represent numbers. As the numbers get larger in value, the symbols are simply repeated. This set of symbols represents 241 513.

1. What number is represented by the following Egyptian symbol?

2. Show how the ancient Egyptians would have represented the number 2147.

The ancient Egyptian system of numbers was not limited to representing numbers. Addition and subtraction could easily be performed when it was required. Perform the following additions and subtractions, giving your answers as Egyptian numbers.

3.

4. 

5.

6.

7. Comment on any difficulties you encountered performing the calculations above.

The ancient Greeks used an alphabet that had 27 letters (the current Greek alphabet has only 24 letters). These letters were also used to represent numbers. The first nine letters represented the numbers 1 to 9, the next nine letters represented the tens from 10 to 90, and the last nine letters represented the hundreds from 100 to 900. Numbers were written by combining the letters as needed.

8. Use the library or internet to find the 27 symbols used in the ancient Greek number system and the number that each symbol represented.

9. How did the ancient Greeks represent numbers greater than or equal to 1000?

10. Research number systems used by other ancient civilisations, using the library or internet. Try to find out what symbols were used to represent numbers and how they were used in calculations. Discuss any advantages for the civilisation using this system and why the system may have fallen out of use. Present your finding on a separate sheet of paper.

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Complete this digital doc: Code puzzle: How many bones are there in your skeleton? (doc-14313)

Answers

Topic 2 Positive integers

Exercise 2.2 Place value

1. a. 900 + 20 + 5 **b.** 1000 + 60 + 2 **c.** 20000 + 8000 + 400 + 60 + 9
d. 40 + 3 **e.** 500000 + 2000 + 30 + 9 **f.** 800000 + 2
g. 1 000000 + 80000 + 100 **h.** 20000 + 2000 + 200 + 20 + 2

2. a. Seven hundred and sixty-five **b.** Nine thousand, one hundred and five
c. Ninety thousand, four hundred and fifty **d.** One hundred thousand, two hundred and thirty-six

3. a. 495 **b.** 2670 **c.** 24000 **d.** 109605

4. a. i. Two hundred **ii.** Forty **iii.** Eight
b. i. Two **ii.** One hundred **iii.** Thirty
c. i. Zero **ii.** Two hundred
d. i. Three hundred **ii.** Fifty **iii.** Seven

5. a. E
b. All 5 numbers have 4 in the thousands place; A, C, D and E have 8 in the hundreds place; A and E have 8 in the tens place and only E has 8 in the units place. E has the largest number.

6. B

7. a. 48 987, 8569, 742, 647, 28 **b.** 72 167, 58 625, 47 890, 32 320, 12 947
c. 7647, 7476, 6747, 6477, 4776 **d.** 8880, 8808, 8800, 8088, 8080, 8008

8. a. 9, 58, 743, 68 247, 1 258 647 **b.** 34 108, 58 610, 60000, 78 645, 84 364
c. 1290, 1902, 2019, 2910, 9021, 9201 **d.** 111, 211, 212, 221, 222, 1112

9. a. i. $6000 **ii.** $340000 **iii.** $58000
b. i. $430K **ii.** $7K **iii.** $800K
c. Check with your teacher.

10. Half a minute **11.** 98641

12. a. i. 2 **ii.** 4 **b. i.** 6 **ii.** 27
c. i. In part **a**, the number of two-digit numbers is the same as the number of digits. In part **b** the number of three-digit numbers is double the number of digits because the number can begin with any one of the three digits, and then there are two digits to choose from for the second place and one digit for the third place; this makes six possible numbers.
ii. In part **a**, the number of two-digit numbers is 2^2. In part **b** the number of three-digit numbers is 3^3.

13. About 10 days at a rate of 1 number per second.

Exercise 2.3 Adding and subtracting positive integers

1. a. 15 **b.** 24 **c.** 37 **d.** 121 **e.** 210 **f.** 240 **g.** 472
h. 1730 **i.** 7458 **j.** 18220 **k.** 175000 **l.** 20 **m.** 40 **n.** 50

2. a. 99 **b.** 999 **c.** 4183 **d.** 72648 **e.** 3485551 **f.** 13629

3. a. 978 **b.** 1155 **c.** 766 **d.** 257 **e.** 1335
f. 11747 **g.** 14462 **h.** 332941 **i.** 1701553 **j.** 1053944

4. a. 180 **b.** 200 **c.** 280 **d.** 460 **e.** 790 **f.** 827

5. a. 6 **b.** 8 **c.** 23 **d.** 5 **e.** 60 **f.** 125
g. 814 **h.** 900 **i.** 700 **j.** 10500 **k.** 70 **l.** 35
m. 500 **n.** 63 **o.** 5 **p.** 26 **q.** 39

6. a. 4 **b.** 20 **c.** 26 **d.** 55
e. 40 **f.** 0 **g.** 73 **h.** 1150

7. a. 44 **b.** 35 **c.** 35123 **d.** 86 **e.** 584599
f. 1831 **g.** 267 **h.** 4413 **i.** 65439

8. a. 40672 **b.** 2634 **c.** 69397 **d.** 56176
e. 1553 **f.** 517526 **g.** 349818 **h.** 19088744

9. $247 **10.** 3032 pages **11.** $71

12. a. 8 days **b.** 2003 km **c.** 3492 km **d.** 2754 km

13. 6670 km

14. a. 7, 6, 14 **b.** 15, 10, 15 **c.** 8, 20, 36

15. 15 **16.** 12 149 **17.** 31

18. a. 215 million **b.** 220 million

19. a. 26 m **b.** 26 m **c.** 52 m **d.** Check with your teacher.

20. a. 172 km **b.** 353 km

21. a. 638 321 − 488 417 = 149 904 **b.** 3492 − 1685 = 1807 (Other answers possible)

22. 5 849 318 **23.** 7 days

24. 735 + 1689 = 2424 or 641 + 8752 = 9393

25. Trixie

26. Three students attend both swimming practice and lifesaving.

27. Three people do not work Monday to Sunday.

28. 6

Exercise 2.4 Multiplying and dividing positive integers

1. a. 12 **b.** 45 **c.** 22 **d.** 56 **e.** 96
f. 110 **g.** 54 **h.** 132 **i.** 72

2. a. 26 **b.** 45 **c.** 50 **d.** 39 **e.** 100 **f.** 90
g. 32 **h.** 70 **i.** 42 **j.** 63 **k.** 108 **l.** 75
m. 72 **n.** 90 **o.** 27 **p.** 90 **q.** 100 **r.** 80

3. a. 128 **b.** 116 **c.** 1233 **d.** 2571 **e.** 24600 **f.** 111888
g. 63104 **h.** 1888 **i.** 16120 **j.** 63582 **k.** 374055 **l.** 492720

4. a. 2288 **b.** 3007 **c.** 1652 **d.** 912 **e.** 7093 **f.** 64 600
g. 21571 **h.** 97478 **i.** 2161485 **j.** 4870460 **k.** 27529460 **l.** 1725805

5. a. 80 **b.** 240 **c.** 1900 **d.** 4500 **e.** 900 **f.** 67000

6. a. 900 **b.** 1830 **c.** 3100 **d.** 16800 **e.** 9500 **f.** 172000

7. a. 8000 **b.** 21000 **c.** 480000 **d.** 7200 **e.** 48000 **f.** 5500000
g. 6300000000 **h.** 144000000 **i.** 5600000 **j.** 54000000 **k.** 440000 **l.** 13200000

8. a. 306 **b.** 747 **c.** 5652 **d.** 7425 **e.** 456 **f.** 208

9. a. 5600 **b.** 560 **c.** 5040

10. a. 4320 **b.** 6660 **c.** 11 250 **d.** 28800 **e.** 3600 **f.** 57600

11. a. 150 **b.** 300 **c.** 300 **d.** Multiply first by 6 then double the answer.

12. a. 180 **b.** 840 **c.** 216 **d.** 1260
e. 350 **f.** 640 **g.** 198 **h.** 680

13. a. 90 **b.** 27 **c.** 117 **d.** Multiply by 10 and by 3 then add the answers.

14. a. 325 **b.** 390 **c.** 312 **d.** 1326

15. a. 4 **b.** 3 **c.** 4 **d.** 9 **e.** 7 **f.** 8
g. 12 **h.** 8 **i.** 4 **j.** 4 **k.** 20 **l.** 13
m. 3 **n.** 2 **o.** 3 **p.** 2 **q.** 2 **r.** 4
s. 2 **t.** 5 **u.** 15

16. a. 10 **b.** 6 **c.** 56 **d.** 35 **e.** 84
f. 60 **g.** 3 **h.** 44 **i.** 54

17. a. 485 **b.** 6942 **c.** 6208
d. 57319 **e.** 2761 **f.** 80045
g. 85064 remainder 2 **h.** 86047 remainder 3 **i.** 8643 remainder 1
j. 904075 remainder 5 **k.** 93155 remainder 2 **l.** 60074 remainder 2

18. a. 700 **b.** 10 **c.** 7 **d.** 900 **e.** 9
f. 500 **g.** 5000 **h.** 7 **i.** 800

19. a. \$72 **b.** \$1728

20. \$132 000

21. a. \$675 **b.** \$1200 **c.** \$2700

22. 490 km

23. a. 4420 litres **b.** 22 100 litres

24. 524 seconds

25. a. 34 hours **b.** \$1105 **c.** \$1885

26. a. 10 km **b.** \$175 **c.** 120 km **d.** \$9100 **e.** 400 papers **f.** 20 800 papers

27. a. 58 **b.** 29 **c.** 29 **d.** Divide by 10 and halve the answer.

28. a. 214 **b.** 92 **c.** $39\frac{1}{2}$ **d.** 32

29. a. 28 km **b.** 14 km

30. a. \$208 **b.** \$48

31. 186 cows

32. Number of tables required: 19; total number of courses to be served: 608; number of waiters required: 8; total charge for catering: \$8360

33. a. 950 m^2 **b.** \$794 750

34. \$16 per night

35. a. 37 **b.** 3

36. a. 94 cents **b.** \$32.90 **c.** \$5146.50 **d.** \$3 **e.** \$10

37. 260 **38.** 46 **39.** 52 GB

40. The two numbers are 34 and 35. Their sum is 69.

Challenge 2.1

1089 2178 3267

a. 4356 **b.** 5445 **c.** 6534 **d.** 7623 **e.** 8712 **f.** 9801

Exercise 2.5 Long division

1. a. 15 **b.** 14 **c.** 32 **d.** 19 **e.** 421

f. 658 **g.** 325 **h.** 362 **i.** 62 **j.** 5502

2. a. 65 r 2 or $65\frac{2}{13}$ **b.** 63 r 6 or $63\frac{2}{5}$ **c.** 71 r 3 or $71\frac{3}{17}$ **d.** 84 r 4 or $84\frac{4}{19}$

e. 123 r 7 or $123\frac{7}{41}$ **f.** 251 r 6 or $251\frac{2}{11}$ **g.** 325 r 6 or $325\frac{2}{7}$ **h.** 249 r 2 or $249\frac{2}{31}$

i. 652 r 1 or $652\frac{1}{23}$ **j.** 741 r 4 or $741\frac{4}{27}$

3. a. $\begin{array}{r} 2\boxed{3}6 \\ 19\overline{)448\boxed{4}} \end{array}$ **b.** $\begin{array}{r} 48\boxed{7} \\ 17\overline{)82\boxed{7}9} \end{array}$ **c.** $\begin{array}{r} 2\boxed{4}3 \text{ r } 17 \\ 21\overline{)512\boxed{0}} \end{array}$

4. a. 37 cartons **b.** 29 cartons **c.** Yes

d. 435 is not divisible by 12 but divisible by 15.

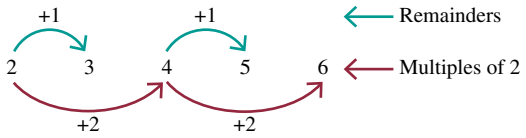
5. Various answers possible.

Dividend	Divisor		
	2	4	8
	Remainder		
2371	1	3	3
2373	1	1	5
2375	1	3	7
2377	1	1	1
2379	1	3	3
2381	1	1	5
2383	1	3	7
2385	1	1	1
2387	1	3	3
2389	1	1	5

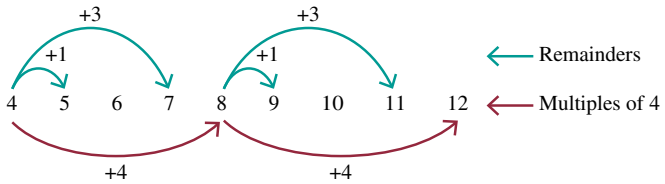
a. When divided by 2, the remainder is always 1.
When divided by 4, the remainder alternates between 1 and 3.
When divided by 8, the remainders repeat 1, 3, 5, 7.

b. All odd numbers leave a remainder of 1 when divided by 2 because the difference between an odd number and an even number is 1.

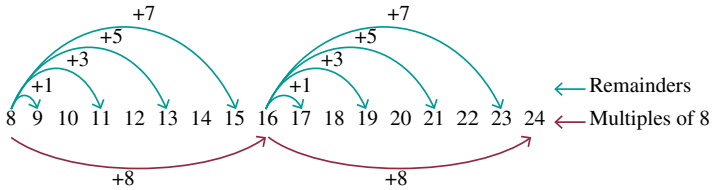
There is only one odd number between each multiple of 2. Therefore, when an odd number is divided by 2, the remainder is always 1.



There are two odd numbers between each multiple of 4. Therefore when an odd number is divided by 4, the remainders are always 1 or 3.



There are four odd numbers between each multiple of 8. Therefore, when an odd number is divided by 8, the remainders are always 1 or 3 or 5 or 7.



6. a. One of the following: 111 111, 222 222, 333 333, 444 444, 555 555, 666 666, 777 777, 888 888, 999 999

b. One of the following: 3367, 6737, 10101, 13468, 16835, 20202, 23569, 26936, 30303.

c. One of the following: 91, 182, 273, 364, 455, 546, 637, 728, 819.

d. One of the following: 1, 2, 3, 4, 5, 6, 7, 8, 9.

e. The final answer is the original digit used to make the number.

7. See answer to question 6; e.g.

$111111 = 1 \times 33 \times 37 \times 91$

$222222 = 2 \times 33 \times 37 \times 91$

Exercise 2.6 Order of operations

1. a. 5 **b.** 9 **c.** 3 **d.** 30 **e.** 59 **f.** 25
g. 7 **h.** 96 **i.** 15 **j.** 24 **k.** 32 **l.** 18
m. 76 **n.** 27 **o.** 26 **p.** 7 **q.** 12 **r.** 96
s. 27 **t.** 0 **u.** 18 **v.** 3

2. a. $(12 - 8) \div 4 = 1$ **b.** $(4 + 8) \times 5 - 4 \times 5 = 40$ **c.** $3 + 4 \times (9 - 3) = 27$
d. $3 \times (10 - 2) \div 4 + 4 = 10$ **e.** $12 \times (4 + 2) - 12 = 60$ **f.** $17 - 8 \times 2 + 6 \times (11 - 5) = 37$
g. $10 \div (5 + 5) \times 9 \times 9 = 81$ **h.** $(18 - 3) \times 3 \div 5 = 9$

3. C

4. B

5. a. $6 + 2 \times (4 - 3) \times 2 = 10$ **b.** $(6 + 2) \times 4 - 3 \times 2 = 26$
c. $6 + (2 \times 4 - 3) \times 2 = 16$ **d.** $6 + 2 \times 4 - 3 \times 2 = 8$, no brackets required

6. James **7.** 43 pebbles **8.** $229.20

9. Two possible solutions: 2 packets with 3 ice-creams and 5 packets with 5 ice-creams, or 7 packets with 3 ice-creams and 2 packets with 5 ice-creams.
With further investigation of the number pattern obtained by summing 3s and 5s, it appears that there are combinations of 3 packs and 5 packs that will amount to any quantity of ice-creams equal to or larger than 8.

10. $1 = (4 - 3) \div (2 - 1)$; $3 = 21 \div (3 + 4)$; $5 = 3 + 4 - (2 \times 1)$

11. $5 = 4 + 3 - (2 \times 1)$; $6 = 3 \times 4 \div (2 \times 1)$
$8 = 4 \times (3 - 2 + 1)$; $9 = 4 + 3 + 2 \div 1$

Exercise 2.7 Factors and multiples

1. a. 3, 6, 9, 12, 15 **b.** 6, 12, 18, 24, 30 **c.** 100, 200, 300, 400, 500 **d.** 11, 22, 33, 44, 55
e. 15, 30, 45, 60, 75 **f.** 4, 8, 12, 16, 20 **g.** 21, 42, 63, 84, 105 **h.** 25, 50, 75, 100, 125
i. 14, 28, 42, 56, 70 **j.** 12, 24, 36, 48, 60 **k.** 50, 100, 150, 200, 250 **l.** 30, 60, 90, 120, 150
m. 33, 66, 99, 132, 165 **n.** 120, 240, 360, 480, 600 **o.** 45, 90, 135, 180, 225 **p.** 72, 144, 216, 288, 360

2. 10, 20, 100, 70 **3.** 21, 7, 70, 35 **4.** 16, 64, 160, 32, 80 **5.** 70, 35, 140, 105, 700

6. Three of: 8, 24, 32, 48, 56, 72, 80, 88, 96

7. 9, 18, 27, 36, 45, 54, 63, 72, 81, 90, 99

8. 102, 108, 114, 120, 126, 132, 138, 144, 150, 156

9. a. C **b.** A **c.** $15 \times 1 = 15$, $15 \times 2 = 30$, $15 \times 3 = 45$

10. a. 1, 2, 3, 4, 6, 12 **b.** 1, 2, 4, 8
c. 1, 2, 4, 5, 8, 10, 20, 40 **d.** 1, 5, 7, 35
e. 1, 2, 4, 7, 14, 28 **f.** 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60
g. 1, 2, 4, 5, 10, 20, 25, 50, 100 **h.** 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72
i. 1, 3, 13, 39 **j.** 1, 5, 17, 85
k. 1, 2, 4, 19, 38, 76 **l.** 1, 3, 23, 69
m. 1, 3, 9, 11, 33, 99 **n.** 1, 2, 5, 10, 25, 50, 125, 250
o. 1, 3, 17, 51 **p.** 1, 3, 5, 7, 15, 21, 35, 105

11. a. 20: 1, 20; 2, 10; 4, 5
b. 18: 1, 18; 2, 9; 3, 6
c. 36: 1, 36; 2, 18; 3, 12; 4, 9; 6, 6
d. 132: 1, 132; 2, 66; 3, 44; 4, 33; 6, 22; 11, 12

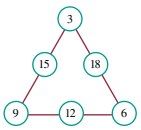
12. 15

13. a. B **b.** A

14. 3, 4

15. a. 6, 12, 18 (or any other multiple of 6) **b.** 6 **c.** 30

16. 4 rows of 5 students or 5 rows of 4 students; 2 rows of 10 students or 10 rows of 2 students; 1 row of 20 students or 20 rows of 1 student

17.


18. a. D **b.** $4 \times 8 \neq 24$, $2 \times 12 = 24$, $4 \times 6 = 24$, $6 \times 5 \neq 24$, $3 \times 8 = 24$

19. 84 **20.** 60 **21.** 12 **22.** 60 **23.** 12 or 30

24. a. 25 cm **b.** 22 **c.** 13 **d.** 286

25. $A = 4$, $B = 8$, $C = 16$, $D = 13$

26. Boy 6, grandfather 66

27. 312 digits

Exercise 2.8 Lowest common multiple and highest common factor

1. a. 4, 8, 12, 16, 20, 24, 28, 32, 36, 40 **b.** 6, 12, 18, 24, 30, 36, 42, 48, 54, 60
c. 12, 24, 36 **d.** 12

2. a. 3, 6, 9, 12, 15, 18 **b.** 9, 18, 27, 36, 45, 54 **c.** 9, 18 **d.** 9

3. a. 3 **b.** 30 **c.** 70 **d.** 48 **e.** 60

4. a. F **b.** T **c.** T **d.** F

5. a. 1, 3, 7, 21 **b.** 1, 2, 4, 7, 8, 14, 28, 56 **c.** HCF = 7

6. a. 1, 3, 9, 27 **b.** 1, 3, 5, 15 **c.** HCF = 3

7. a. 1, 7 **b.** 1, 2, 4, 7, 14, 28 **c.** HCF = 7

8. a. 1, 2, 3, 4, 6, 8, 12, 16, 24, 48

b. 1, 2, 3, 5, 6, 10, 15, 30

c. HCF = 6

9. 9 **10.** 7 **11.** 4 **12.** 9

13. a. 300 **b.** 420 **c.** 660 **d.** 176 **e.** 154

f. 165 **g.** 300 **h.** 360 **i.** 630

14. a. 900 **b.** 1100 **c.** 600 **d.** 1800 **e.** 3100

f. 3300 **g.** 2800 **h.** 6000 **i.** 13000

15. a. 6 days **b.** 6 is the smallest multiple of both 2 and 3.

16. a. 35 minutes **b.** 35 is the smallest multiple of both 5 and 7.

17. a. 12 hours **b.** 12 is the smallest multiple of both 4 and 6.

18. 20 seconds

19. a. 12 **b.** 6 **c.** 7

20. a. 3 metres **b.** 10

21. a. Daniella **b.** 10 minutes

22. a. Two semicircles

b. The starting line for a 100 m race has been positioned in the extended section so the athletes can all start from the same starting line.

c. The finish line for a 100 m race has been positioned so that all athletes can run in a direct line without having to have staggered starting positions.

d. There is a need to stagger the starting blocks in the 200 m, 400 m and 800 m track events so that the outside runners will not be running further than the inside runners.

e. and **f**

Lane number	Distance travelled	Difference
1	400 m	0
2	408 m	8
3	416 m	16
4	424 m	24
5	432 m	32
6	440 m	40
7	448 m	48
8	456 m	56

g. The runner in lane 8 is running 56 metres further than the runner in lane 1.

23. 2 hours

24. a. 4 minutes and any multiple of 4 thereafter. **b.** 3 minutes and any multiple of 3 thereafter.

25. 30 minutes

26. a. 168 **b.** Various methods possible.

Exercise 2.9 Estimation

1. 28000000

2. Estimation table:

	Estimate	Estimated answer	Actual answer
a	500 + 1000	1500	1449
b	30000 + 80000	110000	115670
c	200000 + 700000	900000	907448
d	90000 − 40000	50000	55880

	Estimate	Estimated answer	Actual answer
e	300 − 200	100	127
f	500 000 − 200 000	300 000	296 049
g	40 × 200	8000	7128
h	600 × 10 000	6 000 000	7 894 033
i	30 000 × 40	1 200 000	1 149 954
j	30 000 ÷ 1000	30	32.907 58
k	60 000 ÷ 3000	20	24.325 483
l	70 000 ÷ 1000	70	53.385 505

3. a. D **b.** B **c.** B

4. a. 8000 **b.** 200 **c.** 100 **d.** 400 000
e. 700 000 **f.** 11 000 **g.** 6000 **h.** 40
i. 8 000 000 **j.** 100 **k.** 40 000 000 **l.** 500

5. a. B **b.** D **c.** A **d.** D

6. a. 1500 tickets **b.** $11 250

7. a.

Item	Estimated cost
Bread rolls	$75
Hamburgers	$50
Tomato sauce	$4
Margarine	$4
Onions	$4
Tomatoes	$8
Lettuce	$5

b. $150 **c.** $600 **d.** $450

8. Answers will vary. **9.** 600 students **10.** $7270

Challenge 2.2

a.

b. Turn the page upside down.

2.10 Review

1. a. 7 **b.** 40000 **c.** 1000000 **d.** 90

2. a. 300 + 90 + 2 **b.** 4000 + 100 + 9 **c.** 40000 + 2000 + 1 **d.** 100000 + 20000

3. 349, 394, 934, 943, 3499, 3994

4. 1101, 1100, 1011, 110, 101

5. a. 127 **b.** 3187 **c.** 7641 **d.** 4 217 656

6. a. 2 **b.** 35 **c.** 150 **d.** 14

7. a. 415 **b.** 217 **c.** 60322
d. 37758 **e.** 92957 **f.** 95545

8. a. 150 **b.** 8400 **c.** 1240
d. 16800 **e.** 603 **f.** 589

9. a. 596 **b.** 8420 **c.** 5021

10. a. 8 **b.** 3 **c.** 84
d. 45 **e.** 1 **f.** 22

11. a. 700 **b.** 7 **c.** 410

12. a. 362 **b.** 327 **c.** 632 **d.** 198

13. a. 335 r 7 or $335\frac{7}{17}$ **b.** 301 r 9 or $301\frac{9}{23}$ **c.** 911 r 9 or $911\frac{9}{13}$ **d.** 454 r 21 or $454\frac{21}{29}$

14. Brackets
Indices
Multiplication and division
Addition and subtraction

15. a. 5 **b.** 38 **c.** 6 **d.** 5
e. 70 **f.** 34 **g.** 54 **h.** 29

16. a. 14000 **b.** 5000 **c.** 50 **d.** 420000
e. 120000 **f.** 10000 **g.** 3 **h.** 210000

17. a. 11, 22, 33, 44, 55 **b.** 100, 200, 300, 400, 500 **c.** 5, 10, 15, 20, 25 **d.** 20, 40, 60, 80, 100
e. 13, 26, 39, 52, 65 **f.** 35, 70, 105, 140, 175

18. a. 12 **b.** 30 **c.** 28 **d.** 40

19. a. 1, 2, 4, 8, 16 **b.** 1, 3, 9, 27 **c.** 1, 2, 5, 10, 25, 50 **d.** 1, 2, 3, 6, 7, 14, 21, 42
e. 1, 2, 3, 4, 6, 9, 12, 18, 36 **f.** 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72

20. a. 1, 24; 2, 12; 3, 8; 4, 6 **b.** 1, 40; 2, 20; 4, 10; 5, 8 **c.** 1, 48; 2, 24; 3, 16; 4, 12; 6, 8
d. 1, 21; 3, 7 **e.** 1, 99; 3, 33; 9, 11 **f.** 1, 100; 2, 50; 4, 25; 5, 20; 10, 10

21. a. 306 km **b.** 322 km **c.** 482 km

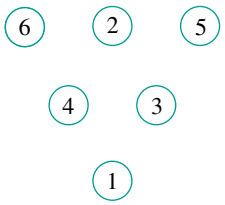
22. a. 448 **b.** 3136 **c.** 98

23. a. 200 s **a.** Hung 8 laps
Frank 5 laps

24. As long as one of the factors in a multiplication is even, the result will be even. Otherwise, the answer will be odd.

25. a. 12, 24, 36 **b.** 12 **c.** 60

26. Six possible answers
Example



27. The sum of its factors (except for 496) add to give 496.

28.
$$\begin{array}{r} 8240 \\ -3681 \\ \hline 4559 \end{array}$$

29. 95

30. 20458

31. 120 seconds or 2 minutes

32. The numbers bounce up and down like hailstones.

33. a. $8 \times 3 + 12 \times 2$ **b.** $48

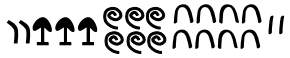
34. 20499

Investigation — Rich task

1. 1 530 430

2.

3.

4. 

5.

6.

7. Answers will vary. Teacher to check.

8.

Symbol	Value	Symbol	Value
α	1	ξ	60
β	2	o	70
γ	3	π	80
δ	4	φ or ς	90
ε	5	ρ	100
ϛ or F	6	σ	200
ξ	7	τ	300
η	8	υ	400
θ	9	ϕ	500
ι	10	χ	600
κ	20	Ψ	700
λ	30	ω	800
μ	40	ϡ	900
ν	50		

9. Adding a subscript or superscript *iota* (Greek letter for 10) with the symbols used for 1 to 9 represented the numbers 1000 to 9000. M was used to represent 10 000. When symbols were placed on top of the M, this meant that the value of the symbols was multiplied by 10 000. This enabled the ancient Greeks to represent larger numbers.

10. Answer will vary. Teacher to check.

TOPIC 3
Indices and primes

3.1 Overview

Numerous **videos** and **interactivities** are embedded just where you need them, at the point of learning, in your learnON title at www.jacplus.com.au. They will help you to learn the content and concepts covered in this topic.

3.1.1 Why learn this?

Indices are very useful in everyday life because they allow us to write very large and very small numbers more simply. For calculations involving such numbers we can use indices to simplify the process. Astronomy is a branch of science where very large distances are involved.

The stars we see in the southern hemisphere night sky are different to the stars people see in the northern hemisphere night sky. Alpha Crusis, the brightest and closest star to us in the Southern Cross constellation, is approximately 3000 million billion kilometres from Earth.

3.1.2 What do you know?

assessON

1. **THINK** List what you know about indices and prime numbers. Use a thinking tool such as a concept map to show your list.
2. **PAIR** Share what you know with a partner and then with a small group.
3. **SHARE** As a class, create a thinking tool such as a large concept map that shows your class's knowledge of indices and prime numbers.

LEARNING SEQUENCE

3.1 Overview
3.2 Index notation
3.3 Prime numbers and composite numbers
3.4 Squares and square roots
3.5 Cubes and cube roots
3.6 Review

3.2 Index notation

3.2.1 Index notation

- An **index** (or **power**) is a shorthand way of writing a repeated multiplication.
- The **base** is the number that is being multiplied and the index (plural *indices*) is the number of times it is multiplied.

$$2 \times 2 \times 2 \times 2 = 2^4$$

(4: index (or power); 2: base)

WORKED EXAMPLE 1

Write the following using index notation.

a $5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5$ **b** $3 \times 3 \times 3 \times 3 \times 7 \times 7$

THINK	WRITE
a 1 Write the multiplication.	**a** $5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5$
2 Write the number being multiplied as the base and the number of times it is written as the index.	$= 5^7$
b 1 Write the multiplication.	**b** $3 \times 3 \times 3 \times 3 \times 7 \times 7$
2 Write the number being multiplied as the base and the number of times it is written as the index.	$= 3^4 \times 7^2$

- By using place value, you can write numbers in expanded form with index notation.
  $2700 = 2000 + 700 = 2 \times 10 \times 10 \times 10 + 7 \times 10 \times 10 = 2 \times 10^3 + 7 \times 10^2$

WORKED EXAMPLE 2

Write the following numbers in expanded form using index notation.

a 59 176 **b** 108 009

THINK	WRITE
a 1 Read the number to yourself, stating the place values.	**a**
2 Write the number as the sum of each place value.	$59\,176 = 50\,000 + 9000 + 100 + 70 + 6$
3 Write each place value in multiples of 10.	$59\,176 = 5 \times 10 \times 10 \times 10 \times 10 + 9 \times 10 \times 10 \times 10 + 1 \times 10 \times 10 + 7 \times 10 + 6$
4 Write each place value in index notation.	$59\,176 = 5 \times 10^4 + 9 \times 10^3 + 1 \times 10^2 + 7 \times 10^1 + 6$
b 1 Read the number to yourself stating the place values.	**b**
2 Write the number as the sum of each place value.	$108\,009 = 100\,000 + 8000 + 9$
3 Write each place value in multiples of 10.	$108\,009 = 1 \times 10 \times 10 \times 10 \times 10 \times 10 + 8 \times 10 \times 10 \times 10 + 9$
4 Write each place value in index notation.	$108\,009 = 1 \times 10^5 + 8 \times 10^3 + 9$

WORKED EXAMPLE 3

Simplify each of the following by writing first as a repeated multiplication and then in index notation.

a $3^4 \times 3^6$ **b** $(4^3)^3$ **c** $(3 \times 5)^2$

THINK	WRITE
a 1 Write the question.	**a** $3^4 \times 3^6$
2 Write the question using repeated multiplication (that is, in expanded form).	$= (3 \times 3 \times 3 \times 3) \times (3 \times 3 \times 3 \times 3 \times 3 \times 3)$ $= 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$
3 Write the repeated multiplication using index notation. *Note:* $3^4 \times 3^6 = 3^{10}$.	$= 3^{10}$
b 1 Write the question.	**b** $(4^3)^3$
2 Write the question using repeated multiplication (that is, in expanded form).	$= 4^3 \times 4^3 \times 4^3$ $= (4 \times 4 \times 4) \times (4 \times 4 \times 4) \times (4 \times 4 \times 4)$ $= 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4$
3 Write the repeated multiplication using index notation. *Note:* $(4^3)^3 = 4^9$.	$= 4^9$
c 1 Write the question.	**c** $(3 \times 5)^2$
2 Write the question using repeated multiplication (that is, in expanded form).	$= (3 \times 5) \times (3 \times 5)$ $= 3 \times 5 \times 3 \times 5$ $= 3 \times 3 \times 5 \times 5$
3 Write the repeated multiplication using index notation. *Note:* $(3 \times 5)^2 = 3^2 \times 5^2$.	$= 3^2 \times 5^2$

Exercise 3.2 Index notation

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1, 2a, b, d, 3, 4a, c, e, g, 5, 6, 7a, c, e, 8a, c, e, 9a, 10, 11, 12b, 16	Questions: 1, 2a, c, e, g, 3, 4a, c, e, g, 5, 6, 7, 8, 9a, 10, 11, 12a, c, e, 13, 15, 17	Questions: 1, 3, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 17, 18

Individual pathway interactivity: int-4319

learnON ONLINE ONLY

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Fluency

1. WE1a Write $7 \times 7 \times 7 \times 7$ in index notation.

2. Write each of the following in index notation.

a. $2 \times 2 \times 2 \times 2$ **b.** $8 \times 8 \times 8 \times 8 \times 8 \times 8$

c. $10 \times 10 \times 10 \times 10 \times 10$ **d.** $3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$

e. $6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6$ **f.** $13 \times 13 \times 13$
g. $12 \times 12 \times 12 \times 12 \times 12 \times 12$ **h.** $9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9$

3. WE1b Write $4 \times 4 \times 4 \times 4 \times 4 \times 6 \times 6 \times 6$ in index notation.

4. Write the following in index notation.
a. $2 \times 2 \times 3$ **b.** $3 \times 3 \times 3 \times 3 \times 2 \times 2$
c. $5 \times 5 \times 2 \times 2 \times 2 \times 2$ **d.** $7 \times 2 \times 2 \times 2$
e. $5 \times 11 \times 11 \times 3 \times 3 \times 3$ **f.** $13 \times 5 \times 5 \times 5 \times 7 \times 7$
g. $2 \times 2 \times 2 \times 3 \times 3 \times 5$ **h.** $3 \times 3 \times 2 \times 2 \times 5 \times 5 \times 5$

5. Write 6^5 using a repeated multiplication (that is, in expanded form).

6. Write each of the following using repeated multiplication (that is, in expanded form).
a. 11^3 **b.** 4^9 **c.** 5^6

7. WE2 Write the following numbers in expanded form using index notation.
a. 300 **b.** 4500 **c.** 6785
d. 10000 **e.** 31702 **f.** 200301

8. WE3 Simplify each of the following by first writing as a repeated multiplication and then using index notation.
a. $5^8 \times 5^3$ **b.** $7^2 \times 7^6$ **c.** $4^8 \times 4^5$
d. $(2^7)^3$ **e.** $(11^3)^8$ **f.** $(9^5)^3$
g. $(3 \times 13)^6$ **h.** $(2 \times 5)^4$ **i.** $(5 \times 7)^5$

Understanding

9. a. MC Which of the following expressions has the greatest value?
A. 2^8 **B.** 8^2 **C.** 3^4 **D.** 4^3 **E.** 9^2
b. Justify your answer using mathematical reasoning.

10. MC The value of 4^4 is:
A. 8 **B.** 16 **C.** 64 **D.** 256 **E.** 484

11. Find the value of $2^3 \times 3^2$.

12. Evaluate each of the following.
a. $3^4 \times 4^3$ **b.** $3^5 + 9^3$ **c.** $8^3 \div 2^5$
d. $6^4 - 9^3$ **e.** $5^3 + 2^5 \times 9^2$ **f.** $2^7 - 4^5 \div 2^6$

Reasoning

13. We know that $12^2 = 144$ and $21^2 = 441$. It is also true that $13^2 = 169$ and $31^2 = 961$. If $14^2 = 196$ will $41^2 = 691$? Try to justify your answer without calculating 41^2.

14. There is a shortcut that allows you to square any number that ends in 5. Look at the following examples and see if you can identify the rule. Use your rule to evaluate 75^2 and 95^2.
$15^2 = 225$, $25^2 = 625$, $35^2 = 1225$, $45^2 = 2025$

Problem solving

15. You received a SMS from your friend. After 5 minutes, you forward the SMS to two of your other friends. After 5 more minutes, those two friends forward it to two more friends. If the SMS is sent every five minutes in this way, how many people have received the SMS in 30 minutes?

16. Jessica has a 'clean-up' button on her mobile phone that will clear half of the read messages in her inbox. She pressed the button six times and the result was just one message left in her inbox. How many messages were in her inbox before the 'clean-up'?

17. A knock-out tennis competition ends with 2 players playing the grand final. If the grand final is the 8th round of the competition, and half the players are knocked out in each round, how many players were there at the start of the competition?

18. The same rule applies to each of these three sets of numbers.
Set A: 1, 2, 5
Set B: 2, 5, 26
Set C: 3, 10, ?
Explain how you can determine the missing number.

Reflection

Why do we use index notation?

3.3 Prime numbers and composite numbers

3.3.1 Prime numbers

- A **prime number** is a counting number that has exactly 2 factors: itself and 1. (Counting numbers are 1, 2, 3, 4, ….)
- The number 3 is a prime number. Its only factors are 3 and 1.
- The number 2 is the only even prime number. Its only factors are 2 and 1.
- A **composite number** is one which has more than two factors.
- The number 4 is composite. Its factors are 1, 2 and 4.
- The number 1 is a special number. It is neither a prime number nor a composite number because it has only one factor — 1.

WORKED EXAMPLE 4

List the prime numbers between 50 and 70.

THINK	WRITE
1 The only even prime number is 2. The prime numbers between 50 and 70 will be odd. Numbers ending in 5 are divisible by 5 so 55 and 65 are not primes.	
2 Check the remaining odd numbers between 50 and 70: $51 = 3 \times 17$ 53 is prime $57 = 3 \times 19$ 59 is prime 61 is prime $63 = 7 \times 9$ 67 is prime $69 = 3 \times 23$	The prime numbers are 53, 59, 61, 67.

WORKED EXAMPLE 5

State whether the following numbers are prime or composite.
a 45 **b 37** **c 86**

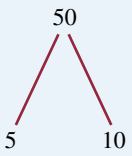
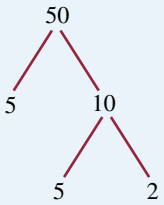
THINK	WRITE
a Factors of 45 are 1, 3, 5, 9, 15 and 45.	**a** 45 is composite.
b The only factors of 37 are 1 and 37.	**b** 37 is prime.
c All even numbers except 2 are composite.	**c** 86 is composite.

3.3.2 Composite numbers and factor trees

- Every composite number can be written as the product of powers of prime numbers; for example, $12 = 2 \times 2 \times 3 = 2^2 \times 3$.
- A **factor tree** shows the **prime factors** of a composite number.
- Each branch shows a factor of all the numbers above it.
- The last numbers in a factor tree are all prime numbers.

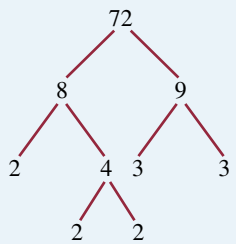
WORKED EXAMPLE 6

a Find the prime factors of 50 by drawing a factor tree.
b Write 50 as a product of its prime factors in index form.

THINK	WRITE
a 1 Find a factor pair of the given number and begin the factor tree ($50 = 5 \times 10$).	**a** 
2 If a branch is prime, no other factors can be found (5 is prime). If a branch is composite, find factors of that number: 10 is composite so $10 = 5 \times 2$.	
3 Continue until all branches end in a prime number, then stop.	
4 Write the prime factors.	The prime factors of 50 are 2 and 5.
b Write 50 as a product of prime factors in index form found in part **a**.	**b** $50 = 5 \times 5 \times 2 = 5^2 \times 2$

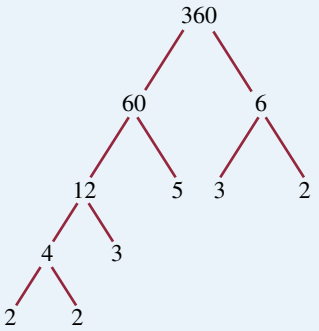
WORKED EXAMPLE 7

Write 72 as a product of its prime factors in index form.

THINK	WRITE
1 Draw a factor tree. When all factors are prime numbers you have found the prime factors.	
2 Write 72 as a product of its prime factors in index form.	$72 = 2^3 \times 3^2$

WORKED EXAMPLE 8

Write 360 as a product of prime factors using index notation.

THINK	WRITE
1 Find a factor pair and begin the factor tree. If the number on the branch is a prime number, stop. If not, continue until a prime number is reached.	
2 Write the number as a product of prime factors.	$360 = 2 \times 2 \times 2 \times 3 \times 3 \times 5$
3 Write your answer using index notation.	$360 = 2^3 \times 3^2 \times 5$

3.3.3 Lowest common multiple and highest common factor using prime factorisation

- It can be difficult to find the lowest common multiple (LCM) or the highest common factor (HCF) of large numbers by inspection alone.
- By expressing large numbers as products of their prime factors, it can be easier to determine their LCM or HCF.
- The HCF of two numbers expressed as the products of their prime factors is the product of the prime factors common to both numbers.
- The LCM of two numbers expressed as the products of their prime factors is the product of the prime factors that are factors of either number.

WORKED EXAMPLE 9

Determine the highest common factor and lowest common multiple of 270 and 900.

THINK	WRITE
1 Write 270 and 900 as products of their prime factors.	$270 = 2 \times 3 \times 3 \times 3 \times 5$ $900 = 2 \times 2 \times 3 \times 3 \times 5 \times 5$
2 Circle the prime factors common to both numbers.	$270 = (2) \times (3) \times (3) \times 3 \times (5)$ $900 = 2 \times (2) \times (3) \times (3) \times 5 \times (5)$
3 For the HCF, multiply the prime factors common to both numbers.	$2 \times 3 \times 3 \times 5 = 90$ The HCF of 270 and 900 is 90.
4 For the LCM, multiply the prime factors that are factors of either number.	$2 \times 3 \times 3 \times 5 \times 2 \times 3 \times 5 = 2700$ The LCM of 270 and 900 is 2700.

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Complete this digital doc: Spreadsheet: Sieve of Eratosthenes (doc-1689)

Complete this digital doc: WorkSHEET: Indices and primes I (doc-1687)

Exercise 3.3 Prime numbers and composite numbers

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1, 2a–f, 3a–f, 4a–f, 5a–f, 6a–d, 7	Questions: 1, 2d–i, 3d–i, 4d–i, 5d–i, 6c–f, 7	Questions: 1, 2g–l, 3g–l, 4g–l, 5g–l, 6e–h, 7, 8

Individual pathway interactivity: int-4320

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Fluency

1. WE4 Find four prime numbers that are between 20 and 40.

2. WE5 State whether each of the following numbers is prime or composite.

a. 9	**b.** 13	**c.** 27	**d.** 55
e. 41	**f.** 64	**g.** 49	**h.** 93
i. 51	**j.** 79	**k.** 243	**l.** 101

3. WE6 **i.** Find the prime factors of each of the following numbers by drawing a factor tree.
ii. Write each one as a product of its prime factors in index form.

a. 15	**b.** 30	**c.** 24	**d.** 100
e. 49	**f.** 72	**g.** 18	**h.** 56
i. 45	**j.** 84	**k.** 98	**l.** 112

4. WE7 **i.** Find the prime factors of the following numbers by drawing a factor tree.
ii. Express the number as a product of its prime factors in index form.

a. 40	**b.** 35	**c.** 32	**d.** 121
e. 110	**f.** 150	**g.** 3000	**h.** 64
i. 96	**j.** 196	**k.** 90	**l.** 75

5. Find the prime factors of each of the following numbers.

a. 48	**b.** 200	**c.** 42	**d.** 81
e. 18	**f.** 39	**g.** 27	**h.** 300
i. 60	**j.** 120	**k.** 50	**l.** 80

6. WE8 Write the following as a product of prime factors using index notation.

a. 60	**b.** 50	**c.** 75	**d.** 220
e. 192	**f.** 72	**g.** 124	**h.** 200

Understanding

7. WE9 By expressing the following pairs of numbers as products of their prime factors, determine their lowest common multiple and their highest common factor.

a. 36 and 84	**b.** 48 and 60	**c.** 120 and 400	**d.** 220 and 800

8. Can you find four prime numbers that are even? Explain.

9. Answer True (T) or False (F) for each of the following.
a. All odd numbers are prime numbers.
b. No even numbers are prime numbers.

c. 1, 2, 3 and 5 are the first four prime numbers.
d. A prime number has two factors only.
e. 2 is the only even prime number
f. The sum of two prime numbers is always even.
g. The product of two prime numbers is always odd.
h. There are no consecutive prime numbers.

10. MC **a.** The number of primes less than 10 is:
A. 4 **B.** 3 **C.** 5 **D.** 2 **E.** 1

b. The first three prime numbers are:
A. 1, 3, 5 **B.** 2, 3, 4 **C.** 2, 3, 5 **D.** 3, 5, 7 **E.** 2, 5, 7

c. The number 15 can be written as the sum of two prime numbers. These are:
A. 3 + 12 **B.** 1 + 14 **C.** 13 + 2 **D.** 7 + 8 **E.** 9 + 6

d. Factors of 12 that are prime numbers are:
A. 1, 2, 3, 4 **B.** 2, 3, 6 **C.** 2, 3
D. 2, 4, 6, 12 **E.** 1, 2, 3, 4, 6, 12

11. Twin primes are pairs of primes that are separated from each other by one even number. For example, 3 and 5 are twin primes. Find two more pairs of twin primes.

12. a. Which of the numbers 2, 3, 4, 5, 6 and 7 cannot be the difference between two consecutive prime numbers? Explain.
b. For each of the numbers that can be a difference between two consecutive primes, give an example of a pair of primes less than 100 with such a difference.

13. The following numbers are not primes. Each of them is the product of two primes. Find the two primes in each case.
a. 365 **b.** 187

14. An easy way to find prime numbers is to use the Sieve of Eratosthenes. Eratosthenes discovered a simple method of sifting out all of the composite numbers so that only prime numbers are left. Alternatively, you can use the Excel file in your learnON title.
You can follow the following steps to find all prime numbers between 1 and 100.
a. Write the numbers from 1 to 100 in a grid as shown.

χ	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

b. Cross out 1 as shown. It is not a prime number.
c. Circle the first prime number, 2. Then cross out all of the multiples of 2.
d. Circle the next prime number, 3. Now cross out all of the multiples of 3 that have not already been crossed out.

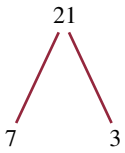
e. The number 4 is already crossed out. Circle the next prime number, 5. Cross out all of the multiples of 5 that are not already crossed out.

f. The next number that is not crossed out is 7. Circle 7 and cross out all of the multiples of 7 that are not already crossed out.

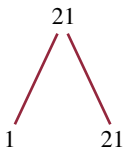
g. Do you need to check the multiples of any primes greater than 7? Why or why not?

15. MC **a.** A factor tree for 21 is:

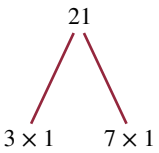
A.



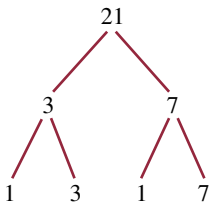
B.



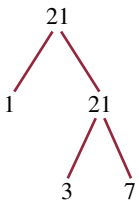
C.



D.

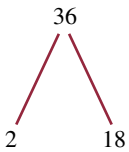


E.

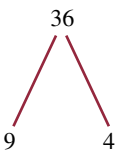


b. A factor tree for 36 is:

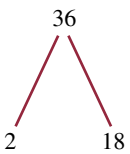
A.



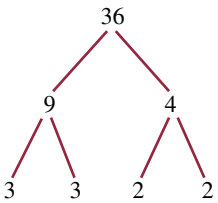
B.



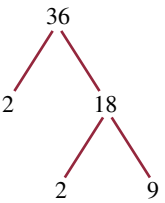
C.



D.



E.



c. The prime factors of 16 are:

A. 1, 2 **B.** 1, 2, 4 **C.** 2 **D.** 1, 2, 4, 8, 16 **E.** 2, 4, 8

d. The prime factors of 28 are:

A. 1, 28 **B.** 2, 7 **C.** 1, 2, 14 **D.** 1, 2, 7 **E.** 2, 7, 14

Reasoning

16. What is the largest three-digit prime number in which each digit is a prime number? Prove that this number is a prime number.

17. Find a prime number greater than 10 where the sum of the digits equals 11. Show your working.

18. My age is a prime number. I am older than 50. The sum of the digits in my age is also a prime number. If you add a multiple of 13 to my age the result is 100. How old am I?

Problem solving

19. Twin primes are pairs of prime numbers that differ by 2. Except for the pair of primes 2 and 3, this is the smallest difference between two prime numbers. The first twin primes are 3 and 5, followed by 5 and 7, then 11 and 13. What other twin primes are there below 100?

20. Find two prime numbers with a product of:

a. 21 **b.** 26 **c.** 323

Reflection

What strategies will you use to help recall the difference between prime and composite numbers?

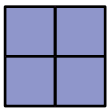
3.4 Squares and square roots

3.4.1 Square numbers

- **Square numbers** are numbers that can be arranged in a square, as shown in the following points.
- The first square number, 1, equals 1×1.

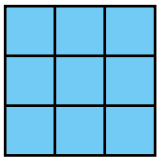
$1^2 = 1$

- The second square number, 4, equals 2×2.



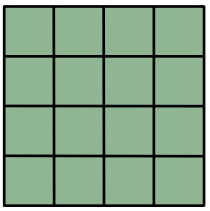
$2^2 = 4$

- The third square number, 9, equals 3×3.



$3^2 = 9$

- The fourth square number, 16, equals 4×4.



$4^2 = 16$

- If this pattern is continued, any square number can be found by multiplying the position of the square number by itself. This is known as *squaring a number* and is written using an index (or power) of 2. For example, 3×3 can be written as 3^2.

WORKED EXAMPLE 10

Find the square numbers between 90 and 150.

THINK	WRITE
1 Use your knowledge of tables to find the first square number after 90.	$10^2 = 10 \times 10 = 100$
2 Find the square numbers which come after that one but before 150.	$11^2 = 11 \times 11 = 121$ $12^2 = 12 \times 12 = 144$ $13^2 = 13 \times 13 = 169$ (too big)
3 Write the answer in a sentence.	The square numbers between 90 and 150 are 100, 121 and 144.

WORKED EXAMPLE 11

Write the two whole square numbers between which 5.7^2 will lie.

THINK	WRITE
1 Write the whole numbers either side of 5.7.	5.7 is between 5 and 6.
2 Consider the square of each whole number.	5.7^2 is between 5^2 and 6^2.
3 Simplify 5^2 and 6^2 then write the answer in a sentence.	So 5.7^2 is between 25 and 36.
4 Verify your answer with a calculator.	

3.4.2 Square roots

- Evaluating the **square root** of a number is the opposite of squaring a number.
- The square root symbol is written as $\sqrt{\ }$.
- By writing $4^2 = 16$, we mean that 4 multiplied by itself is equal to 16. The opposite of this is to say that the square root of 16 is 4, or $\sqrt{16} = 4$.
- Using the illustrations of the squares above, a square of area 16 square units must have a side length of 4 units.
- To find the square roots of larger numbers, it helps to break the number up as a product of two smaller square roots with which we are more familiar. For example:

$$\begin{aligned}\sqrt{900} &= \sqrt{9} \times \sqrt{100} \\ &= 3 \times 10 \\ &= 30\end{aligned}$$

WORKED EXAMPLE 12

Find: **a** $\sqrt{49}$ **b** $\sqrt{3600}$.

THINK	WRITE
a Find a number which when multiplied by itself gives 49.	**a** $\sqrt{49} = 7$ $(7 \times 7 = 49)$.
b 1 Write 3600 as the product of two smaller numbers for which we can find the square root.	**b** $\sqrt{3600} = \sqrt{36} = \sqrt{100}$
2 Take the square root of each of these numbers.	$= 6 \times 10$
3 Find the product.	$= 60$

WORKED EXAMPLE 13

Between which two numbers will $\sqrt{74}$ lie?

THINK	WRITE
1 Write the square numbers either side of 74.	74 is between 64 and 81.
2 Consider the square root of each number.	$\sqrt{74}$ is between $\sqrt{64}$ and $\sqrt{81}$.
3 Simplify $\sqrt{64}$ and $\sqrt{81}$.	So $\sqrt{74}$ is between 8 and 9.
4 Verify your answer with a calculator.	

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Complete this digital doc: SkillSHEET: Finding the square root of a number (doc-6428)

Exercise 3.4 Squares and square roots

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1, 2a, 3a, c, d, 4, 5, 6, 7a, b, c, 9, 10, 11a, b, e, 12, 15	Questions: 1, 2, 3b, d, 4, 5, 6, 7, 8, 9, 11, 12, 13, 14, 15, 16	Questions: 1, 2, 3a, c, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17

Individual pathway interactivity: int-4321

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Fluency

1. Evaluate and verify your answers with a calculator.
a. 8^2 **b.** 11^2 **c.** 15^2 **d.** 25^2

2. WE10 **a.** Find the square numbers between 50 and 100.
b. Find the square numbers between 160 and 200.

3. WE11 Write two whole square numbers between which each of the following will lie.
a. 6.4^2 **b.** 7.8^2 **c.** 9.2^2 **d.** 12.5^2

4. WE12 Find:
a. $\sqrt{25}$ **b.** $\sqrt{81}$ **c.** $\sqrt{144}$ **d.** $\sqrt{400}$

5. Find and verify your answers with a calculator.
a. $\sqrt{4900}$ **b.** $\sqrt{14\,400}$ **c.** $\sqrt{360\,000}$ **d.** $\sqrt{160\,000}$

6. WE13 Between which two numbers will $\sqrt{60}$ lie?

7. Write the two whole numbers between which each of the following will lie.
a. $\sqrt{14}$ **b.** $\sqrt{90}$ **c.** $\sqrt{200}$ **d.** $\sqrt{2}$

Understanding

8. **a.** Find the even square numbers between 10 and 70.
b. Find the odd square numbers between 50 and 120.

9. MC For which of the following square roots can we calculate an exact answer?
A. $\sqrt{10}$ **B.** $\sqrt{25}$ **C.** $\sqrt{50}$ **D.** $\sqrt{75}$ **E.** $\sqrt{82}$

10. MC For which of the following square roots can we not calculate the exact value?
A. $\sqrt{160}$ **B.** $\sqrt{400}$ **C.** $\sqrt{900}$ **D.** $\sqrt{2500}$ **E.** $\sqrt{3600}$

11. Evaluate the following. Verify your answers with a calculator.
a. $2^2 + \sqrt{25}$ **b.** $9^2 - \sqrt{36}$ **c.** $5^2 \times 2^2 \times \sqrt{49}$
d. $3^2 + 2^2 \times \sqrt{16}$ **e.** $3^2 - 2^2 \div \sqrt{4} + \sqrt{49}$ **f.** $\sqrt{9} \times 4^2 - \sqrt{144} \div 2^2$

Reasoning

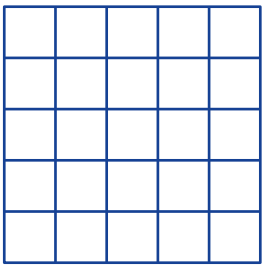
12. Explain why we cannot find the exact value of $\sqrt{10}$.

13. Megan has 3 game scores that happen to be square numbers. The first 2 scores have the same three digits. The total of the 3 scores is 590. What are the 3 scores? Explain how you solved this question.

14. Calculate values for:
a. **i.** 1.5^2 **ii.** 2.5^2 **iii.** 3.5^2
b. Describe the pattern.
c. Explain how you could use this pattern to calculate 8.5^2 without the aid of a calculator.

Problem solving

15. Can you guess the number that matches the following?
It is odd, and it has an odd number of factors.The sum of the digits is a two-digit prime.
The number is less than $\sqrt{10\,000}$ but greater than $\sqrt{100}$.
What number could it be?



16. How many squares of any size can be found in the following diagram?
Hint: Count the number of single squares, count the number of 2×2 squares, then count the number of 3×3 squares. Look for a pattern in these numbers, and use the pattern to help you.

17. Hypatia was an Egyptian mathematician born in AD 370. In her paper on Diophantus, Hypatia posed the following problem. Find a number that satisfies these two conditions:
1. it is the sum of two squares
2. its square is also the sum of two squares.
Can you find 4 numbers that satisfy Hypatia's conditions?

Reflection

Because square numbers form a shape (or figure), they are called figurate numbers. Can you think of any other figurate numbers?

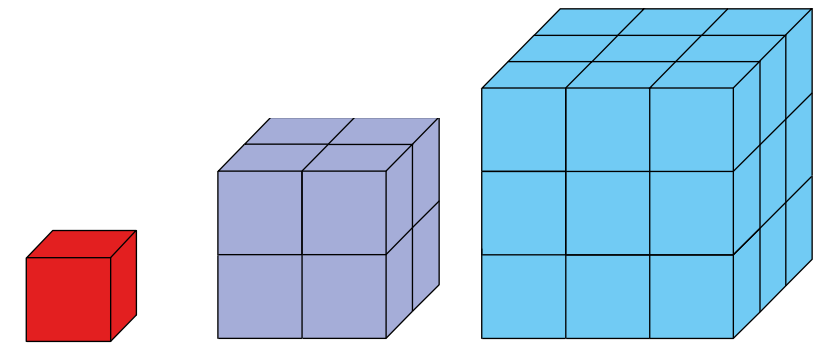
CHALLENGE 3.1

The sum of two numbers is 20, and the difference between their squares is 40. Find the two numbers.

3.5 Cubes and cube roots

3.5.1 Cube numbers

- **Cube numbers** are numbers that can be arranged in a cube, as shown.



- The first cube number, 1, equals $1 \times 1 \times 1$.
- The second cube number, 8, equal $2 \times 2 \times 2$.
- The third cube number, 27, equals $3 \times 3 \times 3$.

- If this pattern is continued, any cube number can be found by multiplying the number by itself three times. This is known as *cubing a number* and is written using an index (or power) of 3. For example, $4^3 = 4 \times 4 \times 4 = 64$.

WORKED EXAMPLE 14

Find the value of 5^3.

THINK	WRITE
1 Write 5^3 as the product of three lots of 5.	$5^3 = 5 \times 5 \times 5$
2 Evaluate.	$= 125$

3.5.2 Cube roots

- The opposite of cubing a number is finding the **cube root** of a number.
- The cube root is found by looking for a number that can be written three times and multiplied to produce the given number.
- The cube root symbol is similar to the square root symbol but with a small 3 written in front, and written as $\sqrt[3]{\ }$.
- From the worked example, we can see that $\sqrt[3]{125} = 5$ ($5 \times 5 \times 5 = 125$).

WORKED EXAMPLE 15

Find $\sqrt[3]{27}$.

THINK	WRITE
Look for a number that when written three times and multiplied gives 27.	$27 = 3 \times 3 \times 3$ $\sqrt[3]{27} = 3$

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Complete this digital doc: SkillSHEET: Finding the cube root of a number (doc-6430)

Complete this digital doc: Indices and primes II (doc-1688)

Exercise 3.5 Cubes and cube roots

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1, 2, 3, 4, 5, 7	Questions: 1, 2, 3, 4, 5, 6, 7	Questions: 1, 2, 3, 4, 5, 6, 7, 8

Individual pathway interactivity: int-4322

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Fluency

1. WE14 Find the value of 4^3.

2. Find the value of:

a. 2^3 **b.** 3^3 **c.** 6^3 **d.** 10^3

3. Write the first 10 cube numbers.

4. WE15 Find $\sqrt[3]{8}$.

Understanding

5. Find each of the following. Verify your answers with a calculator.

a. $\sqrt[3]{64}$ **b.** $\sqrt[3]{216}$ **c.** $\sqrt[3]{343}$ **d.** $\sqrt[3]{729}$

(*Hint:* Use your answer to question **3**.)

Reasoning

6. The first 5 square numbers are 1, 4, 9, 16, 25. If we find the difference between these numbers, we get $4 - 1 = 3$, $9 - 4 = 5$, $16 - 9 = 7$ and $25 - 16 = 9$. These numbers all differ by 2. Representing this in a table, we get:

Square numbers	1	4	9	16	25
First difference	3	5	7	9	
Second difference	2	2	2		

Repeat this process for the first 6 cube numbers. How many times did you need to find the difference until they were equal?

If you look at $1^4, 2^4, 3^4, 4^4, \ldots$, how many differences would you need to find until they were equal?

Problem solving

7 a. Jack was organising a get-together of all his football mates from the last 3 years. He decided to phone 4 footballer mates and ask them to also phone 4 footballers each, who in turn would also phone 4 more footballers. Each team member was given a list of 4 names so that no one received more than one call. How many footballers would receive a phone call about the party?

b. Jack's friend Karin liked the idea, so decided to contact 258 people about a 10-year reunion using Jack's method. Assuming there were three rounds of calls, how many people should each person call?

8. It's an interesting fact that the sum of the digits of a cube of an integer, when reduced to a single digit, can only be 1, 8 or 9.

For example, $12^3 = 1728$

$1 + 7 + 2 + 8 = 18$

$1 + 8 = 9$

There is a pattern in the sequence of these numbers. Try this rule with the cubes of the first 10 integers, then explain how you could determine the single-digit sum of 35^3 without performing the actual calculation.

Reflection

What would be the first 4 numbers that could be arranged as a triangle-based pyramid (all triangles equilateral)?

CHALLENGE 3.2

What is the number? It is greater than 15^2. It is not a multiple of 3 or 5, but a multiple of 4. It is less than 20^2. It is not a multiple of 17. It is a square number.

3.6 Review

3.6.1 Review questions

Fluency

1. Evaluate:
a. 6^2 **b.** 14^2 **c.** 19^2 **d.** 80^2

2. Write the two whole square numbers between which each of the following will lie.
a. 3.8^2 **b.** 5.1^2 **c.** 10.6^2 **d.** 15.2^2

3. Find:
a. $\sqrt{49}$ **b.** $\sqrt{256}$ **c.** $\sqrt{900}$ **d.** $\sqrt{1369}$

4. Evaluate:
a. 4^3 **b.** 7^3 **c.** 15^3 **d.** 30^3

5. Find each of the following.
a. $\sqrt[3]{27}$ **b.** $\sqrt[3]{125}$ **c.** $\sqrt[3]{1000}$ **d.** $\sqrt[3]{8000}$

6. Write the following using index notation.
a. $2 \times 3 \times 3 \times 3$ **b.** $5 \times 5 \times 6 \times 6 \times 6 \times 6$ **c.** $2 \times 5 \times 5 \times 5 \times 9 \times 9 \times 9$

7. Use your calculator to evaluate:
a. 3^5 **b.** 7^3 **c.** 8^4 **d.** 11^5

8. List all of the prime numbers less than 30.

9. How many single digit prime numbers are there?

10. Find the prime number which comes next after 50.

11. Find the prime factors of:
a. 99 **b.** 63 **c.** 125 **d.** 124

12. Express 280 as a product of its prime factors, in index form.

13. Express 144 as a product of its prime factors, in index form.

14. Write the following numbers in expanded form using index notation.
a. 1344 **b.** 30 601

15. Determine the highest common factor and lowest common multiple of the following pairs of numbers by first expressing each number as a product of its prime numbers.
a. 120 and 384 **b.** 72 and 224

Problem solving

16. I am a three-digit number. I am divisible by 6. My middle digit is a prime number. The sum of my digits is 9. I am between 400 and 500. My digits are in descending order. What number am I?

17. Explain why we cannot find the exact value of $\sqrt{20}$.

18. Write down the following numbers in the order of size, starting with the smallest.
$5, \sqrt{64}, 2, \sqrt[3]{27}$

19. Find the smallest possible number which, when multiplied by 120, will give a result that is a square number. Now find the smallest possible number that will give a result that is a cube number.

20. List the prime numbers up to 100. Some prime numbers are separated by a difference of 2 or 6. (For example, $5 - 3 = 2$ and $19 - 17 = 2$.)
a. Find the prime number pairs up to 100 that have a difference of 2.
b. Find the prime number pairs up to 100 that have a difference of 6.
c. Why are there no prime number pairs with a difference of 7?

21. What is the smallest natural number that is both a square number and a cube number? What is the next smallest number?

22. The last digit of 4^2 is 6. What is the last digit of 4^{100}?

23. Complete the following sequence 2, 3, 5, 7, 11, 13, ___, ___, ___.

24. Write each of the numbers 250, 375 and 625 as a product of its prime factors and

a. find the highest common *prime* factor. **b.** find the HCF.

25. A motor boat requires an engine service every 5000 nautical miles, refuelling every 300 nautical miles and an oil change after 2250 nautical miles. How many nautical miles will the motor boat have travelled before all three services are required simultaneously?

26. By folding a sheet of paper in half, you divide it into 2 regions.

a. If you fold it in half again, how many regions are there?

b. Set up a table similar to the one below and fill it in.

c. How does the number of regions increase with each new fold?

d. Write a formula for the number of regions (R) for n folds.

***n* (number of folds)**	***R* (number of regions made)**
0	1
1	2
2	4
3	
4	
5	
6	

27. Look at the table on the next page. As you read down each column, you'll notice that the equations follow a pattern. Continue the patterns to complete the table.

What do you suppose 5^0 equals?

What does 2^2 equal?

$2^5 = 32$	$3^5 = 243$
$2^4 = 16$	$3^4 = 81$
$2^3 = 8$	$3^? = 27$
$2^2 =$	$3^2 =$
$2^1 =$	$3^1 =$
$2^0 =$	$3^0 =$
$2^{-1} =$	$3^{-1s} =$

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Try out this interactivity: Crossword: Topic 3 (int-2588)
Try out this interactivity: Sudoku: Topic 3 (int-3164)
Complete this digital doc: Concept map: Topic 3 (doc-10723)

Language

It is important to learn and be able to use correct mathematical language in order to communicate effectively. Create a summary of the topic using the key terms below. You can present your summary in writing or using a concept map, a poster or technology.

base
composite number
cube
cube root
factor tree
highest common factor
index (indices)
lowest common multiple
prime factors
prime number
square number
square root

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Investigation | Rich task

Alphabet sizes

Bob's business, Reading Resources, produces individual letters of the alphabet printed on cardboard. He caters for the market of visually impaired students who are learning to read. Teachers use the cards to form words for students to read. Because the students have varying difficulties with their sight, the letters are printed in heights of 1 cm, 2 cm, 3 cm,…, 10 cm. Bob has a set of all the letters at 1 cm and uses these as the basis for forming the larger letters.

In Bob's factory, there are 10 machines. Each machine performs only one specific job as indicated in the following information.

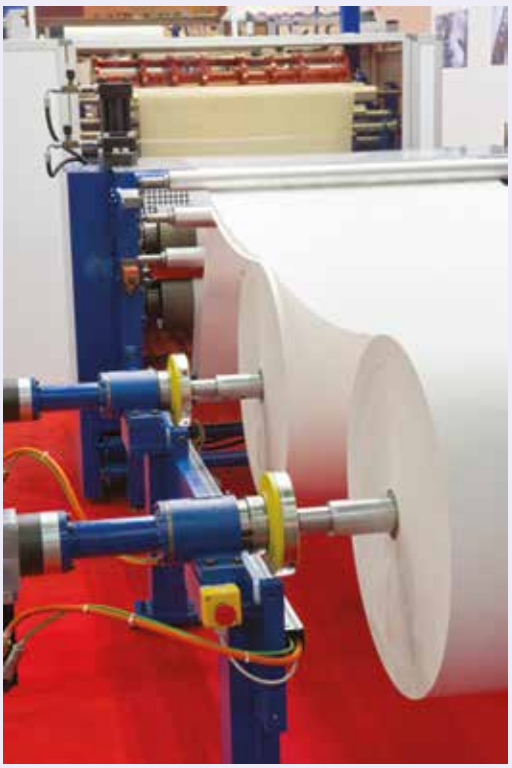
- **Machine 1 makes letters 1 cm high.**
- **Machine 2 enlarges the letters, so they are twice as high.**
- **Machine 3 enlarges the letters, so they are 3 times as high.**
- **Machine 4 enlarges the letters, so they are 4 times as high.**

.
.
.

- **Machine 10 enlarges the letters, so they are 10 times as high.**

Bob has a thriving business and relies heavily on all his machines working. This morning he arrived to process a large order of 6-cm-high letters only to find that machine 6 was not working. He gathered his staff together to discuss the problem.

'No problem!' said Ken. 'As long as the other machines are working we can still get this order done'.



1. Suggest a solution that Ken might have proposed.
2. If Bob can do without machine 6, are there others he can also do without?
3. What is the minimum number of machines that Bob needs to make letters up to 10 cm high?

As Bob's business has become more successful, there has been a demand for letters of a greater height.

4. For Bob to make letters of every whole number up to a height of 20 cm, would he need twice as many machines as he currently has? Explain.
5. Having become very excited about needing fewer machines than he first thought, Bob considered expanding his business to make advertising signs using letters of all whole number heights up to 100 cm. To succeed in this new market, what is the minimum number of machines he would need?
6. Explain why it is necessary to have some specific machine numbers, while some others are not necessary.

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Complete this digital doc: Code puzzle: What does the word Australia mean? (doc-14315)

Answers

Topic 3 Indices and primes

Exercise 3.2 Index notation

1. 7^4

2. a. 2^4 **b.** 8^6 **c.** 10^5 **d.** 3^9
e. 6^7 **f.** 13^3 **g.** 12^6 **h.** 9^8

3. $4^5 \times 6^3$

4. a. $2^2 \times 3$ **b.** $2^2 \times 3^4$ **c.** $2^4 \times 5^2$ **d.** $2^3 \times 7$
e. $3^3 \times 5 \times 11^2$ **f.** $5^3 \times 7^2 \times 13$ **g.** $2^3 \times 3^2 \times 5$ **h.** $2^2 \times 3^2 \times 5^3$

5. $6 \times 6 \times 6 \times 6 \times 6$

6. a. $11 \times 11 \times 11$ **b.** $4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4$ **c.** $5 \times 5 \times 5 \times 5 \times 5 \times 5$

7. a. $300 = 3 \times 10^2$

b. $4500 = 4 \times 10^3 + 5 \times 10^2$

c. $6785 = 6 \times 10^3 + 7 \times 10^2 + 8 \times 10^1 + 5$

d. $10\,000 = 1 \times 10^4$

e. $31\,702 = 3 \times 10^4 + 1 \times 10^3 + 7 \times 10^2 + 0 \times 10^1 + 2$

f. $200\,301 = 2 \times 10^5 + 0 \times 10^4 + 0 \times 10^3 + 3 \times 10^2 + 0 \times 10^1 + 1$

8. a. 5^{11} **b.** 7^8 **c.** 4^{13} **d.** 2^{21} **e.** 11^{24}
f. 9^{15} **g.** $3^6 \times 13^6$ **h.** $2^4 \times 5^4$ **i.** $5^5 \times 7^5$

9. a. A

b. At first the base number 2 would appear to be the smallest but it has the highest power. Although the other numbers have a higher base they have relatively small powers — therefore 2^8 will be the largest number. This can easily be verified by evaluating each number.

10. D

11. 72

12. a. 5184 **b.** 972 **c.** 16
d. 567 **e.** 2717 **f.** 112

13. $41^2 \neq 691$. Notice that $31^2 = 961$, therefore 41^2 will need to be larger than 961.

14. $75^2 = 5625$, $95^2 = 9025$

15. 63

16. 64

17. 256

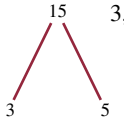
18. Square the first number and add one to calculate the second number. Square the second number and add one to calculate the third number. The missing number is 101.

Exercise 3.3 Prime numbers and composite numbers

1. 23, 29, 31, 37

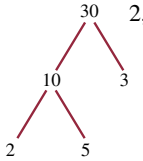
2. a. Composite **b.** Prime **c.** Composite
d. Composite **e.** Prime **f.** Composite
g. Composite **h.** Composite **i.** Composite
j. Prime **k.** Composite **l.** Prime

3. a. i.

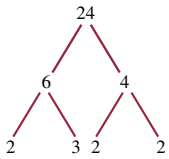


3, 5 **ii.** $15 = 3 \times 5$

b. i.

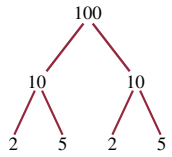


2, 3, 5 **ii.** $30 = 2 \times 3 \times 5$

c. i. 

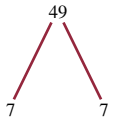
2, 3

ii. $24 = 2^3 \times 3$

d. i. 

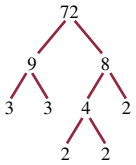
2, 5

ii. $100 = 2^2 \times 5^2$

e. i. 

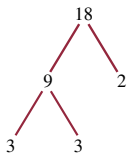
7

ii. $49 = 7^2$

f. i. 

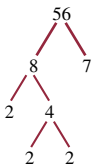
2, 3

ii. $72 = 2^3 \times 3^2$

g. i. 

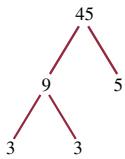
2, 3

ii. $18 = 2 \times 3^2$

h. i. 

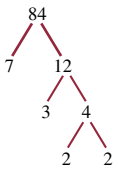
2, 7

ii. $56 = 2^3 \times 7$

i. i. 

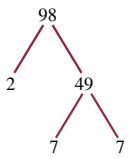
3, 5

ii. $45 = 3^2 \times 5$

j. i. 

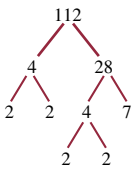
2, 3, 7

ii. $84 = 2^2 \times 3 \times 7$

k. i. 

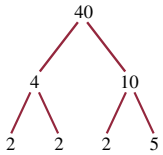
2, 7

ii. $98 = 2 \times 7^2$

l. i. 

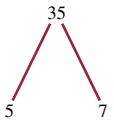
2, 7

ii. $112 = 2^4 \times 7$

4. a. i. 

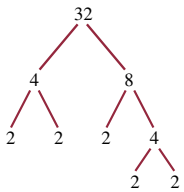
2, 5

ii. $40 = 2^3 \times 5$

b. i. 

5, 7

ii. $35 = 7 \times 5$

c. i. 

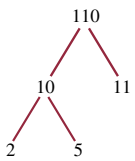
2

ii. $32 = 2^5$

d. i.

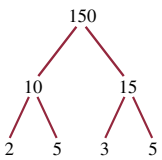
11

ii. $121 = 11^2$

e. i. 

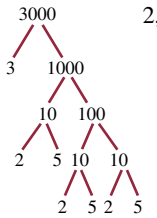
2, 5, 11

ii. $110 = 2 \times 5 \times 11$

f. i. 

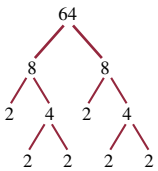
2, 3, 5

ii. $150 = 2 \times 3 \times 5^2$

g. i. 

2, 3, 5

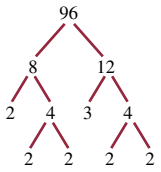
ii. $3000 = 2^3 \times 3 \times 5^3$

h. i. 

2

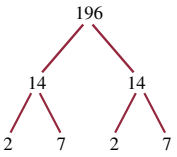
ii. $64 = 2^6$

i. i.



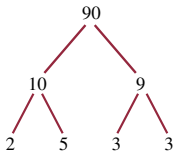
2, 3 **ii.** $96 = 2^5 \times 3$

j. i.



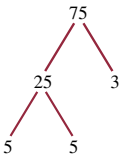
2, 7 **ii.** $196 = 2^2 \times 7^2$

k. i



2, 3, 5 **ii.** $90 = 2 \times 3^2 \times 5$

l. i.



3, 5 **ii.** $75 = 3 \times 5^2$

5. a. 2, 3 **b.** 2, 5 **c.** 2, 3, 7 **d.** 3
e. 2, 3 **f.** 3, 13 **g.** 3 **h.** 2, 3, 5
i. 2, 3, 5 **j.** 2, 3, 5 **k.** 2, 5 **l.** 2, 5

6. a. $2^2 \times 3 \times 5$ **b.** 2×5^2 **c.** 3×5^2 **d.** $2^2 \times 5 \times 11$
e. $2^6 \times 3$ **f.** $2^3 \times 3^2$ **g.** $2^2 \times 31$ **h.** $2^3 \times 5^2$

7. a. LCM is 252; HCF is 12 **b.** LCM is 240; HCF is 12
c. LCM is 1200; HCF is 40 **d.** LCM is 8800; HCF is 20

8. No. The number 2 would always be a factor if the number is even.

9. a. F **b.** F **c.** F **d.** T
e. T **f.** F **g.** F **h.** F

10. a. A **b.** C **c.** C **d.** C

11. 11 and 13; 17 and 19

12. a. 3, 5, 7. Prime numbers greater than 2 are always odd numbers and the difference between 2 odd numbers is an even number. So 2, 4 and 6 can be differences between consecutive primes.

b. 2:17 and 19; 4:19 and 23; 6:23 and 29; other answers possible

13. a. $365 = 5 \times 73$ **b.** $187 = 11 \times 17$

14. a–f

1	(2)	(3)	4	(5)	6	(7)	8	9	10
(11)	12	(13)	14	15	16	(17)	18	(19)	20
21	22	(23)	24	25	26	27	28	(29)	30
(31)	32	33	34	35	36	(37)	38	39	40
(41)	42	(43)	44	45	46	(47)	48	49	50
51	52	(53)	54	55	56	57	58	(59)	60
(61)	62	63	64	65	66	(67)	68	69	70
(71)	72	(73)	74	75	76	77	78	(79)	80
81	82	(83)	84	85	86	87	88	(89)	90
(91)	92	93	94	95	96	(97)	98	99	100

g. It is not necessary to check primes larger than 7. When you check multiples of 7, you know that 7×2 to 7×6 have already been crossed out. The first number to be crossed out will be 49. The next prime is 11. All composite numbers from 11×2 to 11×10 will have already been crossed out. The first composite number not crossed out will be $11 \times 11 = 121 > 100$.

15. a. A **b.** D **c.** C **d.** B

16. 773

17. 29 or 47 or 83 or 137. Others may be possible.

18. 61

19. 17 and 19, 29 and 31, 41 and 43, 59 and 61, 71 and 73

20. a. 3, 7 **b.** 2, 13 **c.** 17, 19

Exercise 3.4 Squares and square roots

1. a. 64 **b.** 121 **c.** 225 **d.** 625

2. a. 64, 81 **b.** 169, 196

3. a. 36 and 49 **b.** 49 and 64 **c.** 81 and 100 **d.** 144 and 169

4. a. 5 **b.** 9 **c.** 12 **d.** 20

5. a. 70 **b.** 120 **c.** 600 **d.** 400

6. 7 and 8

7. a. 3 and 4 **b.** 9 and 10 **c.** 14 and 15 **d.** 1 and 2

8. a. 16, 36, 64 **b.** 81

9. B

10. A

11. a. 9 **b.** 75 **c.** 700 **d.** 25 **e.** 14 **f.** 45

12. An exact value of $\sqrt{10}$ cannot be found because 10 is not a square number.

13. 169, 196, 225

14. a. i. 2.25 **ii.** 6.25 **iii.** 12.25

b. Add the units digit to the square of the units digit and then add 0.25 to the answer.

c. 72.25

15. 49

16. 55

17. 5, 10, 13, 17

Challenge 3.1

9 and 11

Exercise 3.5 Cubes and cube roots

1. 64

2. a. 8 **b.** 27 **c.** 216 **d.** 1000

3. 1, 8, 27, 64, 125, 216, 343, 512, 729, 1000

4. 2

5. a. 4 **b.** 6 **c.** 7 **d.** 9

6. For cube numbers, it was necessary to find the third difference. If we look at power 4, it will be necessary to find the fourth difference.

7. a. 84 footballers would receive a call.

b. Each person would call 6 people.

8. The single-digit sums repeat of cube numbers repeat in the order 1, 8, 9, so the single-digit sum of 35^3 is 8.

Challenge 3.2

256

3.6 Review

1. a. 36 **b.** 196 **c.** 361 **d.** 6400

2. a. 9 and 16 **b.** 25 and 36 **c.** 100 and 121 **d.** 225 and 256

3. a. 7 **b.** 16 **c.** 30 **d.** 37

4. a. 64 **b.** 343 **c.** 3375 **d.** 27 000

5. a. 3 **b.** 5 **c.** 10 **d.** 20

6. a. 2×3^3 **b.** $5^2 \times 6^4$ **c.** $2 \times 5^3 \times 9^3$

7. a. 243 **b.** 343 **c.** 4096 **d.** 161 051

8. 2, 3, 5, 7, 11, 13, 17, 19, 23, 29

9. 4 **10.** 53

11. a. 3, 11 **b.** 3, 7 **c.** 5 **d.** 2, 31

12. $280 = 2 \times 2 \times 2 \times 5 \times 7 = 2^3 \times 5 \times 7$

13. $144 = 2^4 \times 3^2$

14. a. $1344 = 1 \times 10^3 + 3 \times 10^2 + 4 \times 10^1 + 4$

b. $30\,601 = 3 \times 10^4 + 0 \times 10^3 + 6 \times 10^2 + 0 \times 10^1 + 1$

15. a. HCF is 24; LCM is 1920

b. HCF is 8; LCM is 2016

16. 432

17. 20 is not a perfect square, so we can only find an approximate answer. It will be between 4 and 5.

18. 2, $\sqrt[3]{27}$, 5, $\sqrt{64}$

19. 30, 225

20. a. 3, 5; 5, 7; 11, 13; 17, 19; 29, 31; 41, 43; 59, 61; 71, 73

b. 5, 11; 7, 13; 11, 17; 13, 17; 3, 23; 23, 29; 31, 37; 37, 43; 41, 47; 47, 53; 53, 59; 61, 67; 67, 73; 73, 79; 83, 89

c. For 2, the number 7 higher is 9, which is not prime. For odd primes, the number 7 higher is even and therefore not prime.

21. 1, 64 **22.** 6

23. 2, 3, 5, 7, 11, 13, 17, 19, 23

24. a. 5 **b.** 125

25. 45 000

26. a. 4 regions

b.

***n* (number of folds)**	0	1	2	3	4
***R* (number of regions made)**	0	1	4	8	16

c. Each time the page is folded the number of regions is multiplied by 2 (doubled).

d. $R = 2^n$

27.

$2^5 = 32$	$3^5 = 243$
$2^4 = 16$	$3^4 = 81$
$2^3 = 8$	$3^3 = 27$
$2^2 = 4$	$3^2 = 9$
$2^1 = 2$	$3^1 = 3$
$2^0 = 1$	$3^0 = 1$
$2^{-1} = \frac{1}{2}$	$3^{-1} = \frac{1}{3}$
$5^0 = 1$	$2^{-2} = \frac{1}{4}$

Investigation — Rich task

1. Enlarge the letters using machine 2 then machine 3.

2. 4, 6, 8, 9 and 10

3. 5

4. No. Numbers not required can be made using their prime factors.

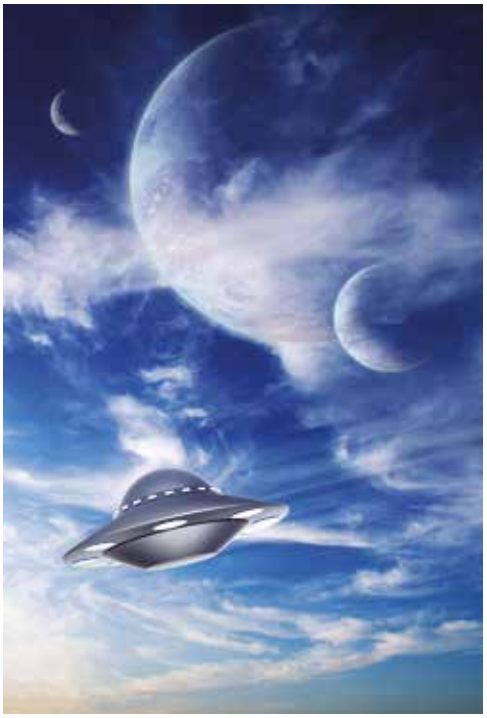
5. Machine 1 and the prime numbers less than 100. Total of 26 machines.

6. Machine 1 is needed to make letters one centimetre high. Machines whose numbers are prime numbers can make all the sizes up to 100 centimetres.

learnon

Project: We are not alone!

Scenario



'ALERT! ALERT! Those alien Earthlings have finally discovered our planet! We must send them a message of peace immediately! WHAT? They do not use our language? Find one that they will understand. Look at their number systems, surely they must have one!'

The year is AD 2123. Earthlings have colonised the Moon and Mars. Earth scientists have now discovered that we are not alone in the Universe. Not much is known about the inhabitants of the planet, but new images from the planet indicate that the planet's inhabitants have two hands like Earthlings. At this stage the images are not clear enough to show how many fingers they have. Scientists are assuming that they may have developed a number system based on the number of fingers on each hand, just like the Earthlings did centuries ago.

Your research centre has the best mathematical team on Earth, so it will be up to you to communicate with these inhabitants and the best way is by numbers. Until more detailed images arrive you must research and prepare for whatever number system may be required.

Task

You will need to research the development of number systems as part of your background knowledge leading to an understanding of these number systems. From your research, you will create an e-poster that will inform your partner of your findings. You will then create a worksheet to ensure your partner's understanding of the number systems you have researched.

Process

Watch the introductory video lesson.

- Team member 1 will research the ancient number systems of the Mayans, Romans and Egyptians and report back to your partner.

- Team member 2 will research the modern era number systems of the Chinese, Hindu/Arabic and Binary and report back to your partner.
- Visit your Media Centre and view the selection of images of other number systems.
- Navigate to your Research Forum.
- Follow the weblinks given to find out how various cultures throughout history developed and used their number systems.
- Navigate to the Worksheets, and complete a Worksheet for each of the different number systems. Submit these completed Worksheets to your teacher for assessment.
- Create an e-poster for each number system that helps explain that number system.
- Create a quiz of 20 questions and solutions to test your class's understanding of these number systems.
- Complete the quiz provided by your partner.
- Write a report that includes your quiz, the results of your quiz and a critical evaluation of the strengths and weaknesses of your quiz. Submit this to your teacher for assessment.
- Remember, Earth needs to be safe — and to do that, communication is vital!

SUGGESTED SOFTWARE

- Microsoft Word
- Microsoft PowerPoint
- Microsoft Publisher
- Microsoft FrontPage
- Crossword maker

TOPIC 4
Rational numbers

4.1 Overview

Numerous **videos** and **interactivities** are embedded just where you need them, at the point of learning, in your learnON title at www.jacplus.com.au. They will help you to learn the content and concepts covered in this topic.

4.1.1 Why learn this?

There are many everyday situations where we do not work with whole numbers. Fractions are a way of expressing parts of whole things, amounts or quantities. They are important in everyday situations such as measuring, cooking, shopping, telling the time and sport. Did you know that approximately $\frac{1}{4}$ of the population of Australia lives in Victoria?

4.1.2 What do you know?

assessON

1. **THINK** List what you know about fractions. Use a thinking tool such as a concept map to show your list.
2. **PAIR** Share what you know with a partner and then with a small group.
3. **SHARE** As a class, create a thinking tool such as a large concept map that shows your class's knowledge of fractions.

LEARNING SEQUENCE

4.1 Overview
4.2 What are rational numbers?
4.3 Simplifying rational numbers
4.4 Mixed numbers and improper fractions
4.5 Adding and subtracting rational numbers
4.6 Multiplying rational numbers
4.7 Dividing rational numbers
4.8 Fractions as ratios
4.9 Working with mixed numbers
4.10 Review

4.2 What are rational numbers?

4.2.1 Understanding fractions

- **Rational numbers** are numbers that can be written as fractions.
- **Fractions** are used to describe parts of a whole.
- The **numerator**, or top number of the fraction, shows how many parts are required. The numerator must be an integer.
- The **denominator**, or bottom number, shows the number of parts into which the whole can be divided. The denominator must be an integer, but cannot be zero.
- The horizontal bar separating the numerator from the denominator is called the **vinculum**.

$$\frac{2}{3}$$

↙ numerator
← vinculum
↖ denominator

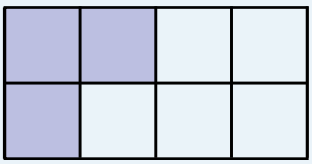
- A **proper fraction** has a numerator that is smaller than the denominator, for example, $\frac{2}{3}$.
- An **improper fraction** has a numerator that is larger than the denominator, for example, $\frac{5}{4}$.
- A **mixed number** is made up of a number and a fraction, for example, $1\frac{3}{5}$.

4.2.2 Expressing one quantity as a fraction of another

- To express one quantity as a fraction of another, write the first quantity as a fraction of the second quantity.

WORKED EXAMPLE 1

a What fraction of the rectangle shown has been shaded?
b Express the number of unshaded squares as a fraction of the number of squares in total.



THINK	WRITE
a 1 Count how many equal parts the rectangle has been divided into.	**a** Total number of parts = 8
2 State how many parts are shaded.	3 parts are shaded.
3 Write the answer in a sentence.	$\frac{3}{8}$ of the rectangle has been shaded.
b 1 Count the number of unshaded squares.	**b** There are 5 unshaded squares.
2 State the number of squares in total.	There are 8 squares in total.
3 Write the first quantity as a fraction of the second quantity and write the answer in words.	The number of unshaded squares as a fraction of the total number of squares in the rectangle is $\frac{5}{8}$.

4.2.3 Equivalent fractions

- Equivalent fractions are equal fractions. For example, the fractions $\frac{1}{2}$, $\frac{5}{10}$, $\frac{10}{20}$, $\frac{8}{16}$ can all be simplified to $\frac{1}{2}$.

4.2.4 Fractions on a number line

- The space between each whole number on a number line is divided into equal parts. The number of equal parts is determined by the denominator of the fraction.
- The position of the fraction is found by counting on the number of parts shown in the numerator.

WORKED EXAMPLE 2

Show the positions of $\frac{5}{6}$ and $1\frac{1}{6}$ on a number line.

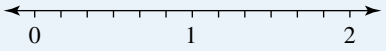
THINK

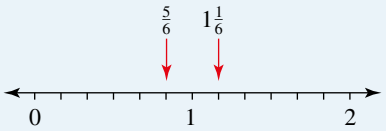
1 Draw a number line from 0 to 2.

2 The denominator of both fractions is 6. Divide the sections between 0 and 1 and between 1 and 2 into 6 equal parts each.

3 To place $\frac{5}{6}$, count on 5 marks from 0. To place $1\frac{1}{6}$ count on 1 mark from the whole number 1 on the number line.

WRITE





WORKED EXAMPLE 3

For the equivalent fractions $\frac{4}{5} = \frac{8}{10}$ find the number which has been used to multiply both numerator and denominator.

THINK

1 What number is 4 multiplied by to equal 8?
$(4 \times _ = 8)$
What number is 5 multiplied by to equal 10?
$(5 \times _ = 10)$

2 Write the answer.

WRITE

$\frac{4}{5} = \frac{4 \times 2}{5 \times 2} = \frac{8}{10}$

The number used is 2.

WORKED EXAMPLE 4

From the list, find those fractions which are equivalent to $\frac{1}{2}$: $\frac{3}{9}, \frac{3}{6}, \frac{9}{18}, \frac{10}{15}, \frac{7}{14}, \frac{17}{34}$.

THINK

1 Multiply the numerator and denominator of $\frac{1}{2}$ by the numerator of the first fraction (3) to check whether the new fraction is in the list.

WRITE

$\frac{1 \times 3}{2 \times 3} = \frac{3}{6}$

2 Multiply the numerator and denominator of $\frac{1}{2}$ by the next different numerator (9) and check whether the new fraction is in the list.

$$\frac{1 \times 9}{2 \times 9} = \frac{9}{18}$$

3 Continue until all fractions have been considered.

$$\frac{1 \times 10}{2 \times 10} = \frac{10}{20}$$

$$\frac{1 \times 7}{2 \times 7} = \frac{7}{14}$$

$$\frac{1 \times 17}{2 \times 17} = \frac{17}{34}$$

4 Write the equivalent fractions.

From the given list the equivalent fractions of $\frac{1}{2}$ are $\frac{3}{6}, \frac{9}{18}, \frac{7}{14}, \frac{17}{34}$.

WORKED EXAMPLE 5

Write the sequence of the first three equivalent fractions for $\frac{2}{3}$.

THINK

1 Find the first three equivalent fractions in the sequence by multiplying both the numerator and denominator by 2, 3 and 4.

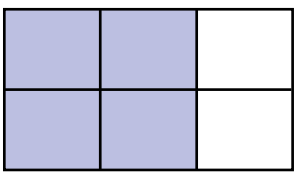
WRITE

$$\frac{2}{3} = \frac{2 \times 2}{3 \times 2}$$
$$= \frac{2 \times 3}{3 \times 3}$$
$$= \frac{2 \times 4}{3 \times 4}$$

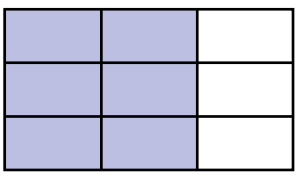
2 Write the sequence of equivalent fractions.

$$\frac{2}{3} = \frac{4}{6} = \frac{6}{9} = \frac{8}{12}$$

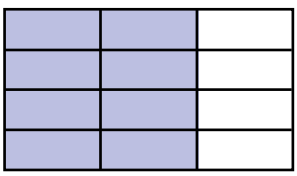
- There are other fractions that are equivalent to $\frac{2}{3}$. These can be found by multiplying the numerator and denominator by other numbers.
- Equivalent fractions can also be shown using diagrams. Equivalent fractions for $\frac{2}{3}$ are shown in the following diagrams.



$\frac{4}{6} = \frac{2}{3}$



$\frac{6}{9} = \frac{2}{3}$



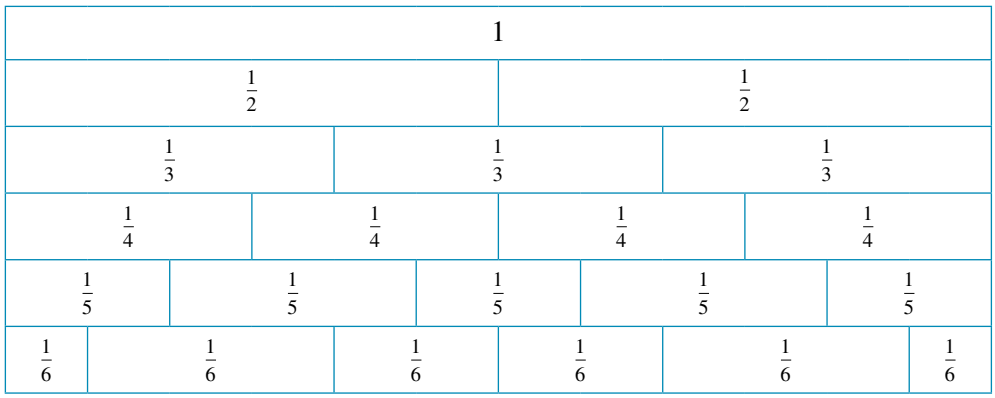
$\frac{8}{12} = \frac{2}{3}$

- Note that the same amount of the rectangle has been shaded in each case.

4.2.5 Comparing fractions

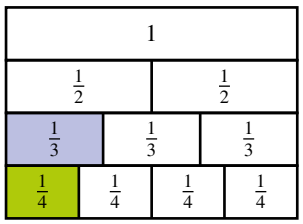
- If we cut a block into 4 pieces, then 1 piece is less than 2 pieces. This means that $\frac{1}{4}$ of the block is smaller than $\frac{2}{4}$.

- Fraction walls, or number lines, are useful tools when comparing fractions. The following figure shows an example of a fraction wall.

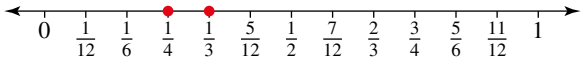


- With a diagram, such as a fraction wall or a number line, we can also see that $\frac{1}{4}$ is less than $\frac{1}{3}$. That is, if we divided a block into 4 equal parts, each part would be smaller than if we divided it into 3 equal parts.

Fraction wall



Number line



- If the numerators are the same, the smaller fraction is the one with the larger denominator. For example, $\frac{1}{7}$ is less than $\frac{1}{6}$ and $\frac{3}{10}$ is less than $\frac{3}{7}$.
- Fractions that do not have the same denominator or numerator can be compared by using equivalent fractions.
- The fractions need to be rewritten so that they have the same denominator.
- Look at the multiples of both denominators and choose the lowest multiple that is common to both.
- The symbol '>' means *is bigger than* or *is greater than*.
- The symbol '<' means *is smaller than* or *is less than*.
- When we write $\frac{1}{2} < \frac{3}{4}$, it means that $\frac{1}{2}$ is less than $\frac{3}{4}$.

WORKED EXAMPLE 6

a Which is the bigger fraction, $\frac{2}{3}$ or $\frac{3}{5}$?

b Justify your answer using a fraction wall and a number line.

THINK	WRITE
a 1 Find the lowest common multiple of the denominators. First list the multiples of 3 and 5. Identify the lowest number that is common to both lists.	**a** Multiples of 3 are 3, 6, 9, 12, 15, 18, … Multiples of 5 are 5, 10, 15, 20, … The lowest common multiple is 15.

2 Write each fraction as an equivalent fraction using the lowest common multiple (15) as the denominator.

$\frac{2}{3} = \frac{2 \times 5}{3 \times 5} = \frac{10}{15}$ and $\frac{3}{5} = \frac{3 \times 3}{5 \times 3} = \frac{9}{15}$

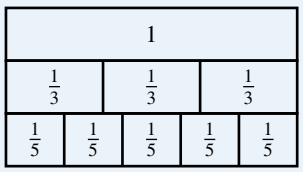
3 Decide which is bigger by comparing the numerators of the equivalent fractions $\left(\frac{10}{15} \text{ and } \frac{9}{15}\right)$.

4 Answer the question in words.

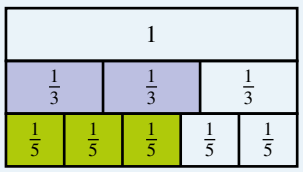
$\frac{2}{3}$ is bigger than $\frac{3}{5}$.

b 1 Create a fraction wall showing thirds and fifths.

b



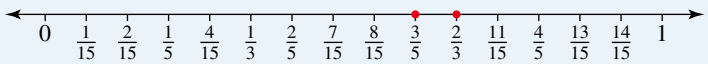
2 Shade $\frac{2}{3}$ and $\frac{3}{5}$.



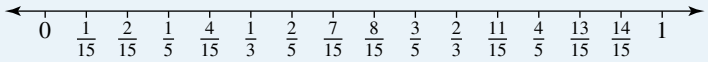
3 Compare the lengths of the shaded areas to compare the fractions. Answer the question in words.

$\frac{2}{3}$ is bigger than $\frac{3}{5}$.

4 Draw a number line showing 0 to 1 in intervals of $\frac{1}{15}$ (found by using the lowest common multiple of the denominators).



5 Mark $\frac{2}{3}$ and $\frac{3}{5}$.



6 Answer the question in words.

$\frac{2}{3}$ is bigger than $\frac{3}{5}$.

WORKED EXAMPLE 7

Insert the appropriate symbol, < or >, between each pair of fractions to make a true statement.

a $\frac{6}{7}$ $\frac{7}{8}$ **b** $\frac{7}{12}$ $\frac{5}{9}$

THINK	WRITE
a 1 Find the lowest common multiple of the denominators.	**a** Multiples of 7 are: 7, 14, 21, 28, 35, 42, 49, 56, 63, … Multiples of 8 are: 8, 16, 24, 32, 40, 48, 56, 64, … The lowest common multiple is 56.

2 Write each fraction as an equivalent fraction using the lowest common multiple (56) as the denominator.

$\frac{6}{7} = \frac{6 \times 8}{7 \times 8} = \frac{48}{56}$ and $\frac{7}{8} = \frac{7 \times 7}{8 \times 7} = \frac{49}{56}$

3 Decide which is bigger by comparing the numerators of the equivalent fractions.

$\frac{6}{7}$ is less than $\frac{7}{8}$.

4 Answer the question.

$\frac{6}{7} < \frac{7}{8}$

b 1 Find the lowest common multiple of the denominators.

b Multiples of 12 are: 12, 24, 36, 48, …
Multiples of 9 are: 9, 18, 27, 36, 45, …
The lowest common multiple is 36.

2 Write each fraction as an equivalent fraction using the lowest common multiple (36) as the denominator.

$\frac{7}{12} = \frac{7 \times 3}{12 \times 3} = \frac{21}{36}$ and $\frac{5}{9} = \frac{5 \times 4}{9 \times 4} = \frac{20}{36}$

3 Decide which is bigger by comparing the numerators of the equivalent fractions.

$\frac{7}{12}$ is greater than $\frac{5}{9}$.

4 Answer the question.

$\frac{7}{12} > \frac{5}{9}$

learnON RESOURCES — ONLINE ONLY

- **Watch this eLesson:** Types of fractions (eles-0002)
- **Complete this digital doc:** SkillSHEET: Types of fractions (doc-6437)
- **Complete this digital doc:** SkillSHEET: Equivalent fractions I (doc-6438)
- **Complete this digital doc:** SkillSHEET: Equivalent fractions II (doc-6439)
- **Complete this digital doc:** SkillSHEET: Lowest common multiple (doc-6440)
- **Complete this digital doc:** SkillSHEET: Ordering fractions (doc-6441)
- **Try out this interactivity:** Fraction walls (int-0002)

Exercise 4.2 What are rational numbers?

assessON

Individual pathways

PRACTISE
Questions:
1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13a, c, 18

CONSOLIDATE
Questions:
2, 3, 4, 5, 6, 7, 8b, d, 9b, c, e, f, 10, 11, 12, 13, 16, 18

MASTER
Questions:
3, 5, 6a, e, i, 7b, d, 10, 11, 13, 14, 15, 16, 17, 18

Individual pathway interactivity: int-4323

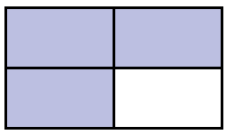
learnON ONLINE ONLY

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

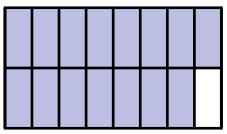
Fluency

1. WE1 What fraction of each of the following rectangles has been shaded?

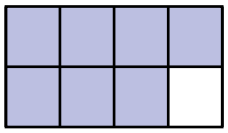
a.



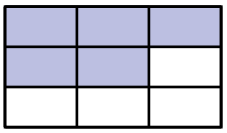
b.



c.



d.



2. What fraction of each of these flags is coloured red?

a.

b.

c. Do you recognise these flags? Can you name the country to which each flag belongs?

3. a. What fraction of the total number of pieces of fruit is made up of:

i. bananas? **ii.** oranges?

b. What fraction of chocolate biscuits in this packet has been eaten?

4. WE2 Draw a number line from 0 to 2. Show the position of each of the following fractions on the number line.

a. $\frac{1}{5}$ **b.** $\frac{4}{5}$ **c.** $1\frac{3}{5}$ **d.** $\frac{7}{5}$

5. Draw a number line from 0 to 3. Show the position of each of the following fractions on the number line.

a. $\frac{1}{3}$ **b.** $\frac{4}{3}$ **c.** $\frac{8}{3}$ **d.** $\frac{3}{3}$

6. WE3 For the following equivalent fractions, find the number which has been used to multiply both numerator and denominator.

a. $\frac{3}{8} = \frac{9}{24}$ **b.** $\frac{1}{4} = \frac{3}{12}$ **c.** $\frac{3}{5} = \frac{12}{20}$

d. $\frac{2}{3} = \frac{10}{15}$ **e.** $\frac{5}{6} = \frac{30}{36}$ **f.** $\frac{9}{10} = \frac{81}{90}$

g. $\frac{7}{8} = \frac{77}{88}$ **h.** $\frac{7}{8} = \frac{84}{96}$ **i.** $\frac{2}{5} = \frac{14}{35}$

7. WE4 From the list, find those fractions which are equivalent to the fractions marked in red.

a. $\frac{2}{3}$ $\frac{7}{9}$ $\frac{20}{30}$ $\frac{5}{8}$ $\frac{12}{16}$ $\frac{14}{21}$ $\frac{40}{60}$

b. $\frac{4}{5}$ $\frac{12}{15}$ $\frac{15}{20}$ $\frac{36}{45}$ $\frac{16}{20}$ $\frac{28}{35}$ $\frac{80}{100}$

c. $\frac{7}{8}$ $\frac{17}{18}$ $\frac{40}{45}$ $\frac{56}{64}$ $\frac{14}{18}$ $\frac{18}{19}$ $\frac{21}{24}$

d. $\frac{7}{10}$ $\frac{18}{25}$ $\frac{35}{50}$ $\frac{14}{21}$ $\frac{21}{30}$ $\frac{14}{20}$ $\frac{140}{200}$

8. Fill in the gaps.

a. $\frac{3}{5} = \frac{9}{_} = \frac{_}{20} = \frac{_}{35}$

b. $\frac{3}{4} = \frac{_}{40} = \frac{18}{_} = \frac{_}{36}$

c. $\frac{1}{4} = \frac{_}{28} = \frac{6}{_} = \frac{3}{_}$

d. $\frac{5}{6} = \frac{15}{_} = \frac{_}{42} = \frac{40}{_}$

Understanding

9. WE5 Write a sequence of the first three equivalent fractions for each of the following.

a. $\frac{5}{6}$ **b.** $\frac{5}{8}$ **c.** $\frac{3}{10}$ **d.** $\frac{2}{3}$ **e.** $\frac{1}{8}$ **f.** $\frac{7}{8}$

10. WE6 Which is the bigger fraction? Justify your answer using a fraction wall or number line.

a. $\frac{2}{5}$ or $\frac{3}{5}$ **b.** $\frac{5}{8}$ or $\frac{7}{8}$ **c.** $\frac{1}{5}$ or $\frac{1}{6}$

d. $\frac{1}{8}$ or $\frac{1}{10}$ **e.** $\frac{1}{2}$ or $\frac{3}{10}$ **f.** $\frac{3}{4}$ or $\frac{3}{5}$

g. $\frac{2}{5}$ or $\frac{5}{8}$ **h.** $\frac{3}{5}$ or $\frac{5}{8}$ **i.** $\frac{4}{5}$ or $\frac{5}{8}$

11. WE7 Insert the appropriate symbol, $<$ or $>$, between each pair of fractions to make a true statement.

a. $\frac{5}{8} \; \frac{3}{8}$ **b.** $\frac{5}{6} \; \frac{7}{6}$ **c.** $\frac{3}{5} \; \frac{1}{4}$

d. $\frac{3}{4} \; \frac{3}{5}$ **e.** $\frac{1}{5} \; \frac{1}{4}$ **f.** $\frac{3}{10} \; \frac{2}{5}$

12. MC **a.** Which fraction is smaller than $\frac{5}{8}$?

A. $\frac{7}{8}$ **B.** $\frac{17}{24}$ **C.** $\frac{11}{12}$ **D.** $\frac{3}{5}$ **E.** $\frac{13}{16}$

b. Which fraction is equivalent to $\frac{3}{4}$?

A. $\frac{3}{8}$ **B.** $\frac{6}{8}$ **C.** $\frac{9}{16}$ **D.** $\frac{12}{24}$ **E.** $\frac{15}{24}$

13. Write the following fractions with the same denominator and then write them in ascending order. (Ascending means from smallest to largest.)

a. $\frac{3}{10}, \frac{1}{2}, \frac{1}{5}$ **b.** $\frac{3}{8}, \frac{1}{2}, \frac{1}{3}$ **c.** $\frac{2}{3}, \frac{4}{5}, \frac{6}{15}$ **d.** $\frac{3}{4}, \frac{2}{3}, \frac{7}{15}$

14. Four hungry teenage girls ordered pizzas, but none could eat the whole pizza. Sally ate $\frac{6}{8}$ of her seafood pizza. Hanna ate $\frac{1}{2}$ of her Hawaiian pizza. Sonya ate $\frac{3}{4}$ of her special pizza. Vanessa ate $\frac{5}{6}$ of her vegetarian pizza.

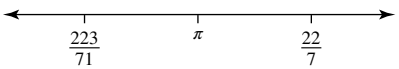
a. Draw the four pizzas and shade the amount that each girl ate.

b. Which girl ate the most pizza?

c. Who ate more pizza, Sally or Sonya?

Reasoning

15. π is a Greek letter that represents the ratio between the circumference of a circle and its diameter. This symbol was introduced by the British mathematician William Jones in 1706. Its approximate value has been known for almost 4000 years. Ancient Babylonians used 3 and later 3.125 as values for π. The famous mathematician Archimedes of Syracuse (287–212 BCE), worked out that $\pi > \frac{223}{71}$ and $\pi < \frac{22}{7}$.



With the development of computers, mathematicians all over the world have been interested in computing π to as many decimal places as possible. In December 2013, Shigeru. Kondo and Alexander. Yee computed π to 12000000000000 decimal places.

The approximation $\pi \simeq 3.14$ is quite commonly used in schools for calculations.

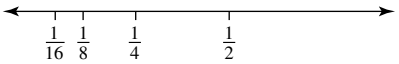
a. Is π a rational number? Explain your answer.

b. Although $\pi \simeq 3.14 \simeq \frac{22}{7}$, when making calculations it is recommended to use the π button on a calculator. Explain the reasoning behind this recommendation.

c. Why are $\frac{223}{71}$ and $\frac{22}{7}$ rational numbers?

d. After how many digits do the numbers $\frac{223}{71}$, π and $\frac{22}{7}$ become different?

16. On the number line shown, the fractions have the same numerator but the denominator is always double the previous one. Explain, with reasons, the relationship between these denominators and the distance between the numbers.



Problem solving

17. Stephanie puts four equally spaced points on a number line. The first point is at the $\frac{3}{10}$ mark and the last point is at the $\frac{7}{20}$ mark. Where has she placed the two points in between?

18. Shirly is sitting $2\frac{5}{7}$ m from the classroom door and Brent is $2\frac{2}{11}$ m from the classroom door. Place Shirly's and Brent's seats on the number line given.

Reflection

Why is it important to express one quantity as a fraction of another?

4.3 Simplifying rational numbers

4.3.1 Simplifying rational numbers

- Generally, fractions are written in their simplest form. In other words, we reduce the fraction to its lowest equivalent form.
- To simplify a fraction means to divide it by the highest common factor of the numerator and the denominator.
- The highest common factor (HCF) of two numbers is the largest factor of both numbers.
- Dividing the numerator and denominator by the same number is called *cancelling*.

WORKED EXAMPLE 8

Write $\frac{3}{6}$ in simplest form.

THINK	WRITE
1 Write the fraction and find the highest common factor (HCF) or the largest number that is a factor of both the numerator and the denominator (3).	$\frac{3}{6}$
2 Cancel by dividing the numerator and denominator by this factor ($3 \div 3 = 1, 6 \div 3 = 2$).	$= \frac{\cancel{3}^{1}}{\cancel{6}^{2}}$
3 Write the answer in simplest form.	$= \frac{1}{2}$

- If the largest factor is not used, the process can be repeated to find the simplest fraction.
- To simplify a mixed number, leave the whole number part and simplify the fraction part.

WORKED EXAMPLE 9

Simplify $7\frac{40}{64}$.

THINK	WRITE
1 Write the mixed number and think of the HCF or largest number that divides into both numerator and denominator; that is, the number 8.	$7\frac{40}{64}$
2 Divide both numerator and denominator by that factor ($40 \div 8 = 5, 64 \div 8 = 8$).	$= 7\frac{\cancel{40}^{5}}{\cancel{64}^{8}}$
3 Write your answer as a mixed number in simplest form.	$= 7\frac{5}{8}$

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Complete this digital doc: SkillSHEET: Highest common factor (doc-6442)

Complete this digital doc: SpreadSHEET: Simplifying a fraction (doc-1729)

Exercise 4.3 Simplifying rational numbers

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1, 2, 3, 4, 6, 11	Questions: 1, 2, 3, 5, 7, 10, 11	Questions: 1a, f, k, 2a, c, e, h, j, 3, 4–12

Individual pathway interactivity: int-4324

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Fluency

1. WE8 Write the following fractions in simplest form.

a. $\frac{5}{10}$	**b.** $\frac{8}{12}$	**c.** $\frac{21}{24}$	**d.** $\frac{48}{60}$	**e.** $\frac{28}{35}$
f. $\frac{18}{24}$	**g.** $\frac{81}{90}$	**h.** $\frac{49}{56}$	**i.** $\frac{100}{120}$	**j.** $\frac{48}{50}$
k. $\frac{63}{72}$	**l.** $\frac{49}{70}$	**m.** $\frac{33}{36}$	**n.** $\frac{22}{50}$	**o.** $\frac{21}{56}$

2. WE9 Simplify.

a. $4\frac{21}{35}$	**b.** $7\frac{45}{54}$	**c.** $10\frac{10}{20}$	**d.** $5\frac{7}{42}$	**e.** $3\frac{56}{64}$
f. $1\frac{44}{48}$	**g.** $6\frac{21}{28}$	**h.** $5\frac{16}{48}$	**i.** $3\frac{11}{55}$	**j.** $2\frac{16}{64}$

3. MC The fraction $\frac{26}{28}$ is the same as:

A. $\frac{2}{3}$	**B.** $\frac{2}{6}$	**C.** $\frac{3}{8}$	**D.** $\frac{9}{12}$	**E.** $\frac{13}{14}$

Understanding

4. Kylie's netball team scored 28 goals. Kylie scored 21 of her team's goals. What fraction of the team's goals did Kylie score? Simplify the answer.

5. Angelo's basketball team scored 36 goals. Angelo scored 20 of the team's goals.

a. What fraction of the team's goals did Angelo score? Simplify the answer.

b. What fraction of the team's goals did the rest of the team score? Simplify the answer.

6. Year 7 students at Springfield High School ran a car wash to raise money for the local hospital. They raised a total of $1000 and drew up a table showing how much money each class raised.

7A	$200	7B	$150	7C	$320
7D	$80	7E	$250		

Express as a simple fraction how much of the total each class raised.

7. Below are the results of a book-reading competition.

Mark	5 books
Ken	10 books
Samantha	8 books
Jules	6 books
Ahmed	4 books
Darren	7 books

a. How many books were read in total?

b. For each contestant, record the number of books read as a fraction of the total number of books read. Where possible, reduce the fractions to their simplest forms.

8. a. Can you arrange the digits 1, 2, 3, 4, 5, 6, 7, 8 and 9 to form a fraction which is equivalent to $\frac{1}{2}$? For example, one way is $\frac{7329}{14\,658}$. Can you find two others?

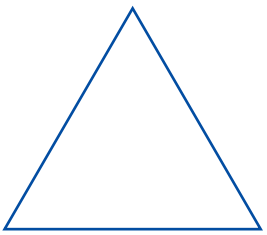
Hint: Try starting with a numerator beginning with 72 for one of them and 67 for the other.

b. Can you find a way to arrange the nine digits so that they form a fraction equivalent to $\frac{1}{3}$?

9. The Geelong train to Melbourne runs on time 3 out of every 5 trains. The Melbourne train to Geelong runs on time 4 out of every 7 trains. Which train is more reliable?

Reasoning

10. a. Draw three lines to divide the triangle shown in 4 equal parts.



b. What fraction of the original triangle is one of the small triangles formed?

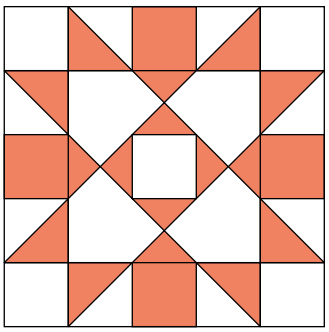
c. Halve each smaller triangle.

d. What fraction of the initial triangle is the new triangle formed?

e. Using all the triangle available explain why $\frac{2}{8} = \frac{1}{4}$.

Problem solving

11. One of the designs for a patchwork motif is made up of squares and right-angled isosceles triangles as shown. What fraction of the whole has been shaded?



12. The fraction $\frac{13}{n}$ is not written in its simplest form. What number could the denominator be? Explain your answer.

Reflection

What strategies will you use to write a fraction in simplest form?

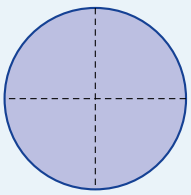
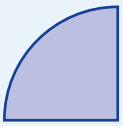
4.4 Mixed numbers and improper fractions

4.4.1 Change improper fractions to mixed numbers

- An improper fraction has a numerator larger than the denominator, for example, $\frac{3}{2}$.
- A mixed number is made up of a whole number and a fraction, for example, $1\frac{3}{4}$, $2\frac{1}{2}$.
- An improper fraction can be changed to a mixed number by dividing the denominator into the numerator and writing the remainder over the same denominator.

WORKED EXAMPLE 10

Draw a diagram to show $\frac{5}{4}$ as parts of a circle, then write the improper fraction as a mixed number.

THINK	WRITE
1 Draw a whole circle and divide it into the number of parts shown by the denominator.	 This is 4 quarters or $\frac{4}{4}$.
2 Find the number of parts left over and draw them.	 Extra $\frac{1}{4}$ needed.
3 Write the improper fraction as a mixed number.	$\frac{5}{4} = 1\frac{1}{4}$

- If the improper fraction had a larger numerator, it might be necessary to draw more circles and the whole number part would then be greater than 1.

WORKED EXAMPLE 11

Express $\frac{11}{5}$ as a mixed number.

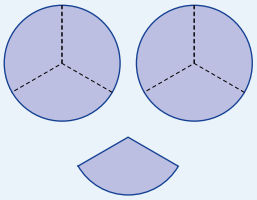
THINK	WRITE
1 Write the improper fraction.	$\frac{11}{5}$
2 Determine how many times the denominator can be divided into the numerator and what the remainder is. The whole number is part of the answer and the remainder becomes the numerator of the fractional part with the same denominator as the original improper fraction.	$= 11 \div 5$ $= 2$ remainder 1
3 Write the answer.	$= 2\frac{1}{5}$

4.4.2 Changing mixed numbers to improper fractions

- A mixed number can be changed to an improper fraction by first multiplying the whole number by the denominator then adding the numerator. The denominator stays the same.

WORKED EXAMPLE 12

Draw a diagram to show $2\frac{1}{3}$ as pieces of a circle and then write $2\frac{1}{3}$ as an improper fraction.

THINK	WRITE
1 Draw two whole circles and $\frac{1}{3}$ of a circle.	
2 Divide the whole circles into thirds.	
3 Count the number of thirds and write the mixed number as an improper fraction.	$2\frac{1}{3} = \frac{7}{3}$

WORKED EXAMPLE 13

Express $2\frac{3}{4}$ as an improper fraction.

THINK	WRITE
1 Write the mixed number.	$2\frac{3}{4}$
2 Multiply the whole number with the denominator then add the numerator. The result becomes the numerator and the denominator stays the same.	$= \frac{2 \times 4 + 3}{4}$
3 Evaluate the top line of the fraction.	$= \frac{8 + 3}{4}$
4 Write the answer.	$= \frac{11}{4}$

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Complete this digital doc: SkillSHEET: Converting an improper fraction to a mixed number (doc-6443)

Complete this digital doc: SkillSHEET: Converting a mixed number to an improper fraction (doc-6444)

Complete this digital doc: Spreadsheet: Converting improper fractions to mixed numbers (doc-1725)

Exercise 4.4 Mixed numbers and improper fractions

assessON

Individual pathways

PRACTISE
Questions:
1, 2, 3a–f, 4a–e, 5, 6, 7, 8, 9, 10, 12, 13a, d, g, 18

CONSOLIDATE
Questions:
1, 2, 3d–i, 4c–g, 5, 6, 7, 8, 9, 11, 13, 16, 18

MASTER
Questions:
1, 2, 3g–l, 4f–j, 5, 6, 7a, e, i, 8a, e, i, 9–18

Individual pathway interactivity: int-4325

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Fluency

1. WE10 Draw a diagram to show the following improper fractions as pieces of a circle, then write each improper fraction as a mixed number.

a. $\frac{5}{2}$ **b.** $\frac{4}{3}$ **c.** $\frac{11}{6}$ **d.** $\frac{13}{8}$ **e.** $\frac{12}{4}$ **f.** $\frac{7}{6}$

2. a. Aamira ate $\frac{3}{2}$ or 3 halves of an apple. Draw the amount of apple that Aamira ate and express $\frac{3}{2}$ as a mixed number.

b. Dean ate $\frac{9}{4}$ or 9 quarters of pizza. Draw the amount of pizza that Dean ate and express $\frac{9}{4}$ as a mixed number.

3. WE11 Express these improper fractions as mixed numbers.

a. $\frac{7}{5}$ **b.** $\frac{11}{4}$ **c.** $\frac{21}{2}$

d. $\frac{39}{10}$ **e.** $\frac{51}{12}$ **f.** $\frac{92}{11}$

g. $\frac{29}{13}$ **h.** $\frac{23}{8}$ **i.** $\frac{100}{3}$

j. $\frac{25}{2}$ **k.** $\frac{20}{3}$ **l.** $\frac{10}{3}$

4. Change these improper fractions to mixed numbers.

a. $\frac{8}{5}$ **b.** $\frac{27}{8}$ **c.** $\frac{11}{2}$ **d.** $\frac{97}{9}$ **e.** $\frac{58}{7}$

f. $\frac{117}{10}$ **g.** $\frac{67}{8}$ **h.** $\frac{74}{9}$ **i.** $\frac{31}{12}$ **j.** $\frac{89}{6}$

5. MC **a.** Which of the following is the same as $\frac{61}{8}$?

A. $5\frac{5}{8}$ **B.** $6\frac{5}{8}$ **C.** $7\frac{5}{8}$ **D.** $8\frac{5}{8}$ **E.** $9\frac{5}{8}$

b. Which of the following is the same as $\frac{74}{10}$?

A. $3\frac{7}{10}$ **B.** $7\frac{2}{5}$ **C.** $7\frac{7}{10}$ **D.** $7\frac{4}{5}$ **E.** $7\frac{4}{10}$

c. Which of the following is the same as $\frac{48}{5}$?

A. $8\frac{4}{5}$ **B.** $5\frac{5}{9}$ **C.** $9\frac{5}{9}$ **D.** $9\frac{3}{5}$ **E.** $9\frac{4}{9}$

6. WE12 Draw a diagram to show the following mixed numbers as pieces of a circle and then write each one as an improper fraction.

a. $1\frac{1}{8}$ **b.** $3\frac{3}{4}$ **c.** $2\frac{1}{6}$ **d.** $1\frac{1}{6}$ **e.** $5\frac{1}{4}$ **f.** $1\frac{3}{5}$

7. WE13 Express the following mixed numbers as improper fractions.

a. $1\frac{1}{7}$ **b.** $1\frac{4}{5}$ **c.** $2\frac{2}{3}$ **d.** $3\frac{1}{3}$ **e.** $6\frac{1}{4}$

f. $4\frac{1}{9}$ **g.** $11\frac{1}{2}$ **h.** $3\frac{7}{8}$ **i.** $7\frac{4}{5}$ **j.** $9\frac{2}{5}$

8. Change these mixed numbers to improper fractions.

a. $3\frac{1}{2}$ **b.** $4\frac{1}{5}$ **c.** $5\frac{4}{5}$ **d.** $6\frac{5}{7}$ **e.** $2\frac{9}{10}$

f. $3\frac{7}{12}$ **g.** $5\frac{3}{5}$ **h.** $9\frac{4}{7}$ **i.** $1\frac{10}{11}$ **j.** $8\frac{5}{6}$

9. MC **a.** Which of the following is the same as $10\frac{2}{7}$?

A. $\frac{27}{10}$ **B.** $\frac{72}{10}$ **C.** $\frac{10}{27}$ **D.** $\frac{72}{7}$ **E.** $\frac{72}{2}$

b. Which of the following is the same as $8\frac{5}{6}$?

A. $\frac{85}{6}$ **B.** $\frac{58}{6}$ **C.** $\frac{56}{8}$ **D.** $\frac{53}{6}$ **E.** $\frac{68}{5}$

c. Which of the following is the same as $7\frac{9}{11}$?

A. $\frac{86}{11}$ **B.** $\frac{99}{7}$ **C.** $\frac{79}{11}$ **D.** $\frac{97}{11}$ **E.** $\frac{77}{9}$

Understanding

10. Kim and Carly arrived home from school quite hungry and cut up some fruit. Kim ate 9 quarters of an apple and Carly ate 11 quarters. How many apples did they each eat?

11. Daniel was selling pieces of quiche at the school fete. Each quiche was divided into 8 pieces. If Daniel sold 52 pieces, how many quiches did Daniel sell?

12. It was Cecilia's responsibility to supply oranges for her soccer team at half-time. If 19 players ate $\frac{1}{4}$ orange each, how many oranges were eaten?

13. Insert the appropriate $<$ or $>$ sign between each pair of fractions to make a true statement.

a. $\frac{8}{5} \quad 1\frac{2}{5}$ **b.** $\frac{19}{2} \quad 8\frac{1}{2}$ **c.** $\frac{33}{32} \quad 1\frac{3}{32}$

d. $\frac{15}{2} \quad 5\frac{1}{2}$ **e.** $3\frac{3}{4} \quad \frac{19}{4}$ **f.** $7\frac{1}{7} \quad \frac{65}{7}$

g. $2\frac{1}{3} \quad \frac{8}{3}$ **h.** $1\frac{16}{17} \quad \frac{35}{17}$ **i.** $4\frac{3}{8} \quad \frac{36}{8}$

14. a. Nasira made apple pies for her Grandma's 80th birthday party. She divided each pie into 6 pieces, and after the party she noted that $4\frac{1}{6}$ pies had been eaten. How many pieces had been eaten?

b. Nasira's cousin Ria provided cordial for the same party, and calculated that she could make 20 drinks from each bottle. At the end of the party $3\frac{17}{20}$ bottles had been used. How many drinks had been consumed?

15. A catering company provided 12 quiches which were each cut into 8 pieces for a luncheon. At the end of the function there were 12 pieces of quiche left over. How many whole quiches were eaten?

Reasoning

16. a. Convert the following numbers to improper fractions.

i. $3\frac{2}{7}$ **ii.** $5\frac{3}{4}$

b. Explain with the use of diagrams why $3\frac{2}{7} = \frac{23}{7}$.

17. A bakery makes birthday cakes to serve 24 people. Arabella takes one to school to celebrate her birthday, but there are 30 people in her class. What fraction of the original slices will each student receive?

Problem solving

18. Baljinder walks to school four days a week and her father drives her to school on Fridays. If she lives $2\frac{1}{3}$ km from school, calculate:

a. the distance Baljinder walks to school and back home each day

b. the total distance walked by Baljinder over the four days

c. the total distance she travels to school either walking or by car over the whole week.

State your answers in improper fractions and mixed numbers if possible.

19. The following mixed numbers are missing their denominators. The missing denominators are 3, 4, 6, 8, and 14. The mixed numbers are placed in order from smallest to largest. Find the missing numbers.

$$1\frac{1}{a}, 2\frac{1}{b}, 2\frac{1}{c}, 3\frac{1}{d}, 3\frac{1}{e}$$

Reflection

What strategies will you use to write an improper fraction as a mixed number?

4.5 Adding and subtracting rational numbers

4.5.1 Adding and subtracting proper fractions

- Fractions can be added and subtracted if they have the same denominator.
- When fractions have the same denominator we can add them by adding the numerators and we can subtract them by subtracting the numerators. Do not add or subtract the denominators.
- Simplify the answer by cancelling, if possible.

WORKED EXAMPLE 14

Find $\frac{5}{9} + \frac{1}{9}$.

THINK	WRITE
1 Write the question.	$\frac{5}{9} + \frac{1}{9}$
2 Add the numerators since the denominators are the same.	$= \frac{6}{9}$ $= \frac{\cancel{6}^{2}}{\cancel{9}^{3}}$
3 Simplify the answer if possible by cancelling. Divide by the highest common factor ($6 \div 3 = 2, 9 \div 3 = 3$).	$= \frac{2}{3}$

- If denominators are different, make equivalent fractions by finding the lowest common denominator, LCD. The LCD is the smallest multiple of all the denominators in a set of fractions.

WORKED EXAMPLE 15

Find $\frac{5}{6} - \frac{1}{12}$, expressing the answer in simplest form.

THINK	WRITE
1 Find multiples of each denominator.	Multiples of 6 are 6, 12 … Multiples of 12 are 12 …
2 Find the lowest common multiple (LCM) of 6 and 12. Therefore, 12 is the lowest common denominator, LCD.	LCD is 12.
3 Rewrite the fractions as equivalent fractions with 12 as the LCD.	$\frac{5}{6} - \frac{1}{12}$ $= \frac{5 \times 2}{6 \times 2} - \frac{1}{12}$ $= \frac{10}{12} - \frac{1}{12}$
4 Subtract the numerators.	$= \frac{9}{12}$
5 Simplify the answer if possible by cancelling. Divide by the highest common factor ($9 \div 3 = 3, 12 \div 3 = 4$).	$= \frac{\cancel{9}^{3}}{\cancel{12}^{4}}$ $= \frac{3}{4}$

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Try out this interactivity: Adding and subtracting fractions (int-2357)

Complete this digital doc: SkillSHEET: Adding and subtracting fractions with the same denominator (doc-6445)

Complete this digital doc: Spreadsheet: Adding and subtracting fractions (doc-1726)

Complete this digital doc: WorkSHEET: Rational numbers I (doc-1721)

Exercise 4.5 Adding and subtracting rational numbers

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1a–f, 2a–f, 3a–f, 4a–f, 5a–f, 6a–f, 7, 8a–f, 9, 11, 14	Questions: 1d–i, 2d–i, 3d–i, 4d–i, 5d–i, 6e–j, 8c–h, 10, 12, 13, 14	Questions: 1d–i, 2d–i, 3d–i, 4d–i, 5d–i, 6e–j, 8c–h, 9–15

Individual pathway interactivity: int-4326

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To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Fluency

1. WE14 Find:

a. $\frac{1}{5} + \frac{3}{5}$ **b.** $\frac{9}{11} - \frac{3}{11}$ **c.** $\frac{3}{6} + \frac{2}{6}$

d. $\frac{20}{50} + \frac{11}{50}$ **e.** $\frac{15}{25} - \frac{6}{25}$ **f.** $\frac{12}{12} - \frac{1}{12}$

g. $\frac{5}{8} - \frac{1}{8} + \frac{3}{8}$ **h.** $\frac{4}{7} + \frac{5}{7} - \frac{3}{7}$ **i.** $\frac{33}{100} + \frac{7}{100} - \frac{11}{100}$

2. Find the following, expressing the answers in simplest form.

a. $\frac{5}{12} + \frac{1}{12}$ **b.** $\frac{7}{16} - \frac{3}{16}$ **c.** $\frac{9}{15} - \frac{4}{15}$

d. $\frac{37}{100} + \frac{13}{100}$ **e.** $\frac{9}{4} - \frac{7}{4}$ **f.** $\frac{4}{7} + \frac{5}{7} - \frac{2}{7}$

g. $\frac{13}{16} + \frac{9}{16} - \frac{10}{16}$ **h.** $\frac{11}{28} + \frac{10}{28}$ **i.** $\frac{21}{81} + \frac{21}{81} + \frac{30}{81}$

3. WE15 Find the following, expressing the answers in simplest form.

a. $\frac{1}{2} + \frac{1}{4}$ **b.** $\frac{1}{2} - \frac{1}{4}$ **c.** $\frac{1}{3} + \frac{2}{21}$

d. $\frac{1}{8} + \frac{20}{32}$ **e.** $\frac{9}{18} + \frac{1}{6}$ **f.** $\frac{17}{40} + \frac{1}{2}$

g. $\frac{7}{9} - \frac{8}{27}$ **h.** $\frac{31}{35} - \frac{3}{7}$ **i.** $\frac{8}{10} - \frac{28}{70}$

4. Find the lowest common multiple of each of the following pairs of numbers.

a. 2 and 3 **b.** 3 and 5 **c.** 5 and 10 **d.** 4 and 6 **e.** 5 and 6

f. 9 and 6 **g.** 4 and 8 **h.** 6 and 8 **i.** 7 and 5

5. Use the answers that were found in question **4** as the lowest common denominators to add or subtract these fractions.

a. $\frac{1}{2} - \frac{1}{3}$ **b.** $\frac{2}{3} - \frac{2}{5}$ **c.** $\frac{3}{5} + \frac{3}{10}$ **d.** $\frac{3}{4} - \frac{1}{6}$ **e.** $\frac{4}{5} - \frac{1}{6}$

f. $\frac{1}{9} + \frac{5}{6}$ **g.** $\frac{1}{4} + \frac{3}{8}$ **h.** $\frac{5}{6} - \frac{3}{8}$ **i.** $\frac{5}{7} - \frac{2}{5}$

6. Answer the following questions by first finding the lowest common denominator. Simplify your answer if necessary.

a. $\frac{4}{5} - \frac{3}{4}$ **b.** $\frac{7}{10} + \frac{2}{15}$ **c.** $\frac{5}{12} + \frac{2}{9}$ **d.** $\frac{7}{11} + \frac{1}{4}$ **e.** $\frac{3}{10} - \frac{1}{20}$

f. $\frac{7}{8} - \frac{3}{5}$ **g.** $\frac{5}{9} - \frac{2}{7}$ **h.** $\frac{2}{3} - \frac{1}{13}$ **i.** $\frac{4}{5} - \frac{2}{3}$ **j.** $\frac{6}{11} - \frac{1}{3}$

7. a. MC The lowest common denominator of $\frac{1}{3}$, $\frac{1}{4}$ and $\frac{1}{6}$ is:

A. 16 **B.** 24 **C.** 18 **D.** 12 **E.** 72

b. The lowest common denominator of $\frac{1}{2}$, $\frac{1}{4}$ and $\frac{1}{5}$ is:

A. 8 **B.** 10 **C.** 20 **D.** 30 **E.** 40

Understanding

8. Answer the following:

a. $\frac{1}{2} + \frac{1}{3} - \frac{1}{4}$ **b.** $\frac{3}{10} + \frac{4}{15} - \frac{1}{6}$ **c.** $\frac{7}{8} - \frac{3}{4} + \frac{4}{6}$

d. $\frac{11}{12} - \frac{8}{15} - \frac{3}{20}$ **e.** $\frac{1}{16} + \frac{1}{8} + \frac{1}{4}$ **f.** $\frac{5}{6} - \frac{7}{18} - \frac{1}{9}$

g. $\frac{1}{2} + \frac{2}{3} + \frac{3}{4}$ **h.** $\frac{3}{4} + \frac{1}{3} + \frac{5}{6}$

9. Melissa and Antonio went berry picking. Melissa picked $\frac{3}{10}$ of a bucket of berries and Antonio picked $\frac{1}{6}$ of a bucket.

a. How much did they pick altogether?

b. If they needed 1 full bucket of berries, what fraction more do they need?

10. Bread needed to be cooked at 200°C for $\frac{3}{4}$ hour then a further $\frac{1}{2}$ hour at 150°C. What is the total cooking time in hours?

11. Robert, Jason and Luke play football in the same team. Last Saturday, Robert kicked $\frac{1}{4}$ of the team's goals, Jason kicked $\frac{1}{3}$ and Luke $\frac{5}{12}$. What fraction of the team's goals were kicked by:

a. Jason and Luke

b. Robert and Jason

c. the three boys together?

12. Suraya and her friend decided to climb the tree in her Nan's front yard. It took Suraya 10 minutes to climb halfway up, 10 more minutes to climb another sixth of the way and yet another 10 minutes to reach a further tenth of the way. Here she sat admiring the view for 5 minutes before beginning her descent.

a. What fraction of the height of the tree had Suraya climbed after 20 minutes?

b. What fraction of the tree did Suraya climb in total?

Reasoning

13. Without using a calculator perform the following additions:

a. $\frac{1}{2} + \frac{1}{3}$

b. $\frac{1}{3} + \frac{1}{4}$

c. $\frac{1}{4} + \frac{1}{5}$

d. Do you notice any pattern in these additions?

e. Using your answer to part **d**, state the answer to the addition $\frac{1}{n} + \frac{1}{n+1}$.

14. Without using a calculator perform the following additions:

a. $\frac{1}{2} - \frac{1}{3}$

b. $\frac{1}{3} - \frac{1}{4}$

c. $\frac{1}{4} - \frac{1}{5}$

d. Do you notice any pattern in these additions?

e. Using your answer to part **d**, state the answer to the subtraction $\frac{1}{n} - \frac{1}{n+1}$

Problem solving

15. Jane sold $\frac{2}{3}$ of the cupcakes she made. She had only 45 cupcakes left. How many cupcakes did she make?

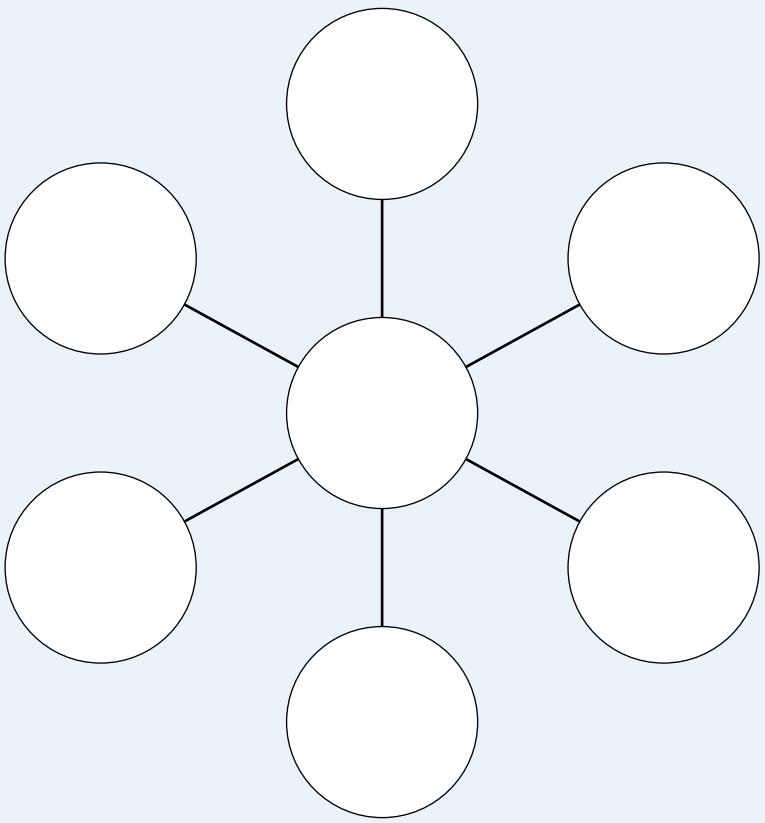
16. Chris lives on a farm and has a horse he rides every day. He fills up the horse's trough with two-thirds apples, $\frac{1}{6}$ carrots, $\frac{1}{12}$ oats and the rest maize. What fraction of the trough is filled with maize?

Reflection

What difficulties might you experience when adding and subtracting fractions with different denominators? How could you overcome them?

CHALLENGE 4.1

Arrange $\frac{1}{12}, \frac{1}{4}, \frac{1}{2}, \frac{1}{6}, \frac{3}{4}, \frac{1}{3}$ and $\frac{2}{3}$ in the circles so that each row adds up to the same value.



4.6 Multiplying rational numbers

4.6.1 Multiplying rational numbers

- It has been seen in an earlier chapter that multiplication by a whole number can be thought of as repeated addition.
  For example, $5 \times 6 = 5 + 5 + 5 + 5 + 5 + 5 = 30$
  The same principle applies to multiplying a whole number by a fraction.
  For example, $\frac{1}{5} \times 6 = \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} = 1\frac{1}{5}$
- To multiply two fractions we need to use an alternative approach.
- To multiply fractions, multiply the numerators and then multiply the denominators.
- It is important to note that the denominators do not need to be the same.
- Simplify the answer if possible.

WORKED EXAMPLE 16

Find $\frac{1}{3} \times 12$ by using the method of repeated addition.

THINK	WRITE
1 Write the question.	$\frac{1}{3} \times 12$
2 Write the expression as a repeated addition, that is, sum 12 thirds.	$= \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3}$
3 Add the numerators.	$= \frac{12}{3}$
4 Simplify the answer.	$= 4$

WORKED EXAMPLE 17

Find $\frac{1}{5} \times \frac{3}{4}$.

THINK	WRITE
1 Write the question.	$\frac{1}{5} \times \frac{3}{4}$
2 Write as one fraction.	$= \frac{1 \times 3}{5 \times 4}$
3 Multiply the numerators, then the denominators and simplify the answer if appropriate.	$= \frac{3}{20}$

- Before being multiplied, fractions can be simplified by dividing both the numerator and denominator by their highest common factor. This is often called *cancelling*.
- When multiplying fractions, cancelling can occur:
  - vertically, for example, $\frac{\cancel{3}^{1}}{\cancel{24}^{8}} \times \frac{5}{7}$
  - diagonally, for example, $\frac{2^{1}}{3} \times \frac{5}{\cancel{6}^{3}}$.

WORKED EXAMPLE 18

Find $\frac{2}{3} \times \frac{9}{10}$.

THINK	WRITE
1 Write the question.	$\frac{2}{3} \times \frac{9}{10}$
2 Write the two fractions as one keeping the multiplication signs.	$= \frac{{}^{1}\cancel{2} \times \cancel{9}^{3}}{{}_{1}\cancel{3} \times \cancel{10}^{5}}$
3 Divide the numbers in the numerator and denominator by their highest common factor. *Note:* This may be done vertically or diagonally.	
4 Multiply the numerators, then the denominators and simplify the answer if appropriate.	$= \frac{3}{5}$

4.6.2 Using the word *of*

- Sometimes the word *of* means 'multiply'.
- It might take a third of an hour (60 minutes) to cook a pizza. A third of an hour, or $\frac{1}{3}$ of 60 minutes, is the same as $\frac{1}{3} \times \frac{60}{1} = 20$ minutes. It takes 20 minutes to cook the pizza.
- If a customer wants to buy $\frac{1}{4}$ of the remaining 8 pizzas, it is the same as $\frac{1}{4} \times \frac{8}{1} = 2$ pizzas. The customer buys 2 pizzas.

WORKED EXAMPLE 19

Find $\frac{2}{3}$ of $\frac{1}{4}$.

THINK	WRITE
1 Write the question.	$\frac{2}{3}$ of $\frac{1}{4}$
2 Change 'of' to $\times$ and cancel if appropriate.	$= \frac{\cancel{2}^{1}}{3} \times \frac{1}{\cancel{4}^{2}}$
3 Perform the multiplication.	$= \frac{1}{6}$

WORKED EXAMPLE 20

Mum put a 2-litre carton of flavoured milk in the refrigerator. When the children came home from school, Joanna drank one quarter of the milk, Kieren drank one half of it and Daisy drank one sixth of it. What fraction of the milk, in litres, did each person drink?

THINK	WRITE
1 Write the fraction of the milk that Joanna drank and simplify the answer.	Joanna: $\frac{1}{4}$ of $2 = \frac{1}{4} \times 2$
2 Perform the multiplication.	$= \frac{1}{\cancel{2}\cancel{4}} \times \frac{\cancel{2}^{1}}{1}$ $= \frac{1}{2}$

3 Write the fraction of the milk that Kieren drank and simplify the answer.

Kieren: $\frac{1}{2}$ of $2 = \frac{1}{2} \times 2$

4 Perform the multiplication.

$= \frac{1}{1\cancel{2}} \times \frac{\cancel{2}^{1}}{1}$
$= 1$

5 Write the fraction of the milk that Daisy drank and simplify the answer.

Daisy: $\frac{1}{6}$ of $2 = \frac{1}{6} \times 2$

6 Perform the multiplication.

$= \frac{1}{\cancel{3}} \times \frac{\cancel{2}^{1}}{1}$
$= \frac{1}{3}$

7 Write a sentence giving the answer in litres.

Joanna drank half a litre, Kieren drank 1 litre and Daisy drank a third of a litre of milk.

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Try out this interactivity: Repeated addition (int-2738)

Complete this digital doc: Spreadsheet: Multiplying fractions (doc-1727)

Exercise 4.6 Multiplying rational numbers

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 2a–h, 3a–h, 4a–f, 5, 6, 7, 11, 13	Questions: 2e–l, 3e–l, 4f–k, 5, 6, 7, 11, 13	Questions: 2i–p, 3i–o, 4k–p, 5–14

Individual pathway interactivity: int-4327

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Fluency

1. WE16 Find:

a. $\frac{2}{3} \times 3$ **b.** $\frac{5}{14} \times 7$ **c.** $\frac{15}{22} \times 11$ **d.** $\frac{5}{16} \times 4$

2. WE17 Find:

a. $\frac{1}{2} \times \frac{1}{4}$ **b.** $\frac{1}{3} \times \frac{2}{3}$ **c.** $\frac{2}{5} \times \frac{3}{5}$ **d.** $\frac{5}{9} \times \frac{4}{3}$

e. $\frac{11}{13} \times \frac{1}{2}$ **f.** $\frac{6}{7} \times \frac{9}{11}$ **g.** $\frac{11}{12} \times \frac{11}{12}$ **h.** $\frac{8}{9} \times \frac{5}{9}$

i. $\frac{5}{8} \times \frac{1}{3}$ **j.** $\frac{2}{3} \times \frac{2}{3}$ **k.** $\frac{5}{6} \times \frac{11}{12}$ **l.** $\frac{5}{11} \times \frac{5}{12}$

m. $\frac{7}{8} \times \frac{7}{8}$ **n.** $\frac{2}{3} \times \frac{4}{13}$ **o.** $\frac{3}{5} \times \frac{3}{10}$ **p.** $\frac{4}{7} \times \frac{2}{3}$

3. WE18 Find:

a. $\frac{4}{8} \times \frac{3}{9}$ **b.** $\frac{2}{5} \times \frac{1}{2}$ **c.** $\frac{3}{7} \times \frac{5}{6}$ **d.** $\frac{3}{8} \times \frac{4}{15}$

e. $\frac{10}{11} \times \frac{22}{25}$ **f.** $\frac{24}{27} \times \frac{9}{8}$ **g.** $\frac{18}{32} \times \frac{64}{72}$ **h.** $\frac{48}{56} \times \frac{24}{60}$

i. $\frac{15}{27} \times \frac{36}{45}$ **j.** $\frac{8}{49} \times \frac{14}{16}$ **k.** $\frac{4}{9} \times \frac{6}{8}$ **l.** $\frac{6}{12} \times \frac{8}{10}$

m. $\frac{42}{54} \times \frac{12}{49}$ **n.** $\frac{5}{25} \times \frac{12}{48}$ **o.** $\frac{18}{24} \times \frac{12}{48}$

4. WE19 Find:

a. $\frac{1}{2}$ of $\frac{1}{4}$ **b.** $\frac{3}{4}$ of $\frac{2}{3}$ **c.** $\frac{5}{6}$ of $\frac{5}{6}$ **d.** $\frac{9}{10}$ of $\frac{5}{7}$

e. $\frac{8}{9}$ of $\frac{1}{4}$ **f.** $\frac{5}{7}$ of $\frac{7}{25}$ **g.** $\frac{4}{5}$ of 20 **h.** $\frac{7}{8}$ of 32

i. $\frac{9}{10}$ of 50 **j.** $\frac{2}{5}$ of 45 **k.** $\frac{3}{5}$ of 25

l. $\frac{8}{9}$ of 81 **m.** $\frac{3}{4}$ of 6 **n.** $\frac{9}{10}$ of 15

o. $\frac{1}{12}$ of 10 **p.** $\frac{3}{7}$ of 12

Understanding

5. WE20 Joshua's mum made 6 litres of cordial for his birthday party. Emily drank $\frac{1}{3}$ of the cordial, Tracy drank $\frac{1}{12}$ and Jonathan drank $\frac{1}{18}$. How much cordial, in litres, did each person drink?

6. Zoe and Sarah play basketball with the Sharp Shooters. The games are played for 40 minutes. Zoe played $\frac{4}{5}$ of last week's game and Sarah played $\frac{7}{8}$. How many minutes of the game did:

a. Zoe play? **b.** Sarah play?

7. Mark's uncle gave him a family-size block of chocolate. He divided it into thirds to share with his two sisters. His friends Tom and Nick then arrived to visit, and Mark shared his portion of the chocolate with them, so that they each had the same amounts.

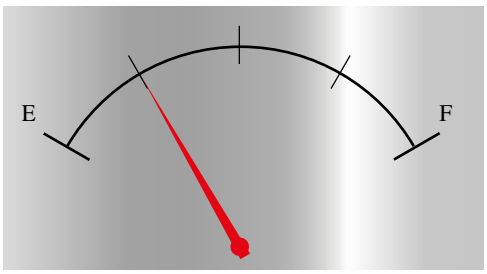
a. Draw a block of chocolate with 9 × 6 pieces, and shade Mark's third of the block. Then draw lines to represent Tom's share.

b. What fraction of the block of chocolate did Nick receive?

8. A car's petrol tank holds 48 litres of fuel. The tank was full at the start of a trip.

a. What fraction of the tank does the petrol gauge show has been used on the trip?

b. How many litres of petrol have been used?



9. a. Draw a quarter of a pie.

b. Shade half of this piece of pie.

c. What fraction of the whole pie is shaded?

d. Complete the mathematical sentence $\frac{1}{4} \times \frac{1}{2} =$ __ to show how the answer was found.

10. a. Find how many half pizzas in 1 pizza.

b. Find how many half pizzas in 2 whole pizzas.

c. Find how many half pizzas in 5 whole pizzas.

Reasoning

11. Gustave's monthly take-home pay is $2400. From this he spends a quarter on his home loan payments, one half on food and drink and one sixth on clothing. One half of the remainder goes into his savings account. How much money does Gustave put into his savings account each month?

12. To make a loaf of fruit bread in a breadmaker the fruit needs to be added after the second rise stage and the top of the bread needs to be glazed two-thirds of the way through the baking stage. The recipe shows how long each stage of the total cycle takes to make.

Stage	Length of time
First knead	5 minutes
Second knead	20 minutes
First rise then punch down	40 minutes
Second rise then punch down	25 minutes
Third rise	50 minutes
Bake	40 minutes

a. After what length of time should the fruit be added?
b. Express this as a fraction of the total time in the bread maker.
c. After what length of time should the top of the bread be glazed?

Problem solving

13. On a given map, one kilometre is represented by $\frac{2}{9}$ cm. If a distance on the map is 27 cm, how many km does this represent?

14. Andrea requires $5\frac{2}{7}$ m of fabric to make a pattern on a quilt. One-third of this fabric has to be red and $\frac{1}{4}$ green. How many metres of red and green fabric does she need?

Reflection

Is repeated addition an efficient method of multiplying fractions? Explain your reasoning.

4.7 Dividing rational numbers

4.7.1 Dividing rational numbers

- The **reciprocal** of a fraction is found by turning the fraction upside down. So $\frac{3}{5}$ is the reciprocal of $\frac{5}{3}$.
- A whole number can be written as a fraction by putting it over 1. This means that $\frac{1}{4}$ is the reciprocal of 4 or $\frac{4}{1}$.

WORKED EXAMPLE 21

Find the reciprocal of $\frac{2}{3}$.

THINK

Turn the fraction upside down and write the answer in a sentence.

WRITE

The reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$.

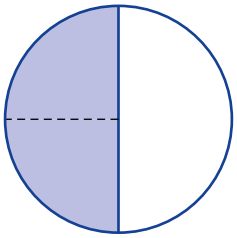
- To find the reciprocal of a mixed number, express it as an improper fraction first.

WORKED EXAMPLE 22

Find the reciprocal of $1\frac{2}{3}$.

THINK	WRITE
1 Write $1\frac{2}{3}$ as an improper fraction.	$1\frac{2}{3} = \frac{5}{3}$
2 Turn it upside down and write the answer in a sentence.	The reciprocal of $1\frac{2}{3}$ is $\frac{3}{5}$.

- To find out how many quarters there are in one half, we could draw a diagram and find that there are 2 quarters.
- This means that $\frac{1}{2} \div \frac{1}{4} = 2$.
- The answer could also have been found by changing the division sign to multiplication and tipping the second fraction upside down.
- Division by a number is the same as multiplication by its reciprocal.



WORKED EXAMPLE 23

Find $\frac{2}{3} \div \frac{4}{9}$.

THINK	WRITE
1 Write the question.	$\frac{2}{3} \div \frac{4}{9}$
2 Change the division sign to a multiplication sign and tip the second fraction.	$= \frac{2}{3} \times \frac{9}{4}$
3 Cancel if possible, vertically or diagonally.	$= \frac{\cancel{2}^{1}}{\cancel{3}^{1}} \times \frac{\cancel{9}^{3}}{\cancel{4}^{4}}$
4 Perform the multiplication.	$= \frac{3}{2}$
5 Simplify your answer.	$= 1\frac{1}{2}$

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Watch this eLesson: Dividing fractions (eles-0003)

Try out this interactivity: Dividing fractions (int-2358)

Complete this digital doc: Spreadsheet: Dividing fractions (doc-1728)

Exercise 4.7 Dividing rational numbers

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1a–h, 2a–e, 3a–d, 4a–f, 6, 7, 9	Questions: 1e–l, 2d–h, 3b–e, 4g–l, 6, 7, 9	Questions: 1m–p, 2g–j, 3c–f, 4l–o, 5, 6, 7, 8, 9

Individual pathway interactivity: int-4328

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Fluency

1. WE21 Find the reciprocals of each of the following.

a. $\frac{3}{4}$ **b.** $\frac{2}{7}$ **c.** $\frac{5}{3}$ **d.** $\frac{5}{12}$

e. $\frac{6}{5}$ **f.** $\frac{9}{2}$ **g.** $\frac{5}{3}$ **h.** $\frac{10}{3}$

i. $\frac{1}{5}$ **j.** $\frac{1}{12}$ **k.** $\frac{3}{15}$ **l.** $\frac{2}{22}$

m. 5 **n.** 20 **o.** 13 **p.** 1

2. WE22 Find the reciprocals of these mixed numbers.

a. $1\frac{2}{3}$ **b.** $4\frac{3}{7}$ **c.** $7\frac{1}{2}$ **d.** $4\frac{2}{5}$ **e.** $9\frac{2}{7}$

f. $3\frac{6}{7}$ **g.** $3\frac{3}{8}$ **h.** $5\frac{1}{3}$ **i.** $10\frac{2}{9}$ **j.** $6\frac{9}{10}$

3. Multiply each of these numbers by its reciprocal.

a. $\frac{5}{7}$ **b.** $\frac{2}{9}$ **c.** $\frac{3}{5}$ **d.** $3\frac{2}{7}$ **e.** $2\frac{1}{8}$ **f.** $3\frac{3}{4}$

4. WE23 Find:

a. $\frac{1}{4} \div \frac{2}{3}$ **b.** $\frac{8}{9} \div \frac{7}{6}$ **c.** $\frac{9}{10} \div \frac{1}{2}$ **d.** $\frac{1}{12} \div \frac{2}{3}$ **e.** $\frac{4}{11} \div \frac{5}{6}$

f. $\frac{7}{12} \div \frac{9}{11}$ **g.** $\frac{10}{3} \div \frac{15}{21}$ **h.** $\frac{2}{7} \div \frac{20}{21}$ **i.** $\frac{8}{15} \div \frac{16}{15}$ **j.** $\frac{4}{9} \div \frac{4}{9}$

k. $\frac{4}{7} \div 10$ **l.** $\frac{5}{8} \div 20$ **m.** $\frac{4}{3} \div 12$ **n.** $\frac{15}{11} \div 5$ **o.** $\frac{7}{9} \div 30$

Understanding

5. What is the value of this expression?

$$\frac{1}{2 + \frac{1}{2\frac{1}{2}}}$$

Reasoning

6. Mai is making sushi. The recipe requires $\frac{1}{4}$ cup of rice. She has a 2 kg bag of rice. If 1 cup of rice weighs 250 g, how many batches of the recipe can she make?

7. Without using a calculator evaluate the following multiplications.

a. $\frac{3}{2} \times \frac{4}{3}$ **b.** $\frac{3}{2} \times \frac{4}{3} \times \frac{5}{4}$

c. $\frac{3}{2} \times \frac{4}{3} \times \frac{5}{4} \times \frac{6}{5}$ **d.** $\frac{3}{2} \times \frac{4}{3} \times \frac{5}{4} \times \frac{6}{5} \times \frac{7}{6}$

e. Convert the answers in parts **a, b, c** and **d** to mixed numbers. Explain the pattern.

f. $\frac{3}{2} \times \frac{4}{3} \times \frac{5}{4} \times \frac{6}{5} \times \frac{7}{6} \ldots \times \frac{n+1}{n}$. Explain the reasoning behind your answer.

Problem solving

8. Find the average of $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{10}$ and $\frac{1}{20}$.

9. Maytreyi bought $1\frac{1}{2}$ kg of chocolates for her birthday party. She is going to give each child attending her birthday $\frac{1}{10}$ kg. How many people are attending her birthday party?

Reflection

What strategies will you use to divide fractions?

CHALLENGE 4.2

A proper fraction has a numerator that is twice the numerator of a second proper fraction. Both numerators add up to 6. If the denominators of the two fractions add up to 8 and have a difference of 2, find the two fractions.

4.8 Fractions as ratios

4.8.1 Introduction to ratios

- **Ratios** are used in many aspects of everyday life. They are used to compare *quantities of the same kind*.
- Where might you hear the following comparisons?
- What are the quantities being compared?

1. Sebastian Vettel's F1 car is twice as fast as Mick's delivery van.

2. You will need 2 buckets of water for every $\frac{1}{2}$ bucket of sand.
3. The fertiliser contains 3 parts of phosphorus to 2 parts of potassium.
4. The Tigers team finished the season with a win : loss ratio of 5 to 2.
5. Mix 1 teaspoon of salt with 4 teaspoons of flour.

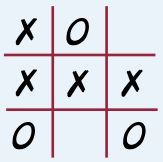
- In the last example we are considering the ratio of salt (1 teaspoon) to flour (4 teaspoons). We *write* 1 : 4 and *say* 'one is to four'; we are actually mixing a total of 5 teaspoons. Ratios can also be written in fraction form:

$$1:4 \Leftrightarrow \frac{1}{4}$$

- *Note:* Since the ratios compare quantities of the same kind, they do not have a name or unit of measurement. That is, we write the ratio of salt to flour as 1 : 4, not 1 teaspoon : 4 teaspoons. The order of the numbers in a ratio is important. In the example of the ratio of salt and flour, 1 : 4 means 1 unit (for example a teaspoon) of salt to 4 units of flour. The amount of flour is 4 times as large as the amount of salt. On the contrary, the ratio 4 : 1 means 4 units of salt to 1 unit of flour, which means the amount of salt is 4 times as large as the amount of flour.

WORKED EXAMPLE 24

Look at the completed game of noughts-and-crosses at right and write down the ratios of:
a crosses to noughts
b noughts to unmarked spaces.



THINK	WRITE
a Count the number of crosses and the number of noughts. Write the 2 numbers as a ratio (the number of crosses must be written first).	**a** 4 : 3
b Count the number of noughts and the number of unmarked spaces. Write the 2 numbers as a ratio, putting them in the order required (the number of noughts must be written first).	**b** 3 : 2

- Before ratios are written, the numbers must be expressed in *the same units of measurement*. When choosing which of the quantities to convert, remember that ratios *contain only whole numbers*.

WORKED EXAMPLE 25

Rewrite the following statement as a ratio: **7 mm to 1 cm.**

THINK	WRITE
1 Express both quantities in the same units. To obtain whole numbers, convert 1 cm to mm (rather than 7 mm to cm).	7 mm to 1 cm 7 mm to 10 mm
2 Omit the units and write the 2 numbers as a ratio.	7 : 10

4.8.2 Simplifying ratios

- Equivalent ratios are when the numbers in a ratio are multiplied or divided by the same number.
  For instance, the ratios 2 : 3 and 4 : 6 are equivalent, as the second ratio can be obtained by multiplying both numbers of the first ratio by 2.
- Ratios 10 : 5 and 2 : 1 are also equivalent, as the second ratio is obtained by dividing both numbers of the first ratio by 5.
- Ratios are usually written in simplest form, that is, reduced to lowest terms. This is achieved by dividing each number in the ratio by the highest common factor (HCF).

WORKED EXAMPLE 26

Express the ratio 16 : 24 in simplest form.

THINK	WRITE
1 What is the HCF of both 16 and 24? It is 8.	16 : 24
2 Divide both 16 and 24 by 8 to obtain an equivalent ratio in simplest form.	2 : 3

WORKED EXAMPLE 27

Write the ratio of 1.5 m to 45 cm in simplest form.

THINK	WRITE
1 Write down the question.	1.5 m to 45 cm
2 Express both quantities in the same units by changing 1.5 m into cm. (1 m = 100 cm)	150 cm to 45 cm
3 Write both numbers as a ratio.	150 : 45
4 Simplify the ratio by dividing both numbers by the HCF 15.	10 : 3

WORKED EXAMPLE 28

Simplify the following ratios.
a $\frac{2}{5} : \frac{7}{10}$ **b** $\frac{5}{6} : \frac{5}{8}$

THINK	WRITE
a **1** Write the fractions in ratio form.	**a** $\frac{2}{5} : \frac{7}{10}$
2 Write equivalent fractions using the lowest common denominator: in this case 10.	$= \frac{4}{10} : \frac{7}{10}$
3 Multiply both fractions by 10.	$= 4 : 7$
4 Simplify the ratio, if possible. In this case it cannot be simplified.	
b **1** Write the fractions in ratio form.	**b** $= \frac{5}{6} : \frac{5}{8}$
2 Write equivalent fractions using the lowest common denominator. In this case 24.	$= \frac{20}{24} : \frac{15}{24}$
3 Multiply both fractions by 24.	$= 20 : 15$
4 Simplify the ratio by dividing both numbers by the HCF, which is 5.	$= 4 : 3$

4.8.3 Dividing in a given ratio

- Consider the following situation:

  Isabel and Rachel decided to buy a $10 lottery ticket. Isabel had only $3 so Rachel put in the other $3 so Rachel put in the other $7. The girls won the first prize of $500 000. How are the girls going to share the prize? Is it fair that they would get equal shares?

  In all fairness, it should be shared in the ratio 3 : 7.

WORKED EXAMPLE 29

Share the amount $500 000 in the ratio 3 : 7.

THINK	WRITE
1 Calculate the total number of parts in the ratio.	Total number of parts $= 3 + 7$ $= 10$
2 The first share represents 3 parts out of a total of 10, so find $\frac{3}{10}$ of the total amount.	First share $= \frac{3}{10} \times 500\,000$ $= \$150\,000$
3 The second share represents 7 parts out of a total of 10, so find $\frac{7}{10}$ of the total amount.	Second share $= \frac{7}{10} \times 500\,000$ $= \$350\,000$

- We can mentally check the answer by adding the two shares together. The total should be the original amount to be shared.

WORKED EXAMPLE 30

Concrete mixture for a footpath was made up of 1 part of cement, 2 parts of sand and 4 parts of blue metal. How much sand was used to make 4.2 m^2 of concrete?

THINK	WRITE
1 Write in ratio form	1: 2: 4
2 Find the total number of parts.	Total number of parts $= 1 + 2 + 4$ $= 7$
3 There are 2 parts of sand to be used in the mixture, so find $\frac{2}{7}$ of the total amount of concrete made.	Amount of sand $= \frac{2}{7} \times 4.2\ m^2$ $= 1.2\ m^2$

Exercise 4.8 Fractions as ratios

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–4, 5i–j, 6, 7a–j, 8, 9, 10, 11, 12, 13, 14, 16, 19, 21	Questions: 1–4, 5f–o, 6, 7f–o, 8, 9, 10, 11, 12, 13, 14, 16, 18, 19, 21, 22	Questions: 1–4, 5k–t, 6, 7k–t, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 22, 23

Individual pathway interactivity: int-4329

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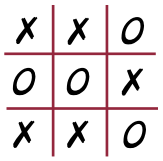
Fluency

1. WE24 Look at the completed game of noughts-and-crosses and write down the ratios of:

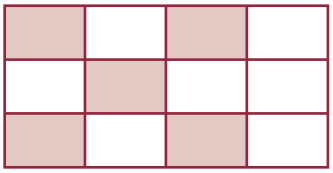


a. noughts to crosses
b. crosses to noughts
c. crosses to total number of spaces
d. total number of spaces to noughts
e. noughts in the top row to crosses in the bottom row.

2. Look at the coloured circles on the right then write the following ratios.

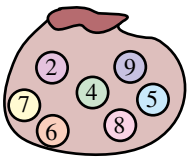
a. Black : red
b. Red : black
c. Aqua : black
d. Black : aqua
e. Aqua : red
f. Black : (red and aqua)
g. Aqua : (black and red)
h. Black : total circles
i. Aqua : total circles
j. Red : total circles

3. For the diagram shown, write down the following ratios.



a. Shaded parts : unshaded parts
b. Unshaded parts : shaded parts
c. Shaded parts : total parts

4. In the bag of numbers shown to the right, write down the ratios of:

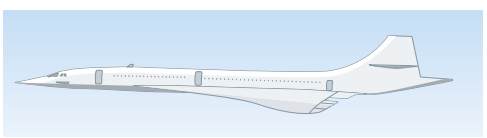


a. even numbers to odd numbers
b. prime numbers to composite numbers
c. numbers greater than 3 to numbers less than 3
d. multiples of 2 to multiples of 5
e. numbers divisible by 3 to numbers not divisible by 3.

5. WE25 Rewrite each of the following statements as a ratio.

a. 3 mm to 5 mm
b. 6 s to 19 s
c. $4 to $11
d. 7 teams to 9 teams
e. 1 goal to 5 goals
f. 9 boys to 4 boys
g. 3 weeks to 1 month
h. 3 mm to 1 cm
i. 17 seconds to 1 minute
j. 53 cents to $1
k. 11 cm to 1 m
l. 1 g to 1 kg
m. 1 L to 2 kL
n. 7 hours to 1 day
o. 5 months to 1 year
p. 1 km to 27 m
q. 7 apples to 1 dozen apples
r. 13 pears to 2 dozen pears
s. 3 females to 5 males
t. 1 teacher to 22 students

6. MC In each of the following situations, choose the best ratio *approximation*.



a. Of the 90 000 people who attended the test match, 23 112 were females. The number of males to females is approximately:

A. 1 : 4 **B.** 4 : 1
C. 3 : 1 **D.** 1 : 3

b. A Concorde jet travels at 1050 km/h and a Cessna travels at 210 km/h. Compare their speeds.

A. 2 : 1 **B.** 3 : 1
C. 4 : 1 **D.** 5 : 1

c. A house-and-land package is sold for $200 000. If the land was valued at $80 000, compare the land and house values.
A. 1 : 2 **B.** 1 : 3 **C.** 2 : 5 **D.** 2 : 3

d. In 1.65 kilogram of fertiliser, there are 550 g of phosphorus. Compare the amount of phosphorus to other components of the fertiliser.
A. 1 : 2 **B.** 4 : 3 **C.** 3 : 1 **D.** 5 : 1

e. Sasha saves $120 out of his take-home pay of $720 each fortnight. Compare his savings with his expenses.
A. 7 : 12 **B.** 1 : 5 **C.** 1 : 3 **D.** 1 : 7

7. WE26 Express each ratio in simplest form.

a. 2 : 4	**b.** 3 : 9	**c.** 5 : 10	**d.** 6 : 18	**e.** 12 : 16
f. 15 : 18	**g.** 24 : 16	**h.** 21 : 14	**i.** 25 : 15	**j.** 13 : 26
k. 15 : 35	**l.** 27 : 36	**m.** 36 : 45	**n.** 42 : 28	**o.** 45 : 54
p. 50 : 15	**q.** 56 : 64	**r.** 75 : 100	**s.** 84 : 144	**t.** 88 : 132

8. Complete the patterns of equivalent ratios.

a. 1 : 3	**b.** 2 : 1	**c.** 2 : 3	**d.** 64 : 32	**e.** 48 : 64
2 : 6	4 : 2	4 : 6	__ : 16	24 : __
_ : 9	_ : 4	6 : _	__ : 8	12 : __
_ : 12	_ : 8	_ : 12	8 : __	__ : 8
5 : __	20 : _	_ : 24	__ : 1	__ : __

9. WE27 Write the following ratios in simplest form.

a. 8 cm to 12 cm	**b.** $6 to $18	**c.** 50 s to 30 s
d. 80 cm to 2 m	**e.** 75 cents to $3	**f.** 2 h to 45 min
g. 300 mL to 4 L	**h.** 500 g to 2.5 kg	**i.** 45 mm to 2 cm
j. $4 to $6.50	**k.** 2500 L to 2 kL	**l.** 2500 m to 2 km
m. 30 cents to $1.50	**n.** 2 h 45 min to 30 min	**o.** 200 m to 0.5 km
p. 0.8 km to 450 m	**q.** $1\frac{1}{2}$ min to 300 s	**r.** 1.8 cm to 12 mm
s. 3500 mg to 1.5 g	**t.** $1.75 to $10.50	

10. Compare the following, using a mathematical ratio (in simplest form):
a. The Hawks won 8 games and the Lions won 10 games.
b. This jar of coffee costs $4 but that one costs $6.
c. While Joanne made 12 hits, Holly made 8 hits.
d. In the first innings, Ian scored 48 runs and Adam scored 12 runs.
e. During the race, Rebecca's average speed was 200 km/h while Donna's average speed was 150 km/h.
f. In the basketball match, the Tigers beat the Magic by 105 points to 84 points.
g. The capacity of the plastic bottle is 250 mL and the capacity of the glass container is 2 L.
h. Joseph ran the 600 m in 2 minutes but Maya ran the same distance in 36 seconds.
i. In the movie audience, there were 280 children and 35 adults.
j. On a page in the novel *Moby Dick* there are 360 words. Of these, 80 begin with a vowel.

11. WE28 Write the following ratios in simplest form.

a. $\frac{1}{3}$ to $\frac{2}{3}$	**b.** $\frac{5}{7}$ to $\frac{6}{7}$	**c.** $\frac{1}{4} : \frac{1}{2}$	**d.** $\frac{2}{5} : \frac{5}{6}$
e. $\frac{4}{5}$ to $\frac{9}{20}$	**f.** $\frac{2}{3} : \frac{3}{5}$	**g.** $\frac{3}{10} : 1$	**h.** $1\frac{2}{3}$ to $\frac{1}{3}$
i. $1\frac{1}{4} : 1\frac{1}{2}$	**j.** $3\frac{1}{3}$ to $2\frac{1}{2}$	**k.** $1 : 1\frac{3}{5}$	**l.** $3\frac{1}{4}$ to $2\frac{4}{5}$

12. WE29 Share the amount of $1000 in the following ratios.

a. 2 : 3	**b.** 3 : 1	**c.** 1 : 4	**d.** 1 : 1	**e.** 3 : 5
f. 5 : 3	**g.** 3 : 7	**h.** 9 : 1	**i.** 7 : 13	**j.** 9 : 11

13. Rosa and Mila bought a lottery ticket costing $10. How should they share the first prize of $50000 if their respective contributions were:

a. $2 and $8 **b.** $3 and $7 **c.** $4 and $6
d. $5 and $5 **e.** $2.50 and $7.50?

Understanding

14. WE30 Concrete mixture is made up of 1 part cement, 2 parts sand and 4 parts blue metal.

a. How much sand is needed for 7 m^3 of concrete?
b. How much cement is needed for 3.5 m^3 of concrete?
c. How much blue metal is required for 2.8 m^3 of concrete?
d. How much sand is used for 5.6 m^3 of concrete?
e. How much cement is needed to make 8.4 m^3 of concrete?

15. Three friends buy a Lotto ticket costing $20. How should they share the first prize of $600000, if they each contribute:

a. $3, $7 and $10 **b.** $6, $6 and $8 **c.** $1, $8 and $11
d. $5, $6 and $9 **e.** $5, $7.50 and $7.50?

16. Three angles of a triangle are in the ratio 1 : 2 : 3. What is the magnitude of each angle?

17. In a school, the ratio of girls in Years 8, 9 and 10 is 6 : 7 : 11. If there are 360 girls in the school:

a. how many Year 8 girls are there?
b. how many more Year 10 girls are there than Year 8 girls?

18. In a moneybox, there are 5 cent, 10 cent and 20 cent coins in the ratio 8 : 5 : 2. If there are 225 coins altogether:

a. how many 5 cent coins are there?
b. how many more 10 cent coins than 20 cent coins are there?
c. what is the total value of the 5 cent coins?
d. what is the total value of the coins in the moneybox?

Reasoning

19. In a family, 3 children receive their allowances in the ratio of their ages, which are 15 years, 12 years and 9 years. If the total of the allowances is $30, how much does each child receive? Explain your answer.

20. The angles of a quadrilateral are in the ratio 2 : 3 : 4 : 6. What is the difference in magnitude between the smallest and largest angles?

Problem solving

21. You drop a rubber ball from a height of 10 m. The ball rebounds to a height of 8 m and then to a height of 6.4 m. The ball continues to bounce in the same ratio. What is the height of the ball when the top of its bounce is less than 3 m, and on what bounce does this occur?

22. Two brothers are going on a trip. They are each carrying a backpack weighing 2 kg. In these backpacks they will carry 4 kg of books, 6 kg of clothes, 3 kg of toiletries, 3 kg of shoes, 2 kg of snacks and 2 kg of water. The older brother offers to carry double the weight carried by his younger brother. Can the older brother divide the luggage such that his backpack weighs double that of his younger brother's backpack?

23. A rectangular lawn measures 40 m by 20 m. Your lawn mower can cut a path 30 cm wide. If you start along the outer edge of the lawn and mow around the outer portion of the uncut lawn twenty times, what fractional part of the lawn have you mowed?

Reflection

Why is it important to be able to write ratios as fractions?

4.9 Working with mixed numbers

4.9.1 Adding and subtracting mixed numbers

- Change mixed numbers to improper fractions.
- Use equivalent fractions to find the lowest common denominator (LCD).
- Add or subtract the numerators. Do not add or subtract denominators.
- Simplify the answer if possible.
- Estimation can be used to find approximate answers to addition and subtraction of mixed numbers.
- This allows you to check easily whether your answer to a calculation is reasonable.

WORKED EXAMPLE 31

Find $2\frac{3}{4} + 1\frac{1}{2}$.

THINK	WRITE
1 Write the question.	$2\frac{3}{4} + 1\frac{1}{2}$
2 Change each mixed number into an improper fraction.	$= \frac{11}{4} + \frac{3}{2}$
	$= \frac{11}{4} + \frac{3 \times 2}{2 \times 2}$
3 Write both fractions with the lowest common denominator, LCD.	$= \frac{11}{4} + \frac{6}{4}$
4 Add the numerators.	$= \frac{17}{4}$
5 Simplify the answer.	$= 4\frac{1}{4}$

WORKED EXAMPLE 32

Find $3\frac{1}{5} - 1\frac{3}{4}$.

THINK	WRITE
1 Write the question.	$3\frac{1}{5} - 1\frac{3}{4}$
2 Change each mixed number into an improper fraction.	$= \frac{16}{5} - \frac{7}{4}$
	$= \frac{16 \times 4}{5 \times 4} - \frac{7 \times 5}{4 \times 5}$
3 Write both fractions with the lowest common denominator, LCD.	$= \frac{64}{20} - \frac{35}{20}$
4 Subtract the numerators.	$= \frac{39}{20}$
5 Simplify the answer.	$= 1\frac{19}{20}$

WORKED EXAMPLE 33

Find $3\frac{1}{3} - 2\frac{1}{2}$ and check the answer by estimation.

THINK	WRITE
1 Write the question.	$3\frac{1}{3} + 2\frac{1}{2}$

2 Change each mixed number into an improper fraction.	$= \frac{10}{3} + \frac{5}{2}$ $= \frac{10 \times 2}{3 \times 2} + \frac{5 \times 3}{2 \times 3}$
3 Write both fractions with the lowest common denominator, LCD.	$= \frac{20}{6} + \frac{15}{6}$
4 Add the numerators.	$= \frac{35}{6}$
5 Simplify the answer.	$= 5\frac{5}{6}$
6 Check the answer by adding the whole numbers to find an approximation.	$3\frac{1}{3} + 2\frac{1}{2} > 3 + 2$ $3\frac{1}{3} + 2\frac{1}{2} > 5$ The approximation shows that the answer we obtain should be greater than 5.

4.9.2 Multiplying mixed numbers

- Change mixed numbers into improper fractions.
- Simplify fractions by dividing, either vertically or diagonally, by their highest common factor.
- Multiply the numerators. Then multiply the denominators.
- Simplify the answer if possible.

WORKED EXAMPLE 34

Find $1\frac{1}{2} \times 3\frac{3}{5}$.

THINK	WRITE
1 Write the question.	$1\frac{1}{2} \times 3\frac{3}{5}$
2 Change mixed numbers to improper fractions and cancel if possible.	$= \frac{3}{\cancel{2}_{1}} \times \frac{\cancel{18}^{9}}{5}$
3 Multiply the numerators and then the denominators.	$= \frac{27}{5}$
4 Simplify the answer.	$= 5\frac{2}{5}$

4.9.3 Dividing mixed numbers

- To divide mixed numbers, first change them to improper fractions.
- Once the fractions are written as improper fractions, divide as normal.

WORKED EXAMPLE 35

Find $1\frac{1}{2} \div 3\frac{2}{5}$.

THINK	WRITE
1 Write the question.	$1\frac{1}{2} \div 3\frac{2}{5}$

2 Change the mixed numbers to improper fractions. $= \frac{3}{2} \div \frac{17}{5}$

3 Change the $\div$ to $\times$ and tip the second fraction. $= \frac{3}{2} \times \frac{5}{17}$

4 Multiply the numerators, then the denominators and simplify the answer if necessary. $= \frac{15}{34}$

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Exercise 4.9 Working with mixed numbers

assessON

Individual pathways

PRACTISE
Questions:
1, 2, 3, 4, 5, 6, 7, 9, 13, 18, 21

CONSOLIDATE
Questions:
1, 2, 3, 4, 5, 6, 7, 8, 11, 15, 17, 20, 21

MASTER
Questions:
1c, f, 2c, f, 3d, 4c, f, 5i, o, r, 6c, f, i, l, o, 7–20, 22

Individual pathway interactivity: int-4330

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Fluency

1. WE31 Find:

a. $1\frac{1}{5} + 3\frac{1}{2}$ **b.** $3\frac{2}{3} - 2\frac{1}{3}$ **c.** $5\frac{2}{3} - 2\frac{1}{4}$

d. $9\frac{3}{8} + 4$ **e.** $2\frac{3}{5} - 1\frac{1}{2}$ **f.** $3\frac{1}{2} + 2\frac{3}{4}$

2. WE32 Find:

a. $7 - 5\frac{2}{3}$ **b.** $2\frac{1}{3} - \frac{5}{6}$ **c.** $3\frac{1}{2} - 1\frac{7}{8}$

d. $4\frac{3}{5} - 2\frac{9}{10}$ **e.** $5\frac{1}{4} - 1\frac{7}{12}$ **f.** $5\frac{3}{4} - 3\frac{5}{6}$

3. Find:

a. $\frac{1}{5} + 2\frac{2}{7}$ **b.** $\frac{5}{8} + 3\frac{3}{4} - \frac{1}{2}$ **c.** $2\frac{2}{9} + 1\frac{1}{3} + \frac{1}{6}$ **d.** $4\frac{2}{3} + 1\frac{3}{8} - 3$

4. WE33 Find the following and check the answers by estimation.

a. $5\frac{1}{3} + 2\frac{1}{4}$ **b.** $1\frac{1}{6} + 2\frac{1}{2}$ **c.** $3\frac{1}{3} + 1\frac{1}{6}$

d. $6\frac{1}{2} - 2\frac{1}{4}$ **e.** $4\frac{3}{5} - 1\frac{3}{10}$ **f.** $7\frac{5}{6} - 3\frac{1}{3}$

5. WE34 Find:

a. $1\frac{1}{4} \times \frac{2}{3}$ **b.** $\frac{3}{4} \times 1\frac{5}{8}$ **c.** $1\frac{1}{2} \times 1\frac{1}{2}$ **d.** $2\frac{5}{8} \times 1\frac{3}{4}$

e. $2\frac{1}{5} \times 3\frac{1}{2}$ **f.** $10\frac{9}{10} \times \frac{2}{3}$ **g.** $2\frac{1}{4} \times 2\frac{1}{2}$ **h.** $\frac{4}{5} \times 1\frac{6}{7}$

i. $\frac{9}{10} \times 3\frac{2}{3}$ **j.** $4\frac{1}{3} \times \frac{3}{5}$ **k.** $1\frac{1}{2} \times 12$ **l.** $2\frac{3}{4} \times 6$

m. $5 \times 3\frac{1}{2}$ **n.** $8 \times 3\frac{1}{4}$ **o.** $4\frac{3}{5} \times 2\frac{1}{10}$ **p.** $3\frac{5}{6} \times 9\frac{5}{7}$

q. $5\frac{1}{5} \times 6\frac{2}{3}$ **r.** $3\frac{3}{4} \times 2\frac{2}{3}$ **s.** $\frac{7}{8} \times 3\frac{1}{3}$ **t.** $6\frac{2}{5} \times 4\frac{1}{7}$

6. WE35 Find:

a. $1\frac{1}{4} \div \frac{3}{4}$ **b.** $9 \div 4\frac{1}{2}$ **c.** $\frac{9}{4} \div 3\frac{1}{2}$ **d.** $\frac{10}{3} \div 3\frac{1}{3}$

e. $5 \div 2\frac{3}{4}$ **f.** $7\frac{4}{9} \div \frac{1}{3}$ **g.** $\frac{1}{8} \div 2\frac{1}{2}$ **h.** $1\frac{1}{5} \div 2\frac{1}{3}$

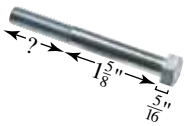
i. $5\frac{6}{7} \div 2\frac{3}{4}$ **j.** $1\frac{1}{2} \div \frac{1}{2}$ **k.** $10\frac{9}{10} \div 4$ **l.** $4\frac{1}{2} \div 3\frac{6}{7}$

m. $3\frac{2}{7} \div 2\frac{1}{7}$ **n.** $4\frac{2}{3} \div \frac{7}{6}$ **o.** $3\frac{5}{8} \div 1\frac{3}{4}$

Understanding

7. One-third of a litre of cordial is mixed with $1\frac{1}{2}$ litres of water. How many litres of drink have been made?

8. The lengths of bolts and nails are often measured in inches. For the bolt shown, find the length of the threaded section if the total length of the bolt is $3\frac{1}{4}$ inches.



9. Julia is planning a holiday to the US. She has 5 months and has worked out the following itinerary. She will be in California for $1\frac{1}{2}$ months, in Colorado for $1\frac{2}{3}$ months and in Florida for $\frac{3}{4}$ of a month. The other state she will be visiting is New York. How much time will she spend in New York?

10. James is building a house. Before the frame can be assembled the footings need to be dug. The digging will take $3\frac{1}{2}$ days. The concrete slab that is then needed will take $2\frac{3}{4}$ days to pour and set. How long will it be before James can start assembling the frame?

11. Lachlan was looking for something to do one Saturday afternoon and his dad suggested he cook something for afternoon tea. Lachlan found a recipe for peanut butter muffins.

Ingredients:

$\frac{1}{4}$ cup sugar, $\frac{1}{4}$ cup margarine,

$\frac{1}{2}$ cup peanut butter, 2 eggs,

$1\frac{1}{2}$ cups milk, $2\frac{1}{2}$ cups self-raising flour,

$\frac{1}{4}$ teaspoon baking soda

Method:

Blend the sugar, margarine and peanut butter. Beat in the eggs and milk. Add the self-raising flour and baking soda. Place the mixture in greased muffin pans and sprinkle with cinnamon sugar. Bake for 15–20 minutes in a 200°C oven.

This recipe makes 10 muffins, but Lachlan wants enough mixture to make 15. He needs to multiply the quantities by $1\frac{1}{2}$. Write the quantities of each ingredient Lachlan needs to make 15 muffins.

12. Felicity is dividing $1\frac{1}{2}$ kilograms of mince steak into 8 hamburger portions. How many kilograms of mince will be in each hamburger portion?

13. Ned spends his school holidays helping his father with the shearing.

a. It takes Ned $\frac{1}{4}$ hour to shear one sheep. How many sheep will Ned shear in $5\frac{1}{2}$ hours?

b. Ned's father, Wesley, shears a sheep every $\frac{1}{12}$ of an hour. How many sheep would Wesley shear if he worked continuously for 8 hours?

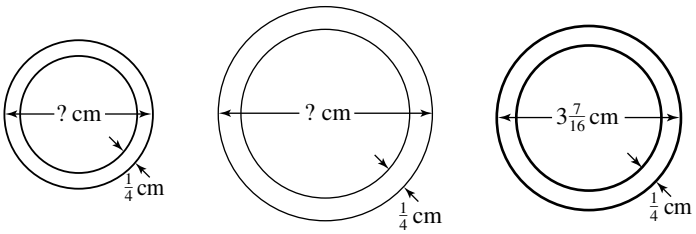
14. Andrew bought $7\frac{3}{4}$ kg of prawns for a party and $\frac{3}{5}$ of them were eaten. How many kilograms of prawns were left over?

15. Teagan buys $8\frac{3}{4}$ kg of sugar. She wants to store it in containers that can hold $1\frac{1}{4}$ kg. How many containers will she need?

16. Max works as a train driver and is paid $21 per hour. If Max works on a public holiday, he is paid double time and a half, which means he earns $2\frac{1}{2}$ times his normal hourly rate. Calculate what Max earns per hour worked on a public holiday.

Reasoning

17. To make the casing for a simple collapsible telescope you need two cylinders, one of which fits inside the other. One cylinder has measurements as shown below left.



a. What is the width of a cylinder (see middle diagram above) which would slide over the one shown above left?

b. What is the width of a cylinder (see diagram above right) which would slide into the one shown above left?

18. Michelle can run one lap of a cross-country course in $\frac{1}{5}$ hour.

a. How many laps could she complete in $1\frac{1}{3}$ hours? (Assume she can keep up the same pace.)

b. How many minutes does it take Michelle to run one lap?

19. Five-eighths of the class were girls and $\frac{2}{3}$ of the girls were brunette. What fraction of the class are brunette girls? If there were 24 people in the class, how many are brunette girls?

Problem solving

20. Worldwide figures indicate that $\frac{1}{10}$ of men are left-handed, while $\frac{2}{25}$ of women are left-handed. If there are 400 men in a room, how many women would need to be there so that there are the same number of left-handed men as left-handed women in the room?

21. A skydiver opens his parachute at 846 m above ground level. He has already fallen five-sevenths of the distance to the ground. How high was the plane from which he jumped?

22. A new vending machine stocking water and sports drinks was installed at the local gymnasium. The number of bottles sold each day over the first 5 weeks after its installation is shown.

4, 39, 31, 31, 50, 43, 60, 45, 57, 61, 18,
26, 3, 52, 51, 59, 33, 51, 27, 62, 30, 40, 3,
30, 37, 9, 33, 44, 53, 16, 22, 6, 42, 33, 19

The gymnasium realises that the number of daily sales will vary, depending particularly on the weather. The machine is stacked with 60 bottles at the start of each day. The supplier advises that, because of electricity costs and other running expenses, the machine is not worthwhile having unless at least three-quarters of the bottles are sold on average 2 days per week. Round these sales to the nearest 5 and analyse the figures to determine whether the gymnasium should keep the vending machine.

Reflection

How could you convince someone that $\frac{1}{2}$ of $3\frac{1}{3}$ is $1\frac{2}{3}$?

4.10 Review

4.10.1 Review questions

Fluency

1. What fraction of the eggs in this carton are brown?

2. Write the sequence of the first three equivalent fractions for the following.

a. $\frac{3}{8}$ **b.** $\frac{4}{9}$

c. $\frac{5}{11}$ **d.** $\frac{6}{7}$

3. Fill the gaps to make equivalent fractions.

a. $\frac{1}{2} = \frac{}{4} = \frac{3}{}$ **b.** $\frac{1}{3} = \frac{3}{} = \frac{}{15}$ **c.** $\frac{3}{4} = \frac{9}{} = \frac{}{28}$

d. $\frac{2}{5} = \frac{}{10} = \frac{16}{}$ **e.** $\frac{1}{4} = \frac{5}{} = \frac{}{100}$ **f.** $\frac{3}{10} = \frac{}{100} = \frac{300}{}$

4. Ricky, Leo and Gabriel were the highest goal scorers for their local football team throughout the season. Ricky scored $\frac{2}{7}$ of the team's goals, Leo scored $\frac{1}{3}$ of the goals and Gabriel scored $\frac{1}{4}$ of the goals.

a. Who scored most of the goals?

b. Who scored more goals; Ricky or Gabriel?

5. Simplify these fractions.

a. $\frac{42}{49}$ **b.** $\frac{81}{108}$ **c.** $\frac{18}{20}$ **d.** $\frac{21}{28}$

e. $\frac{36}{72}$ **f.** $\frac{50}{150}$ **g.** $\frac{75}{125}$ **h.** $\frac{2}{80}$

6. Simplify the following mixed numbers.

a. $6\frac{8}{10}$ **b.** $3\frac{6}{9}$ **c.** $4\frac{15}{20}$ **d.** $1\frac{20}{100}$

7. What fraction of the months of the year begin with the letter J? Express your answer in simplest form.

8. Year 7 students at Camberwell Secondary College raised a total of $1600 for the Royal Children's Hospital Good Friday appeal. The table below shows how much money each class raised.

Class	7.1	$200	7.2	$400	7.3	$250
Amount raised ($)	7.4	$250	7.5	$280	7.6	$220

Express as a simple fraction how much of the total each class raised.

9. Convert these improper fractions to mixed numbers.

a. $\frac{16}{3}$ **b.** $\frac{21}{5}$ **c.** $\frac{12}{7}$ **d.** $\frac{11}{2}$

e. $\frac{5}{4}$ **f.** $\frac{80}{7}$ **g.** $\frac{55}{9}$ **h.** $\frac{72}{10}$

10. Express these mixed numbers as improper fractions.

a. $2\frac{3}{4}$ **b.** $9\frac{7}{8}$ **c.** $3\frac{5}{7}$ **d.** $5\frac{5}{6}$

e. $3\frac{2}{3}$ **f.** $9\frac{9}{10}$ **g.** $3\frac{11}{12}$ **h.** $8\frac{1}{2}$

11. Marcella was in charge of baking and selling lemon slices at the school fete. Each tray she baked contained 15 lemon slices. If Marcella sold 68 lemon slices, how many empty trays were there?

12. Find:

a. $\frac{5}{6} + \frac{1}{8}$ **b.** $\frac{7}{9} - \frac{2}{5}$ **c.** $\frac{12}{5} - \frac{4}{3}$

13. Find:

a. $\frac{4}{7} \times \frac{5}{9}$ **b.** $\frac{2}{5} \times \frac{3}{5}$ **c.** $\frac{25}{36} \times \frac{18}{20}$ **d.** $\frac{56}{81} \times \frac{18}{64}$

e. $\frac{4}{5}$ of 55 **f.** $\frac{2}{11}$ of 121 **g.** $\frac{7}{9}$ of $\frac{18}{84}$ **h.** $\frac{1}{12}$ of 200

14. Write the reciprocals of:

a. $\frac{5}{2}$ **b.** $\frac{2}{7}$ **c.** $\frac{1}{9}$ **d.** 12

e. 124 **f.** $\frac{1}{4}$ **g.** $5\frac{3}{4}$ **h.** $8\frac{1}{2}$

i. $7\frac{2}{5}$ **j.** $2\frac{4}{9}$ **k.** $3\frac{5}{8}$ **l.** $4\frac{1}{5}$

15. Find:

a. $\frac{4}{5} \div \frac{8}{15}$ **b.** $\frac{9}{10} \div \frac{27}{40}$ **c.** $\frac{7}{9} \div 4$ **d.** $\frac{3}{7} \div 10$

16. Calculate:

a. $1\frac{7}{8} + 2\frac{5}{6}$ **b.** $10\frac{3}{7} - 8\frac{1}{2}$ **c.** $6\frac{1}{3} - 2\frac{5}{9}$ **d.** $5\frac{1}{2} - 2\frac{5}{6}$

e. $3\frac{5}{8} - 2\frac{7}{10}$ **f.** $8\frac{2}{9} - 4\frac{5}{8}$ **g.** $12\frac{4}{7} - 10\frac{9}{10}$ **h.** $12\frac{6}{11} - 11\frac{7}{9}$

17. Calculate the following and verify each answer using a calculator.

a. $1\frac{1}{5} + 3\frac{1}{3} - \frac{13}{15}$ **b.** $4\frac{7}{9} - 1\frac{5}{6} + 3\frac{1}{3}$

18. Calculate:

a. $4\frac{2}{5} \times 3\frac{2}{11}$ **b.** $10\frac{2}{7} \times 6\frac{2}{9}$

19. Calculate:

a. $\frac{4}{9} \times 2\frac{1}{4} \times \frac{3}{5}$ **b.** $3\frac{5}{12} \times 1 \times \frac{3}{5} \times 2\frac{1}{2}$ **c.** $\frac{6}{12} \times 1\frac{1}{4} \times 3\frac{2}{5}$

20. Calculate:

a. $1\frac{1}{2} \div 4\frac{4}{5}$ **b.** $6\frac{2}{3} \div 4\frac{1}{6}$ **c.** $4\frac{1}{5} \div 2\frac{7}{8}$

d. $10\frac{1}{3} \div 12\frac{5}{7}$ **e.** $6\frac{2}{7} \div 4\frac{1}{5}$

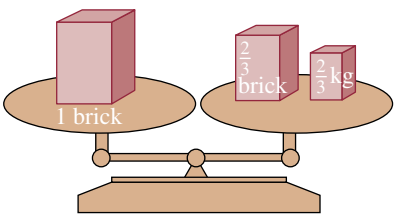
21. Divide \$25 in the ratio 2 : 3.

22. Three people share a lotto prize of \$6600 in the ratio 4 : 5 : 6. What is the difference between the smallest and largest shares?

Problem solving

23. Anthony's monthly take-home pay is \$2800. He spends a quarter on his home loan payments, one-half on food and drink and one-seventh on clothing. One-half of the remainder goes into his savings account. How much money does Anthony put into his savings account each month?

24. On a balance scale, a brick exactly balances $\frac{2}{3}$ of a brick and a $\frac{2}{3}$-kg weight. How heavy is the brick?



25. The temperature rose from $21\frac{1}{2}$°C to $21\frac{3}{4}$°C. What was the temperature rise?

26. You need a piece of trimming $7\frac{5}{8}$ m long to finish an upholstery job. You have a piece that is $\frac{31}{4}$ m left over from another job. Will the trimming be enough to complete the job?

27. Complete the following:

a. $\frac{5}{12} = \frac{1}{4} + ?$ **b.** $\frac{5}{12} = \frac{2}{3} + ?$ **c.** $\frac{5}{12} = \frac{3}{4} + ?$ **d.** $\frac{5}{12} = \frac{4}{5} + ?$

28. James, Jessica, Will, Alice and Alex held a lunchtime sausage sizzle and made a profit of $320. Since some students had worked harder than others, it was agreed the money should be split as follows: Alice gets one quarter and Will gets one fifth. James gets the average of Alice's and Will's shares. If Jessica and Alex split the remainder, determine the amount they each receive.

29. Explain how you can determine a fraction whose numerator and denominator have a product of 252, and which is equivalent to $\frac{4}{7}$.

30. Tomas buys a 24-piece pizza and eats 8 pieces. Drew buys a 16-piece pizza of the same size, and eats 5 pieces. Who ate the most pizza? Explain your reasoning.

31. An Egyptian fraction is one that can be written as the sum of a set of different unit fractions; for example, $\frac{3}{4} = \frac{1}{2} + \frac{1}{4}$. (Note that the numerator of each fraction is 1.) Write $\frac{4}{5}$ as the sum of a set of different unit fractions.

32. Six containers, each holding $5\frac{1}{3}$ litres, are filled from a large tank of chemicals. If the large tank started with 50 litres of chemicals, how much is left in the tank after the six containers are filled?

33. The approximate lifespan of a dollar note is $1\frac{1}{2}$ years. A coin lasts about $16\frac{2}{3}$ times as long. How long is a coin in circulation?

34. A skydiver opens his parachute at 846 m above ground level. He has already fallen five-sevenths of the distance to the ground. How high was the plane from which he jumped?

35. Mr Thompson earned $2400 a month. Recently, he had a pay increase. He now earns $1\frac{1}{4}$ times his old salary. What is his new salary?

Mr Thompson's colleague, Mr Goody, earned $3000 per month and had a pay cut. He now gets $\frac{1}{6}$ less of his old salary. What will be his new salary?

36. The total attendance at a large outdoor concert was 32 200. Of this total, 13 800 were female.

a. How many males attended the concert?

b. Compare the number of females to males by writing the ratio in simplest form.

c. Write the number of females as a fraction of the number of males.

37. A shopping centre has a car park with 250 parking spaces. Of these, $\frac{3}{10}$ are suitable for small cars only. The remaining spaces are for standard cars. Last Friday small cars took up $\frac{14}{15}$ of their allocated spots but overall the car park was $\frac{7}{10}$ full. Determine the fraction of standard car spaces that were filled.

38. Byron needs $\frac{2}{5}$ kg of catalyst mixed with $4\frac{1}{2}$ kg of resin to create one surfboard.

a. If he has $2\frac{1}{4}$ kg of catalyst, and $20\frac{1}{4}$ kg of resin, how many whole surfboards can Byron build?

b. Calculate how much more catalyst and resin Byron needs to build one more surfboard. Explain your reasoning.

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Try out this interactivity: Word search: Topic 4 (int-2589)

Try out this interactivity: Crossword: Topic 4 (int-2590)

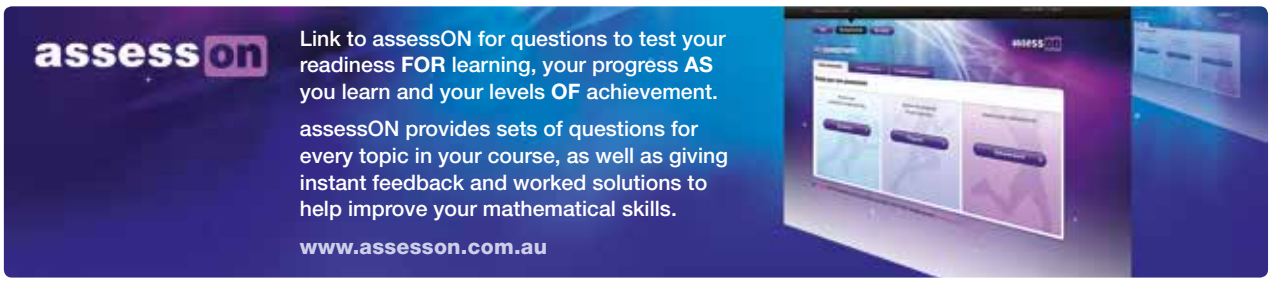
Try out this interactivity: Sudoku: Topic 4 (int-3165)

Complete this digital doc: Concept map: Topic 4 (doc-10725)

Language

It is important to learn and be able to use correct mathematical language in order to communicate effectively. Create a summary of the topic using the key terms below. You can present your summary in writing or using a concept map, a poster or technology.

cancelling
denominator
fraction
improper fraction
lowest common denominator
mixed number
numerator
proper fraction
rational number
reciprocal
simplest form
vinculum



Investigation | Rich Task

The Memory game

The game Memory is played with a standard deck of playing cards. All cards are placed face-side down on a table and each player takes turns to reveal 2 cards. If the 2 cards contain the same number or suit, they are a pair and the player gets another turn. If they are not a pair, they are returned to their original spot and turned face-down. The object of the game is to remember where the same cards are and collect as many pairs as you can. The winner is the person with the most pairs.

Your task is to develop a version of the Memory game with a fractions flavour. Each of your cards will have a fraction written on it and its pair will be an equivalent fraction.

1. As an introduction to the game, match the pairs of equivalent fractions on the cards below.

$\frac{6}{8}$	$\frac{3}{9}$	$\frac{1}{3}$	$\frac{3}{4}$	$\frac{4}{8}$	$\frac{1}{2}$
A	B	C	D	E	F

The requirements for your game are outlined as follows.

- **There are 15 pairs of equivalent fractions in this game — that makes 30 cards.**
- **One of the fractions of each pair must be in simplest form.**
- **For example, $\frac{1}{3}$ and $\frac{4}{12}$ are an acceptable pair, but $\frac{10}{30}$ and $\frac{4}{12}$ are not.**
- **Fractions don't have to be proper fractions. Equivalent mixed numbers and improper fractions are allowed.**

2. Using the above information, write down the 15 pairs of fractions that will be a part of your memory game.

When the game is packaged for sale, additional information accompanies the cards. This information includes a list of the fractions enclosed and instructions for the game. The list of fractions is given in order from smallest to largest.

3. List your fraction pairs in order from smallest to largest.

When items are placed for sale, presentation of the packaging is important. Eye-catching designs will attract attention and mistakes must be avoided at all costs. You are now the designer for a company that is going to market the Memory game.

4. In the space below, design the back of your card and how each fraction will appear on the face of the card.

The playing instructions are very important pieces of information in games. They explain the number of players allowed, how the game is to be played and who wins.

5. Taking into consideration all of the information that has been provided for you throughout this task, write a set of instructions that show how your Memory game is played.
6. Now construct your own set of cards and present them in a suitable package. Remember, presentation of the game is important. When you have finished, try the game out with your friends.

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Complete this digital doc: Code puzzle: Flatback turtle facts (doc-14317)

Answers

Topic 4 Rational numbers

Exercise 4.2 What are rational numbers?

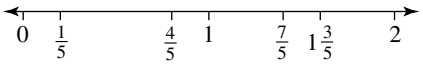
1. a. $\frac{3}{4}$ **b.** $\frac{15}{16}$ **c.** $\frac{7}{8}$ **d.** $\frac{5}{9}$

2. a. $\frac{1}{3}$ **b.** $\frac{3}{4}$

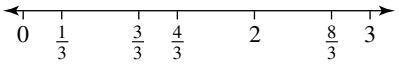
c. The flags belong to Italy and Samoa.

3. a. i. $\frac{3}{11}$ **ii.** $\frac{2}{11}$ **b.** $\frac{3}{11}$

4.



5.



6. a. 3 **b.** 3 **c.** 4 **d.** 5 **e.** 6

f. 9 **g.** 11 **h.** 12 **i.** 7

7. a. $\frac{20}{30}\ \frac{14}{21}\ \frac{40}{60}$ **b.** $\frac{12}{15}\ \frac{36}{45}\ \frac{16}{20}\ \frac{28}{35}\ \frac{80}{100}$ **c.** $\frac{56}{64}\ \frac{21}{24}$ **d.** $\frac{35}{50}\ \frac{21}{30}\ \frac{14}{20}\ \frac{140}{200}$

8. a. $\frac{9}{15} = \frac{12}{20} = \frac{21}{35}$ **b.** $\frac{30}{40} = \frac{18}{24} = \frac{27}{36}$ **c.** $\frac{7}{28} = \frac{6}{24} = \frac{3}{12}$ **d.** $\frac{15}{18} = \frac{35}{42} = \frac{40}{48}$

9. a. $\frac{5}{6} = \frac{10}{12} = \frac{15}{18} = \frac{20}{24}$ **b.** $\frac{5}{8} = \frac{10}{16} = \frac{15}{24} = \frac{20}{32}$ **c.** $\frac{3}{10} = \frac{6}{20} = \frac{9}{30} = \frac{12}{40}$

d. $\frac{2}{3} = \frac{4}{6} = \frac{6}{9} = \frac{8}{12}$ **e.** $\frac{1}{8} = \frac{2}{16} = \frac{3}{24} = \frac{4}{32}$ **f.** $\frac{7}{8} = \frac{14}{16} = \frac{21}{24} = \frac{28}{32}$

10. a. $\frac{3}{5}$ **b.** $\frac{7}{8}$ **c.** $\frac{1}{5}$ **d.** $\frac{1}{8}$ **e.** $\frac{1}{2}$

f. $\frac{3}{4}$ **g.** $\frac{5}{8}$ **h.** $\frac{5}{8}$ **i.** $\frac{4}{5}$

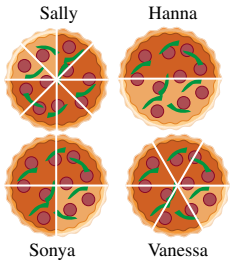
11. a. $>$ **b.** $<$ **c.** $>$ **d.** $>$ **e.** $<$ **f.** $<$

12. a. D **b.** B

13. a. $\frac{3}{10}, \frac{5}{10}, \frac{2}{10}, \frac{1}{5}, \frac{3}{10}, \frac{1}{2}$ **b.** $\frac{9}{24}, \frac{12}{24}, \frac{8}{24}, \frac{1}{3}, \frac{3}{8}, \frac{1}{2}$

c. $\frac{10}{15}, \frac{12}{15}, \frac{6}{15}, \frac{6}{15}, \frac{2}{3}, \frac{4}{5}$ **d.** $\frac{45}{60}, \frac{40}{60}, \frac{28}{60}, \frac{7}{15}, \frac{2}{3}, \frac{3}{4}$

14. a.



b. Vanessa

c. They ate the same amount.

15. a. No, because it cannot be written as a fraction.

b. For accuracy. The calculator uses a more accurate value for π.

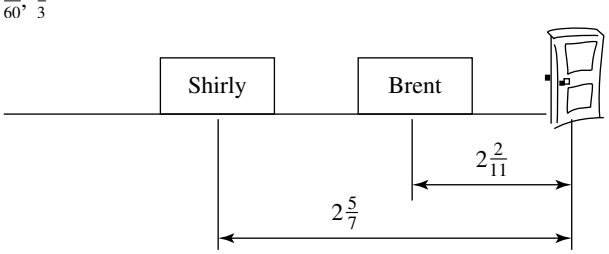
c. Because they can be written as fractions with integers on the numerator and the denominator.

d. After 3 digits.

16. As the denominator doubles the distance between fractions halves, because doubling the denominator is the same as dividing the fraction by 2.

17. $\frac{19}{60}, \frac{1}{3}$

18.



Exercise 4.3 Simplifying rational numbers

1. a. $\frac{1}{2}$ **b.** $\frac{2}{3}$ **c.** $\frac{7}{8}$ **d.** $\frac{4}{5}$ **e.** $\frac{4}{5}$

f. $\frac{3}{4}$ **g.** $\frac{9}{10}$ **h.** $\frac{7}{8}$ **i.** $\frac{5}{6}$ **j.** $\frac{24}{25}$

k. $\frac{7}{8}$ **l.** $\frac{7}{10}$ **m.** $\frac{11}{12}$ **n.** $\frac{11}{25}$ **o.** $\frac{3}{8}$

2. a. $4\frac{3}{5}$ **b.** $7\frac{5}{6}$ **c.** $10\frac{1}{2}$ **d.** $5\frac{1}{6}$ **e.** $3\frac{7}{8}$

f. $1\frac{11}{12}$ **g.** $6\frac{3}{4}$ **h.** $5\frac{1}{3}$ **i.** $3\frac{1}{5}$ **j.** $2\frac{1}{4}$

3. E

4. $\frac{21}{28} = \frac{3}{4}$

5. a. $\frac{20}{36} = \frac{5}{9}$ **b.** $\frac{16}{36} = \frac{4}{9}$

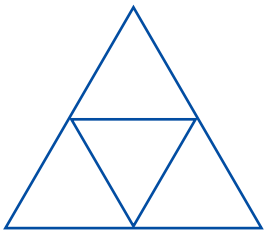
6. 7A:$\frac{1}{5}$, 7B:$\frac{3}{20}$, 7C:$\frac{8}{25}$, 7D:$\frac{2}{25}$, 7E:$\frac{1}{4}$

7. a. 40 **b.** Mark: $\frac{5}{40} = \frac{1}{8}$, Ken: $\frac{10}{40} = \frac{1}{4}$, Samantha: $\frac{8}{40} = \frac{1}{5}$, Jules: $\frac{6}{40} = \frac{3}{20}$, Ahmed: $\frac{4}{40} = \frac{1}{10}$, Darren: $\frac{7}{40}$

8. a. $\frac{7293}{14586}, \frac{6792}{13584}$ **b.** $\frac{5823}{17469}$

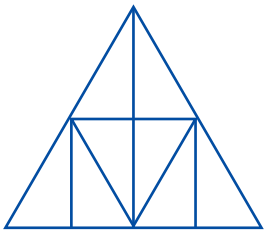
9. Geelong to Melbourne is more reliable.

10. a.



b. $\frac{1}{4}$

c.



d. $\frac{1}{8}$

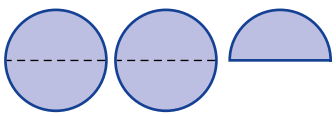
e. Because two smallest triangles form one middle size triangle.

11. $\frac{2}{5}$

12. The denominator could be any multiple of 13 because the fraction is not written in its simplest form and 13 is a prime number.

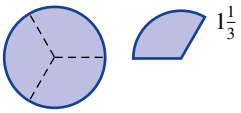
Exercise 4.4 Mixed numbers and improper fractions

1. a.



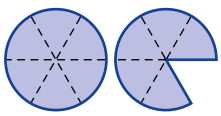
$2\frac{1}{2}$

b.



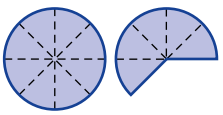
$1\frac{1}{3}$

c.



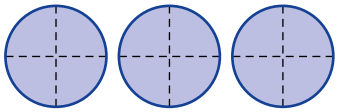
$1\frac{5}{6}$

d.



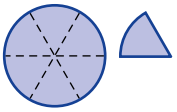
$1\frac{5}{8}$

e.



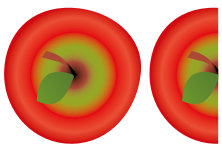
3

f.



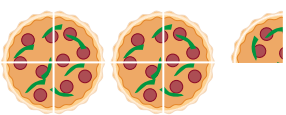
$1\frac{1}{6}$

2. a.



$\frac{3}{2} = 1\frac{1}{2}$

b.



$\frac{9}{4} = 2\frac{1}{4}$

3. a. $1\frac{2}{5}$ **b.** $2\frac{3}{4}$ **c.** $10\frac{1}{2}$ **d.** $3\frac{9}{10}$ **e.** $4\frac{1}{4}$ **f.** $8\frac{4}{11}$

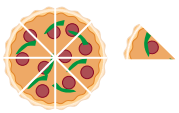
g. $2\frac{3}{13}$ **h.** $2\frac{7}{8}$ **i.** $33\frac{1}{3}$ **j.** $12\frac{1}{2}$ **k.** $6\frac{2}{3}$ **l.** $3\frac{1}{3}$

4. a. $1\frac{3}{5}$ **b.** $3\frac{3}{8}$ **c.** $5\frac{1}{2}$ **d.** $10\frac{7}{9}$ **e.** $8\frac{2}{7}$ **f.** $11\frac{7}{10}$

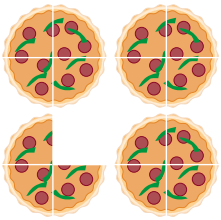
g. $8\frac{3}{8}$ **h.** $8\frac{2}{9}$ **i.** $2\frac{7}{12}$ **j.** $14\frac{5}{6}$

5. a. C **b.** B **c.** D

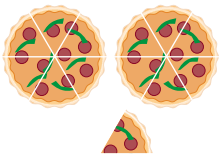
6. a. $\frac{9}{8}$



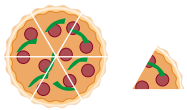
b. $\frac{15}{4}$



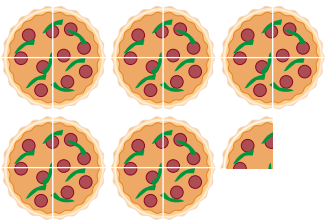
c. $\frac{13}{6}$



d. $\frac{7}{6}$



e. $\frac{21}{4}$



f. $\frac{8}{5}$

7. a. $\frac{8}{7}$ **b.** $\frac{9}{5}$ **c.** $\frac{8}{3}$ **d.** $\frac{10}{3}$ **e.** $\frac{25}{4}$

f. $\frac{37}{9}$ **g.** $\frac{23}{2}$ **h.** $\frac{31}{8}$ **i.** $\frac{39}{5}$ **j.** $\frac{47}{5}$

8. a. $\frac{7}{2}$ **b.** $\frac{21}{5}$ **c.** $\frac{29}{5}$ **d.** $\frac{47}{7}$ **e.** $\frac{29}{10}$

f. $\frac{43}{12}$ **g.** $\frac{28}{5}$ **h.** $\frac{67}{7}$ **i.** $\frac{21}{11}$ **j.** $\frac{53}{6}$

9. a. D **b.** D **c.** A

10. Kim: $2\frac{1}{4}$, Carly: $2\frac{3}{4}$

11. $6\frac{1}{2}$

12. $4\frac{3}{4}$

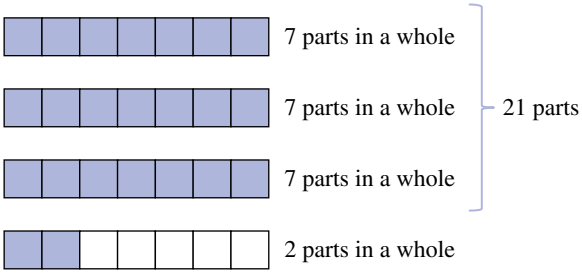
13. a. > **b.** > **c.** < **d.** > **e.** <

f. < **g.** < **h.** < **i.** <

14. a. 25 **b.** 77

15. $10\frac{1}{2}$

16. a. i. $\frac{23}{7}$ **ii.** $\frac{23}{4}$



17. $\frac{4}{5}$

18. a. $\frac{14}{3}$ or $4\frac{2}{3}$ km **b.** $\frac{56}{3}$ or $18\frac{2}{3}$ km **c.** $\frac{70}{3}$ or $23\frac{1}{3}$ km

19. $1\frac{1}{4}, 2\frac{1}{12}, 2\frac{1}{6}, 3\frac{1}{8}, 3\frac{1}{3}$

Exercise 4.5 Adding and subtracting rational numbers

1. a. $\frac{4}{5}$ **b.** $\frac{6}{11}$ **c.** $\frac{5}{6}$ **d.** $\frac{31}{50}$ **e.** $\frac{9}{25}$

f. $\frac{11}{12}$ **g.** $\frac{7}{8}$ **h.** $\frac{6}{7}$ **i.** $\frac{29}{100}$

2. a. $\frac{1}{2}$ **b.** $\frac{1}{4}$ **c.** $\frac{1}{3}$ **d.** $\frac{1}{2}$ **e.** $\frac{1}{2}$

f. 1 **g.** $\frac{3}{4}$ **h.** $\frac{3}{4}$ **i.** $\frac{8}{9}$

3. a. $\frac{3}{4}$ **b.** $\frac{1}{4}$ **c.** $\frac{3}{7}$ **d.** $\frac{3}{4}$ **e.** $\frac{2}{3}$

f. $\frac{37}{40}$ **g.** $\frac{13}{27}$ **h.** $\frac{16}{35}$ **i.** $\frac{2}{5}$

4. a. 6 **b.** 15 **c.** 10 **d.** 12 **e.** 30

f. 18 **g.** 8 **h.** 24 **i.** 35

5. a. $\frac{1}{6}$ **b.** $\frac{4}{15}$ **c.** $\frac{9}{10}$ **d.** $\frac{7}{12}$ **e.** $\frac{19}{30}$

f. $\frac{17}{18}$ **g.** $\frac{5}{8}$ **h.** $\frac{11}{24}$ **i.** $\frac{11}{35}$

6. a. $\frac{1}{20}$ **b.** $\frac{5}{6}$ **c.** $\frac{23}{36}$ **d.** $\frac{39}{44}$ **e.** $\frac{1}{4}$

f. $\frac{11}{40}$ **g.** $\frac{17}{63}$ **h.** $\frac{23}{39}$ **i.** $\frac{2}{15}$ **j.** $\frac{7}{33}$

7. a. D **b.** C

8. a. $\frac{7}{12}$ **b.** $\frac{2}{5}$ **c.** $\frac{19}{24}$ **d.** $\frac{7}{30}$

e. $\frac{7}{16}$ **f.** $\frac{1}{3}$ **g.** $1\frac{11}{12}$ **h.** $1\frac{11}{12}$

9. a. $\frac{7}{15}$ **b.** $\frac{8}{15}$

10. $1\frac{1}{4}$ hours

11. a. $\frac{3}{4}$ **b.** $\frac{7}{12}$ **c.** 1

12. a. $\frac{2}{3}$ **b.** $\frac{23}{30}$

13. a. $\frac{5}{6}$ **b.** $\frac{7}{12}$ **c.** $\frac{9}{20}$

d. When adding two fractions with numerators 1 and denominators consecutive numbers, the answer is the fraction $\frac{\text{sum of denominators}}{\text{product of denominators}}$.

e. $\frac{n + (n + 1)}{n(n + 1)}$

14. a. $\frac{1}{6}$ **b.** $\frac{1}{12}$ **c.** $\frac{1}{20}$

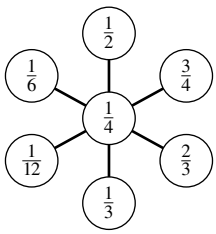
d. When subtracting two fractions with numerators 1 and denominators consecutive numbers, the answer is the fraction $\frac{1}{\text{product of denominators}}$.

e. $\frac{1}{n(n + 1)}$

15. 135

16. $\frac{1}{12}$

Challenge 4.1



Exercise 4.6 Multiplying rational numbers

1. a. 2 **b.** $2\frac{1}{2}$ **c.** $7\frac{1}{2}$ **d.** $1\frac{1}{4}$

2. a. $\frac{1}{8}$ **b.** $\frac{2}{9}$ **c.** $\frac{6}{25}$ **d.** $\frac{20}{27}$

e. $\frac{11}{26}$ **f.** $\frac{54}{77}$ **g.** $\frac{121}{144}$ **h.** $\frac{40}{81}$

i. $\frac{5}{24}$ **j.** $\frac{4}{9}$ **k.** $\frac{55}{72}$ **l.** $\frac{25}{132}$

m. $\frac{49}{64}$ **n.** $\frac{8}{39}$ **o.** $\frac{9}{50}$ **p.** $\frac{8}{21}$

3. a. $\frac{1}{6}$ **b.** $\frac{1}{5}$ **c.** $\frac{5}{14}$ **d.** $\frac{1}{10}$ **e.** $\frac{4}{5}$

f. 1 **g.** $\frac{1}{2}$ **h.** $\frac{12}{35}$ **i.** $\frac{4}{9}$ **j.** $\frac{1}{7}$

k. $\frac{1}{3}$ **l.** $\frac{2}{5}$ **m.** $\frac{4}{21}$ **n.** $\frac{1}{20}$ **o.** $\frac{1}{4}$

4. a. $\frac{1}{8}$ **b.** $\frac{1}{2}$ **c.** $\frac{25}{26}$ **d.** $\frac{9}{14}$

e. $\frac{2}{9}$ **f.** $\frac{1}{5}$ **g.** 16 **h.** 28

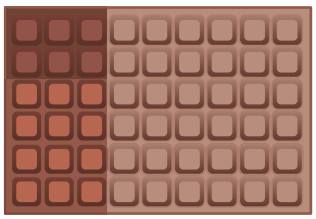
i. 45 **j.** 18 **k.** 15 **l.** 72

m. $4\frac{1}{2}$ **n.** $13\frac{1}{2}$ **o.** $\frac{5}{6}$ **p.** $5\frac{1}{7}$

5. Emily drank 2 litres. Tracy drank $\frac{1}{2}$ litre.
Jonathan drank $\frac{1}{3}$ litre.

6. a. Zoe 32 minutes **b.** Sarah 35 minutes

7. a.



Answers may vary.

b. $\frac{1}{9}$

8. a. $\frac{3}{4}$ **b.** 36 litres

9. a.

b.

c. $\frac{1}{8}$ **d.** $\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$

10. a. 2 **b.** 4 **c.** 10

11. $\frac{1}{24} \times 2400 = \$100$

12. a. 90 minutes **b.** $\frac{90}{180} = \frac{1}{2}$ **c.** $140 + 26\frac{2}{3} = 166\frac{2}{3}$ minutes

13. $121\frac{1}{2}$ km.

14. Red $= 1\frac{16}{21}$ m, green $= 1\frac{9}{28}$ m

Exercise 4.7 Dividing rational numbers

1. a. $\frac{4}{3}$ **b.** $\frac{7}{2}$ **c.** $\frac{3}{5}$ **d.** $\frac{12}{5}$

e. $\frac{5}{6}$ **f.** $\frac{2}{9}$ **g.** $\frac{3}{5}$ **h.** $\frac{3}{10}$

i. 5 **j.** 12 **k.** 5 **l.** 11

m. $\frac{1}{5}$ **n.** $\frac{1}{20}$ **o.** $\frac{1}{13}$ **p.** 1

2. a. $\frac{3}{5}$ **b.** $\frac{7}{31}$ **c.** $\frac{2}{15}$ **d.** $\frac{5}{22}$ **e.** $\frac{7}{65}$

f. $\frac{7}{27}$ **g.** $\frac{8}{27}$ **h.** $\frac{3}{16}$ **i.** $\frac{9}{92}$ **j.** $\frac{10}{69}$

3. a. 1 **b.** 1 **c.** 1 **d.** 1 **e.** 1 **f.** 1

4. a. $\frac{3}{8}$ **b.** $\frac{16}{21}$ **c.** $1\frac{4}{5}$ **d.** $\frac{1}{8}$ **e.** $\frac{24}{55}$

f. $\frac{77}{108}$ **g.** $4\frac{2}{3}$ **h.** $\frac{3}{10}$ **i.** $\frac{1}{2}$ **j.** 1

k. $\frac{2}{35}$ **l.** $\frac{1}{32}$ **m.** $\frac{1}{9}$ **n.** $\frac{3}{11}$ **o.** $\frac{7}{270}$

5. $\frac{5}{12}$

6. 32

7. a. 2 **b.** $\frac{5}{2}$ **c.** 3 **d.** $\frac{7}{2}$

e. 2, $2\frac{1}{2}$, 3, $3\frac{1}{2}$. Every time a new fraction following the pattern is added to the multiplication, the answer increases by $\frac{1}{2}$

f. The answer is $\frac{n+1}{2}$ because all the other numbers cancel down; $\frac{\cancel{3}}{2} \times \frac{\cancel{4}}{\cancel{3}} \times \frac{\cancel{5}}{\cancel{4}} \times \frac{\cancel{6}}{\cancel{5}} \times \frac{\cancel{7}}{\cancel{6}} \times \ldots \times \frac{n+1}{\cancel{n}}$.

8. $\frac{3}{20}$

9. 15 people

Challenge 4.2

$\frac{2}{3}, \frac{4}{5}$

Exercise 4.8 Fractions as ratios

1. **a.** 4:5 **b.** 5:4 **c.** 5:9 **d.** 9:4 **e.** 1:2

2. **a.** 5:3 **b.** 3:5 **c.** 1:5 **d.** 5:1 **e.** 1:3
f. 5:4 **g.** 1:8 **h.** 5:9 **i.** 1:9 **j.** 3:9

3. **a.** 5:7 **b.** 7:5 **c.** 5:12

4. **a.** 4:3 **b.** 3:4 **c.** 6:1 **d.** 4:1 **e.** 2:5

5. **a.** 3:5 **b.** 6:19 **c.** 4:11 **d.** 7:9 **e.** 1:5
f. 9:4 **g.** 3:4 **h.** 3:10 **i.** 17:60 **j.** 53:100
k. 11:100 **l.** 1:1000 **m.** 1:2000 **n.** 7:24 **o.** 5:12
p. 1000:27 **q.** 7:12 **r.** 13:24 **s.** 3:5 **t.** 1:22

6. **a.** C **b.** D **c.** D **d.** A **e.** B

7. **a.** 1:2 **b.** 1:3 **c.** 1:2 **d.** 1:3 **e.** 3:4
f. 5:6 **g.** 3:2 **h.** 3:2 **i.** 5:3 **j.** 1:2
k. 3:7 **l.** 3:4 **m.** 4:5 **n.** 3:2 **o.** 5:6
p. 10:3 **q.** 7:8 **r.** 3:4 **s.** 7:12 **t.** 2:3

8. **a.** 1:3 2:6 3:9 4:12 5:15 **b.** 2:1 4:2 8:4 16:8 20:10 **c.** 2:3 4:6 6:9 8:12 16:24
d. 64:32 32:16 16:8 8:4 2:1 **e.** 48:64 24:32 12:16 6:8 3:4

9. **a.** 2:3 **b.** 1:3 **c.** 5:3 **d.** 2:5 **e.** 1:4
f. 8:3 **g.** 3:40 **h.** 1:5 **i.** 9:4 **j.** 8:13
k. 5:4 **l.** 5:4 **m.** 1:5 **n.** 11:2 **o.** 2:5
p. 16:9 **q.** 3:10 **r.** 3:2 **s.** 7:3 **t.** 1:6

10. **a.** 4:5 **b.** 2:3 **c.** 3:2 **d.** 4:1 **e.** 4:3
f. 5:4 **g.** 1:8 **h.** 10:3 **i.** 8:1 **j.** 9:2

11. **a.** 1:2 **b.** 5:6 **c.** 1:2 **d.** 4:5
e. 16:9 **f.** 10:9 **g.** 3:10 **h.** 5:1
i. 5:6 **j.** 4:3 **k.** 5:8 **l.** 65:56

12. **a.** \$400, \$600 **b.** \$750, \$250 **c.** \$200, \$800 **d.** \$500, \$500 **e.** \$375, \$625
f. \$625, \$375 **g.** \$300, \$700 **h.** \$900, \$100 **i.** \$350, \$650 **j.** \$450, \$550

13. **a.** \$10 000, \$40 000 **b.** \$15 000, \$35 000 **c.** \$20 000, \$30 000
d. \$25 000, \$25 000 **e.** \$12 500, \$37 500

14. **a.** 2m^3 **b.** 0.5m^3 **c.** 1.6m^3 **d.** 1.6m^3 **e.** 1.2m^3

15. **a.** \$90 000, \$210 000, \$300 000 **b.** \$180 000, \$180 000, \$240 000 **c.** \$30 000, \$240 000, \$330 000
d. \$150 000, \$180 000, \$270 000 **e.** \$150 000, \$225 000, \$225 000

16. 30°, 60°, 90°

17. **a.** 90 **b.** 75

18. **a.** 120 **b.** 45 **c.** \$6 **d.** \$19.50

19. \$12.50, \$10, \$7.50

20. 96°

21. 2.6 m, 6th bounce

22. The older brother carries a 2 kg backpack, 4 kg of books, 6 kg of clothes, 2 kg of snacks, 2 kg of water; a total of 16 kg. The younger brother carries a 2 kg backpack, 3 kg of toiletries, 3 kg of shoes; a total of 8 kg.

23. $\frac{18}{25}$

Exercise 4.9 Working with mixed numbers

1. **a.** $4\frac{7}{10}$ **b.** $1\frac{1}{6}$ **c.** $3\frac{5}{12}$ **d.** $13\frac{3}{8}$ **e.** $1\frac{1}{10}$ **f.** $6\frac{1}{4}$

2. **a.** $1\frac{1}{3}$ **b.** $1\frac{1}{2}$ **c.** $1\frac{5}{8}$ **d.** $1\frac{7}{10}$ **e.** $3\frac{2}{3}$ **f.** $1\frac{11}{12}$

3. **a.** $2\frac{17}{35}$ **b.** $3\frac{7}{8}$ **c.** $3\frac{13}{18}$ **d.** $3\frac{1}{24}$

4. **a.** $7\frac{7}{12}$; >7 **b.** $3\frac{2}{3}$; >3 **c.** $4\frac{1}{2}$; >4 **d.** $4\frac{1}{4}$; >4 **e.** $3\frac{3}{10}$; >3 **f.** $4\frac{1}{2}$; >4

5. **a.** $\frac{5}{6}$ **b.** $1\frac{7}{32}$ **c.** $2\frac{1}{4}$ **d.** $4\frac{19}{32}$ **e.** $7\frac{7}{10}$

f. $7\frac{4}{15}$ **g.** $5\frac{5}{8}$ **h.** $1\frac{17}{35}$ **i.** $3\frac{3}{10}$ **j.** $2\frac{3}{5}$

k. 18 **l.** $16\frac{1}{2}$ **m.** $17\frac{1}{2}$ **n.** 26 **o.** $9\frac{33}{50}$

p. $37\frac{5}{21}$ **q.** $34\frac{2}{3}$ **r.** 10 **s.** $2\frac{11}{12}$ **t.** $26\frac{18}{35}$

6. **a.** $1\frac{2}{3}$ **b.** 2 **c.** $\frac{9}{14}$ **d.** 1 **e.** $1\frac{9}{11}$

f. $22\frac{1}{3}$ **g.** $\frac{1}{20}$ **h.** $\frac{18}{35}$ **i.** $2\frac{10}{77}$ **j.** 3

k. $2\frac{29}{40}$ **l.** $1\frac{1}{6}$ **m.** $1\frac{8}{15}$ **n.** 4 **o.** $2\frac{1}{14}$

7. $1\frac{5}{6}$ L

8. $1\frac{5}{16}$ inches

9. $1\frac{1}{12}$ months

10. $6\frac{1}{4}$ days

11. $\frac{3}{8}$ cup sugar

$\frac{3}{8}$ cup margarine

$\frac{3}{4}$ cup peanut butter

3 eggs

$2\frac{1}{4}$ cups milk

$3\frac{3}{4}$ cups self-raising flour

$\frac{3}{8}$ teaspoon baking soda

12. $\frac{3}{16}$ kg

13. **a.** 22 **b.** 96

14. $3\frac{1}{10}$ kg

15. 7 containers

16. $52.50

17. **a.** $3\frac{15}{16}$ cm **b.** $2\frac{15}{16}$ cm

18. **a.** $6\frac{2}{3}$ laps **b.** 12 minutes

19. $\frac{5}{12}$, 10

20. 500 women

21. 2961 m

22. The gymnasium should keep the drink machine.

4.10 Review

1. $\frac{3}{20}$

2. **a.** $\frac{6}{16}, \frac{9}{24}, \frac{12}{32}$ **b.** $\frac{8}{18}, \frac{12}{27}, \frac{16}{36}$ **c.** $\frac{10}{22}, \frac{15}{33}, \frac{20}{44}$ **d.** $\frac{12}{14}, \frac{18}{21}, \frac{24}{28}$

3. **a.** $\frac{2}{4} = \frac{3}{6}$ **b.** $\frac{3}{9} = \frac{5}{15}$ **c.** $\frac{9}{12} = \frac{21}{28}$ **d.** $\frac{4}{10} = \frac{16}{40}$ **e.** $\frac{5}{20} = \frac{25}{100}$ **f.** $\frac{30}{100} = \frac{300}{1000}$

4. **a.** Leo **b.** Ricky

5. **a.** $\frac{6}{7}$ **b.** $\frac{3}{4}$ **c.** $\frac{9}{10}$ **d.** $\frac{3}{4}$

e. $\frac{1}{2}$ **f.** $\frac{1}{3}$ **g.** $\frac{3}{5}$ **h.** $\frac{1}{40}$

6. **a.** $6\frac{4}{5}$ **b.** $3\frac{2}{3}$ **c.** $4\frac{3}{4}$ **d.** $1\frac{1}{5}$

7. a. $\frac{3}{12} = \frac{1}{4}$

8. a. $7.1 = \frac{1}{8}$ **b.** $7.2 = \frac{1}{4}$ **c.** $7.3 = \frac{5}{32}$ **d.** $7.4 = \frac{5}{32}$ **e.** $7.5 = \frac{7}{40}$ **f.** $7.6 = \frac{11}{80}$

9. a. $5\frac{1}{3}$ **b.** $4\frac{1}{5}$ **c.** $1\frac{5}{7}$ **d.** $5\frac{1}{2}$
e. $1\frac{1}{4}$ **f.** $11\frac{3}{7}$ **g.** $6\frac{1}{9}$ **h.** $7\frac{1}{5}$

10. a. $\frac{11}{4}$ **b.** $\frac{79}{8}$ **c.** $\frac{26}{7}$ **d.** $\frac{35}{6}$
e. $\frac{11}{3}$ **f.** $\frac{99}{10}$ **g.** $\frac{47}{12}$ **h.** $\frac{17}{2}$

11. a. 4 trays

12. a. $\frac{23}{24}$ **b.** $\frac{17}{45}$ **c.** $1\frac{1}{15}$

13. a. $\frac{20}{63}$ **b.** $\frac{6}{25}$ **c.** $\frac{5}{8}$ **d.** $\frac{7}{36}$
e. 44 **f.** 22 **g.** $\frac{1}{6}$ **h.** $\frac{50}{3}$ or $16\frac{2}{3}$

14. a. $\frac{2}{5}$ **b.** $\frac{7}{2}$ **c.** 99 **d.** 9 **e.** $\frac{1}{12}$
f. $\frac{1}{124}$ **g.** 4 **h.** $\frac{4}{23}$ **i.** $\frac{2}{17}$ **j.** $\frac{5}{37}$
k. $\frac{9}{22}$ **l.** $\frac{8}{29}$ **m.** $\frac{5}{21}$

15. a. $\frac{3}{2}$ or $1\frac{1}{2}$ **b.** $\frac{4}{3}$ or $1\frac{1}{3}$ **c.** $\frac{7}{36}$ **d.** $\frac{3}{70}$

16. a. $4\frac{17}{24}$ **b.** $1\frac{13}{14}$ **c.** $3\frac{7}{9}$ **d.** $2\frac{2}{3}$
e. $6\frac{13}{40}$ **f.** $3\frac{43}{72}$ **g.** $1\frac{47}{70}$ **h.** $\frac{76}{99}$

17. a. $3\frac{2}{3}$ **b.** $6\frac{5}{18}$

18. a. 14 **b.** 64

19. a. $\frac{3}{5}$ **b.** $13\frac{2}{3}$ **c.** $1\frac{1}{2}$

20. a. $\frac{5}{16}$ **b.** $1\frac{3}{5}$ **c.** $1\frac{53}{115}$ **d.** $\frac{217}{267}$ **e.** $1\frac{73}{147}$

21. \$10, \$15

22. \$880

23. \$150

24. 2 kg

25. $\frac{1}{4}$°C

26. Yes

27. a. $\frac{1}{6}$ **b.** $\frac{1}{4}$ **c.** $\frac{5}{9}$ **d.** $\frac{48}{25} = 1\frac{23}{25}$

28. Alice \$80, Will \$64, James \$72, Jessica and Alex \$52

29. $\frac{12}{21}$

30. Tomas

31. $\frac{1}{2} + \frac{1}{4} + \frac{1}{20}$

32. 18 litres

33. Approx. 25 years

34. 2961 m

35. Mr Thompson \$3000, Mr Goody \$2500

36. a. 18400 **b.** 3:4 **c.** $\frac{3}{4}$

37. $\frac{3}{5}$ of the standard spots were filled.

38. a. 4 complete surfboards **b.** $2\frac{1}{4}$ kg of resin, 0 catalyst

Investigation — Rich task

1. A and D, B and C, E and F

2. 2 to 6 Teacher to check.

TOPIC 5 Geometry

5.1 Overview

Numerous **videos** and **interactivities** are embedded just where you need them, at the point of learning, in your learnON title at www.jacplus.com.au. They will help you to learn the content and concepts covered in this topic.

5.1.1 Why learn this?

Geometry allows us to explore our world in a very precise way. Builders, architects, surveyors and engineers use knowledge of geometry to ensure buildings are stable and visually pleasing. A lot of geometry is also involved in sport. Careful estimation of angles is required when playing pool or shooting for goal in hockey and football.

5.1.2 What do you know?

assessON

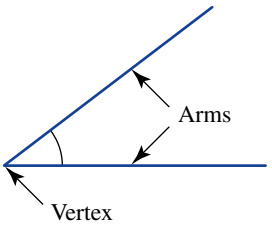
1. **THINK** List what you know about angles, triangles and quadrilaterals. Use a thinking tool such as a concept map to show your list.
2. **PAIR** Share what you know with a partner and then with a small group.
3. **SHARE** As a class, create a thinking tool such as a large concept map that shows your class's knowledge of angles, triangles and quadrilaterals.

LEARNING SEQUENCE

5.1 Overview
5.2 Measuring angles
5.3 Constructing angles with a protractor
5.4 Types of angles and naming angles
5.5 Triangles
5.6 Quadrilaterals and their properties
5.7 Parallel and perpendicular lines
5.8 Review

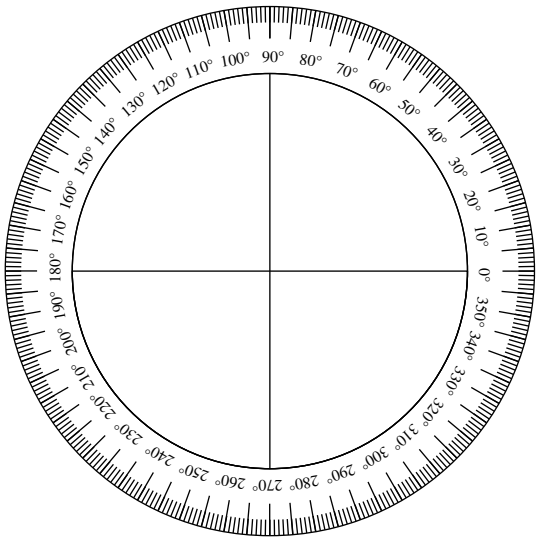
5.2 Measuring angles

- An **angle** is made when two lines meet at a point.
- The point where the two lines meet is called a **vertex** and the lines are called **arms** of the angle.
- Angles are measured in degrees.
- One full revolution is equal to 360 degrees.
- The symbol for degrees is °.
- The device for measuring angles is called a **protractor**. There are two types of protractors: circular and semicircular.



5.2.1 The circular protractor

- Circular protractors are marked from 0° to 360°.

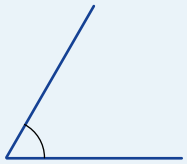


- To measure angles using a circular protractor, follow these steps.
  Step 1: Place the centre of the protractor on the vertex of the angle.
  Step 2: Match the 0° horizontal centre line with one of the arms of the angle.
  Step 3: Read the size of the angle indicated by the other arm.

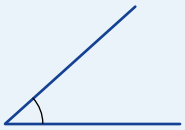
WORKED EXAMPLE 1

First estimate and then measure the size of each of the following angles using a circular protractor. Comment on the accuracy of the estimate.

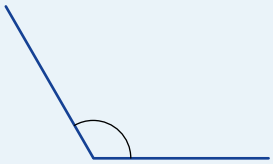
a



b



c



THINK

a 1 Estimate the size of the angle. Write the estimate.
Note: The angle is between 0° and 90° but closer to 90°.

2 Place the centre of the protractor on the vertex of the angle.

3 Match the horizontal line that passes through the centre of the protractor and points to 0° with one of the arms of the angle.

4 Read the size of the angle indicated by the other arm. (You may need to extend the line so that you can read it on the protractor.)

5 Comment on the accuracy of the estimate.

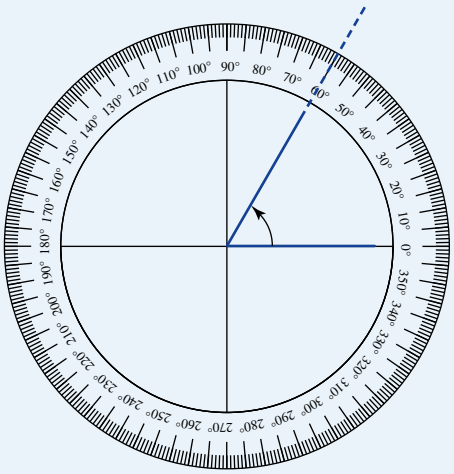
b 1 Estimate the size of the angle. Write the estimate.
Note: The angle appears to be approximately halfway between 0° and 90°.

2 Repeat steps **2** to **5** from part **a**.

c 1 Estimate the size of the angle. Write the estimate.
Note: The angle is between 90° and 180° but closer to 90°.

WRITE

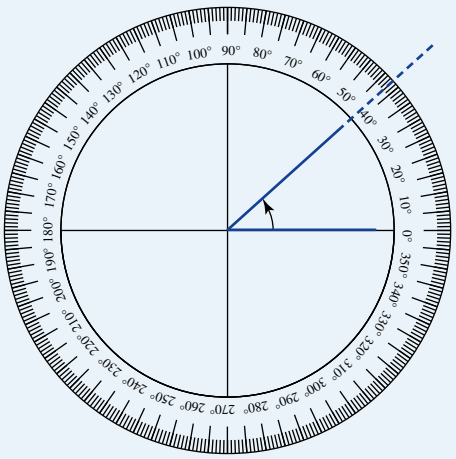
a The estimate is approximately 60°.



The size of this angle is 60°.

The estimate was an accurate one.

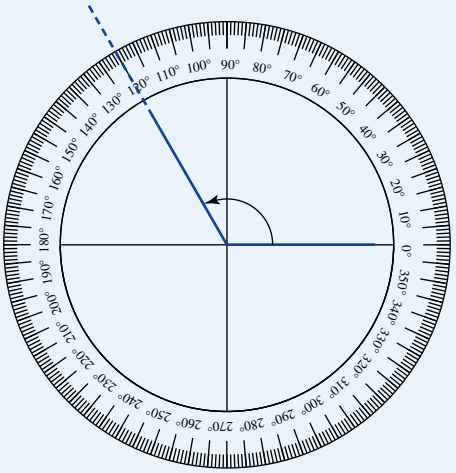
b The estimate is approximately 45°.



The size of this angle is 42°. The estimate was a reasonable one.

c The estimate is approximately 115°.

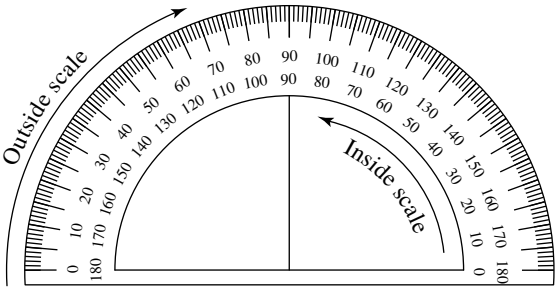
2 Repeat steps **2** to **5** from part **a**.



The size of this angle is 120°.
The estimate was a reasonable one.

5.2.2 The semicircular protractor

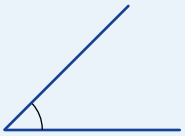
- Semicircular protractors have two scales: inner and outer.
- Each scale is marked from 0° to 180°.
- Either of the two scales can be used for measuring angles.

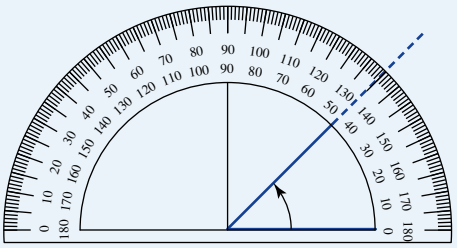


- To measure angles that are less than 180°, follow these steps.
  Step 1: Line up the centre of the protractor's baseline with the vertex (that is, where the vertical 90° line intersects with the horizontal 180° baseline).
  Step 2: Match the base of the protractor with one of the arms of the angle.
  Step 3: Use the scale on the base starting at 0 to read the number of degrees spanned by the angle.
- To measure angles which are greater than 180°, follow these steps.
  Step 1: Measure the angle enclosed by the straight lines (that is, the one that is smaller than 180°).
  Step 2: Subtract the number obtained in step (1) from 360°.

WORKED EXAMPLE 2

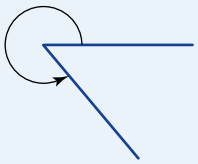
First estimate and then measure the size of the angle shown at right using a semicircular protractor. Comment on the accuracy of the estimate.

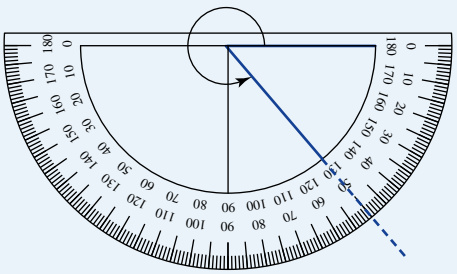


THINK	WRITE
1 Estimate the size of the angle. Write the estimate. *Note:* The angle appears to be approximately halfway between 0° and 90°.	The estimate is approximately 45°.
2 Line up the centre of the protractor's baseline with the vertex (that is, where the vertical 90° line intersects with the horizontal 180° baseline).	
3 Match the base of the protractor with one of the arms of the angle. (You may need to extend the line so that you can read it on the protractor.)	
4 Use the scale on the base starting at 0° (in this case an inner scale) to read the number of degrees spanned by the angle.	The size of this angle is 45°.
5 Comment on the accuracy of the estimate.	The estimate was an accurate one.

WORKED EXAMPLE 3

First estimate and then measure the size of the angle shown at right using a semicircular protractor. Comment on the accuracy of the estimate.



THINK	WRITE
1 Estimate the size of the angle. Write the estimate. *Note:* The angle appears to be approximately halfway between 270° and 360°.	The estimate is approximately 315°.
	
2 Measure the angle enclosed by the straight lines.	The smaller angle is 50°.
3 Subtract 50° from 360° to find the required angle.	Required angle: $360° - 50° = 310°$
4 Comment on the accuracy of the estimate.	The estimate was a reasonable one.

learnON RESOURCES — ONLINE ONLY

Complete this digital doc: SkillSHEET: Reading the scale on a protractor (doc-6447)

Complete this digital doc: SkillSHEET: Comparing the size of angles (doc-6448)

Complete this digital doc: SkillSHEET: Estimating the size of an angle (doc-6449)

Complete this digital doc: SkillSHEET: Measuring angles with a protractor (doc-6450)

Try out this interactivity: Digital protractor (int-2348)

Exercise 5.2 Measuring angles

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1, 2, 3, 4, 5, 6, 7	Questions: 1, 2, 3, 4, 5, 6, 7, 8	Questions: 1, 2, 3, 4, 5, 6, 7, 8

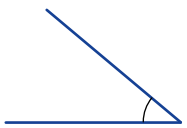
Individual pathway interactivity: int-4331

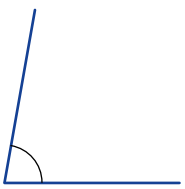
learnON ONLINE ONLY

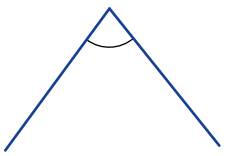
To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

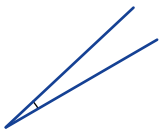
Fluency

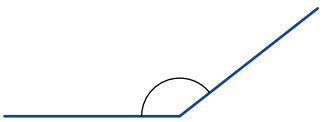
1. WE 1, 2, 3 First estimate and then measure the size of each of the following angles using a protractor. Comment on the accuracy of the estimate.

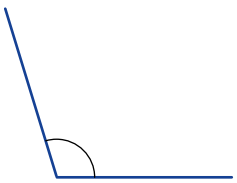
a.


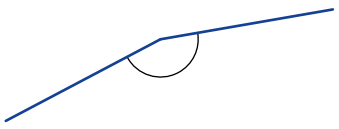
b.


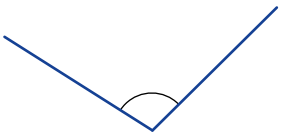
c.


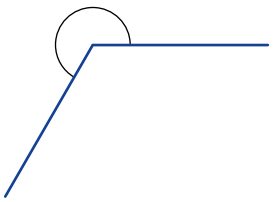
d.


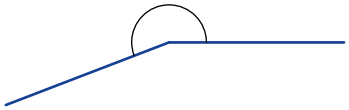
e.


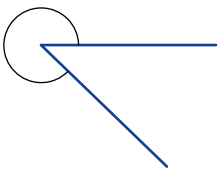
f.


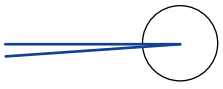
g.


h.


i.


j.


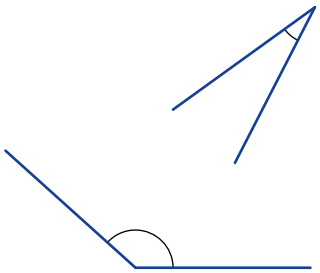
k.


l.


2. Comment on the accuracy of your measurement compared with the answer in the text.

3. a. MC The size of the angle shown is closest to:

A. 20° **B.** 30° **C.** 40°
D. 150° **E.** 140°



b. The size of the angle shown is closest to:

A. 42° **B.** 138° **C.** 142°
D. 145° **E.** 38°

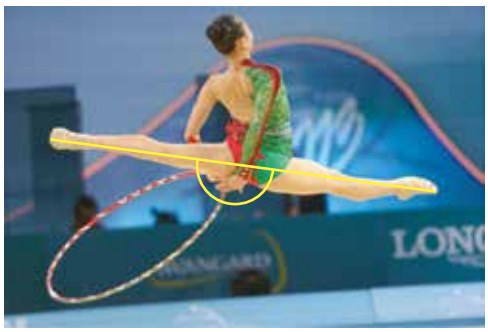
Understanding

4. In each of the photographs, measure the indicated angle.

a.

b.

c.



d.

5. The recommended slope for wheelchair access is 4° from the horizontal. An angle greater than this makes it more difficult for the person in the wheelchair to travel up the ramp.

a. Would you rate the slope of the bottom section of the ramp as 'difficult', 'as recommended' or 'easy'? Why?

b. How would you rate the second part of the ramp?

Reasoning

6. Study the cartoon of the golfer to help you answer the following questions.

1.

2.

3.

4.

5.

6.

7.

8.

a. Copy the following table.

Diagram	Estimate of angle	Measured angle	Difference
1			
2			
3			
4			

b. Without using a protractor, estimate the size of each angle in the first four diagrams of the golf sequence above and complete the second column of the table.

c. Explain how you obtained an estimate. What steps did you follow?

d. Complete the third column by measuring each angle with a protractor.

e. Calculate the difference between your estimate and the actual value of each angle.

f. Describe how you could improve your skills in estimating the size of an angle.

g. Try these improvements for estimating the size of the angles in the remaining four diagrams. Copy and complete the following table.

Diagram	Estimate of angle	Measured angle	Difference
5			
6			
7			
8			

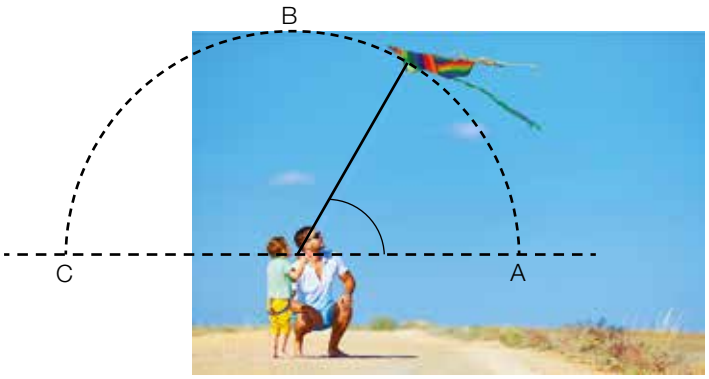
h. Has the difference between the estimated and actual value for each angle become smaller? Have your estimating skills improved?

Problem solving

7. a. Using a protractor, measure the angles between the two clock hands shown in the diagrams.

b. Find a way to calculate the angle between the two arms without using a protractor.

8. Sean is flying a kite. Depending on the position of the kite, the angle between the string and the horizontal line changes.



Fill in the following statements:

a. As the kite moves from point A to point B, the angle increases from _____ to _____.

b. As the kite moves from point B to point C, the angle _____ from _____ to _____.

Reflection

Where in everyday life may I need to use my skills of measuring angles?

5.3 Constructing angles with a protractor

5.3.1 Constructing angles using a circular protractor

- Protractors can be used not only for measuring, but also for constructing angles.
- To construct an angle using a circular protractor, follow the following steps.

  Step 1: Draw a straight line.

  Step 2: Put a dot at one end of the line. (The line now represents one of the arms and the dot represents the vertex of the angle.)

  Step 3: Place the protractor so that its centre is on the vertex and 0° is on the baseline.

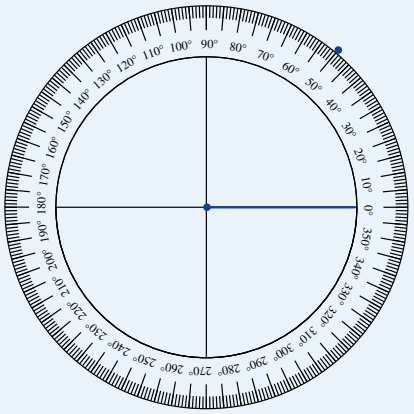
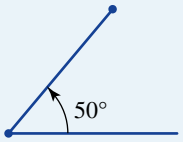
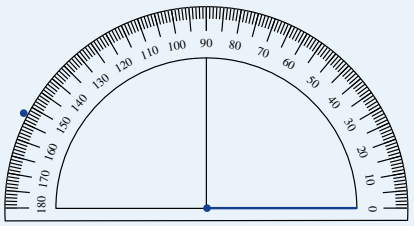
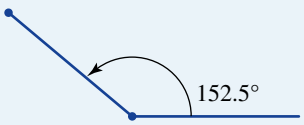
  Step 4: Find the required angle on the scale and mark a small dot at the edge of the protractor.

  Step 5: Join the small dot with the vertex to form the second arm of the angle.

  Step 6: Label the angle.
- If the angle to be constructed is between 0° and 180°, a semicircular protractor can be used in exactly the same way as the circular one.

WORKED EXAMPLE 4

Construct each of the following angles.
a 50° **b** 152.5°

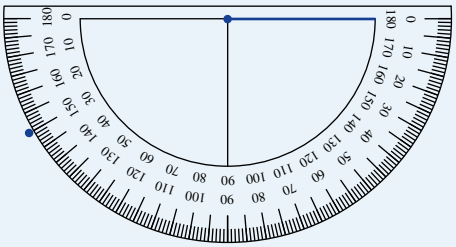
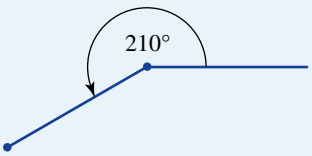
THINK	WRITE/DRAW
a 1 Draw a baseline. Put a dot at one end. This is to be the vertex of the angle.	**a**
2 Place the protractor so that its centre is on the vertex and 0° is on the baseline.	
3 Find 50° on the scale and mark a small dot at the edge of the protractor.	
4 Join the small dot and the vertex with a straight line.	
5 Label the angle.	
b 1 Repeat steps **1** and **2** from part **a**.	**b**
2 Find 152.5° on the scale and mark a small dot at the edge of the protractor. Remember to start at 0° and, using the inside scale, move in an anti-clockwise direction.	
3 Join the vertex and the dot where you have marked the 152.5° with a straight line.	
4 Label the angle.	

5.3.2 Constructing angles greater than 180° using a semicircular protractor

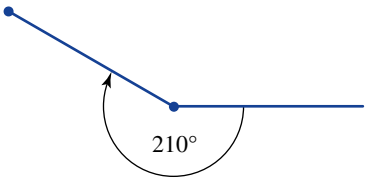
- To construct an angle greater than 180° using a semicircular protractor, follow the following steps.
  Step 1: Subtract the given angle from 360°.
  Step 2: Construct the resulting angle (which will be less than 180°).
  Step 3: Label the required angle (it is 'outside' the one that has been constructed).

WORKED EXAMPLE 5

Construct an angle of 210° using a semicircular protractor.

THINK	WRITE/DRAW
1 Subtract 210° from 360°.	$360° - 210° = 150°$
2 Draw a baseline. Put a dot at one end. This will become the vertex of the angle.	
3 Place the protractor upside down on the baseline so that its centre is on the vertex.	
4 Start at 0° and, using the outside scale, move in a clockwise direction; mark the position of 150° with a small dot.	
5 Remove the protractor and join the vertex to the small dot representing 150°.	
6 Mark the outside angle as 210°.	

- It is not necessary to place the protractor upside down. The diagram at right shows the angle of 210° (the same as in Worked example 5), constructed by placing the protractor in the upright position.



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Exercise 5.3 Constructing angles with a protractor

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1, 2, 3, 4, 5, 6, 8	Questions: 1, 2, 3, 4, 5, 6, 7, 8	Questions: 1, 2, 3, 4, 5, 6, 7, 8, 9

Individual pathway interactivity: int-4332

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Fluency

1. WE4a Construct each of the following angles.

a. 15° **b.** 9° **c.** 53° **d.** 75.5° **e.** 45° **f.** 40.8° **g.** 88° **h.** 76°

2. WE4b Construct each of the following angles.

a. 96° **b.** 92° **c.** 165.2° **d.** 143° **e.** 140° **f.** 156.4° **g.** 127° **h.** 149°

3. WE5 Construct each of the following angles using a semicircular protractor.
a. 185° **b.** 210.5° **c.** 235° **d.** 260° **e.** 243° **f.** 192° **g.** 249.8° **h.** 214°

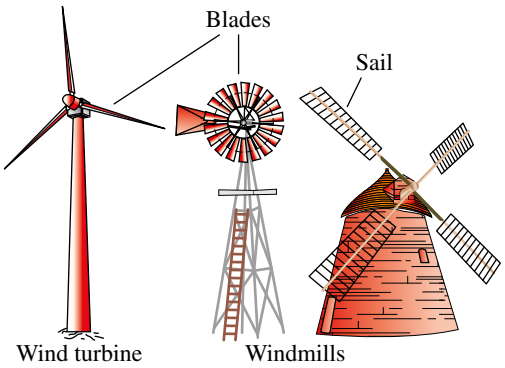
4. MC To construct an angle of 212° using a semicircular protractor, we need to construct an angle less than 180° first and then label the outside angle. The size of the first angle to be constructed is:
A. 212° **B.** 58° **C.** 148° **D.** 112° **E.** none of these.

5. Construct each of the following angles.
a. 295° **b.** 269° **c.** 307° **d.** 349° **e.** 328° **f.** 300° **g.** 345° **h.** 358°

Understanding

6. a. Construct a 20° angle.
b. On one side of this angle draw a 45° angle.
c. On the second arm of the angle in part **b** draw a 108° angle.
d. What is the remaining reflex angle?
e. What is the total sum of the angles?

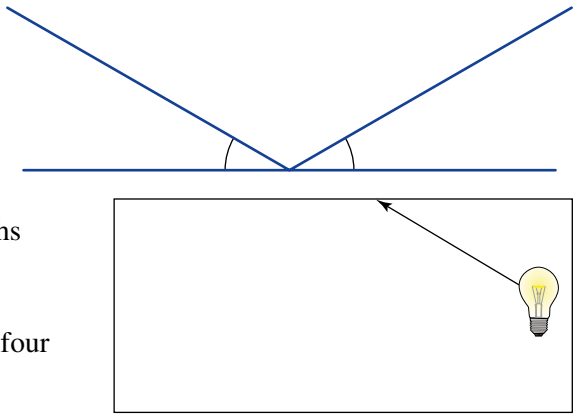
7. Windmills use wind power to produce electricity, pump water and mill grain. The three types of machine shown in the diagram have blades or sails constructed at equal angles. Using a protractor and a ruler first measure the angles between blades and then draw the three types of blades.



Reasoning

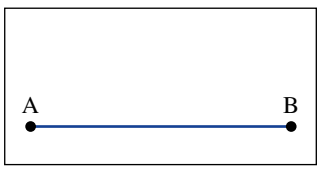
8. Explain the method you would use to construct a reflex angle if you only had a 180° protractor.

9. We see the objects surrounding us because light reflects of the surface of the objects as shown in the diagram.
The two angles shown are always equal. Imagine you are in a room lit by a light globe as shown in the following diagram.
a. Using a protractor and a ruler draw the next four paths followed by the ray of light shown in the diagram.
b. Draw another ray of light in a different direction.
c. Use the same procedure as in part **a** to draw the first four paths of the line drawn in part **b**.

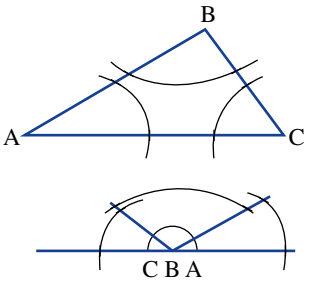


Problem solving

10. On an A4 sheet of paper draw a 20 cm long straight horizontal line close to the bottom of the page. Label this line AB.
a. Using a protractor construct an angle of 30° at point A and an angle of 30° at point B. Extend the two lines until they intersect.
b. Cut out the triangle and compare it with the triangles constructed in this way by other students.
c. Write a statement about your findings.



11. Draw any triangle on an A4 sheet of paper.
a. Use a protractor to measure the three angles.
b. Cut out the triangle and then cut the angles as shown in the diagram.
c. Place the angles next to each other so that their arms touch (see diagram).
d. Measure the new angle formed.
e. Compare your results with other students and write a statement to summarise your findings.



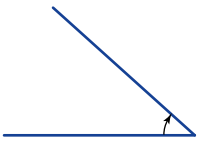
Reflection

In what situations would a semicircular protractor be easier to use than a circular one?

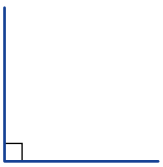
5.4 Types of angles and naming angles

5.4.1 Types of angles

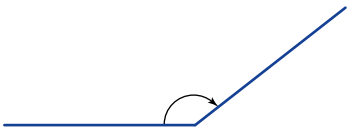
- Angles can be classified according to their size.



An **acute angle** is greater than 0°, but less than 90°.

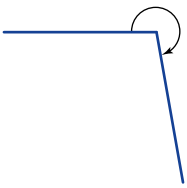


A **right angle** is an angle that equals exactly 90°.



An **obtuse angle** is greater than 90° but less than 180°.

A **straight angle** equals exactly 180°.



A **reflex angle** is greater than 180° but less than 360°.

A **revolution** or a **perigon** is an angle of 360° (a full circle).

- Two angles that add to give 90° are called **complementary angles**. Each angle is the complement of the other.
- Two angles that add to give 180° are called **supplementary angles**. Each angle is the supplement of the other.

WORKED EXAMPLE 6

Classify each of the following angles according to its size.
a 115° **b** 27° **c** 300°

THINK	WRITE
a The given angle is larger than 90°, but smaller than 180°, so classify it accordingly.	**a** 115° is an obtuse angle.
b The given angle is between 0° and 90°, so classify it accordingly.	**b** 27° is an acute angle.
c The given angle is larger than 180°, but less than 360°, so classify it accordingly.	**c** 300° is a reflex angle.

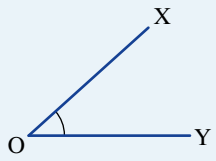
5.4.2 Naming angles

- Angles can be named using capital letters of the English alphabet.
- A common way to name an angle is to use three letters: two letters to represent the arms of the angle, and a third letter to represent its vertex. The letter representing the vertex is always placed in the middle (between the two letters representing the arms).
- Instead of writing the word *angle*, we use the symbol ∠.

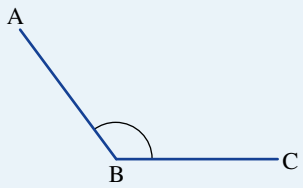
WORKED EXAMPLE 7

Name each of the following angles.

a



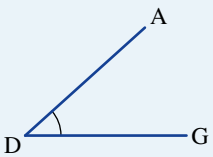
b



THINK	WRITE
a Name the angle by starting either from X, or from Y, and placing O (the letter for the vertex) in the middle. Remember to place the angle symbol (∠) before the letters.	**a** ∠XOY or ∠YOX
b Name the angle by starting either from A, or from C, and placing B (the letter representing the vertex) in the middle. Remember to place the angle symbol (∠) before the letters.	**b** ∠ABC or ∠CBA

WORKED EXAMPLE 8

Draw the acute angle ∠ADG.

THINK	DRAW
1 Construct any acute angle (unless specified otherwise).	
2 Letter D is in the middle, so place it at the vertex.	
3 Place letter A at one arm of the angle and letter G at the other. (Note that it does not matter which arm is represented by AD and GD.)	

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Complete this digital doc: SkillSHEET: Naming angles (doc-6453)

Complete this digital doc: WorkSHEET: Geometry I (doc-1755)

Exercise 5.4 Types of angles and naming angles

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–11, 13, 15	Questions: 1–13, 15	Questions: 1–l5

Individual pathway interactivity: int-4333

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Fluency

1. WE6 Classify each of the following angles according to its size.

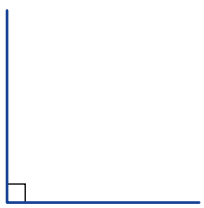
a. 12°	**b.** 215.3°	**c.** 98°	**d.** 156°
e. 180°	**f.** 62°	**g.** 355.2°	**h.** 90°
i. 4.8°	**j.** 360°	**k.** 100°	**l.** 45°
m. 82°	**n.** 270°	**o.** $36\frac{1}{2}^\circ$	**p.** 195°

2. The following list gives values of particular angles.
3°, 45°, 65°, 123°, 69°, 234°, 90°, 360°, 300°, 270°, 165°, 210°, 180°

a. How many of these are acute angles? List them.
b. How many of these are obtuse angles? List them.
c. Is there a right angle in the list?
d. Is there a straight angle?
e. How many reflex angles are there? List them.
f. Is there a full revolution? What is its value?
g. What is the complement of 65°?
h. What is the supplement of 123°?

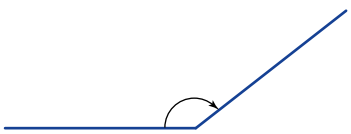
3. Consider the following diagrams and write down the type of angle shown in each case.

a.



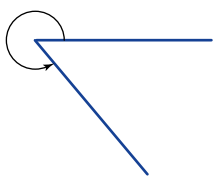
b.

c.



d.

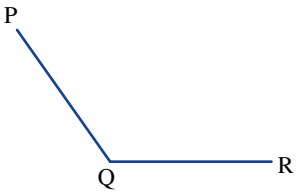
e.

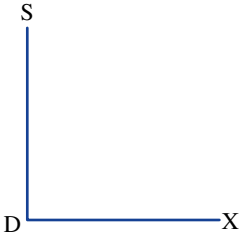


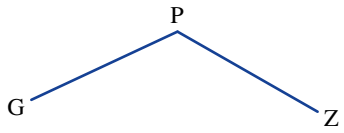
f.

4. WE7 Name each of the following angles.

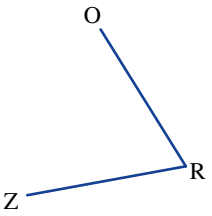
a.

b.


c.


d.


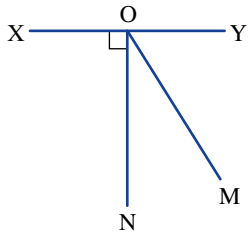
e.

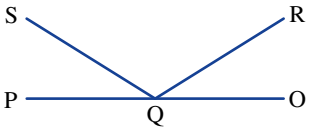
f.


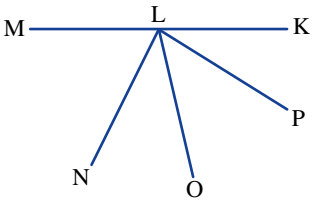
5. WE8 Draw each of the following angles as specified in brackets.

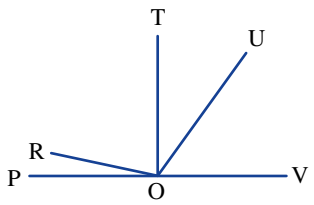
a. ∠CDE (acute)
b. ∠TRE (obtuse)
c. ∠QAS (straight)
d. ∠FGH (reflex)
e. ∠KJF (right)
f. ∠NBV (acute)

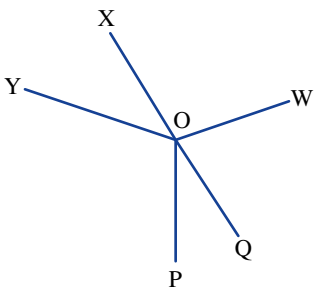
6. Name each acute and obtuse angle in the following diagrams.

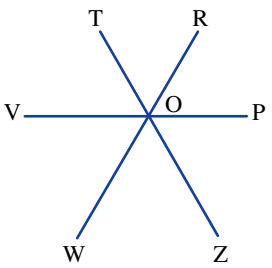
a.


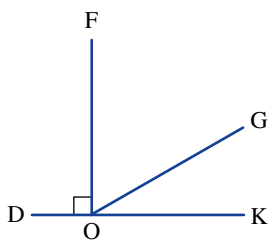
b.


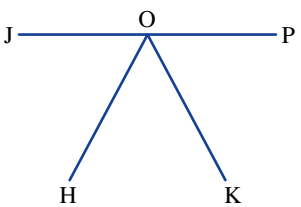
c.


d.


e.


f.


g.


h.


Understanding

7. State the type of angle shown in each photograph.

a.

b.

c.

8. State the type of angle that the minute hand of a clock sweeps while moving for the period of:

a. 10 minutes **b.** 15 minutes **c.** 20 minutes
d. 30 minutes **e.** 45 minutes **f.** 1 hour.

9. At various times of the day the hands of a clock form two angles: the inside angle (the smaller one) and the outside angle (the larger one).

a. State the type of the inside angle formed by the hands of the clock at:

i. 1 o'clock **ii.** 20 minutes to 12 **iii.** 6 o'clock
iv. 9 o'clock **v.** quarter to 4 **vi.** half past 10.

b. Write two different times when the inside angle of the clock is:

i. acute **ii.** obtuse **iii.** right.

10. Which type of angle does the minute hand of a clock sweep while moving for the period of:

a. 10 minutes **b.** 15 minutes **c.** 20 minutes

11. In the photo below, acute, right-angled and obtuse angles have been used.

i. Name an acute angle.

ii. Name two right angles.

iii. Name an obtuse angle.

iv. Can you name a reflex angle?

Reasoning

12. Give reasons for the following statements.

a. The sum of two acute angles can be an acute angle.

b. The sum of two acute angles can be a right angle.

c. The sum of two acute angles can be an obtuse angle.

d. The sum of two acute angles can be a reflex angle.

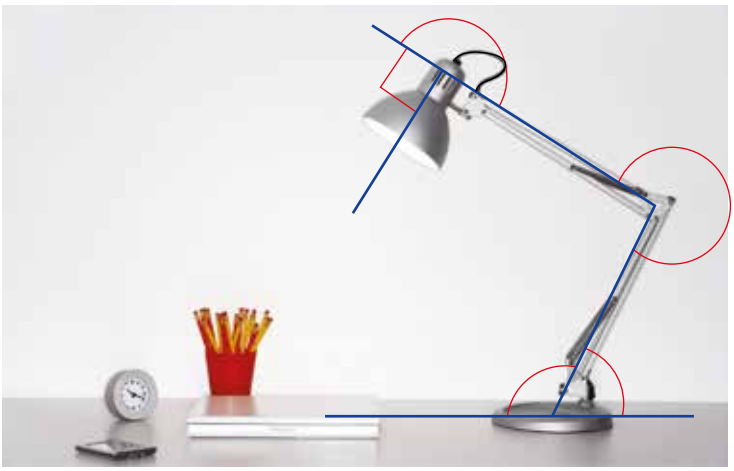
13. a. Can you construct a full revolution using one 90° angle, one straight angle, and two acute angles?

b. Construct a diagram to represent the full revolution in part **a**.

Problem solving

14. Watchmakers take particular care that the angles between the hands of an analogue clock represent the time accurately. What is the smaller angle between the hour and minute hands when an analogue clock displays the time as ten past nine?

15. Label each angle marked in the following diagram with the word describing the type of angle it is.

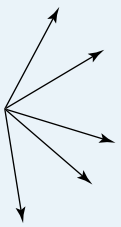


Reflection

Why is it important to always put the letter representing the vertex in the middle of the angle's name?

CHALLENGE 5.1

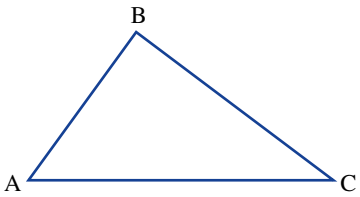
1. How many angles appear in this diagram?
2. At midday, the hour hand, the minute hand and the second hand on a clock are all pointing to the 12. What is the angle between the hour hand and the minute hand after the second hand has completed 30 revolutions? *Hint*: It is not 180° — draw a separate diagram to help.



5.5 Triangles

5.5.1 Types of triangles

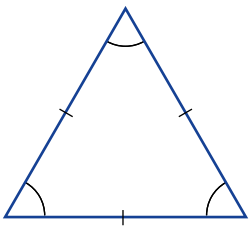
- Every **triangle** has three angles and three sides.
- Capital letters of the English alphabet are used at each vertex (in any order) to name triangles.
- The symbol Δ is used in place of the word *triangle*. This triangle can be named ΔABC. (It can also be named ΔBCA, ΔCAB, ΔACB, ΔBAC or ΔCBA.)



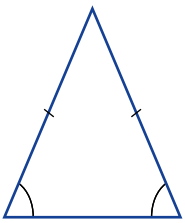
- Identical marks on the sides of the triangle are used to indicate that the sides have the same length. Likewise, identical marks on the angles indicate that the angles are equal in size.

5.5.2 Classifying triangles according to the length of their sides

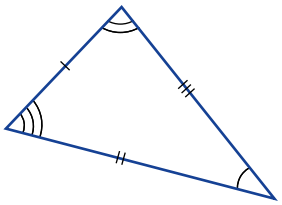
- According to their side lengths, triangles can be classified as **equilateral**, **isosceles** or **scalene**.
- An *equilateral* triangle has all sides equal in length. (All angles of an equilateral triangle are also equal in size.)



- An isosceles triangle has two sides of equal length. (The angles adjacent to the unequal third side are also equal in size.)



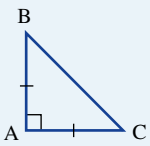
- A scalene triangle has no equal sides. (All angles in a scalene triangle are different in size.)



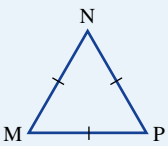
WORKED EXAMPLE 9

Classify each of these triangles according to the lengths of its sides.

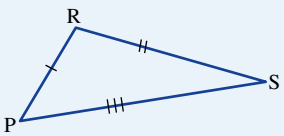
a



b



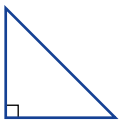
c



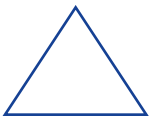
THINK	WRITE
a Sides AB and AC have identical markings on them, which indicate that they are of equal length. So ΔABC has 2 equal sides. Classify it accordingly.	**a** ΔABC is an isosceles triangle.
b The 3 sides of ΔMNP have identical markings on them, which means that all 3 sides are equal in length. Classify this triangle.	**b** ΔMNP is an equilateral triangle.
c All 3 sides of ΔPRS are marked differently. Therefore, no sides in this triangle are equal in length. Use this information to classify the triangle.	**c** ΔPRS is a scalene triangle.

5.5.3 Classifying triangles according to the size of their angles

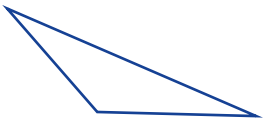
- According to the size of their angles, triangles can be classified as being *right-angled*, *acute-angled*, or *obtuse-angled*.



A right-angled triangle has one of its angles equal to 90° (one of its angles is a right angle). A small square in the corner marks the right angle.



An acute-angled triangle has all angles smaller than 90° (all three angles are acute).



An obtuse-angled triangle has one angle greater than 90° (one angle is obtuse).

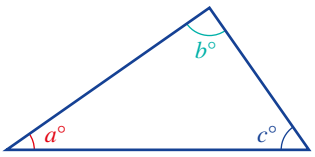
WORKED EXAMPLE 10

Classify each of the triangles in Worked example 9 according to the size of its angles.

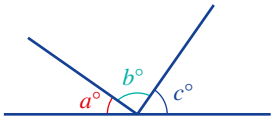
THINK	WRITE
a In ΔABC, ∠CAB is marked as the right angle, so classify it accordingly.	**a** ΔABC is a right-angled triangle.
b In ΔMNP all angles are less than 90°, so classify this triangle.	**b** ΔMNP is an acute-angled triangle.
c In ΔPRS, ∠PRS is greater than 90°; that is, it is obtuse. Use this information to classify the triangle.	**c** ΔPRS is an obtuse-angled triangle.

5.5.4 Angles in a triangle

- The sum of the three angles in any triangle is always 180°.



$a° + b° + c°$ makes a straight angle

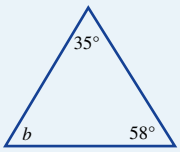


$a° + b° + c° = 180°$

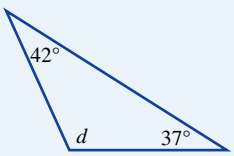
WORKED EXAMPLE 11

Find the value of the pronumeral in each of the following triangles.

a



b

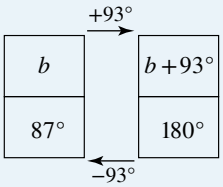


THINK

a 1 The sum of the three angles (b, 35° and 58°) must be 180°. Write this as an equation.

2 Simplify by adding 35° and 58° together.

3 Use inspection or backtracking to solve for b.



WRITE

a $b + 35° + 58° = 180°$

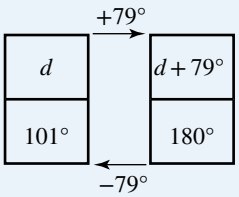
$b + 93° = 180°$

$b = 180° - 93°$
$b = 87°$

b 1 The sum of the three angles (d, 37° and 42°) must be 180°. Write this as an equation.

2 Simplify by adding 37° and 42° together.

3 Use inspection or backtracking to solve for d.



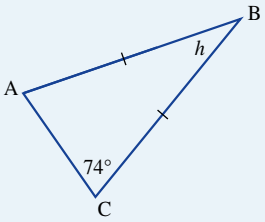
b $d + 37° + 42° = 180°$

$d + 79° = 180°$

$d = 180° - 79°$
$d = 101°$

WORKED EXAMPLE 12

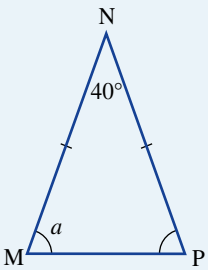
Find the value of the pronumeral in the following triangle.



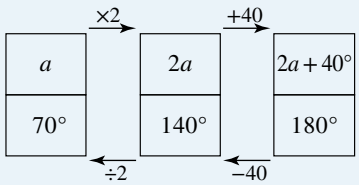
THINK	WRITE
1 The markings on the diagram indicate that ΔABC is isosceles with AB = BC. Therefore, the angles at the base are equal in size; that is, ∠BCA = ∠BAC = 74°.	∠BCA = ∠BAC = 74°
2 All 3 angles in a triangle must add up to 180°.	∠ABC + ∠BAC + ∠BCA = 180° h + 74° + 74° = 180°
3 Simplify.	h + 148° = 180°
4 Solve for h, that is, subtract 148° from 180°.	h = 180° − 148° h = 32°

WORKED EXAMPLE 13

Find the value of the pronumeral in the following triangle.

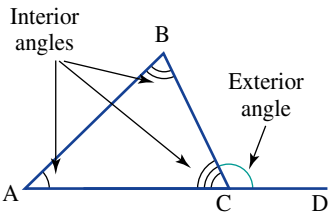


THINK	WRITE
1 From the diagram we can see that ΔMNP is isosceles with MN = NP. Hence, ∠NPM = ∠NMP = a.	∠NMP = ∠NPM = a
2 Form an equation by putting the sum of the angles on one side and 180° on the other side of the equals sign.	∠NMP + ∠NPM + ∠MNP = 180° a + a + 40° = 180°
3 Simplify by collecting like terms.	2a + 40° = 180°
4 Use inspection or backtracking to solve for a.	2a = 180° − 40° 2a = 140° $a = \frac{140°}{2}$ a = 70°

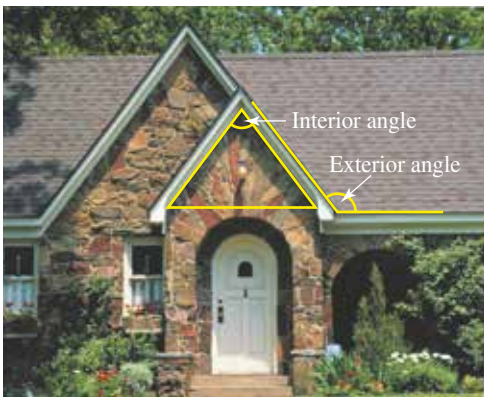


5.5.5 Interior and exterior angles of a triangle

- The angles inside a triangle are called **interior angles**.
- If any side of a triangle is extended outwards, the angle formed is called an **exterior angle**.
- The exterior angle and the interior angle adjacent (next) to it add up to 180°.
- The sum of the two interior opposite angles is equal to the exterior angle.

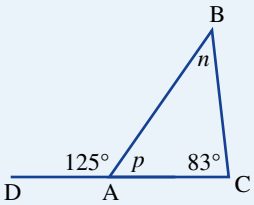


$\angle ACB + \angle BCD = 180°$
and
$\angle BAC + \angle ABC = \angle BCD.$



WORKED EXAMPLE 14

Find the value of the pronumerals in the following diagram.



THINK

1 $\angle BAC$ (angle p) together with its adjacent exterior angle ($\angle DAB$) add up to 180°. Furthermore, $\angle DAB = 125°$. So, form an equation.

2 Solve for p, that is subtract 125° from 180°.

3 The interior angles of ΔABC add up to 180°. Identify the values of the angles and form an equation.

4 Simplify by adding 83° and 55°.

5 Solve for n; that is, subtract 42° from 180°.

WRITE

$\angle BAC = p; \quad \angle DAB = 125°$
$\angle BAC + \angle DAB = 180°$
$p + 125° = 180°$

$p = 180° - 125°$
$p = 55°$

$\angle BCA + \angle BAC + \angle ABC = 180°$
$\angle BCA = 83°; \quad \angle BAC = p = 55°;$
$\angle ABC = n$
$83° + 55° + n = 180°$

$n + 138° = 180°$

$n = 180° - 138°$
$n = 42°$

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Complete this digital doc: SkillSHEET: Classifying triangles according to the length of their sides (doc-6454)

Complete this digital doc: SkillSHEET: Classifying triangles according to the size of their angles (doc-6455)

Exercise 5.5 Triangles

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 15, 16	Questions: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 14, 15, 16a, c, 18, 20, 23, 24	Questions: 1, 2, 3, 4, 5, 6b, e, f, 7c, 8c, 9b, f, 10–24

Individual pathway interactivity: int-4334

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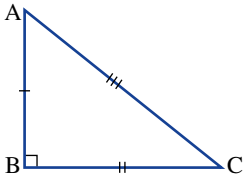
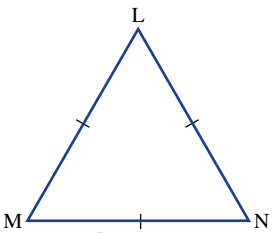
Fluency

1. WE9 Name each of the following triangles using the capital letters, then classify each triangle according to the lengths of its sides.

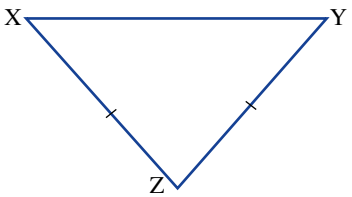
a.



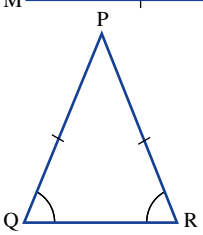
b.



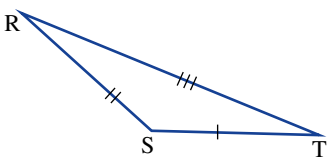
c.



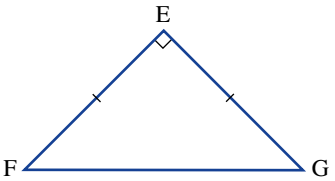
d.



e.



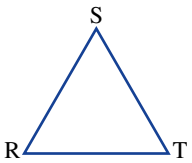
f.



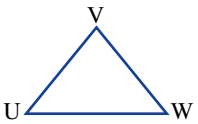
2. WE10 Classify each of the triangles in question **1** according to the size of its angles.

3. Add side and angle markings to these diagrams to show that:

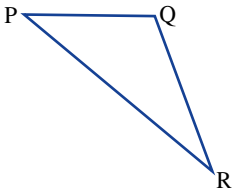
a. ΔRST is an equilateral triangle



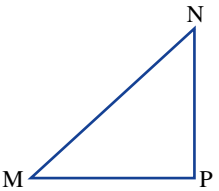
b. ΔUVW is an isosceles triangle



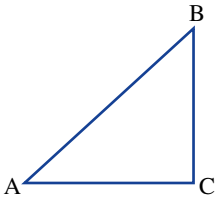
c. ΔPQR is a scalene triangle



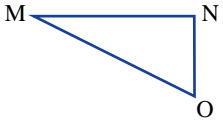
d. ΔMNP is a right-angled triangle



e. ΔABC is a right-angled and isosceles triangle

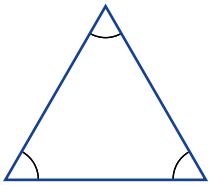


f. ΔMNO is a right-angled and scalene triangle.

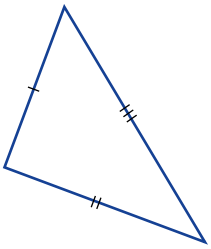


4. MC **a.** Which of these triangles is an equilateral triangle?

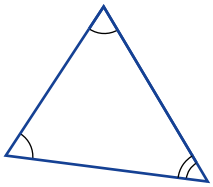
A.



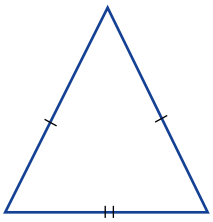
B.



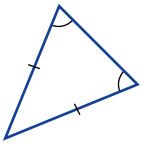
C.



D.

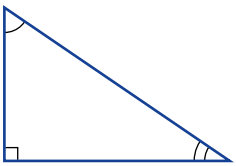


E.

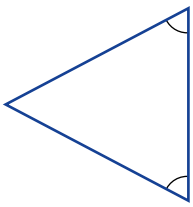


b. Which of these triangles is not a scalene triangle?

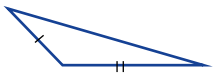
A.



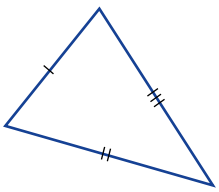
B.



C.



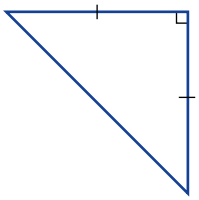
D.



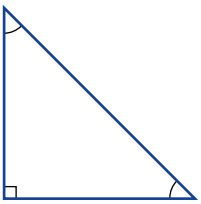
E.

5. MC **a.** Which of these triangles is both right-angled and scalene?

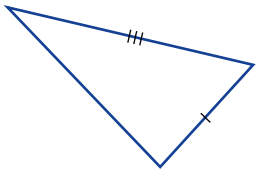
A.



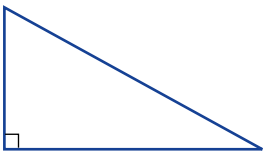
B.



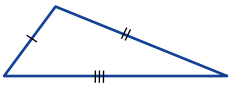
C.



D.

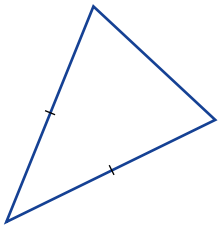


E.

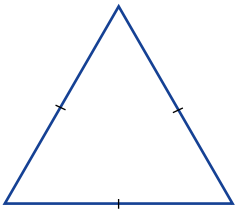


b. Which of these triangles is both acute-angled and isosceles?

A.



B.



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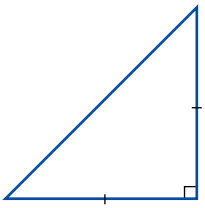
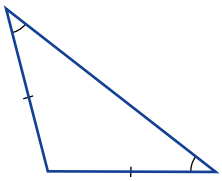


D.



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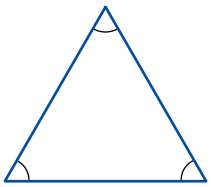
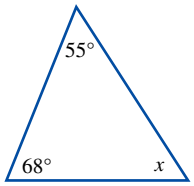
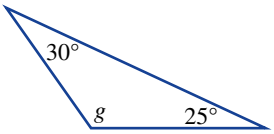


6. WE11 Find the value of the pronumeral in each of the following triangles.

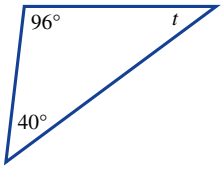
a.



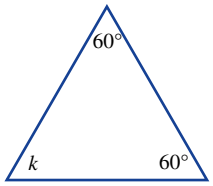
b.



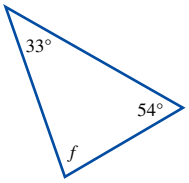
c.



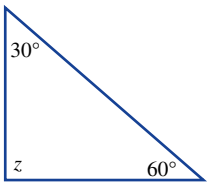
d.



e.

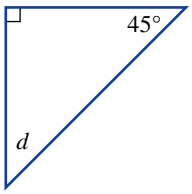


f.

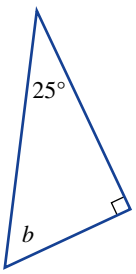


7. Find the value of the pronumeral in each of the following right-angled triangles.

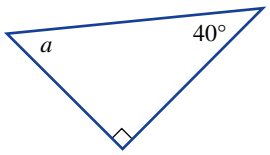
a.



b.

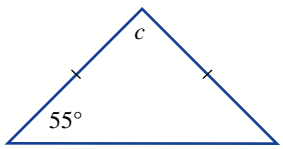


c.

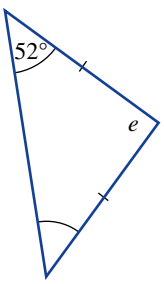


8. WE12 Find the value of the pronumeral in each of the following triangles, giving reasons.

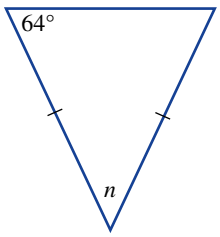
a.



b.

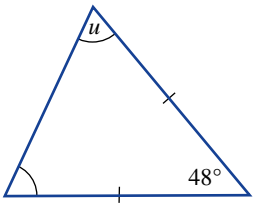


c.

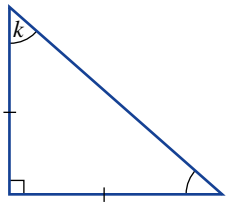


9. WE13 Find the value of the pronumeral in each of the following triangles, giving reasons.

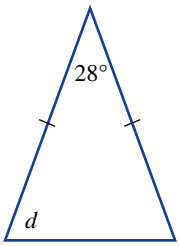
a.



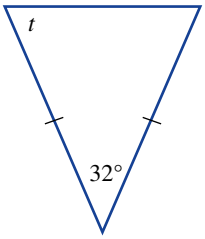
b.



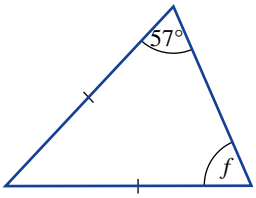
c.



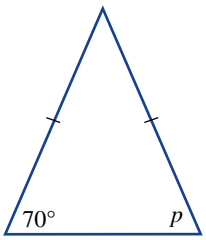
d.



e.

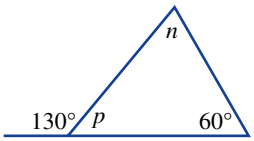


f.

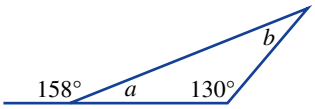


10. WE14 Find the value of the pronumerals in the following diagrams.

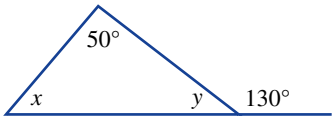
a.



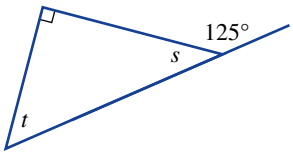
b.



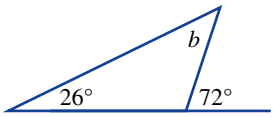
c.



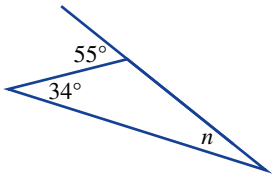
d.



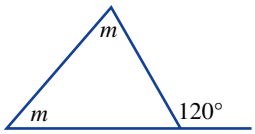
e.



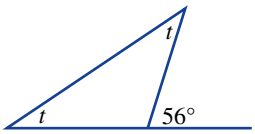
f.



g.



h.



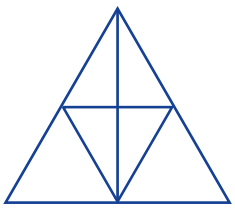
Understanding

11. What types of triangles can you see in the sandwich on the right?

12. Write down three acute triangles you can see around you.

13. Find one example in your classroom or home of each of the six types of triangles described in this chapter. Describe clearly where the triangle occurs, draw the triangle and classify it according to both side and angle types.

14. In the following triangle:



a. how many equilateral triangles can you find?

b. how many right-angled triangles can you find?

c. how many isosceles triangles can you find?

15. Use your ruler, pencil and protractor to accurately draw:

a. an equilateral triangle with side lengths 6cm and all angles 60°

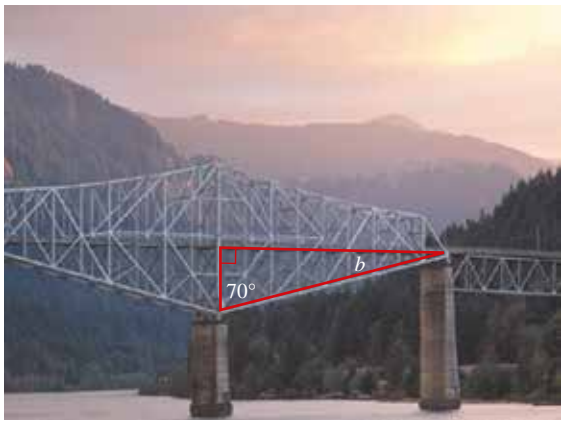
b. an isosceles triangle with two sides which are 6cm each with a 40° angle between them

c. a right-angled triangle whose two short sides are 6cm and 8cm. (How long is the longest side?)

d. a scalene triangle with two of the sides measuring 4cm and 5cm and an angle of 70° between the two sides.

16. Find the missing angle in each of the triangles marked on the following photographs.

a.



b.

17. a. An isosceles triangle has 2 angles of 55° each. Find the size of the third angle.
b. An isosceles triangle has 2 angles of 12° each. Find the size of the third angle.
c. Two angles of a triangle are 55° and 75° respectively. Find the third angle.
d. Two angles of a triangle are 48° and 68° respectively. Find the third angle.

18. a. Use a ruler and a protractor to construct each of the following triangles.
i. An isosceles triangle with a base of 4cm and equal angles of 50° each.
ii. An isosceles triangle with two sides which are 5cm each and two equal angles which are 45° each.
b. On your diagrams label the size of each angle. Classify the triangles according to the size of their angles.

19. For each of the following sets of 3 angles, state whether or not it is possible to construct a triangle with these angles. Give a reason for your answer.
a. 40°, 40°, 100° **b.** 45°, 60°, 70° **c.** 45°, 55°, 85° **d.** 111°, 34.5°, 34.5°

20. Explain in your own words why it is impossible to construct a triangle with 2 obtuse angles.

Reasoning

21. Cut eighteen strips of paper — three each of the following six lengths: 3cm, 4cm, 5cm, 6cm, 8cm and 10cm. Use these strips to help you answer the following questions.
a. Using a combination of three of these strips (without bending them), how many different triangles can you make?
b. What type of triangle is each one? How many are there of each kind?
c. Are there any similar triangles (same shape, but different size)?
d. Measure the size of the angles. What general relationship can you observe between the lengths of the sides of a triangle and the size of the angles?
e. Formulate a general statement which would enable you to determine whether it is possible to form a triangle from three given side lengths. Why is it not possible in some cases?

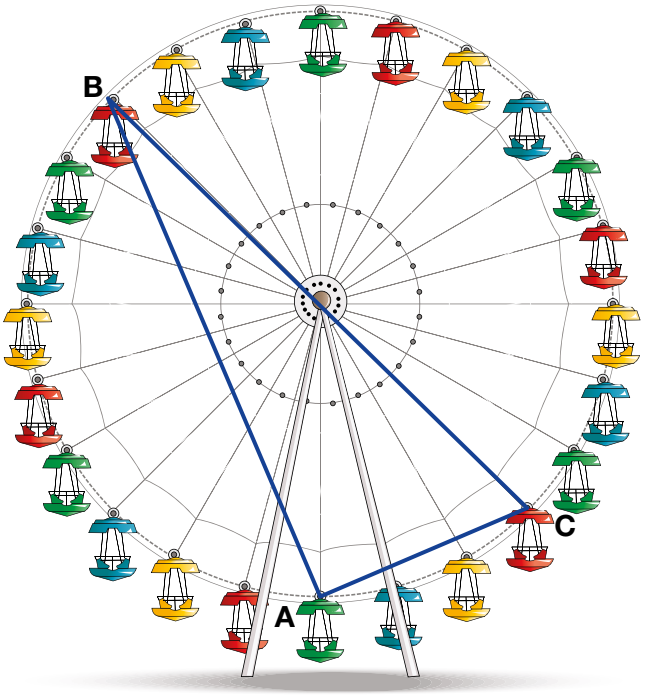
22. a. Construct the isosceles triangle ABC with $\angle B = \angle C = 80°$.
b. Extend the line BC to the right and label the other end D where BD = AB and form the ΔABD.
c. Calculate the magnitude of $\angle D$.
d. Extend the line CD to the right and label the other end E where DE = AD and form the ΔADE.
e. Calculate the magnitude of $\angle E$.
f. Repeat this process again.
g. What is the magnitude of the last angle?
h. Is there a pattern? Explain your answer.

Problem solving

23. a. ΔMNP is an isosceles triangle with MN = MP and ∠MNP = 72°. Calculate the magnitudes of ∠MPN and ∠NMP.

b. ΔMNP is an isosceles triangle with MN = MP and ∠NMP = 72°. Calculate the magnitudes of ∠MPN and ∠MNP.

24. ΔABC has its vertices on a Ferris wheel as shown in the diagram. BC is a diameter of the wheel because it passes through the centre of the wheel.



a. Use a protractor to measure the three angles of the triangle and record these values in a table like the one shown.

Triangle	∠*A*	∠*B*	∠*C*	Type of triangle
1				
2				

b. Construct another triangle by moving point A on the circumference of the circle. Keep BC unchanged.

c. Measure the new angles and record these values in your table.

d. Construct another triangle by moving points B and C on the circumference of the circle to form another diameter. Keep point A unchanged.

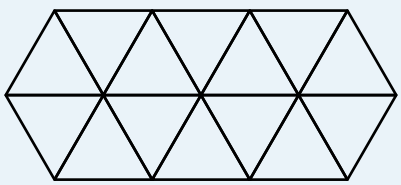
e. Measure the new angles and record these values in your table.

f. Construct another triangle by moving the three points A, B and C on the circumference of the circle. The only condition is to ensure that BC is a diameter of the circle.

g. What conclusion can you draw from this activity?

CHALLENGE 5.2

How many triangles can you see in this shape?



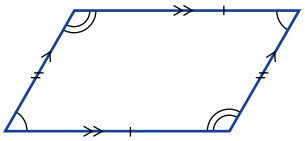
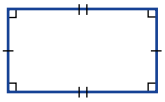
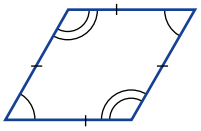
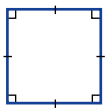
Reflection

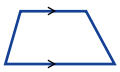
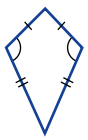
How can you prove the statement: 'The sum of the opposite interior angles is equal to the exterior angle'?

5.6 Quadrilaterals and their properties

5.6.1 Types of quadrilaterals

- A **quadrilateral** is a 2-dimensional closed shape with four straight sides.
- Quadrilaterals can be divided into two major groups: **parallelograms** and other quadrilaterals.
- Parallelograms are quadrilaterals with both pairs of opposite sides being parallel to each other.
- All **rectangles**, **squares** and **rhombi** (diamonds) are parallelograms.
- Other quadrilaterals (non-parallelograms) include **trapeziums**, **kites** and **irregular quadrilaterals**.

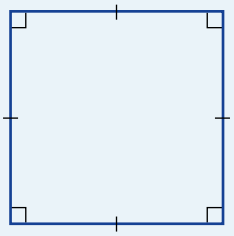
Parallelograms	Shape	Properties
Parallelogram		Opposite sides are equal in length. Opposite angles are equal in size.
Rectangle		Opposite sides are equal in length. All angles are the same and equal 90°.
Rhombus		All sides are equal in length. Opposite angles are equal in size.
Square		All sides are equal in length. All angles are the same and equal 90°.

Other quadrilaterals	Shape	Properties
Trapezium		One pair of opposite sides is parallel.
Kite		Two pairs of adjacent (next to each other) sides are equal in length. One pair of opposite angles (the ones that are between the sides of unequal length) are equal.
Irregular quadrilateral		This shape does not have any special properties.

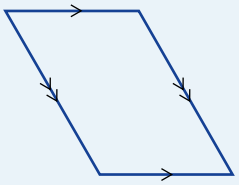
WORKED EXAMPLE 15

Name the following quadrilaterals, giving reasons for your answers.

a



b



THINK

a The markings on this quadrilateral indicate that all sides are equal in length and all angles equal 90°. Classify the quadrilateral by finding the matching description in the table.

b The arrows on the sides of this quadrilateral indicate that there are two pairs of parallel sides and hence it is a parallelogram. Check the descriptions in the table to see if it is a particular type of parallelogram.

WRITE

a The given quadrilateral is a square, since all sides are equal and all angles are 90°.

b The given quadrilateral is a parallelogram, since it has two pairs of parallel sides.

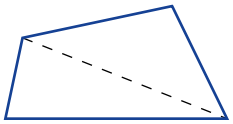
5.6.2 Angles in a quadrilateral

- The sum of angles in any quadrilateral is 360°.

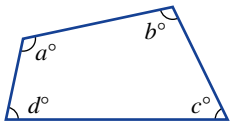
Quadrilaterals are made up of 2 triangles

Sum of angles in 1 triangle = 180°

$\therefore$ 2 triangles = $2 \times 180° = 360°$

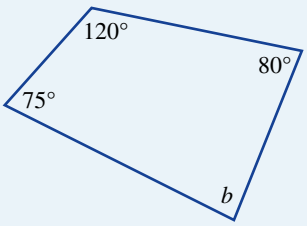


$a° + b° + c° + d° = 360°$



WORKED EXAMPLE 16

Find the value of the pronumeral in the diagram.



THINK

1 The sum of the angles in a quadrilateral is 360°. So, express this as an equation.

2 Simplify by adding 120°, 80° and 75°.

3 Solve to find the value of b; that is, subtract 275° from 360°.

WRITE

$b + 80° + 75° + 120° = 360°$

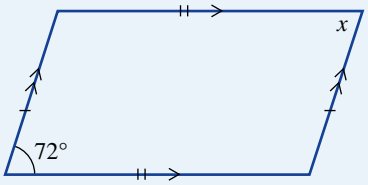
$b + 275° = 360°$

$b = 360° - 275°$

$b = 85°$

WORKED EXAMPLE 17

Find the value of the pronumeral in the diagram, giving a reason for your answer.



THINK

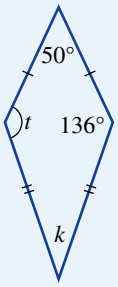
According to the markings, the opposite sides of the given quadrilateral are parallel and equal in length. Therefore, this quadrilateral is a parallelogram. In a parallelogram opposite angles are equal. So, state the value of the pronumeral.

WRITE

Opposite angles in a parallelogram are equal in size. Therefore, $x = 72°$.

WORKED EXAMPLE 18

Find the value of the pronumerals in the diagram at right.



THINK

1 Form an equation by writing the sum of the angles on one side and 360° on the other side of an equals sign.

2 The quadrilateral shown in the diagram is a kite. Angle t and angle 136° are the angles between unequal sides and therefore must be equal in size.

3 Replace t in the equation with 136°.

4 Simplify.

5 Solve to find the value of k, that is, subtract 322° from 360°.

WRITE

$k + t + 50° + 136° = 360°$

$t = 136°$
(The figure is a kite.)

$k + 136° + 50° + 136° = 360°$

$k + 322° = 360°$

$k = 360° - 322°$
$k = 38°$

Exercise 5.6 Quadrilaterals and their properties

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1, 2, 3, 4, 5, 6, 7, 9, 12, 14, 18, 21	Questions: 1, 2, 3a, b, d, g, h, 4b, d, f, 5, 6, 7, 9, 10, 12, 14, 16, 17, 18, 20, 21	Questions: 1, 2, 3a, d, e, h, 4a, d, 5, 6, 7, 8–22

Individual pathway interactivity: int-4335

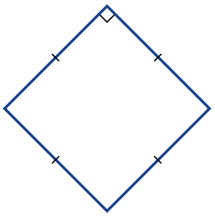
learnON ONLINE ONLY

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

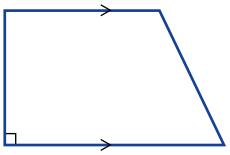
Fluency

1. WE15 Name the following quadrilaterals, giving reasons for your answers.

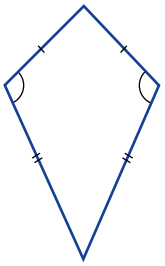
a.



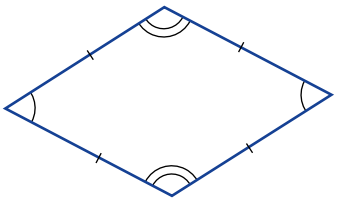
b.



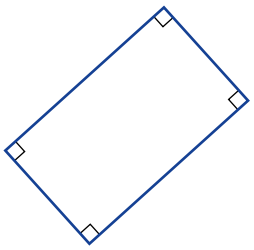
c.



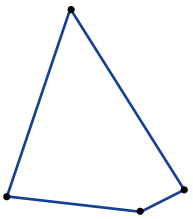
d.



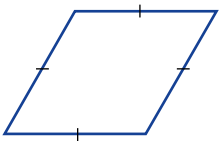
e.



f.



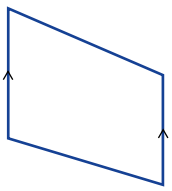
2. MC **a.**



This quadrilateral is a:

A. square
B. rectangle
C. kite
D. parallelogram
E. rhombus

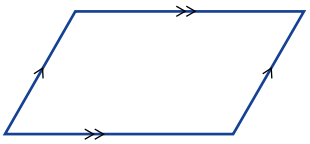
b.



This quadrilateral is a:

A. trapezium
B. parallelogram
C. rhombus
D. kite
E. square

c.

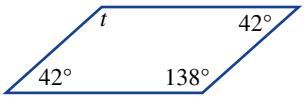


This quadrilateral is a:

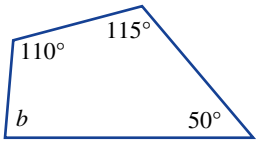
A. trapezium
B. square
C. kite
D. rhombus
E. parallelogram

3. WE16 Find the value of the pronumeral in each of the following diagrams.

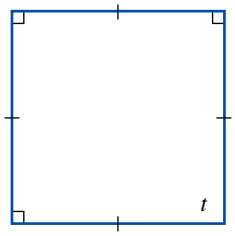
a.



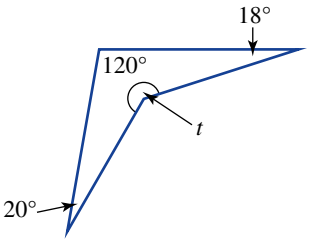
b.



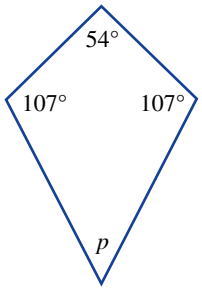
c.



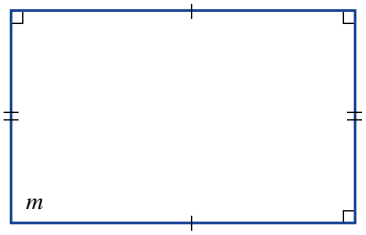
d.



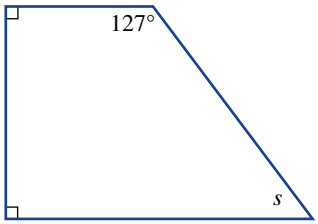
e.



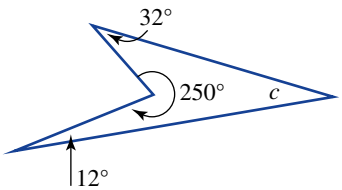
f.



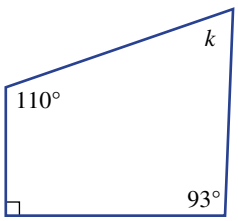
g.



h.

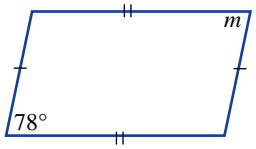


i.

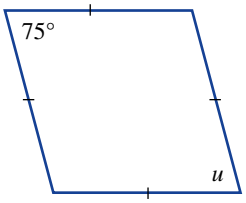


4. WE17 Find the value of the pronumeral in each of the following diagrams, giving reasons for your answers.

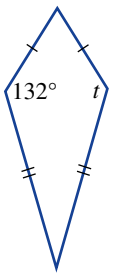
a.



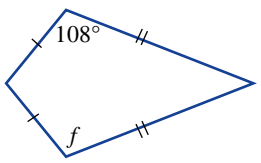
b.



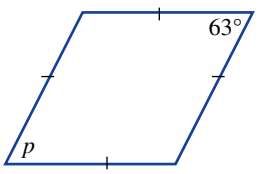
c.



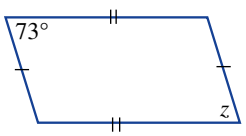
d.



e.

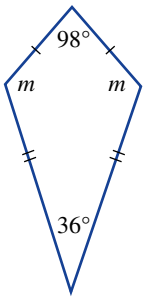


f.

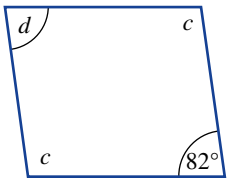


5. WE18 Find the value of the pronumerals in each of the following diagrams.

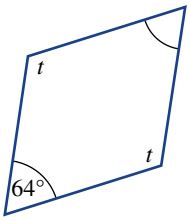
a.



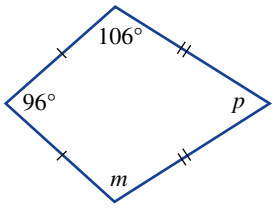
b.



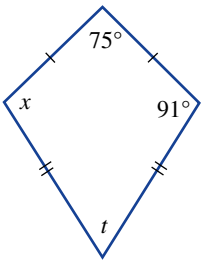
c.



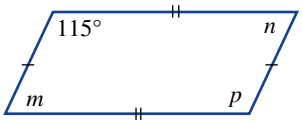
d.



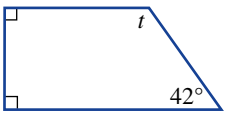
e.



f.

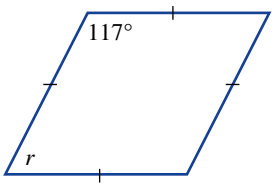


6. MC The value of t in the diagram at right is:



A. 360° **B.** 112° **C.** 222°
D. 138° **E.** 180°

7. MC The value of r in the diagram at right is:



A. 117° **B.** 180° **C.** 234°
D. 126° **E.** 63°

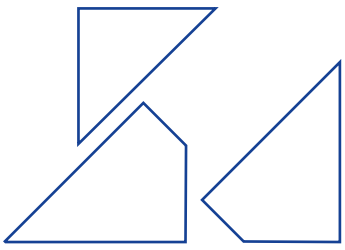
Understanding

8. State whether each of the following statements is true or false.
- **a.** All squares are rectangles.
- **b.** All squares are rhombuses.
- **c.** All rectangles are squares.
- **d.** Any rhombus with at least one right angle is a square.
- **e.** A rectangle is a parallelogram with at least one angle equal to 90°.

9. MC A rectangle is a quadrilateral because:
- **A.** it has 4 right angles
- **B.** it has 2 pairs of parallel sides
- **C.** its opposite sides are equal in length
- **D.** it has 4 straight sides
- **E.** it has 2 pairs of parallel sides and 4 right angles

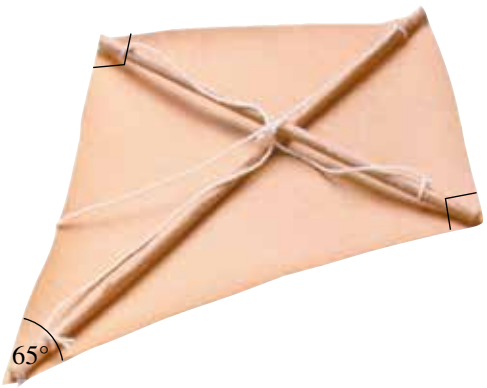
10. Draw 4 equilateral triangles with side lengths 4cm and cut them out.
- **a.** Use 2 of these triangles to make a rhombus. Draw your solution.
- **b.** Use 3 of these triangles to make a trapezium. Draw your solution.
- **c.** Use all 4 triangles to make a parallelogram. Draw your solution.

11. Copy and cut out the set of shapes shown at right. Arrange the shapes to form a square. Draw your solution.



12. In your house, find an example of each type of quadrilateral discussed in this section. Write down the type of quadrilateral and where you found it.

13. This photograph shows the roof of a gazebo. Calculate the value of p.



14. Find the size of the obtuse angle in the following kite.

15. Two angles in a parallelogram are 45° and 135°. Find the other 2 angles.

16. Tom measures 2 angles of a kite at 60° and 110°, but forgets which angle is which. Draw 3 different kites that Tom may have measured, showing the size of all angles in each diagram.

17. For each of the following sets of 4 angles, decide whether it is possible to construct a quadrilateral. Explain your answer.

a. 25°, 95°, 140°, 100°

b. 40°, 80°, 99°, 51°

18. Three angles of a quadrilateral are 60°, 70° and 100°.

a. What is the size of the fourth angle of this quadrilateral?

b. How many quadrilaterals with this set of angles are possible?

c. Construct one quadrilateral with the given angle sizes in your book. (The choice of the length of the sides is yours.)

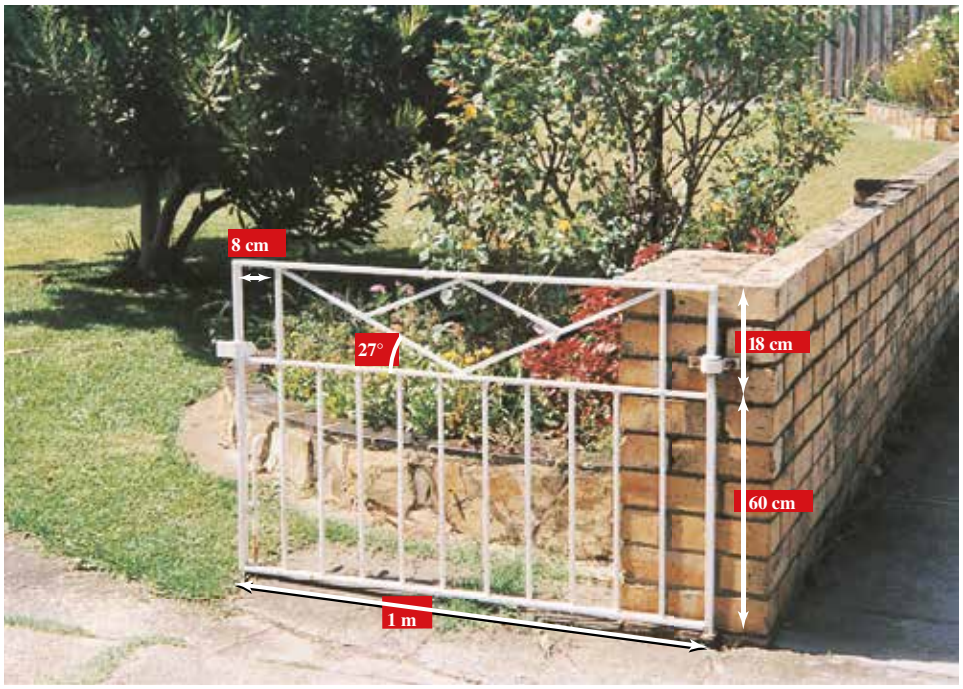
Reasoning

19. Val and Peter want to replace their front gate with another of the same design. To have this gate made, they need to supply a diagram of it with all measurements and angles shown. Study the photograph of Val and Peter's gate and use it to help you answer the following questions.

a. There are 4 different shapes formed by the metal bars of the gate. How many different types of triangles are there? Can you name them?

b. How many types of quadrilaterals are there? Name them.

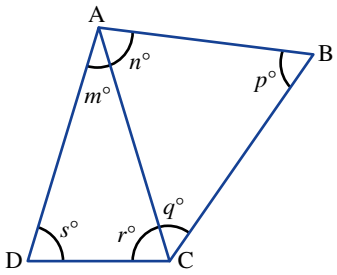
c. Draw a diagram of the gate showing the length measurements and the one angle that is given.



d. Use this angle to calculate all the remaining angles in the diagram.

e. Using a ruler and protractor, design a fence that is to be constructed using metal bars. Include different triangles and quadrilaterals to make your design as interesting as possible. Write a short report describing the shapes and important angles you have used in your design to assist in the construction of the fence.

20. Consider the diagram shown. Prove that the sum of all angles in a quadrilateral is 360°.



Problem solving

21. What is the angle sum (sum of all the angles) of a decagon?

22. Draw a quadrilateral, PQRS. Label angle *P* as 100° and angle *R* as a right angle. Angle *Q* is two-thirds of angle *S*. Determine the measurement of angle *S*.

Reflection

What would be an easy way to remember the sum of angles in quadrilaterals?

5.7 Parallel and perpendicular lines

5.7.1 Vertically opposite and adjacent angles

- When two straight lines intersect, they form four angles with a common vertex.
- The angles that are opposite each other are called **vertically opposite angles**.
- Vertically opposite angles are equal in size.
- The angles next to each other are called **adjacent angles**. (Adjacent angles share an arm.)

In the following diagram there are two pairs of vertically opposite angles.

∠AOB = ∠DOC and ∠BOC = ∠AOD

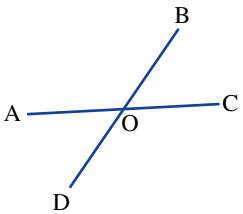
There are 4 pairs of adjacent angles.

∠AOB is adjacent to ∠BOC.

∠BOC is adjacent to ∠COD.

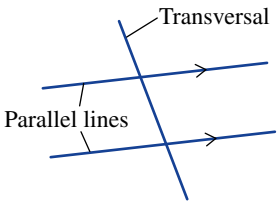
∠COD is adjacent to ∠AOD.

∠AOD is adjacent to ∠AOB.

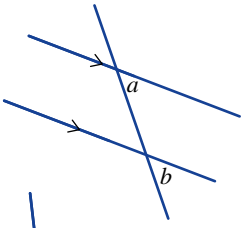


5.7.2 Parallel lines

- Parallel lines are indicated in a diagram by lines with arrows on them.
- A line intersecting a pair (or set) of parallel lines is called a **transversal**.
- Cutting parallel lines by a transversal creates a series of angles that are related in a number of ways.
- The symbol used to mean 'is parallel to' is ||.



5.7.3 Corresponding angles

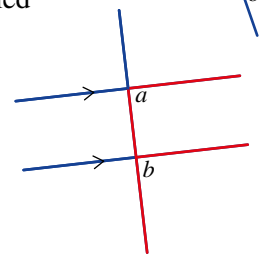


The following two diagrams show two angles, a and b, positioned below the parallel lines to the right of a transversal. When both angles are on the same side of the transversal (both to the left, or both to the right of it) and are either both above, or both below the parallel lines, such angles are called **corresponding angles**.

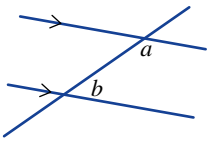
Corresponding angles are equal in size.

Thus, in the diagram at right $\angle a = \angle b$.

The position of corresponding angles is easy to remember by associating it with an F shape.



5.7.4 Co-interior angles (or allied angles)

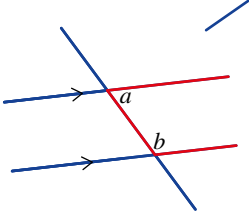


The diagram at right shows two angles, a and b, positioned between the parallel lines, on the same side (to the right) of the transversal. Such angles are called **co-interior angles** or **allied angles**.

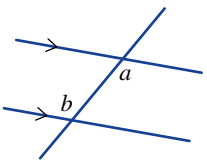
Co-interior angles are supplementary; that is, they add up to 180°.

Thus, in the diagram at right $\angle a + \angle b = 180°$.

The position of the co-interior angles is easy to remember by associating it with a C shape.



5.7.5 Alternate angles

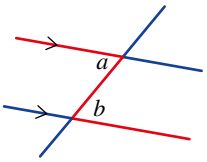


The diagram at right shows two angles, a and b, positioned between the parallel lines and on alternate sides of the transversal. Such angles are called **alternate angles**.

Alternate angles are equal in size.

Thus, in the diagram at right, $\angle a = \angle b$.

The position of alternate angles is easy to remember by associating it with a Z shape.

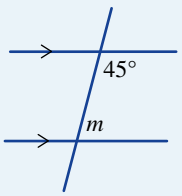


5.7.6 Calculating angles associated with parallel lines

Angle relationships associated with parallel lines can be used to find the size of missing angles, as shown in the following worked examples. It is important to remember that Z, F or C shapes may be upside-down or backwards in some diagrams.

WORKED EXAMPLE 19

For the diagram at right:
a state the type of angle relationship
b find the value of the pronumeral.

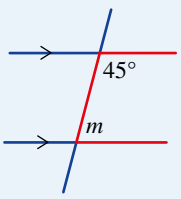


THINK

a 1 Study the diagram: which shape — Z, F or C — would include both angles that are shown? Copy the diagram into your workbook and highlight the appropriate shape.

WRITE

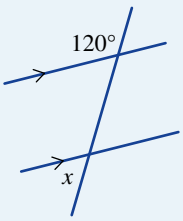
a

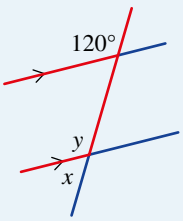


2	State the name of the angles suggested by a C shape.	Shown angles are co-interior.
b 1	Co-interior angles add to 180°. Write this as an equation.	**b** $m + 45° = 180°$
2	Solve for m	$m = 180° - 45°$ $= 135°$

WORKED EXAMPLE 20

Find the value of the pronumeral in the diagram shown, giving reasons.



THINK	WRITE
1 The two angles shown are neither C, Z nor F angles. So we must find some other angle first, that will enable us to find the size of angle x. (This other angle must be in a certain relation with *both* given angles.) Draw an F shape that includes a 120° angle. The other angle in an F shape is related to both given angles: it is corresponding to the 120° angle and it is supplementary to angle x. Call this angle y.	
2 State the size of angle y, specifying the reason (the angle relation).	$y = 120°$ (as corresponding angles are equal)
3 The angles x and y are supplementary (add to 180°). State this as an equation.	$x + y = 180°$ (as angle sum of straight line is 180°)
4 Substitute the value of y into the equation.	$x + 120° = 180°$
5 Solve for x.	$x = 180° - 120°$ $= 60°$

5.7.7 Testing for parallel lines

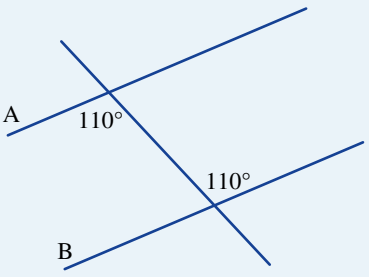
If 2 lines and a transversal intersect to form any of the following angles:

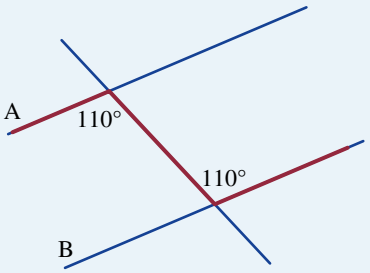
- corresponding angles
- alternate angles
- co-interior angles

then the two lines are parallel.

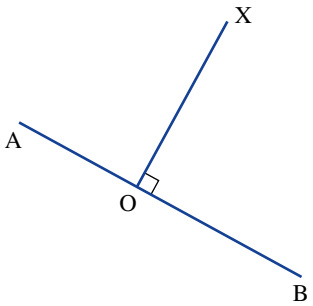
WORKED EXAMPLE 21

Are the following lines A and B parallel?
Give reasons for your answer.



THINK	WRITE
1 The given angles are equal to each other. They are both 110°.	
2 State the name of the angles suggested by a Z shape. These angles are alternate.	
3 Since the 2 angles are alternate and equal, then the lines A and B must be parallel.	A ∥ B

5.7.8 Perpendicular lines



- **Perpendicular** means 'at right angles'.
- When two line segments meet at right angles they are said to be perpendicular. That is, the angle between them is 90°. In the following diagram, the angles ∠AOX and ∠XOB are each right angles, which means that the line segments AB and OX are perpendicular.
- To write 'the line segments AB and OX are perpendicular' using mathematical symbols, simply write AB ⊥ OX.
- The symbol ⊥ means 'is perpendicular to'.

learnON RESOURCES — ONLINE ONLY

- **Try out this interactivity:** Vertically opposite angles (int-0814)
- **Try out this interactivity:** Corresponding angles (int-0815)
- **Try out this interactivity:** Alternate angles (int-0816)
- **Try out this interactivity:** Co-interior angles (int-0817)
- **Complete this digital doc:** Investigation: Investigate parallel and perpendicular lines (doc-6456)
- **Complete this digital doc:** WorkSHEET: Geometry II (doc-6618)

Exercise 5.7 Parallel and perpendicular lines

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 16	Questions: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 14, 16	Questions: 1, 2, 3, 4, 5, 6, 7, 8, 11, 12, 13, 14, 15, 16

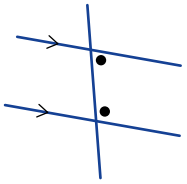
Individual pathway interactivity: int-4336

learnON ONLINE ONLY

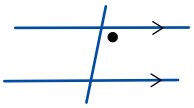
To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Fluency

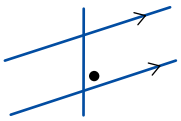
1. Copy and complete this table.

Diagram	Type of angle relation	Associated shape	Rule
	Corresponding		Are equal in size.
	Alternate	Z	
			Add up to 180°.

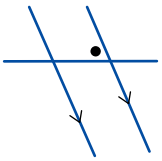
2. a. Copy the following diagram into your workbook. Clearly draw an F shape on your diagram and label the angle corresponding to the one that is marked.



b. Copy the following diagram into your workbook. Clearly draw an Z shape on your diagram and label the angle alternate to the marked angle.



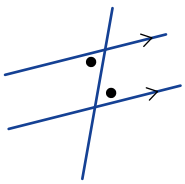
c. Copy the following diagram and label the angle co-interior to the marked angle. Clearly draw a C shape on your diagram.



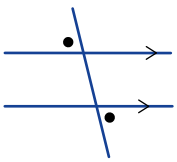
3. Match each diagram with the appropriate name from the four options listed.

Diagram

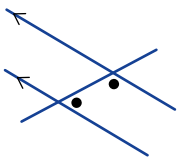
a.



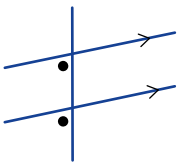
b.



c.



d.



Name

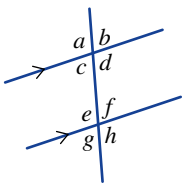
A. Co-interior angles (C)

B. Corresponding angles (F)

C. Alternate angles (Z)

D. None of the above

4. In the following diagram, list all pairs of:



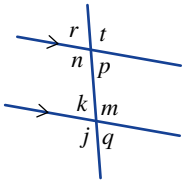
a. vertically opposite angles

b. corresponding angles

c. co-interior angles

d. alternate angles.

5. MC In the following diagram:



a. which angle is vertically opposite to angle p?

A. k **B.** m **C.** r
D. q **E.** t

b. which angle is corresponding to angle p?

A. k **B.** m **C.** r
D. q **E.** t

c. which angle is co-interior to angle p?

A. k **B.** m **C.** r **D.** q **E.** n

d. which angle is alternate to angle p?

A. k **B.** m **C.** r **D.** q **E.** n

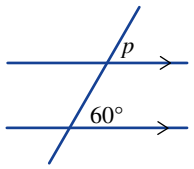
e. name two angles adjacent to angle k.

6. WE19 For each of the following diagrams:

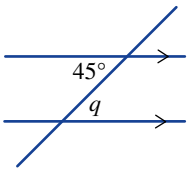
i. state the type of angle relationship

ii. find the value of the pronumeral.

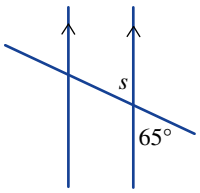
a.



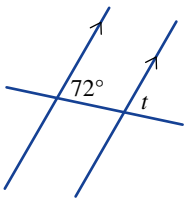
b.



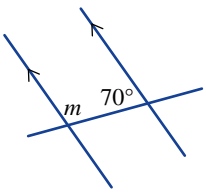
c.



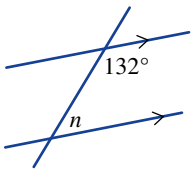
d.



e.

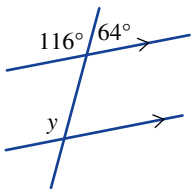


f.

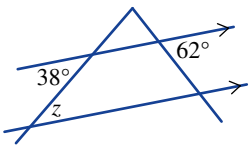


7. WE20 Find the value of the pronumerals in each of the following diagrams, giving reasons.

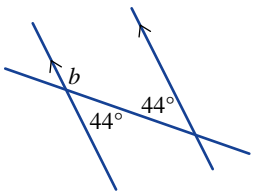
a.



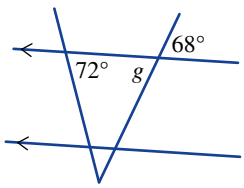
b.



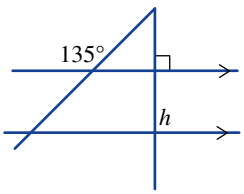
c.



d.



e.



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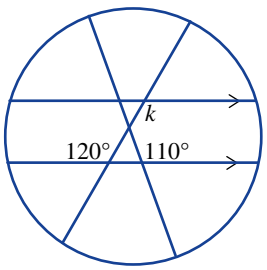
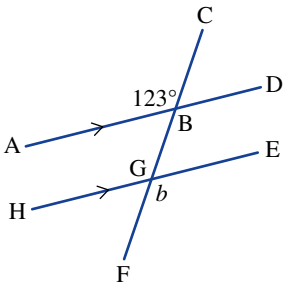
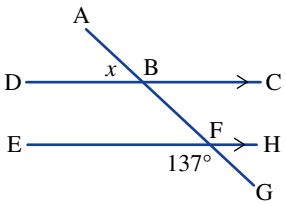


8. a. WE20 Find the value of the pronumerals in the following diagrams.

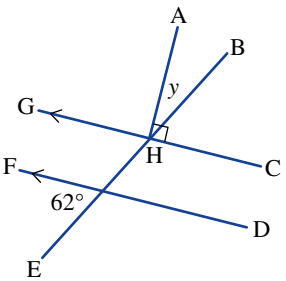
i.



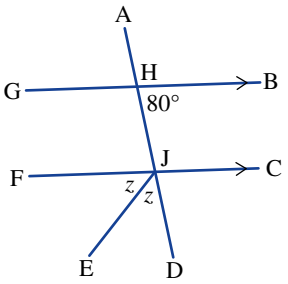
ii.



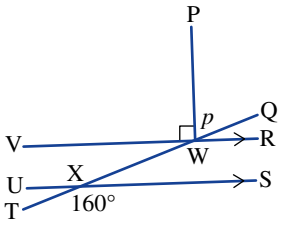
iii.



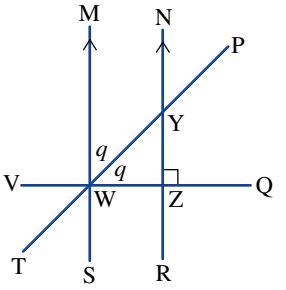
iv.



v.



vi.



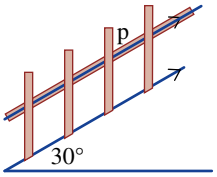
b. Name all the perpendicular lines in the preceding diagrams in part **a**.

Understanding

9. If the angle allied to x is 135°, find the size of angle x.

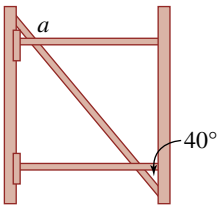
10. If the angle corresponding to y is 55°, find the size of angle y.

11. A hill is at an angle of 30° to the horizontal. A fence is put in, consisting of a railing parallel to the ground and vertical fence posts. Find the angle, p, between the top of the fence post and the rail.

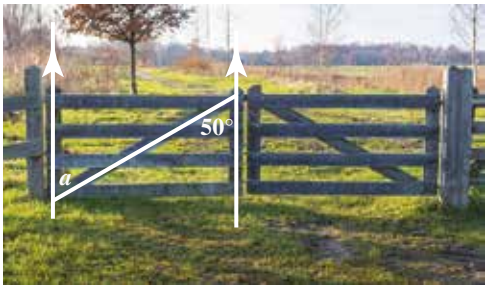


12. Two gates consist of vertical posts, horizontal struts and diagonal beams. Find the angle, a, as shown in the following gates.

a.

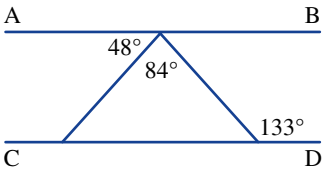


b.

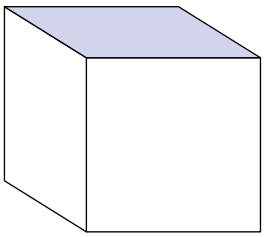


Reasoning

13. WE21 Is the line AB parallel to the line CD? Explain your answer.



14. Perspective drawing also allows us to create the illusion of looking at the same object from a number of different angles. For example, consider the following simple box.



Figures A and B are views of the cube from different angles. In all cases the top of the box has been shaded.

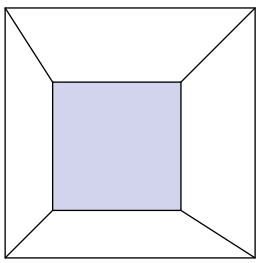


Figure A

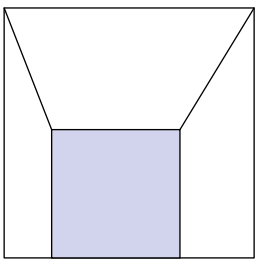


Figure B

a. Find the vanishing point in each diagram, by extending the edges in the diagram until they meet.

b. True or False? Explain your answers.

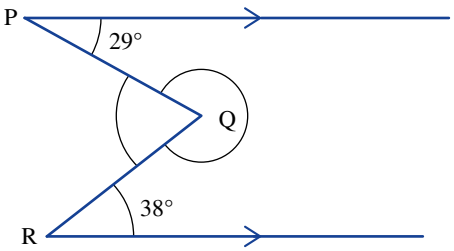
i. The viewer is above the box in Figure A.

ii. The viewer is behind the box in Figure B.

iii. The viewer of Figure A is probably taller than the viewer of Figure B.

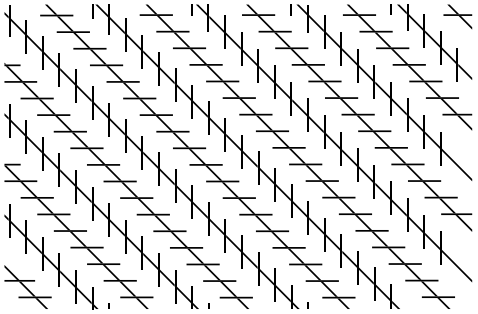
Problem solving

15. In the diagram shown calculate the magnitude of the acute angle ∠PQR and the reflex angle ∠PQR.

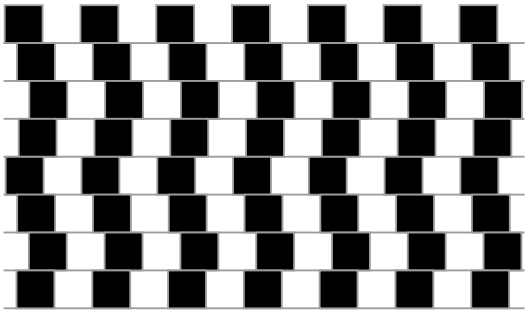


16. Look at the following optical illusions. Are any of the lines parallel? Explain your answer.

a.



b.



Reflection

Can you think of a way to remember the different types of angles associated with parallel lines and their special properties?

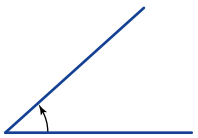
5.8 Review

5.8.1 Review questions

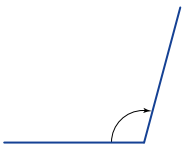
Fluency

1. First estimate, then use a protractor to measure the magnitude of each of the following angles. Comment on the accuracy of your estimate.

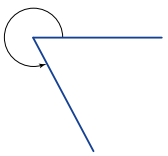
a.



b.



c.



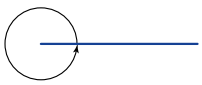
2. Use a protractor to construct each of the following angles.

a. 43° **b.** 68° **c.** 90° **d.** 128° **e.** 170°

f. 226° **g.** 270° **h.** 285° **i.** 300° **j.** 310°

3. State the type of each of the angles shown below.

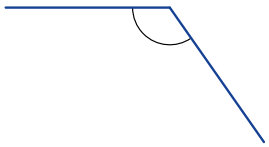
a.



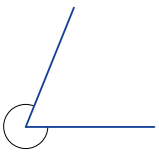
b.

c.

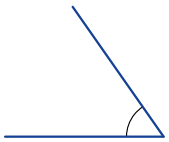
d.



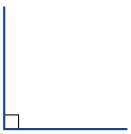
e.



f.



Questions **4** and **5** refer to the following diagram.



4. MC The angle shown in the diagram has a magnitude of:

A. 50° **B.** 40° **C.** 90° **D.** 100° **E.** 190°

5. MC The angle shown is:

A. a right angle **B.** a straight angle **C.** an acute angle

D. an obtuse angle **E.** a reflex angle

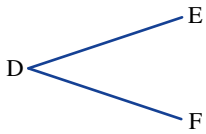
6. MC An acute angle has a magnitude that is:

A. more than 90°
B. less than 90°
C. exactly 90°
D. more than 90° but less than 180°
E. less than 180°

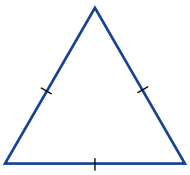
7. MC A straight angle has a magnitude of:

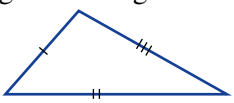
A. 90°
B. 270°
C. 360°
D. 180°
E. 60°

8. Name the angle shown.



9. Name the following triangles according to the length of their sides.

a.


b.


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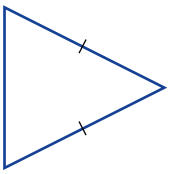
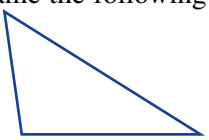
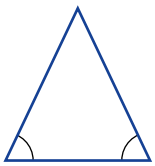


10. Name the following triangles according to the size of their angles.

a.


b.


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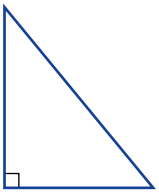
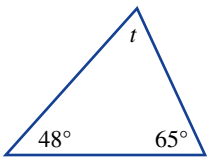
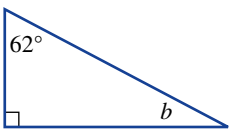


11. Find the value of the pronumeral in each of the following.

a.


b.


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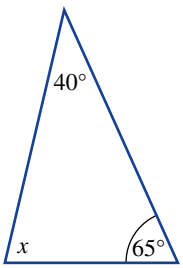
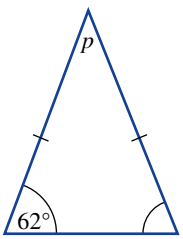


12. Find the value of the pronumeral in each of the following.

a.


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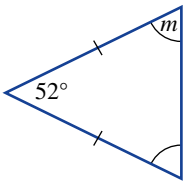

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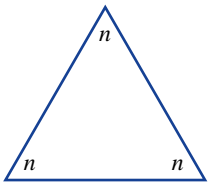
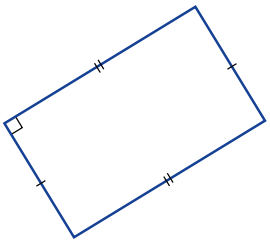


13. Name the following quadrilaterals, giving reasons for your answers.

a.


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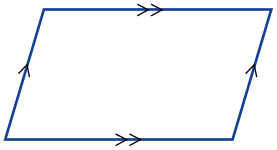

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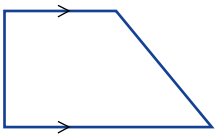

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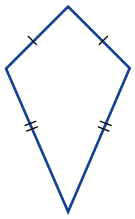

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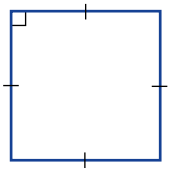

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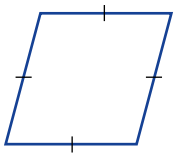
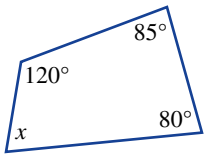


14. Find the value of the pronumeral in each of the following.

a.



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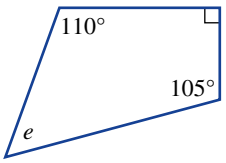

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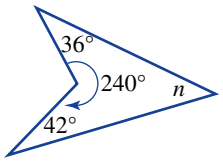
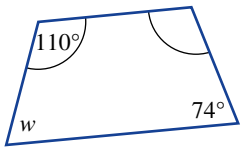
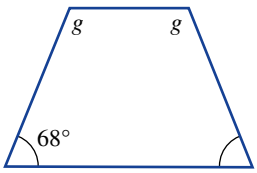


15. Find the value of the pronumeral in each of the following.

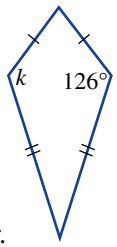
a.



b.

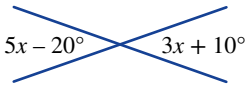


c.

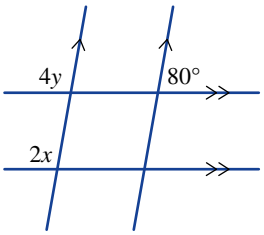


16. Find the value of each of the pronumerals, giving reasons for your answer.

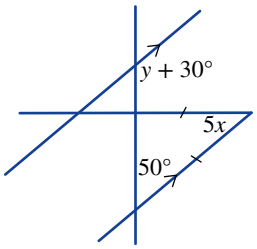
a.



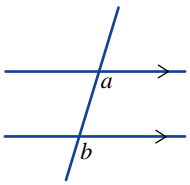
b.



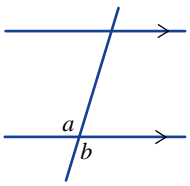
c.



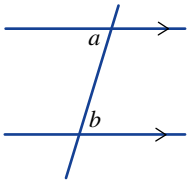
17. MC *Note:* There may be more than one correct answer.
In the diagrams below, angles a and b are:



A. equal **B.** corresponding **C.** allied
D. alternate **E.** supplementary

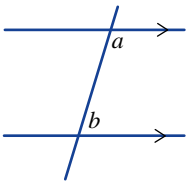


A. vertically opposite **B.** corresponding **C.** co-interior
D. alternate **E.** equal



A. corresponding **B.** co-interior **C.** alternate

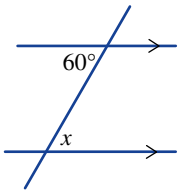
D. supplementary **E.** equal



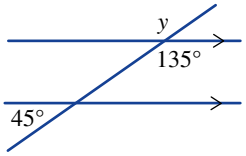
A. vertically opposite **B.** corresponding **C.** allied
D. alternate **E.** supplementary

18. For each diagram:
i. state the type of angle relation that you use
ii. calculate the value of the pronumeral.

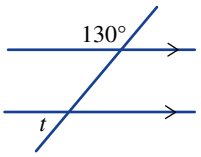
a.



b.

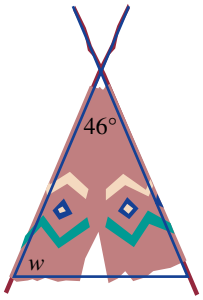


c.

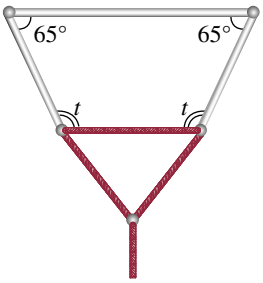


Problem solving

19. The teepee shown has an angle of 46°at its peak. What angle, w, does the wall make with the floor?

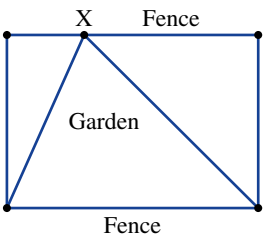


20. A circus trapeze attached to a rope is shown. Find the size of angle t.



21. Triangles can be classified according to the number of congruent sides (scalene, isosceles or equilateral) and by the type of angles (acute, obtuse or right). Decide which of the following combinations are possible, and draw an example.
Right-scalene, right-isosceles, right-equilateral
Acute-scalene, acute-isosceles, acute-equilateral
Obtuse-scalene, obtuse-isosceles, obtuse-equilateral.

22. Mark is building a triangular garden in his rectangular yard (the diagram below). He wants one side of the garden to lie along his fence line. The other corner of the garden (X) is to touch the opposite fence. He's trying to work out how much soil to order. Does his positioning of X change the amount of soil required?
Mark's garden plan.



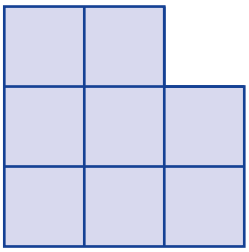
23. Jack wants to lay a rectangular concrete slab 1.8 m × 1.2 m. He has 2 pieces of timber 1.8 m long, and 2 pieces 1.2 m long. He positions the timber so that they look like a rectangle, and he measures the diagonals to check. Explain why it is necessary to measure the diagonals if Jack wants to be sure that his slab is a rectangle.

24. Digital numbers are drawn using a number of horizontal and vertical strokes. Which digital numbers have the same number of strokes as the number itself?

25. What is the total number of right angles on the faces of a normal 6-sided die?

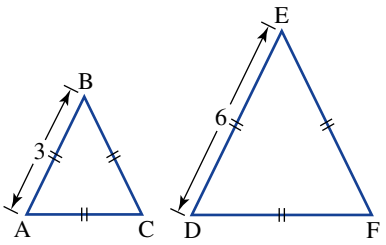
26. Show how you can use 9 matchsticks to construct a figure showing 5 equilateral triangles.

27. This figure has a small square removed from the right-hand corner. Make a copy. Draw two straight lines on the figure, cut along the lines, then rearrange the pieces to form a square.

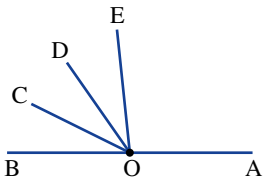


28. At midday, the hour hand, the minute hand and the second hand are all pointing to the 12. What is the angle between the hour hand and the minute hand after the second hand has completed 30 revolutions?

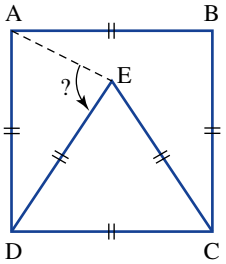
29. The triangles below are both equilateral triangles, but the side length of triangle ABC is 3 units and the side length of triangle DEF is 6 units. How many of the smaller triangles are required to fill the larger triangle?



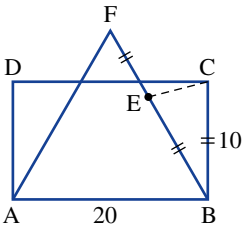
30. If angle AOE is obtuse, name all the acute angles in the diagram below.



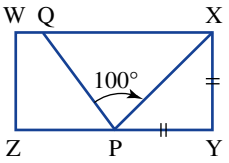
An equilateral triangle EDC is drawn inside a square ABCD as shown below. Determine the magnitude of the unknown angle below:



31. ABCD is a rectangle with a length of 20 and width of 10; that is, AB = 20 and AB = 10.
ABF is an equilateral triangle that overlays the rectangle as shown. E is the midpoint of the side BF.
a. Determine the magnitude of ∠ECD.
b. Name all pairs of perpendicular line segments.



32. WXYZ is a rectangle as shown. Points P and Q are drawn so that XY = PY and ∠XPQ = 100°.
Find the size of ∠PQX.



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- **Try out this interactivity:** Crossword: Topic 5 (int-2592)
- **Try out this interactivity:** Sudoku: Topic 5 (int-3166)
- **Complete this digital doc:** Concept map: Topic 5 (doc-10727)

Language

It is important to learn and be able to use correct mathematical language in order to communicate effectively. Create a summary of the topic using the key terms below. You can present your summary in writing or using a concept map, a poster or technology.

acute angle
acute-angled triangle
adjacent angles
alternate angles
angle
arms
co-interior angles (allied)
complementary angles
corresponding angles
equilateral triangle
exterior angle
interior angle
irregular quadrilateral
isosceles triangle
kite
line segment
obtuse angle
obtuse-angled triangle
parallel
parallelogram
perpendicular
perigon
protractor
quadrilateral
rectangle
reflex angle
revolution
rhombus
right angle
right-angled triangle
scalene
square
straight angle
supplementary angles
transversal
trapezium
vertex
vertically opposite angles

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Investigation | Rich task

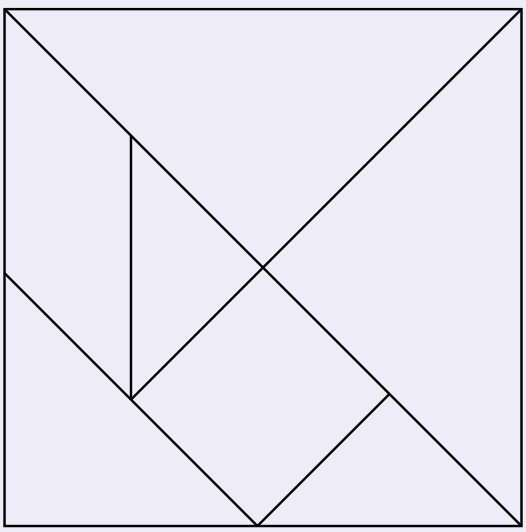
Tangrams

Tangrams are an ancient Chinese puzzle game called *ch'i ch'ae pan* or Seven-board of Cunning. The first tangram books were printed in 1813 but the word *ch'i ch'ae* dates back to 740–330 BC. One Chinese legend tells:

A servant of a Chinese emperor was carrying a very expensive square ceramic tray and, when he tripped and fell, it was shattered into seven pieces (called tans). He was unable to arrange the pieces back into the original shape, but he realised that there were many other shapes that could be built from the pieces.

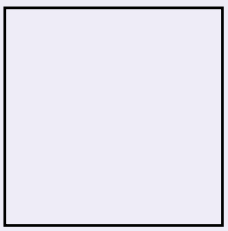
Instructions

Trace the following tangram pieces onto cardboard and cut out each piece accurately.



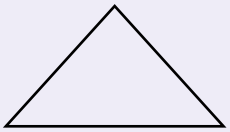
1. Using only the two small triangles, make and draw the following shapes, showing the joins in your book.

a.



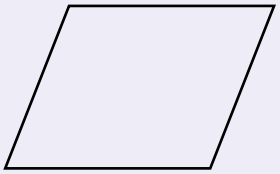
Square

b.



Triangle

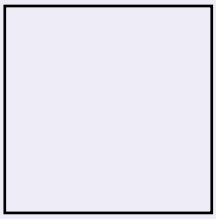
c.



Parallelogram

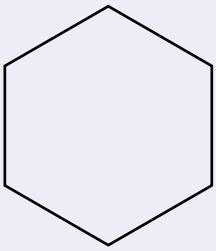
2. Using the square, the two small triangles, the parallelogram and the medium triangle, make and draw the following shapes.

a.



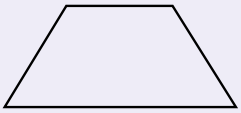
Square

b.



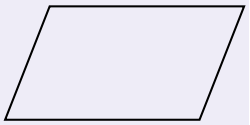
Hexagon

c.



Trapezium

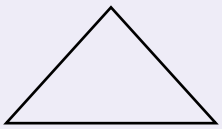
d.



Parallelogram

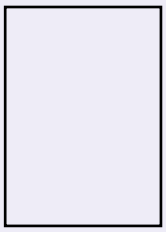
3. Using all the pieces, make and draw the following shapes.

a.



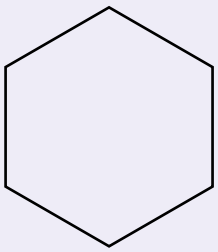
Triangle

b.



Rectangle

c.



Hexagon

4. Use all of the pieces to make the following shapes and the other shapes shown above.

a. Rocket **b.** The letter E

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Answers

Topic 5 Geometry

Exercise 5.2 Measuring angles

1. a. 40° **b.** 81° **c.** 75° **d.** 13° **e.** 142° **f.** 107°
g. 162° **h.** 103° **i.** 240° **j.** 201° **k.** 316° **l.** 356°

2. Discuss with your teacher as answers will vary.

3. a. B **b.** B

4. a. 45° **b.** 155° **c.** 180° **d.** 63°

5. a. Difficult, as angle is approximately 7° **b.** Difficult

6. a, b, c, e, f, h — various answers
d, g

Diagram	Measured angle
1	190°
2	130°
3	110°
4	40°
5	300°
6	200°
7	140°
8	290°

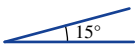
7. a. 210° (reflex angle) and 150° (obtuse angle), 180° (straight angle) and 180° (straight angle), 150° (obtuse angle) and 210° (reflex angle), 15° (acute angle) and 345° (reflex angle), 108° (obtuse) and 252° (reflex angle), 114° (obtuse angle) and 246° (reflex angle).

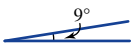
b. 360° ÷ 60 minutes = 6°; 360° ÷ 12 hours = 30°

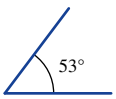
8. a. As the kite moves from point A to point B, the angle increases from 0° to 90°.

b. As the kite moves from point B to point C, the angle decreases from 90° to 0°.

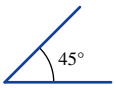
Exercise 5.3 Constructing angles with a protractor

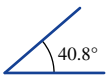
1. a. 

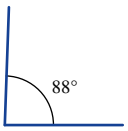
b. 

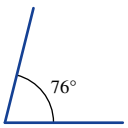
c. 

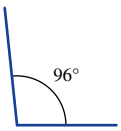
d.

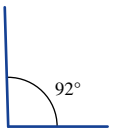
e. 

f. 

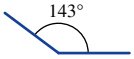
g. 

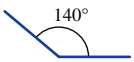
h. 

2. a. 

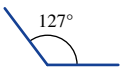
b. 

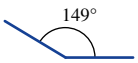
c.

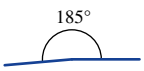
d. 

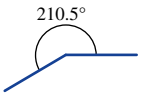
e. 

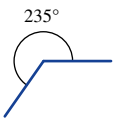
f.

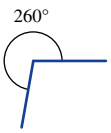
g. 

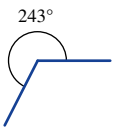
h. 

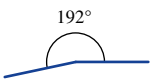
3. a. 

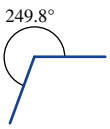
b. 

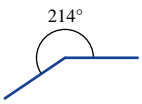
c. 

d. 

e. 

f. 

g. 

h. 

4. C

5. a. 295°

b. 269°

c. 307°

d. 349°

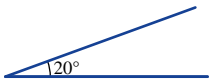
e. 328°

f. 300°

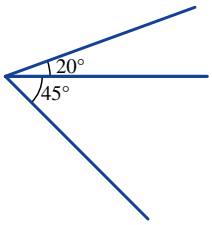
g. 345°

h. 358°

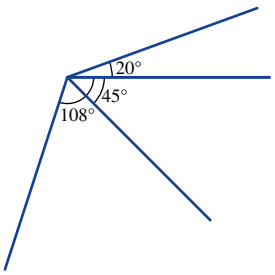
6. a.



b.



c.



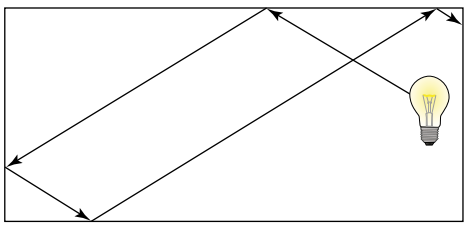
d. 187°

e. 360°

7. 120°, 12°and 90°

8. Construct the corresponding acute angle and then label the reflex angle.

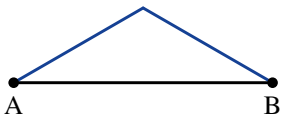
9. a.



b. Various answers possible.

c. Various answers possible.

10. a.



b. All triangles should be identical.

c. If two equal angles are drawn at the ends of a straight line of set length, there is only one size triangle that forms.

11. Draw any triangle on an A4 sheet of paper.

a. Various magnitudes possible.

d. 180°

e. The sum of the angles of a triangle is 180°.

Exercise 5.4 Types of angles and naming angles

1. a. Acute **b.** Reflex **c.** Obtuse **d.** Obtuse
e. Straight **f.** Acute **g.** Reflex **h.** Right
i. Acute **j.** Revolution **k.** Obtuse **l.** Acute
m. Acute **n.** Reflex **o.** Acute **p.** Reflex

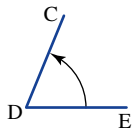
2. a. 4; 3°, 45°, 65°, 69° **b.** 2; 123°, 165° **c.** Yes, 90° **d.** Yes, 180°
e. 4; 234°, 300°, 270°, 210° **f.** Yes, 360° **g.** 25° **h.** 57°

3. a. Right **b.** Straight line **c.** Obtuse
d. Acute **e.** Reflex **f.** Revolution

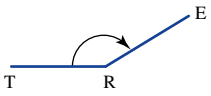
4. a. ∠XUV or ∠VUX **b.** ∠PQR or ∠RQP **c.** ∠SDX or ∠XDS
d. ∠GPZ or ∠ZPG **e.** ∠PFH or ∠HFP **f.** ∠ZRO or ∠ORZ

5. Answers will vary; however, in each case, the middle letter should be at the vertex of the angle.

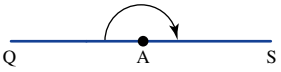
a.



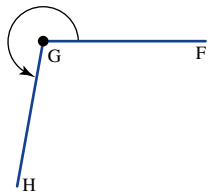
b.



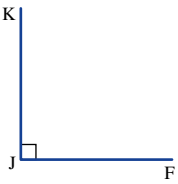
c.



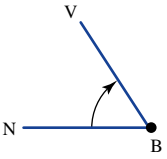
d.



e.



f.



6. a. Acute: ∠NOM, ∠MOY;
obtuse: ∠MOX

b. Acute: ∠PQS, ∠RQO;
obtuse: ∠SQR, ∠SQO, ∠RQP

c. Acute: ∠MLN, ∠NLO, ∠OLP, ∠PLK, ∠NLP, ∠OLK;
obtuse: ∠MLO, ∠MLP, ∠NLK

d. Acute: ∠POR, ∠ROT, ∠TOU, ∠UOV;
obtuse: ∠POU, ∠ROU, ∠ROV

e. Acute: ∠YOX, ∠WOQ, ∠POQ;
obtuse: ∠XOW, ∠YOP, ∠YOW, ∠XOP, ∠YOQ, ∠POW

f. Acute: ∠VOT, ∠TOR, ∠ROP, ∠POZ, ∠ZOW, ∠WOV;
obtuse: ∠VOR, ∠TOP, ∠ROZ, ∠POW, ∠ZOV, ∠WOT

g. Acute: ∠FOG, ∠GOK;
obtuse: ∠DOG

h. Acute: ∠JOH, ∠KOH, ∠POK;
obtuse: ∠JOH, ∠HOP

7. a. Acute **b.** Revolution **c.** Obtuse

8. a. Acute **b.** Right **c.** Obtuse **d.** Straight **e.** Reflex **f.** Revolution

9. a. i. Acute **ii.** Obtuse **iii.** Straight **iv.** Right **v.** Obtuse **vi.** Obtuse

b. Various answers

10. a. Acute **b.** Right **c.** Obtuse

11. In each case the middle letter should be at the vertex of the angle.

i. Acute: ∠HUT **ii.** Right: ∠ABC, ∠GHI **iii.** Obtuse: ∠DEF **iv.** Reflex: ∠MNO

12. a. If the sum of the two angles is less than 90°.

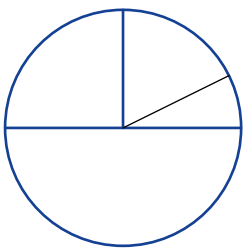
b. If the sum of the two angles is equal to 90°.

c. If the sum of the two angles is greater than 90°.

d. This statement is untrue as two acute angles can't add to more than 180°.

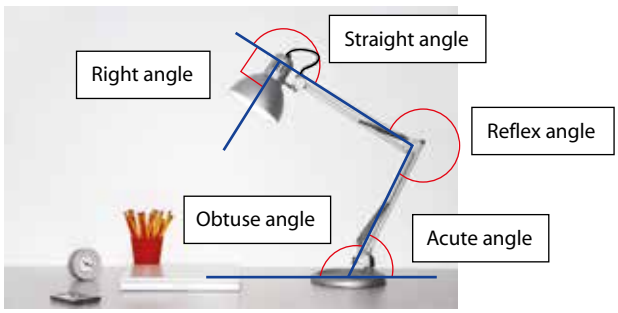
13. a. Yes

b.



14. 145°

15.



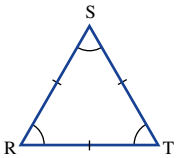
Challenge 5.1

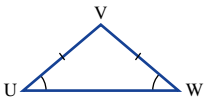
1. 20 **2.** 165°

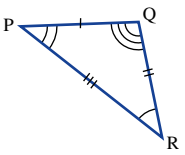
5.5 Triangles

1. a. ΔABC Scalene **b.** ΔLMN Equilateral **c.** ΔXYZ Isosceles
d. ΔPQR Isosceles **e.** ΔRST Scalene **f.** ΔEFG Isosceles

2. a. Right-angled triangle **b.** Acute-angled triangle **c.** Acute-angled triangle
d. Acute-angled triangle **e.** Obtuse-angled triangle **f.** Right-angled triangle

3. a.


b.


c.


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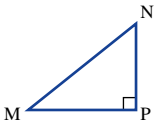
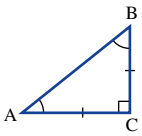
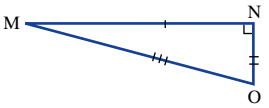


e.


f.


4. a. A **b.** B

5. a. D **b.** A

6. a. $x = 57°$ **b.** $g = 125°$ **c.** $t = 44°$ **d.** $k = 60°$ **e.** $f = 93°$ **f.** $z = 90°$

7. a. $d = 45°$ **b.** $b = 65°$ **c.** $a = 50°$

8. a. $c = 70°$ **b.** $e = 76°$ **c.** $n = 52°$

9. a. $u = 66°$ **b.** $k = 45°$ **c.** $d = 76°$ **d.** $t = 74°$ **e.** $f = 57°$ **f.** $p = 70°$

10. a. $p = 50°$; $n = 70°$ **b.** $a = 22°$; $b = 28°$ **c.** $x = 80°$; $y = 50°$ **d.** $t = 35°$; $s = 55°$
e. $b = 46°$ **f.** $n = 21°$ **g.** $m = 60°$ **h.** $t = 28°$

11. Isosceles/acute-angled triangle

12. Check with your teacher.

13. Check with your teacher.

14. a. 5 **b.** 6 **c.** 2

15. c. 10 cm (**a**, **b**, **d**; check with your teacher)

16. a. $b = 20°$ **b.** $k = 30°$

17. a. 70° **b.** 156° **c.** 50° **d.** 64°

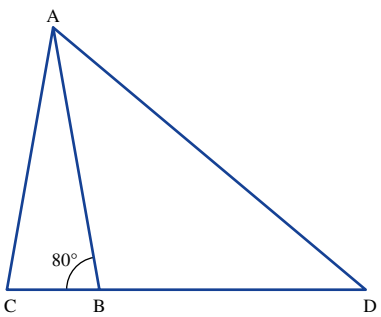
18. Check with your teacher.

19. a. Yes, angles add to 180°. **b.** No, angles add to 175°.
c. No, angles add to 185°. **d.** Yes, angles add to 180°.

20. The sum of the angles will be greater than 180°.

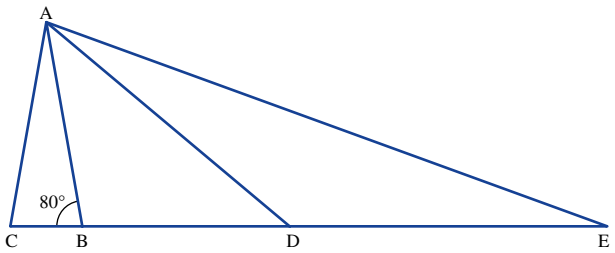
21. a. 42
b. 12 scalene triangles, 6 equilateral triangles and 24 isosceles triangles
c. Yes
d. The largest angle is always opposite the longest side (likewise, the smallest angle is opposite the shortest side and the middle size angle is opposite the middle length side). Also, angles that are opposite to sides of equal length are equal in size.
e. The sum of any two side lengths must be greater than the length of the third side.

22. a.

b.


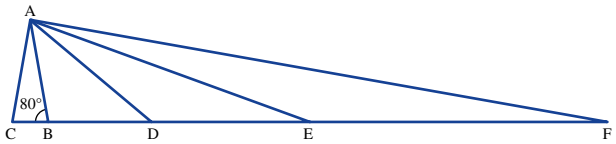
c. 40°

d.



e. 20°

f.



g. 10°

h. The angle is halved every time a new triangle is constructed.

23. a. ∠MPN = 72° and ∠NMP = 36° **b.** ∠MPN = 54° and ∠MNP = 54°

24. a. Use a protractor to measure the three angles of the triangle, and record these values in a table like the one shown.

Triangle	$\angle A$	$\angle B$	$\angle C$	Type of triangle
1	90°	25°	65°	Right-angled
2	90°	Various values possible	Various values possible	Right-angled

b. Various possible diagrams

c. As part **a**. **d.** As part **b**. **e.** As part **a**. **f.** As part **b**.

g. Any triangle formed with one of its sides as the diameter of the circle and the three vertices on the circle is a right-angled triangle.

Challenge 5.2

18

5.6 Quadrilaterals and their properties

1. a. Square **b.** Trapezium **c.** Kite

d. Rhombus **e.** Rectangle **f.** Irregular quadrilateral

2. a. E **b.** A **c.** E

3. a. $t = 138°$ **b.** $b = 85°$ **c.** $t = 90°$

d. $t = 202°$ **e.** $p = 92°$ **f.** $m = 90°$

g. $s = 53°$ **h.** $c = 66°$ **i.** $k = 67°$

4. a. $m = 78°$ (opposite angles of parallelogram equal) **b.** $u = 75°$ (opposite angles of rhombus equal)

c. $t = 132°$ (opposite angles of a kite equal) **d.** $f = 108°$ (opposite angles of a kite equal)

e. $p = 63°$ (opposite angles of rhombus equal) **f.** $z = 73°$ (opposite angles of parallelogram equal)

5. a. $m = 113°$ **b.** $c = 98°; d = 82°$ **c.** $t = 116°$

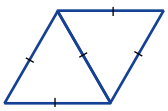
d. $p = 52°; m = 106°$ **e.** $t = 103°; x = 91°$ **f.** $m = 65°; p = 115°; n = 65°$

6. D **7.** E

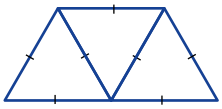
8. a. True **b.** True **c.** False **d.** True **e.** True

9. D

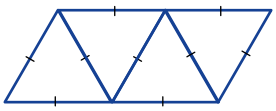
10. a.



b.



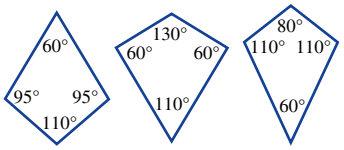
c.



11.

12. Check with your teacher. **13.** $p = 61°$ **14.** 115° **15.** 45° and 135°

16.



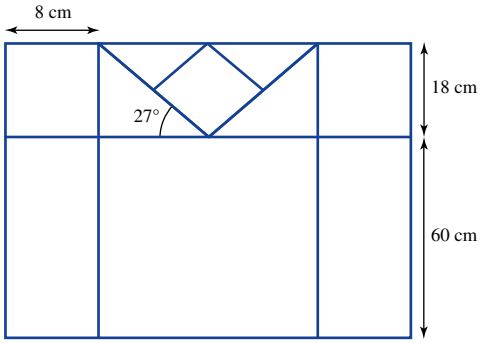
17. a. Yes, angles add up to 360°. **b.** No, angles add up to 270°.

18. a. 130° **b.** Infinite number **c.** Various answers

19. a. Two smaller obtuse-angled isosceles triangles; one larger size obtuse-angled isosceles triangle and two right-angled scalene triangles.

b. Two types: rectangles (of different sizes) and a rhombus.

c.



d. In both right-angled triangles angles are: 27°, 63° and 90°; in all isosceles triangles angles are: 126°, 27°, 27°; in a rhombus angles are: 126°, 54°, 126°, 54°; in all rectangles all angles are 90° each.

e. Various answers

20. Consider the sum of the angles of the two triangles ABC and ACD.

In ΔABC: $n + p + q = 180°$

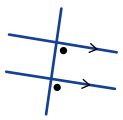
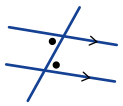
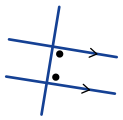
In ΔACD: $m + r + s = 180°$

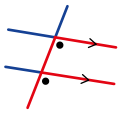
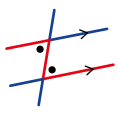
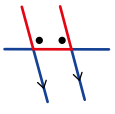
In quadrilateral ABCD: $n + p + q + m + r + s = 360°$

21. 1440° **22.** 102°

Exercise 5.7 Parallel and perpendicular lines

1.

Diagram	Type of angle	Associated shape	Rule
	Corresponding	F	Are equal
	Alternate	Z	Are equal
	Co-interior	C	Add to 180°

2. a.  **b.**  **c.** 

3. a. C **b.** D **c.** A **d.** B

4. a. a and d, c and b, e and h, f and g **b.** a and e, c and g, b and f, d and h

c. c and e, d and f **d.** c and f, d and e

5. a. C **b.** D **c.** B **d.** A **e.** D

6. a. i. Corresponding angles **ii.** $p = 60°$

b. i. Alternate angles **ii.** $q = 45°$

c. i. Vertically opposite angles **ii.** $s = 65°$

d. i. Corresponding angles **ii.** $t = 72°$

e. i. Co-interior angles **ii.** $m = 110°$

f. i. Co-interior angles **ii.** $n = 48°$

7. a. $y = 116°$, corresponding **b.** $z = 38°$, alternate

c. $b = 136°$, supplementary **d.** $g = 68°$, vertically opposite

e. $h = 90°$, corresponding **f.** $k = 120°$, alternate

8. a. i. $b = 123°$ **ii.** $x = 43°$ **iii.** $y = 28°$ **iv.** $z = 50°$ **v.** $p = 70°$ **vi.** $q = 45°$

b. In **iii**, AH $\perp$ GC **v**, PW $\perp$ VR **vi**, NR $\perp$ QV, MS $\perp$ VQ

9. $x = 45°$

10. $y = 55°$

11. 60°

12. a. 130° **b.** 50°

13. No, as $48° + 133° \neq 180°$. For parallel lines, co-interior angles need to add to 180° (or alternate angles must be equal).

14. Check with teacher.

15. Acute $\angle PQR = 67°$ and reflex $\angle PQR = 293°$.

16. a. The vertical lines are parallel, the horizontal lines are parallel and the oblique lines are parallel because they are equally distant from each other.

b. The vertical lines are parallel and the horizontal lines are parallel because they are equally distant from each other.

5.8 Review

1. a. 42° **b.** 105° **c.** 298°

2. Constructions

3. a. Revolution **b.** Right angle **c.** Straight line

d. Obtuse **e.** Reflex **f.** Acute

4. C **5.** A **6.** B **7.** D

8. $\angle EDF$ or $\angle FDE$

9. a. Equilateral triangle **b.** Scalene triangle **c.** Isosceles triangle

10. a. Obtuse-angled triangle **b.** Acute-angled triangle **c.** Right-angled triangle

11. a. $t = 67°$ **b.** $b = 28°$ **c.** $x = 75°$

12. a. $p = 56°$ **b.** $m = 64°$ **c.** $n = 60°$

13. a. Rectangle **b.** Parallelogram **c.** Trapezium

d. Kite **e.** Square **f.** Rhombus

14. a. 75° **b.** 55° **c.** 42°

15. a. 66° **b.** 112° **c.** 126°

16. a. $x = 150°$, vertically opposite angles are equal

b. $x = 50°$, corresponding angles are equal, angle sum of a straight line is 180°; $y = 25°$, corresponding angles are equal

c. $y = 100°$, co-interior angles add to 180°; $x = 16°$, angles opposite the equal sides of an isosceles triangle are equal, angle sum of a triangle is 180°

17. a. A, B **b.** A, E **c.** C, E **d.** C,E

18. a. i. Alternate angles

ii. $x = 60°$

b. i. Vertically opposite angles

ii. $y = 135°$

c. i. Vertically opposite angles then co-interior angles, then vertically opposite angles

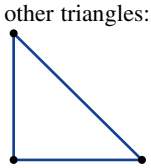
ii. $t = 50°$

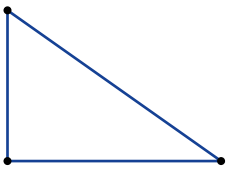
Problem solving

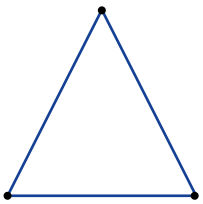
19. 67°

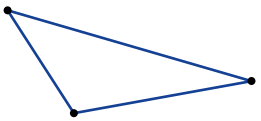
20. 115°

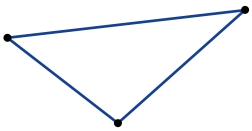
21. As an equilateral triangle has 3 congruent angles, right-equilateral and obtuse-equilateral are not possible. Examples of the other triangles:

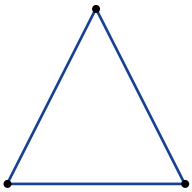

Right-isosceles

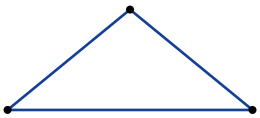

Right-isosceles


Acute-isosceles


Obtuse-scalene


Acute-scalene


Acute-equilateral


Obtuse-isosceles

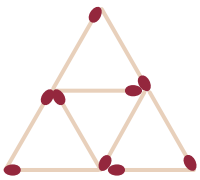
22. As the base and height of the triangle do not change as X moves along the fence line, the area of the triangle won't change. The volume of soil needed won't change.

23. When he first arranges the timber, the opposite sides are congruent. He has made a parallelogram. A rectangle can be thought of as a parallelogram with equal diagonals. Hence, it is necessary to measure the diagonals to verify that a rectangle has been created.

24. 4, 5 and 6

25. 24

26.



27.

28. 165°

29. 4

30. ∠BOE, ∠BOD, ∠BOC, ∠COE, ∠COD, ∠DOE

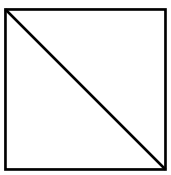
31. 35°

32. a. 15° **b.** AD ⊥ CD, CD ⊥ BC, AB ⊥ AD, AB ⊥ BC

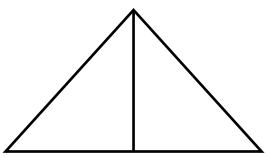
33. 35°

Investigation — Rich task

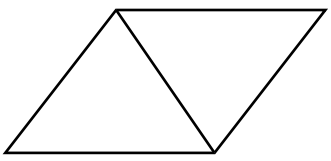
1. a.



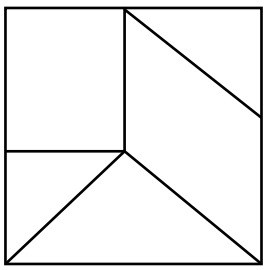
b.



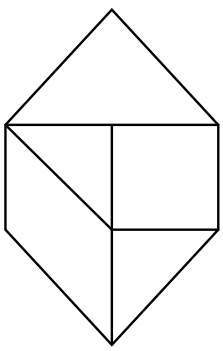
c.



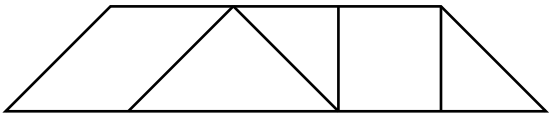
2. a.



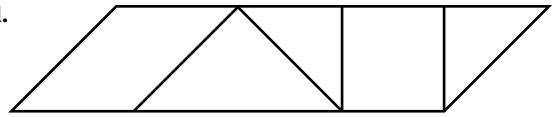
b.



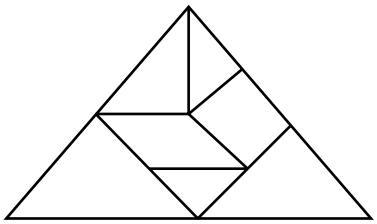
c.



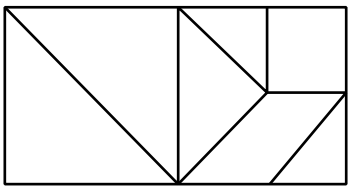
d.



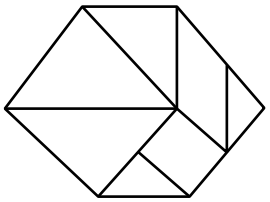
3. a.



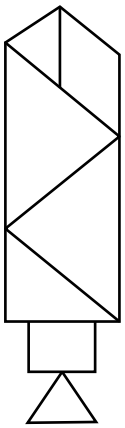
b.



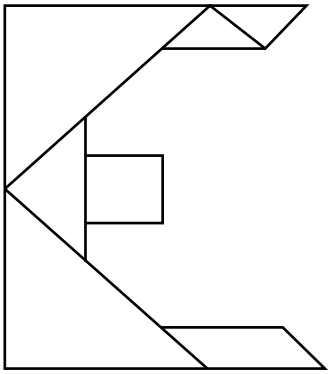
c.



4. a.



b.



TOPIC 6 Decimals

6.1 Overview

Numerous **videos** and **interactivities** are embedded just where you need them, at the point of learning, in your learnON title at www.jacplus.com.au. They will help you to learn the content and concepts covered in this topic.

6.1.1 Why learn this?

Decimals allow us to represent fractions and percentages in a different way. They are important in calculations with parts of a whole. Decimals are commonly used in business, shopping, measuring and sport. When Cathy Freeman won a gold medal at the Sydney Olympics, the winning margin was just 0.02 of a second.

6.1.2 What do you know?

assessON

1. **THINK** List what you know about decimals. Use a thinking tool such as a concept map to show your list.
2. **PAIR** Share what you know with a partner and then with a small group.
3. **SHARE** As a class, create a thinking tool such as a large concept map that shows your class's knowledge of decimals.

LEARNING SEQUENCE

6.1 Overview
6.2 Place value and comparing decimals
6.3 Converting decimals to fractions and fractions to decimals
6.4 Rounding and repeating decimals
6.5 Adding and subtracting decimals
6.6 Multiplying decimals (including by multiples of 10)
6.7 Dividing decimals (including by multiples of 10)
6.8 Review

6.2 Place value and comparing decimals

6.2.1 Whole numbers

- The position of a digit within a number indicates the value of the digit.
- The further the digit is to the left of the number, the larger the place value.

Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Units
100 000	10 000	1000	100	10	1

- Each place to the left of another has a value which is 10 times larger.
- Each place to the right of another has a value which is $\frac{1}{10}$ of the previous position.
- In the following numbers, look at the value of the 3.

Number	Value of 3 in number
132	3 tens or 30
3217	3 thousands or 3000
4103	3 units (ones) or 3

6.2.2 Decimal parts

- Whole numbers have units as their smallest place value.
- To show values smaller than units, a decimal is placed after the units.
- The value of the positions to the left and right of the decimal point are shown in the table below.

Thousands	Hundreds	Tens	Units	.	Tenths	Hundredths	Thousandths	Ten thousandths
1000	100	10	1	.	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$	$\frac{1}{10\,000}$

- In the following numbers, look at the value of the 3.

Number	Value of 3 in number
14.32	The 3 is 3 tenths or $\frac{3}{10}$
106.013	The 3 is 3 thousandths or $\frac{3}{1000}$
0.000 03	The 3 is 3 hundred thousandths or $\frac{3}{100\,000}$

WORKED EXAMPLE 1

Find the value of the 7 in each of the following
a 10.74 **b 0.173** **c 321.037**

THINK	WRITE
a The value of the first place to the right of the decimal point is tenths, so the digit is tenths.	**a** $\frac{7}{10}$
b The second place after the decimal point is hundredths, so the digit is hundredths.	**b** $\frac{7}{100}$
c The digit is in the thousandths place, so the digit is thousandths.	**c** $\frac{7}{1000}$

WORKED EXAMPLE 2

For the number 76.204 write the value of each digit in words and numbers

THINK	WRITE
1 7 is in the tens position.	Seventy, 70
2 6 is in the units position.	Six, 6
3 2 is in the first position after the decimal point, so it is tenths.	Two tenths, $\frac{2}{10}$
4 0 is in the hundredths position.	Zero hundredths, $\frac{0}{100}$
5 4 is in the thousandths position.	Four thousandths, $\frac{4}{1000}$

- The number 76.204 can be written in expanded notation as:

$$(7 \times 10) + (6 \times 1) + \left(2 \times \frac{1}{10}\right) + \left(4 \times \frac{1}{1000}\right)$$

WORKED EXAMPLE 3

Write 3.4501 in expanded notation.

THINK	WRITE
1 Write the decimal.	3.4501

2 Find the place value of each digit.

3:3 units = 3, 4:4 tenths = $\frac{4}{10}$,

5:5 hundredths = $\frac{5}{100}$,

0:0 thousandths = $\frac{0}{1000}$,

1:1 ten thousandth = $\frac{1}{10\,000}$.

= 3 units + 4 tenths + 5 hundredths + 0 thousandths + 1 ten thousandth

$= 3 + \frac{4}{10} + \frac{5}{100} + \frac{0}{1000} + \frac{1}{10000}$

3 Write the number in expanded notation.

$= (3 \times 1) + (4 \times \frac{1}{10}) + (5 \times \frac{1}{100}) + (1 \times \frac{1}{10\,000})$

6.2.3 Comparing decimals

- To compare the size of numbers that include decimal digits ('decimal numbers' or 'decimals'), it is necessary to compare the size of the digits with the same place value.
- First, compare the whole number part (digits to the left of the decimal point). If they are the same for each number, move to the first digit after the decimal point, then the second, the third and so on until the digits are different. The larger digit will belong to the larger number.

WORKED EXAMPLE 4

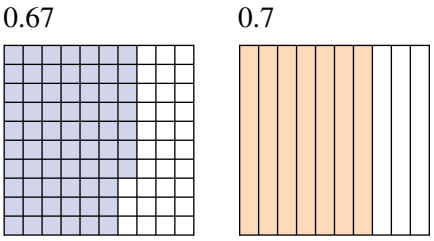
Find the largest number in each of the following.

a 0.126, 0.216, 0.122 **b 2.384, 2.388, 2.138** **c 0.506, 0.605, 0.612**

THINK	WRITE
a 1 As the units digit is 0 in each number, compare the tenths. The number 0.216 has 2 tenths, the others have 1 tenth so 0.216 is the largest number.	**a** 0.216 is larger than 0.126 and 0.122.
2 Answer the question.	The largest number is 0.216.
b 1 As the units digits are the same, compare the tenths and eliminate the smallest number.	**b** 2.384 and 2.388 are both larger than 2.138.
2 The hundredths are the same so compare the thousandths and decide which number is bigger.	2.388 is larger than 2.384 and 2.138.
3 Answer the question.	The largest number is 2.388.
c 1 As the unit digit is 0 compare the tenths and eliminate the smallest number.	**c** 0.605 and 0.612 are larger than 0.506.
2 Compare the hundredths and find the biggest number.	0.612 is larger than 0.605 and 0.506.
3 Answer the question.	The largest number is 0.612.

- Decimals can also be compared using the *area model*. For example, we could use the area model to determine whether 0.67 or 0.7 is larger. From the following area model, it can be seen that 0.7 covers a greater area than 0.67 and is therefore larger.

- When comparing two numbers, it is easier to use symbols instead of words, as shown in Topic 4 on fractions.
- In Worked example 4, the result '0.216 is larger than 0.126' could be written as $0.216 > 0.126$. We could also say that 0.126 is less than 0.216, or write it as $0.126 < 0.216$.



WORKED EXAMPLE 5

Insert the appropriate < or > sign between the following pairs of numbers to make true statements.

a **0.312 0.318** **b** **0.0246 0.0168**

THINK	WRITE
a Compare the numbers. Both numbers have the same number of tenths and the same number of hundredths, so compare thousandths and insert the correct sign.	**a** $0.312 < 0.318$
b Compare the numbers. Both numbers have no tenths, so compare hundredths and insert the correct sign.	**b** $0.0246 > 0.0168$

learnON RESOURCES — ONLINE ONLY

Watch this eLesson: Place value (eles-0004)

Complete this digital doc: SkillSHEET: Writing decimals (doc-6458)

Complete this digital doc: SkillSHEET: Comparing decimals (doc-6459)

Exercise 6.2 Place value and comparing decimals

assessON

Individual pathways

PRACTISE
Questions:
1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 18

CONSOLIDATE
Questions:
1b, c, f, g, j, k, 2b, c, f, g, j, k, 3c, d, f 4c, d, g, h, k, l, 5c, d, g, h, k, l, 6, 7, 8a, c, e, g, i, k, 9d, e, j, 10a, g, l, 11a, g, l, 12, 13, 14, 16, 18

MASTER
Questions:
1a, e, l, 2b, f, j, 4d, h, i, 5d, h, l, 6, 8b, k, 9c, l, 10g, l, 11g, l, 12–19

Individual pathway interactivity: int-4337

learnON ONLINE ONLY

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Fluency

1. WE1 Find the value of the 2 in each of the following.

a. 5.2	**b.** 19.12	**c.** 0.02	**d.** 100.29
e. 0.982 047	**f.** 491.7521	**g.** 6.1342	**h.** 90.0002
i. 27.003	**j.** 12.14	**k.** 1.8902	**l.** 897.014 12

2. Find the value of the 9 in each of the following.

a. 0.9 **b.** 14.98 **c.** 6.1937 **d.** 18.89
e. 12.090 **f.** 0.009 14 **g.** 3.4629 **h.** 1.276 89
i. 39.214 **j.** 9 **k.** 900.76 **l.** 90.612

3. WE2 For the following numbers write the value of each digit in *words* and *numbers*.

a. 4.1 **b.** 1.85 **c.** 0.271 **d.** 9.020
e. 16.001 **f.** 3.402 07

4. For each of the following numbers write the value of each digit in *numbers*.

a. 0.4 **b.** 2.7 **c.** 6.80 **d.** 5.23 **e.** 0.763
f. 2.108 **g.** 19.910 **h.** 0.1101 **i.** 7.2964 **j.** 0.330 24
k. 300.03 **l.** 12.276 403

5. WE3 Write the following numbers in expanded notation.

a. 2.47 **b.** 3.69 **c.** 1.25 **d.** 56.01 **e.** 39.01
f. 16.07 **g.** 7.123 **h.** 5.987 **i.** 13.482 **j.** 0.3062
k. 0.1305 **l.** 0.5002

6. MC **a.** Four tenths, 1 hundredth and 3 thousandths equals:

A. 4.13 **B.** 3.14 **C.** 0.413 **D.** 0.314 **E.** 314

b. Five hundredths, 2 thousandths and 7 ten thousandths equals:

A. 527 **B.** 52.7 **C.** 5.27 **D.** 0.0527 **E.** 0.527

7. Copy and complete the table by putting only one digit in each box.

		Tens	Units		Tenths	Hundredths	Thousandths
Example	37.684	3	7	.	6	8	4
a	0.205			.			
b	1.06			.			
c	74.108			.			
d	0.108			.			
e	50.080			.			

8. WE4 Find the largest number in each of the following.

a. 0.24, 0.32, 0.12 **b.** 0.76, 0.68, 0.91
c. 0.57, 0.51, 0.59 **d.** 0.92, 0.99, 0.93
e. 0.192, 0.191, 0.901 **f.** 0.660, 0.666, 0.662
g. 0.1440, 0.1436, 0.1433 **h.** 0.0392, 0.039 90, 0.0039
i. 2.506, 2.305, 2.599 **j.** 10.0023, 10.0231, 10.0233
k. 0.110 43, 0.110 49, 0.110 40 **l.** 0.102 36, 0.100 23, 0.101 09

9. WE5 Insert the appropriate $<$ or $>$ sign between each of the following pairs of numbers to make true statements.

a. 3.2 2.9 **b.** 8.6 8.9 **c.** 1.27 1.28 **d.** 0.64 0.67
e. 1.37 1.41 **f.** 0.29 0.39 **g.** 13.103 13.112 **h.** 0.427 0.424
i. 0.580 0.508 **j.** 0.0101 0.0120 **k.** 0.048 01 0.4801 **l.** 1.3830 1.3824

10. Write the following in order from smallest to largest (ascending order).

a. 0.21, 0.39, 0.17, 0.45, 0.33 **b.** 0.16, 0.19, 0.27, 0.12, 0.22
c. 0.314, 0.413, 0.420, 0.391, 0.502 **d.** 0.613, 0.624, 0.690, 0.710, 0.677
e. 0.821, 0.803, 0.811, 0.807, 0.902 **f.** 0.1164, 0.1245, 0.1033, 0.1002, 0.1196
g. 0.9864, 0.9812, 0.9943, 0.9087, 0.9189 **h.** 0.4004, 0.4139, 0.4826, 0.4100, 0.4076
i. 4.6249, 4.5097, 4.802, 4.6031, 4.0292 **j.** 13.0294, 13.0291, 13.0229, 13.0299, 13.0929
k. 0.004 65, 0.005 02, 0.003, 0.0056, 0.009 **l.** 0.507, 0.61, 0.595, 0.5079, 0.617

11. Write the following in order from largest to smallest (descending order).

a. 0.36, 0.31, 0.39, 0.48, 0.19
b. 0.27, 0.38, 0.16, 0.02, 0.35
c. 0.91, 0.97, 0.90, 0.95, 0.99
d. 0.02, 0.29, 0.07, 0.13, 0.09
e. 1.264, 1.279, 1.273, 1.291, 1.288
f. 0.442, 0.437, 0.491, 0.406, 0.433
g. 0.372, 0.318, 0.390, 0.309, 0.317
h. 0.502, 0.556, 0.573, 0.602, 0.591
i. 0.8207, 0.8889, 0.8823, 0.8217, 0.8448
j. 0.7657, 0.6024, 0.0307, 0.1079, 0.7695
k. 1.349 54, 1.486 59, 1.702 96, 1.843 21, 1.486 13
l. 12.289 50, 12.208 64, 12.392, 12.002 36, 12.9092

12. MC The largest number in the following list; 0.4261, 0.4265, 0.4273, 0.4199, 0.3999 is:

A. 0.4261 **B.** 0.4199 **C.** 0.4265 **D.** 0.3999 **E.** 0.4273

13. MC The smallest number in the following list; 0.4261, 0.4265, 0.4273, 0.4199, 0.3999 is:

A. 0.4261 **B.** 0.4199 **C.** 0.4265 **D.** 0.3999 **E.** 0.4273

14. MC The following list; 0.4261, 0.4265, 0.4273, 0.4199, 0.3999 when arranged from smallest to largest is:

A. 0.4273, 0.4265, 0.4261, 0.4199, 0.3999
B. 0.4273, 0.4261, 0.4265, 0.4199, 0.3999
C. 0.3999, 0.4199, 0.4265, 0.4261, 0.4273
D. 0.3999, 0.4199, 0.4261, 0.4273, 0.4265
E. 0.3999, 0.4199, 0.4261, 0.4265, 0.4273

Understanding

15. For each of the following numbers:

i. state the place value of the zero.
ii. would the value of the number change if the zero wasn't there? (Write yes or no.)

a. 6.02 **b.** 10.49 **c.** 7.360 **d.** 13.10 **e.** 4.0
f. 133.027 **g.** 0.65 **h.** 17.809 **i.** 20 **j.** 108.62

16. Write True (T) or False (F) for each of the following.

a. 76.34 has 4 decimal places.
b. $\frac{6}{10} + \frac{3}{100} + \frac{4}{10\,000}$ is the same as 0.6304.
c. 4.03 has the same value as 4.3.
d. 29.60 has the same value as 29.6.
e. 1.2804 could be written as $1 + \frac{3}{10} + \frac{8}{100} + \frac{4}{1000}$.
f. 1090.264 51 has 5 decimal places.

Reasoning

17. Year 7 girls competing in their school swimming sports recorded the following times in the 50-metre freestyle, backstroke and breaststroke events.

	Time (seconds) recorded by contestants						
Event	**Carolyn**	**Jessica**	**Mara**	**Jenika**	**Robyn**	**Shelley**	**Kyah**
Freestyle	37.23	39.04	40.90	38.91	37.45	37.02	37.89
Backstroke	40.23	43.87	44.08	42.65	41.98	40.29	41.05
Breaststroke	41.63	42.70	41.10	41.21	42.66	41.33	41.49

a. Who won the freestyle event? How much did she win it by?
b. Who won the backstroke event? How much did she win it by?
c. Who won the breaststroke event? How much did she win it by?

d. List the first 3 placings of the freestyle event.
e. List the first 3 placings of the backstroke event.
f. List the first 3 placings of the breaststroke event.
g. Did any students obtain a placing in all three events?

Problem solving

18. What is my number?
My number contains four different even digits and has three decimal places. The digit in the thousandths position is half the value of the digit in the units position. The sum of the digits in the units and tenths positions is the same as the sum of the digits in the hundredths and thousandths positions. The digit in the thousandths position is bigger than the digit in the tenths position.

19. a. Calculate the decimal values of the following fractions.

$$\frac{1}{7}, \frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}$$

Write as many decimals as possible.
b. Is there a pattern? Explain.

Reflection

What strategies will you use to find the smallest decimal?

6.3 Converting decimals to fractions and fractions to decimals

6.3.1 Converting decimals to fractions

- Decimals can be written as single fractions by using place values.

Number	Ones	Tenths	Hundredths
0.3	0.	3	0
1.25	1.	2	5

- The decimal 0.3 can be written as $\frac{3}{10}$.
- The decimal 1.25 can be thought of as $1 + \frac{2}{10} + \frac{5}{100} = 1 + \frac{20}{100} + \frac{5}{100} = 1\frac{25}{100}$.
- In practice, the digits after the decimal point become the numerator and the place value of the last digit gives the denominator.
- Once decimals have been written as fractions or mixed numbers, they can be simplified by dividing the denominator and numerator by the highest common factor.
- Sometimes the highest common factor may not be obvious, and extra steps may be required to obtain the simplest form. For example, when simplifying $1\frac{25}{100}$ the following steps could be used: $1\frac{25}{100} = 1\frac{5}{20} = 1\frac{1}{4}$.

WORKED EXAMPLE 6

Write the following decimals as fractions, then simplify where appropriate.
a 0.2 **b 0.86** **c 0.6021**

THINK	WRITE
a 1 Write the decimal.	**a** 0.2
2 The numerator is 2 and the last decimal place is tenths so the denominator is 10.	$= \frac{2}{10}$
3 Divide the numerator and the denominator by the highest common factor.	$= \frac{1}{5}$
4 Simplify the fraction.	
b 1 Write the decimal.	**b** 0.86
2 The numerator is 86. The last decimal place is hundredths so the denominator is 100.	$= \frac{86}{100}$
3 Repeat steps **3** and **4** of part **a**.	$= \frac{43}{50}$
c 1 Write the decimal.	**c** 0.6021
2 The numerator is 6021. The last place is tens of thousandths so the denominator is 10 000.	$= \frac{6021}{10\,000}$

WORKED EXAMPLE 7

Write each of the following as a mixed number in its simplest form.
a 3.041 **b 7.264**

THINK	WRITE
a 1 Write the decimal.	**a** 3.041
2 Write the whole number part and change the decimal part to a fraction. The numerator is 41. The last decimal place is thousandths so the denominator is 1000.	$= 3\frac{41}{1000}$
b 1 Write the decimal.	**b** 7.264
2 Write the whole number part and change the decimal part to a fraction. The numerator is 264 and the denominator is 1000.	$= 7\frac{264}{1000}$
3 Divide the numerator and the denominator by the highest common factor.	$= 7\frac{264 \div 8}{1000 \div 8}$
4 Simplify the fraction.	$= 7\frac{33}{125}$

6.3.2 Converting fractions to decimals

- To change any fraction into a decimal, divide the denominator into the numerator. For example, to change $\frac{1}{4}$ into a decimal, divide 4 into 1:

  $4\overline{)1.00}$ add a decimal point and as many zeros as required.

WORKED EXAMPLE 8

Change the following fractions into finite decimals.

a $\frac{2}{5}$ **b** $\frac{1}{8}$

THINK	WRITE
a 1 Set out the question as for division of whole numbers, adding a decimal point and the required number of zeros. *Note:* $\frac{2}{5} = 2 \div 5$.	**a** $5\overline{)2.0}$
2 Divide, writing the answer with the decimal point exactly in line with the decimal point in the question.	$\frac{2}{5} = 0.4$
b 1 Set out the question as for division of whole numbers, adding a decimal point and the required number of zeros. *Note:* $\frac{1}{8} = 1 \div 8$.	**b** $8\overline{)1.000}$
2 Divide, writing the answer with the decimal point exactly in line with the decimal point in the question.	$\frac{1}{8} = 0.125$

- By knowing the decimal equivalent of any fraction, it is possible to determine the equivalent of any multiple of that fraction. The following worked example illustrates this.

WORKED EXAMPLE 9

Use the results of Worked example 8 to find decimal equivalents for:

a $\frac{3}{8}$ **b** $4\frac{5}{8}$

THINK	WRITE
a 1 Write the decimal equivalent for the fraction with 1 as the numerator.	**a** $\frac{1}{8} = 0.125$
2 Multiply both sides of this equation by the appropriate multiple (3 in this case).	$\frac{1}{8} \times 3 = 0.125 \times 3$
3 Write the answer.	$\frac{3}{8} = 0.375$
b 1 Consider only the fraction part of the mixed number. Write the decimal equivalent of this fraction with 1 as the numerator.	**b** $\frac{1}{8} = 0.125$
2 Multiply both sides of this equation by the appropriate multiple (5 in this case).	$\frac{1}{8} \times 5 = 0.125 \times 5$
3 Simplify both sides.	$\frac{5}{8} = 0.625$
4 Combine with the whole number and write the answer.	$4\frac{5}{8} = 4.625$

Exercise 6.3 Converting decimals to fractions and fractions to decimals

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Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–6, 8	Questions: 1–8	Questions: 1–9

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Fluency

1. WE6 Write the following decimals as fractions, then simplify where appropriate.

a. 0.3	**b.** 0.5	**c.** 0.9	**d.** 0.21	**e.** 0.4
f. 0.8	**g.** 0.24	**h.** 0.44	**i.** 0.49	**j.** 0.63
k. 0.502	**l.** 0.617	**m.** 0.12	**n.** 0.30	**o.** 0.64
p. 0.28	**q.** 0.882	**r.** 0.9456	**s.** 0.9209	**t.** 0.4621
u. 0.75	**v.** 0.120	**w.** 0.286	**x.** 0.468	

2. WE7 Write the following decimals as mixed numbers in simplest form.

a. 1.3	**b.** 1.6	**c.** 2.7	**d.** 9.4	**e.** 1.2
f. 2.8	**g.** 4.2	**h.** 8.5	**i.** 2.13	**j.** 6.48
k. 5.27	**l.** 19.182	**m.** 12.42	**n.** 3.15	**o.** 6.25
p. 9.140	**q.** 12.843	**r.** 16.682	**s.** 2.4917	**t.** 4.3386
u. 37.205	**v.** 18.645	**w.** 24.345	**x.** 100.0048	

3. MC **a.** 0.13 as a fraction is:

A. $\frac{13}{10}$ **B.** $\frac{13}{100}$ **C.** $\frac{13}{1000}$ **D.** $\frac{1.3}{100}$ **E.** $1\frac{3}{10}$

b. 0.207 as a fraction is:

A. $\frac{207}{1000}$ **B.** $\frac{207}{100}$ **C.** $2\frac{7}{10}$ **D.** $20\frac{7}{10}$ **E.** 207

c. 0.52 as a fraction in simplest form is:

A. $\frac{52}{100}$ **B.** $\frac{26}{50}$ **C.** $\frac{13}{25}$ **D.** $\frac{26}{100}$ **E.** $\frac{13}{50}$

d. 0.716 as a fraction in simplest form is:

A. $\frac{716}{10\,000}$ **B.** $\frac{368}{500}$ **C.** $\frac{716}{1000}$ **D.** $\frac{179}{250}$ **E.** $\frac{358}{1000}$

e. 5.325 as a fraction in simplest form is:

A. $\frac{5325}{1000}$ **B.** $\frac{325}{1000}$ **C.** $5\frac{325}{1000}$ **D.** $5\frac{65}{200}$ **E.** $5\frac{13}{40}$

4. WE8, 9 Change the following fractions to decimals.

a. $\frac{3}{4}$	**b.** $\frac{1}{2}$	**c.** $\frac{4}{5}$	**d.** $\frac{1}{20}$	**e.** $\frac{3}{15}$
f. $\frac{3}{12}$	**g.** $\frac{3}{8}$	**h.** $\frac{1}{50}$	**i.** $\frac{8}{25}$	

Understanding

5. Write $\frac{1}{8}$ as a decimal. Using this value, find:

a. $\frac{3}{8}$ as a decimal **b.** $\frac{7}{8}$ as a decimal **c.** $\frac{1}{16}$ as a decimal.

Reasoning

6. Consider these fractions:

$$\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{7}, \frac{1}{8}, \frac{1}{9}$$

Without performing any division, you can see that $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{5}$ and $\frac{1}{8}$ will produce terminating decimals, while $\frac{1}{3}$, $\frac{1}{6}$, $\frac{1}{7}$ and $\frac{1}{9}$ will produce repeating decimals.

Explain how this can be seen, and write a general statement to determine whether a fraction will produce a terminating or repeating decimal.

Problem solving

7. The decimal equivalents of $\frac{1}{7}, \frac{2}{7}, \frac{3}{7} \ldots \frac{6}{7}$ can be found in the sequence 142857142857142857…

If $\frac{1}{7} = 0.\dot{1}42857\dot{}$ and $\frac{2}{7} = 0.\dot{2}85714\dot{}$, without using a calculator, write $\frac{3}{7}$, $\frac{4}{7}$, $\frac{5}{7}$ and $\frac{6}{7}$ as decimals.

8. You are competing in a long jump contest. Your first jump was 3.78 m, your second jump was $3\frac{8}{9}$ m and your third jump is $3\frac{4}{5}$ m. You are congratulated on your third jump, as being your best. Do you agree? Explain.

9. In a survey, 24 people were asked: 'What is the maximum distance you would walk to a train station?' A person who says '800 m' will walk up to 800 m, but no more. The survey responses are:

100 m	200 m	250 m	1 km	$\frac{1}{2}$ km	$\frac{3}{4}$ km
600 m	500 m	450 m	100 m	1.2 km	800 m
1.5 km	1.4 km	200 m	300 m	1.2 km	350 m
900 m	750 m	300 m	650 m	320 m	100 m.

What is the longest distance (as a multiple of 100 m) that at least $\frac{3}{4}$ of the people would walk to the train station?

Reflection

If you need to choose between a fraction and a decimal, when is a fraction a better choice and when is a decimal a better choice?

6.4 Rounding and repeating decimals

6.4.1 Rounding

- Rounding means to give your answer correct to a certain number of decimal places.
- When **rounding** decimals, look at the first digit after the number of decimal places required.
- If this digit is *less than 5*, write the number without any change.
- If this digit is *5 or more*, add one to the digit in the last decimal place.

WORKED EXAMPLE 10

Round the following to 2 decimal places.
a 3.641 883 **b 18.965 402 0**

THINK

a 1 Write the number and underline the required decimal place.

2 Circle the next digit and round according to the rule.
Note: Since the circled digit is less than 5, we leave the number as it is.

WRITE

a 3.6$\underline{4}$1 883

= 3.6$\underline{4}$(1)883
≈ 3.64

b	**1** Write the number and underline the required decimal place.	**b** 18.9$\underline{6}$5 402 0
	2 Circle the next digit and round according to the rule. *Note:* Since the circled digit is greater than or equal to 5, add 1 to the last decimal place that is being kept.	= 18.96(5)402 0 ≈ 18.97

- If you need to add 1 to the last decimal place and the digit in this position is a 9, the result is 10. The 0 is put in the last required place and the 1 is added to the digit in the next place to the left.
- 0.298 rounded to 2 decimal places is 0.30.

WORKED EXAMPLE 11

Round the following to the number of decimal places shown in the brackets.
a 27.462 973 (4) **b 0.009 94 (3)**

THINK	WRITE
a 1 Write the number and underline the required decimal place.	**a** 27.462$\underline{9}$73
2 Circle the next digit and round according to the rule. *Note:* Since the circled digit is greater than 5, add 1 to the last decimal place that is being kept. As 1 is being added to 9, write 0 in the last place and add 1 to the previous digit.	= 27.462$\underline{9}$(7)3 ≈ 27.4630
b Repeat steps **1** and **2** of part **a**.	**b** 0.009$\underline{9}$4 = 0.00$\underline{9}$(9)4 ≈ 0.010

WORKED EXAMPLE 12

Round 8.672 to the nearest unit.

THINK	WRITE
1 Write the decimal and think of the number with the whole number part only.	8.672
2 Look at the first digit after the decimal point and, if it is greater than or equal to 5, add 1 to the whole number.	≈ 9

- When trying to answer Worked example 12, you can think of the question as: 'Is 8.672 closer to 8 or 9?'

WORKED EXAMPLE 13

Melinda had $51.67 in her bank account. She wanted to withdraw all her money so the bank rounded the amount to the nearest 5 cents. How much money did the teller give to Melinda?

THINK	WRITE
1 Write the actual amount she had in her account.	$51.67
2 Determine whether the last digit is closer to 5 or closer to 10, then rewrite the approximate value. *Note:* Alternatively it can be seen that 67 cents is closer to 65 cents than 70 cents.	≈ $51.65
3 Write a sentence.	Melinda will receive $51.65 from the bank.

6.4.2 Repeating decimals

- When a decimal number ends (terminates) after any number of decimal places, it is said to be a **finite decimal**. The decimal 0.124 is an example of a finite decimal.
- Sometimes when we divide the denominator into the numerator, the answer keeps repeating and the amount left over each time keeps repeating too. When this happens, the answer is called a **recurring decimal** or a repeating decimal.

WORKED EXAMPLE 14

Convert $\frac{1}{11}$ to a decimal. Continue dividing until a pattern emerges, then round the answer to 2 decimal places.

THINK	WRITE
1 Set out the question as for division of whole numbers, adding a decimal point and enough zeros to see a pattern emerging.	$11\overline{)1.^{1}0^{10}0^{1}0^{10}0^{1}0^{10}0^{1}0^{10}0}$ $0.\ 0\ \ 9\ \ 0\ \ 9\ \ 0\ \ 9\ \ 0\ \ 9\ \ 0\ \ 9\ ...$
2 Divide, writing the answer with the decimal point exactly in line with the decimal point in the question. (The amount left over each time is 10 then 1 then 10 then 1 again. The decimal answer is also repeating.)	
3 Write the approximate answer rounded to 2 decimal places.	$\frac{1}{11} \approx 0.09$

- Recurring decimals can be written in one of the following shorter ways for an exact answer.
  - 4.6666 … could be written as $4.\dot{6}$ (with a dot above the repeating part of the decimal).
  - 3.512 512 … could be written as $3.\dot{5}1\dot{2}$ (with a dot above the first and last digits of the repeating part).
  - 6.121 212 … could be written as $6.\overline{12}$ (with a line above the repeating part of the decimal).
  - Like finite decimals, the decimal equivalent of a fraction can be used to determine the decimal equivalent of any multiple of that fraction.

WORKED EXAMPLE 15

Use the result from Worked example 14 to find the decimal equivalent for $\frac{6}{11}$.

THINK	WRITE
1 Write the decimal equivalent for the fraction with 1 as the numerator. In this case it is an infinite recurring decimal therefore 0.090 909 … can be written as $0.\overline{09}$.	$\frac{1}{11} = 0.090\,909\ldots$
2 Multiply both sides of this equation by the appropriate multiple (6 in this case).	$\frac{1}{11} \times 6 = 0.090\,909\ \ldots \times 6$ $= 0.545\,454\ \ldots$
3 Simplify and write the answer.	$\frac{6}{11} = 0.\overline{54}$

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Exercise 6.4 Rounding and repeating decimals

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Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–14, 16, 20	Questions: 1–10, 11, 12, 13 columns 1 and 2, 14, 16, 18, 20, 21	Questions: 1–3 column 3, 4, 5, 6, 7 column 3, 8–22

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Fluency

1. WE10 Round the following to 2 decimal places.

a. 0.3241	**b.** 0.863	**c.** 1.246 10
d. 13.049 92	**e.** 7.128 63	**f.** 100.813 82
g. 71.260 39	**h.** 0.0092	**i.** 0.185 00
j. 19.6979	**k.** 0.3957	**l.** 0.999

2. Round the following to 1 decimal place.

a. 0.410	**b.** 0.87	**c.** 9.27
d. 25.25	**e.** 300.06	**f.** 12.82
g. 99.91	**h.** 8.88	**i.** 17.610 27
j. 0.8989	**k.** 93.994	**l.** 0.959 027

3. WE11 Round the following to the number of decimal places shown in the brackets.

a. 2.386 214	(2)	**b.** 14.034 59	(1)	**c.** 0.027 135	(2)
d. 0.876 4903	(4)	**e.** 64.295 18	(4)	**f.** 0.382 04	(3)
g. 96.28 049	(1)	**h.** 3.0409	(2)	**i.** 8.902	(2)
j. 47.879 69	(3)	**k.** 0.099 498 632	(2)	**l.** 0.486 2590	(2)

4. MC **a.** 13.179 rounded to 2 decimal places is equal to:

A. 13.17 **B.** 13.20 **C.** 13.18 **D.** 13.27 **E.** 13.19

b. 0.2465 rounded to 1 decimal place is equal to:

A. 0.3 **B.** 0.25 **C.** 1.2 **D.** 0.2 **E.** 0.5

c. 1.7688 rounded to 3 decimal places is equal to:

A. 1.768 **B.** 1.770 **C.** 1.778 **D.** 1.769 **E.** 1.800

d. 2.998 rounded to 1 decimal place is equal to:

A. 3.0 **B.** 2.9 **C.** 2.8 **D.** 3.1 **E.** 3.9

5. WE12 Round the following to the nearest unit.

a. 10.7	**b.** 8.2	**c.** 3.6	**d.** 92.7
e. 112.1	**f.** 21.76	**g.** 42.0379	**h.** 2137.50
i. 0.12	**j.** 0.513	**k.** 0.99	**l.** 40.987

6. Write the following infinite recurring decimals using one of the short forms.

a. 2.555 …	**b.** 0.666 …	**c.** 12.888 88 …
d. 49.111 11 …	**e.** 0.262 626 …	**f.** 0.414 141 …
g. 0.913 913 …	**h.** 8.641 864 18 …	**i.** 0.040 121 21 …
j. 133.946 2462 …	**k.** 1.833 333 …	**l.** 0.127 7777 …

7. Convert each of the following to a decimal. Continue dividing until a pattern emerges, then round the answer to the number of decimal places indicated in the brackets.

a. $\frac{1}{6}$ (2)	**b.** $\frac{1}{3}$ (1)	**c.** $\frac{1}{9}$ (1)
d. $\frac{2}{15}$ (2)	**e.** $\frac{2}{11}$ (2)	**f.** $\frac{4}{9}$ (1)
g. $\frac{5}{12}$ (3)	**h.** $\frac{1}{7}$ (6)	**i.** $\frac{7}{15}$ (2)

8. MC **a.** $\frac{3}{5}$ as a decimal is:

A. 0.3 **B.** 0.6 **C.** 0.2 **D.** 0.9 **E.** 0.5

b. 1.8888 … written as an exact answer is:

A. $1.\dot{8}$ **B.** 1.888 **C.** 1.88 **D.** 1.9 **E.** 1.889

c. 12.412 412 … written as an exact answer is:

A. 12.412 **B.** $12.41\dot{2}$ **C.** 12.412 412 **D.** $12.\dot{4}1\dot{2}$ **E.** $12.\overline{412}$

d. $\frac{3}{7}$ as a decimal in exact form is:

A. $0.\dot{4}28571\dot{4}$ **B.** $0.\overline{428}$ **C.** 0.4 **D.** 0.428 517 4 **E.** $2.\dot{3}$

9. Find decimal equivalents for the following fractions. Give your answer in exact form as either a finite decimal or an infinite recurring decimal.

a. $\frac{1}{2}$ **b.** $\frac{1}{3}$ **c.** $\frac{1}{4}$ **d.** $\frac{1}{5}$ **e.** $\frac{1}{9}$

10. Using the answers from question **9**, find decimal equivalents for the following fractions.

a. $\frac{7}{2}$ **b.** $4\frac{2}{3}$ **c.** $\frac{3}{4}$ **d.** $3\frac{4}{5}$ **e.** $\frac{7}{9}$

Understanding

11. Round the following to the nearest ten.

a. 13	**b.** 76	**c.** 47	**d.** 138
e. 262	**f.** 175	**g.** 306.2	**h.** 1484
i. 10 024	**j.** 209 718.5	**k.** 18.6	**l.** 5.92

12. Round the following to the nearest hundred.

a. 320	**b.** 190	**c.** 894	**d.** 138
e. 125	**f.** 6751	**g.** 875.2	**h.** 9750.051
i. 1724	**j.** 1 462 836.5	**k.** 71	**l.** 47

13. Round the following to the nearest thousand.

a. 3426	**b.** 5890	**c.** 12 300	**d.** 18 640
e. 28 000	**f.** 9462	**g.** 1098	**h.** 496 830
i. 12 780	**j.** 862	**k.** 129 980	**l.** 49 899

14. WE13 In the supermarket Christine's shopping bill came to $27.68. As there are no 1- or 2-cent pieces, this amount must be rounded to the nearest 5 cents. How much will Christine pay for her shopping?

15. Using a calculator, Greg worked out that the piece of timber required to finish making a support for a gate should be 3.567 82 metres. Realistically, the timber can be measured only to the nearest millimetre (nearest thousandth of a metre). What measurement should be used for the length of the timber? Explain why 3.567 82 m is unreasonable as a measurement for timber.

16. Rank the following decimals from smallest to largest:

$$0.295, 0.29\dot{5}, 02.\dot{9}\dot{5}, 0.\dot{2}\dot{9}\dot{5}$$

17. Find a decimal that is greater than $0.\dot{5}432\dot{1}$ and less than 0.54322.

18. Barney rounds a decimal to 0.6 correct to 1 decimal place. Fred then says that the number before being rounded off could not have been bigger than 0.64. Is Fred correct? Explain your answer.

Reasoning

19. The maximum temperature was recorded as 24.7 °C. In the news broadcast, the presenter quoted this to the nearest degree. What temperature was quoted?

20. a. Round the decimal number 0.8375 to the nearest:

i. tenth **ii.** hundredth **iii.** thousandth.

b. Do the rules for rounding a decimal up or down depend on the number of decimal places?

Problem solving

21. a. Calculate the decimal values, correct to 6 decimal places, of the following fractions.

$$\frac{1}{6}, \frac{7}{6}, \frac{13}{6}, \frac{19}{6}, \frac{25}{6}$$

b. State any similarities between the five answers.

c. State any differences between the five answers.

d. Without using a calculator, state the decimal values of $\frac{31}{6}, \frac{37}{6}, \frac{43}{6}$ correct to 3 decimal places. Explain the reasoning for your answer.

22. Divide 100 by the first 10 prime numbers. Write the answers correct to 4 decimal places.

Reflection

Which number is bigger, 0.3 or $0.\dot{3}$? How do you know this?

6.5 Adding and subtracting decimals

6.5.1 Adding decimals

- Decimals can be added using a method similar to that for whole numbers.
- Set out the addition in vertical columns and line up the decimal points so that the digits with the same place value are underneath each other.
- If the question is not written in columns, it is necessary to rewrite it with the decimal points lined up.
- Fill the empty places with zeros.

6.5.2 Checking by estimating

- Answers to decimal addition and subtraction can be checked by estimating. Round each decimal to the nearest whole number or to a similar number of decimal places and then add or subtract them.

WORKED EXAMPLE 16

Calculate:

a
$$\begin{array}{r} 1.3 \\ +\ 0.5 \\ \hline \end{array}$$

b
$$\begin{array}{r} 12.84 \\ +\ 2.33 \\ \hline \end{array}$$

c
$$\begin{array}{r} 1.25 \\ 3.146 \\ +\ 7.0 \\ \hline \end{array}$$

THINK	WRITE
a Copy the question exactly and add the digits as for whole numbers, working from right to left. Write the decimal point directly below the decimal points in the question.	**a** 1.3 +0.5 1.8
b Copy the question exactly and add the digits as for whole numbers, working from right to left. Write the decimal point directly below the decimal points in the question.	**b** 12.84 + 2.33 15.17
c **1** Write the question, replacing the spaces with zeros. **2** Add the digits as for whole numbers, working from right to left. Write the decimal point directly below the decimal points in the question.	**c** 1.250 3.146 +7.000 11.396

WORKED EXAMPLE 17

Rewrite in columns, then add 0.26 + 1.8 + 12.214.

THINK	WRITE
1 Write the question in columns with the decimal points directly beneath each other with the zeros included. **2** Add the digits as for whole numbers. Write the decimal point directly below the decimal points in the question. **3** Check the answer by rounding to get an estimate and then adding 0.3 + 2 + 12 = 14.3, which is close to 14.274.	0.260 1.800 +12.214 14.274

6.5.3 Subtracting decimals

- Decimals can be subtracted using a method similar to that for whole numbers.
- Set out the subtraction in vertical columns and line up the decimal points so that the digits with the same place value are underneath each other.
- If the question is not written in columns, it is necessary to rewrite it with the decimal points lined up.
- Fill the empty places with zeros.

WORKED EXAMPLE 18

Calculate: $\begin{array}{r} 0.56 \\ -0.14 \\ \hline \end{array}$

THINK	WRITE
1 Copy the question exactly and subtract the digits as for whole numbers, working from right to left. Write the decimal point directly below the decimal points in the question. **2** Check the answer by rounding to get an estimate; 0.6 − 0.1 = 0.5, which is close to 0.42. (This step can be done in your head.)	0.56 −0.14 0.42

WORKED EXAMPLE 19

Rewrite the following in columns, then subtract.
a 1.82 − 0.57 **b** 2.641 − 0.85

THINK	WRITE
a 1 Write in columns with the decimal points directly under each other. Subtract, and insert the decimal point directly below the other decimal points in the question.	**a** $\begin{array}{r} {}^{7}\not{1}.\not{8}^{1}2 \\ -0.5\,7 \\ \hline 1.2\,5 \end{array}$
2 Check the answer by rounding to get an estimate; 2 − 1 = 1, which is close to 1.25.	
b 1 Write in columns with the decimal points directly under each other, adding zeros as appropriate. Subtract as for whole numbers and insert the decimal point directly below the other decimal points.	**b** $\begin{array}{r} {}^{5}\not{2}.\not{6}^{1}41 \\ -0.8\,50 \\ \hline 1.7\,91 \end{array}$
2 Check the answer by rounding to get an estimate; 3 − 1 = 2, which is close to 1.791.	

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- **Complete this digital doc:** SkillSHEET: Adding decimals (same number of decimal places) (doc-6461)
- **Complete this digital doc:** SkillSHEET: Adding decimals (different number of decimal places) (doc-6462)
- **Complete this digital doc:** SkillSHEET: Subtracting decimals (same number of decimal places) (doc-6463)
- **Complete this digital doc:** SkillSHEET: Subtracting decimals (different number of decimal places) (doc-6464)
- **Try out this interactivity:** Decimal target shoot (int-0003)
- **Complete this digital doc:** WorkSHEET: Decimals I (doc-1769)

Exercise 6.5 Adding and subtracting decimals

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Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–8, 10, 17, 20, 21	Questions: 2c, d, 3b, c, e, f, h, i, 4b, d, f, h, 5, 6, 7, 8, 9, 12, 15, 17, 20, 21, 23	Questions: 4, 5, 6, 7c, f, i, l, o, 8–19, 21–23

Individual pathway interactivity: int-4340

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Fluency

1. WE16a, b Calculate the following.

a. $\begin{array}{r} 1.2 \\ +2.3 \\ \hline \quad \end{array}$ **b.** $\begin{array}{r} 1.67 \\ +1.02 \\ \hline \quad \end{array}$ **c.** $\begin{array}{r} 8.062 \\ +5.177 \\ \hline \quad \end{array}$ **d.** $\begin{array}{r} 10.0364 \\ +92.1494 \\ \hline \quad \end{array}$

2. WE16c Calculate the following, after filling the blank spaces.

a.
$$\begin{array}{r} 6.27 \\ +0.5 \\ \hline \quad\quad \end{array}$$

b.
$$\begin{array}{r} 3.26 \\ +18.6460 \\ \hline \quad\quad \end{array}$$

c.
$$\begin{array}{r} 4.2 \\ 62.013 \\ +1946.12 \\ \hline \quad\quad \end{array}$$

d.
$$\begin{array}{r} 48.12906 \\ 9 \\ +204.32 \\ \hline \quad\quad \end{array}$$

3. WE17 Rewrite the following in columns, then add. Check your answer by rounding to get an estimate.

a. 1.4 + 3.2 **b.** 6.5 + 0.4 **c.** 0.22 + 1.37
d. 3.261 + 0.21 **e.** 15.987 + 1.293 **f.** 8.027 + 0.9415
g. 10.8271 + 6.5 **h.** 1.8 + 18.6329 **i.** 26.29 + 1030.4963

4. Rewrite the following sums, then add. Check your answer by rounding to get an estimate.

a. 0.24 + 3.16 + 8.29 **b.** 14.23 + 1.06 + 86.29 + 3.64
c. 40.271 + 0.36 + 1.4 **d.** 5.27 + 1.381 + 12.3
e. 100 + 4.3 + 0.298 + 1.36 **f.** 82.3 + 100.6 + 0.9949 + 9
g. 3.026 + 5.9938 + 8.7718 + 3.2 **h.** 126 + 372.8 + 100.0264 + 2020.13

5. MC **a.** 1.6 + 4.8 equals:

A. 5.4 **B.** 6.4 **C.** 0.54 **D.** 0.64 **E.** 64

b. 3.26 + 0.458 equals:

A. 3.718 **B.** 0.784 **C.** 0.037 18 **D.** 3.484 **E.** 7.84

c. 1.84 + 0.61 + 4.07 equals:

A. 6.52 **B.** 6.42 **C.** 5.42 **D.** 5.52 **E.** 0.652

d. 216 + 1.38 + 0.002 64 equals:

A. 217.4064 **B.** 0.618 **C.** 217.644 **D.** 217.382 64 **E.** 21.7644

6. WE18 Calculate the following, filling the spaces with zeros as required. Check your answer by rounding to get an estimate.

a.
$$\begin{array}{r} 6.87 \\ -6.27 \\ \hline \quad\quad \end{array}$$

b.
$$\begin{array}{r} 12.231 \\ -\ 8.026 \\ \hline \quad\quad \end{array}$$

c.
$$\begin{array}{r} 0.6301 \\ -0.5495 \\ \hline \quad\quad \end{array}$$

d.
$$\begin{array}{r} 3.0091 \\ -1.6723 \\ \hline \quad\quad \end{array}$$

e.
$$\begin{array}{r} 31.02 \\ -26 \\ \hline \quad\quad \end{array}$$

f.
$$\begin{array}{r} 98.26 \\ -9.07 \\ \hline \quad\quad \end{array}$$

g.
$$\begin{array}{r} 146 \\ -\ 58.91 \\ \hline \quad\quad \end{array}$$

h.
$$\begin{array}{r} 3.2 \\ -0.467 \\ \hline \quad\quad \end{array}$$

7. WE19 Rewrite the following in columns, then subtract. Check your answer by rounding to get an estimate.

a. 5.64 − 2.3 **b.** 12.07 − 6.14 **c.** 13.869 − 10.02
d. 0.687 − 0.36 **e.** 15.226 − 11.08 **f.** 42.83 − 15
g. 6.734 − 4.8 **h.** 12.2 − 8.911 **i.** 13.9009 − 12.65
j. 100.562 − 86.0294 **k.** 38 − 21.234 **l.** 47 − 8.762
m. 5 − 0.8864 **n.** 0.2 − 0.0049 **o.** 3.279 − 2.506 84

8. MC **a.** 0.39 − 0.15 equals:

A. 0.0024 **B.** 0.024 **C.** 0.24 **D.** 2.4 **E.** 24

b. 1.4 − 0.147 would be rewritten as:

A.
$$\begin{array}{r} 1.4 \\ -0.417 \\ \hline \quad\quad \end{array}$$

B.
$$\begin{array}{r} -1.400 \\ -0.147 \\ \hline \quad\quad \end{array}$$

C.
$$\begin{array}{r} 1.40 \\ -1.47 \\ \hline \quad\quad \end{array}$$

D.
$$\begin{array}{r} 1.004 \\ -0.147 \\ \hline \quad\quad \end{array}$$

E.
$$\begin{array}{r} 1.040 \\ -0.147 \\ \hline \quad\quad \end{array}$$

c. 0.3 − 0.024 equals:

A. 0.06 **B.** 0.276 **C.** 0.7 **D.** 0.76 **E.** 76

d. 150.278 − 0.99 equals:

A. 150.728 **B.** 149.288 **C.** 1.492 88 **D.** 159.388 **E.** 1.593 88

Understanding

9. Josh deposited $27.60 into his bank account. If his balance before the deposit was $139.40, what is Josh's new bank balance?

10. Jessica bought the following items at the school canteen: 1 can of Coke for $1.60, 1 sausage roll for $1.20, 1 packet of chips for $1.50 and 2 Redskins for $0.40 (Redskins cost 20 cents each). How much did Jessica spend?

11. A triathlon consists of a 0.5-kilometre swim, a 15.35-kilometre ride and a 4.2-kilometre run. How far do the competitors have to travel altogether?

12. Amy walked 3.6 kilometres to school, 0.8 kilometres from school to the shops, 1.2 kilometres from the shops to a friend's house and finally 2.5 kilometres from her friend's house to her home. How far did Amy walk?

13. For lunch Paula ordered 1 potato cake, 1 dim sim, the minimum of chips and a milkshake from the menu shown below. How much did Paula spend on her lunch?

MENU			
Flake	$3.50	Coffee	$2.20
Whiting	$3.50	Tea	$2.20
Dim sims	$0.60	Soft drinks	$1.80
Potato cakes	$0.50	Milkshakes	$3.00
Minimum chips	$2.50	Water	$1.80

14. Tarak works in a newsagency. A customer buys $9.65 worth of goods and gives Tarak a $20 note. How much change should Tarak give the customer?

15. A jockey has a mass of 52.3 kilograms. After exercising and training for 2 days and spending time in a sauna, the jockey has lost 1.82 kilograms. What is the jockey's mass now?

16. If 1.27 metres is cut from a piece of material that is 13 metres long, how much material is left?

Reasoning

17. Cathy Freeman won a particular 400 metre race in 51.35 seconds. In her next race, her time was 2.97 seconds faster than this. What was Cathy's time for this race?

18. Gary and Liz are replacing the skirting boards in their lounge room. They know the perimeter of the room is 34.28 metres. If there is a door 0.82 metres wide and a fireplace 2.18 metres wide that do not require skirting boards, how much wood will they need to buy for their lounge room?

19. The following table shows the times recorded for each swimmer in the under-13, 50-metre freestyle relay for 6 teams.

	Times for each swimmer (seconds)			
Team	**Swimmer 1**	**Swimmer 2**	**Swimmer 3**	**Swimmer 4**
1	36.7	41.3	39.2	35.8
2	38.1	46.5	38.8	35.9
3	34.6	39.2	39.9	35.2
4	41.6	40.8	43.7	40.5
5	37.9	40.2	38.6	39.2
6	38.3	39.1	40.8	37.6

a. Find the total time for each team. Put your results in a table.

b. Which team won the relay?

c. What was the difference in time between the first and second placed teams?

Problem solving

20. Australian coins, which are minted by the Royal Australian Mint, have the following masses:

5c	2.83 g	10c	5.65 g	20c	11.3 g
50c	15.55 g	$1	9.0 g	$2	6.6 g

I have 6 coins in my pocket, with a total value of $2.45. What could the total mass of these coins be?

21. Jo and Anton made a certain number of telephone calls over a 2-month period. There is a charge of 25 cents for each call, and the monthly rental of the phone line is $13.60. The bill for the 2 months comes to $39.45. How many calls did they make in the 2-month period?

22. You purchased some shares over a 4-month period. The price fell by $4.23 in the first month, rose by $6.67 in the second month, rose by $1.35 in the third month and fell by $3.28 in the fourth month. Did the shares increase or decrease in value over the four months, and by how much?

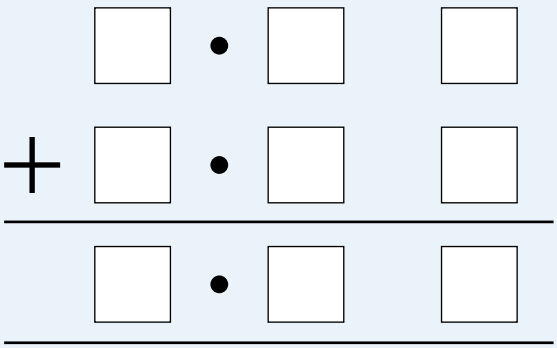
23. Without using a calculator, find the average of the numbers 0.1, 0.11 and 0.111.

Reflection

How is rounding used in estimation?

CHALLENGE 6.1

Arrange the digits 1 to 9 into the boxes on the right to make a true decimal addition.



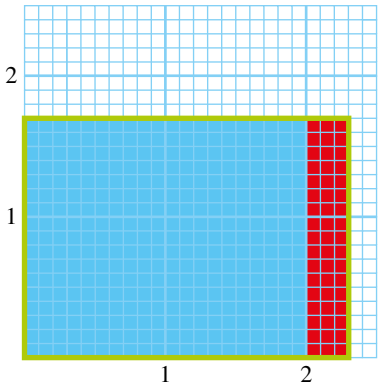
6.6 Multiplying decimals (including by multiples of 10)

6.6.1 Multiplying decimals

- The calculation below shows the multiplication 1.7×2.3. The diagram below at right is a visual representation of each step in the calculation. There are 1.7 rows of 2.3, or 1.7 groups of 2.3.

$$
\begin{array}{rll}
1.7 & & \\
\times\ 2.3 & & \\
\hline
51 & \longleftarrow & 0.3 \times 1.7 = 0.51 \\
3\,40 & \longleftarrow & 2.0 \times 1.7 = 3.40 \\
\hline
3.91 & \longleftarrow & \text{Total}
\end{array}
$$

- The smallest place value in the answer is determined by multiplying the smallest place values from each of the decimal numbers. The first multiplication is $\frac{3}{10} \times \frac{7}{10} = \frac{21}{100}$, so the smallest place value in the answer will be hundredths.
- A quicker method of determining the number of decimal places in the answer is to count the total number of decimal places in the question.



WORKED EXAMPLE 20

Calculate the following:

a 12.6×7

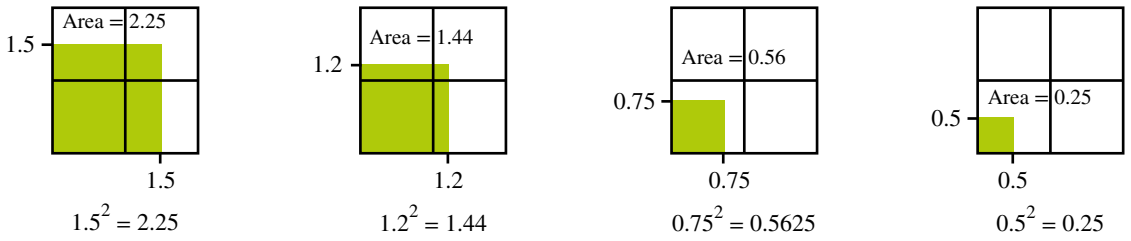
b
$$\begin{array}{r} 3.26 \\ \times 0.4 \\ \hline \end{array}$$

c
$$\begin{array}{r} 0.4629 \\ \times 2.6 \\ \hline \end{array}$$

THINK	WRITE
a 1 Rewrite and multiply digits as for whole numbers, ignoring the decimal point. Count the number of decimal places altogether (1) and put in the decimal point.	**a** $\begin{array}{r} 12.6 \\ \times 7 \\ \hline 88.2 \end{array}$
2 Check the answer by rounding; $10 \times 7 = 70$, which is close to 88.2.	
b 1 Multiply, ignoring the decimal places.	**b** $\begin{array}{r} 326 \\ \times 4 \\ \hline 1304 \end{array}$
2 Count the number of digits after the point in both the decimals being multiplied and insert the decimal point in the answer. There are 2 decimal places in 3.26 and 1 in 0.4 so there will be 3 decimal places in the answer.	$3.26 \times 0.4 = 1.304$
3 Check the answer by rounding; $3 \times 0.4 = 1.2$, which is close to 1.304.	
c 1 Multiply, ignoring the decimal places.	**c** $\begin{array}{r} 4629 \\ \times 26 \\ \hline 27774 \\ 92580 \\ \hline 120354 \end{array}$
2 Count the number of digits after the point in both the decimals being multiplied. Insert the decimal point in that position in the answer.There are 4 decimal places in 0.4629 and 1 decimal place in 2.6, so there will be 5 decimal places in the answer.	$0.4629 \times 2.6 = 1.20354$

6.6.2 Squaring decimals

- To square a decimal, multiple the number by itself. The number of decimal places in the square is twice the number of decimal places in the original number.
- The diagrams below shows how squaring decimal numbers can be represented visually.



$1.5^2 = 2.25$ $1.2^2 = 1.44$ $0.75^2 = 0.5625$ $0.5^2 = 0.25$

WORKED EXAMPLE 21

Calculate the following.
a $\mathbf{0.5^2}$ **b** $\mathbf{1.2^2}$

THINK	WRITE
a 1 Multiply the number by itself, ignoring the decimal places.	**a** $5 \times 5 = 25$
2 Count the number of digits after the point in both the decimals being multiplied and insert the decimal point in the answer. There will be 2 decimal places in the answer.	$0.5 \times 0.5 = 0.25$
3 Write the answer.	$0.5^2 = 0.25$
b 1 Multiply the number by itself, ignoring the decimal places.	**b** $12 \times 12 = 144$
2 Count the number of digits after the point in both the decimals being multiplied and insert the decimal point in the answer. There will be 2 decimal places in the answer.	$1.2 \times 1.2 = 1.44$
3 Write the answer.	$1.2^2 = 1.44$

6.6.3 Finding square roots of decimals

- To find the square root of a decimal, ignore the decimal and find the square root of the number. The number of decimal places in the answer will be half the number of decimal places in the number whose square root you are trying to find.

WORKED EXAMPLE 22

Calculate the following.
a $\mathbf{\sqrt{0.81}}$ **b** $\mathbf{\sqrt{0.0121}}$

THINK	WRITE
a 1 Take the number under the square root symbol, ignoring the decimal places. Find the square root of this number.	**a** $\sqrt{81} = 9$

	2 Count the number of decimal places in the original number (2 in this case). The number which needs to be squared to form this decimal will have half the number of decimal places. So, the final answer will have 1 decimal place. Write the answer.	$\sqrt{0.81} = 0.9$
	3 Check the result using a calculator or by squaring the answer. ($0.9^2 = 0.81$)	
b	**1** Take the number under the square root symbol, ignoring the decimal places. Find the square root of this number.	**b** $\sqrt{121} = 11$
	2 Count the number of decimal places in the original number (4 in this case). The number, which needs to be squared to form this decimal, will have half this number of decimal places. So, the final answer will have 2 decimal places. Write the answer.	$\sqrt{0.0121} = 0.11$
	3 Check the result using a calculator or by squaring the answer. ($0.11^2 = 0.0121$)	

6.6.4 Multiplying by multiples of 10

- The multiples of 10 are 10, 20, 30, 40, … 120, … 1000, …
- When you are multiplying a number by a multiple of 10, factorise the multiple to give a power of 10 and the other factor. Multiply the number by the other factor first, and then by the power of 10. For example, if you are multiplying a number by 1200, first write $1200 = 12 \times 100$, multiply by 12 then by 100.

WORKED EXAMPLE 23

Calculate:
a 5.1×600 **b** 0.0364×24000.

THINK	WRITE
a 1 Multiplying by 600 is the same as first multiplying by 6 then multiplying by 100. Calculate 5.1×6.	**a** $\begin{array}{r} 5.1 \\ \times\ 6 \\ \hline 30.6 \end{array}$
2 Multiply the result by 100. Move the position of the decimal point 2 places to the right.	$30.6 \times 100 = 3060$
3 Write the final answer.	$5.6 \times 600 = 3060$
b 1 Multiplying by 24 000 is the same as first multiplying by 24 then multiplying by 1000. Calculate 0.0364×24.	**b** $\begin{array}{r} 0.0364 \\ \times\ 24 \\ \hline 1456 \\ 7280 \\ \hline 0.8736 \end{array}$
2 Multiply the result by 1000. Move the position of the decimal point 3 places to the right.	$0.8736 \times 1000 = 873.6$
3 Write the answer.	$0.0364 \times 24000 = 873.6$

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Complete this digital doc: SkillSHEET: Multiplying decimals by a single-digit number (doc-6465)

Complete this digital doc: SkillSHEET: Multiplying decimals by 10, 100, 1000 (doc-6466)

Exercise 6.6 Multiplying decimals (including by multiples of 10)

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–8, 9, 11, 15, 17	Questions: 1, 2, 3, 4 (columns 2 and 3), 5, 6, 7 column 2, 8, 10, 13, 14, 17	Questions: 1–4 (column 3), 5, 6, 7b, d, h, i, l, 8–18

Individual pathway interactivity: int-4341

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Fluency

1. Calculate the following.

a. 3.5×4	**b.** 15.7×8	**c.** 16.3×9
d. 10.2×6	**e.** 22.34×5	**f.** 47.63×9
g. 27.18×7	**h.** 64.87×8	**i.** 3.724×7
j. 1.064×6	**k.** $0.264\,81 \times 3$	**l.** $14.192\,683 \times 8$

2. WE20a Calculate the following.

a. 1.4×0.6	**b.** 4.2×0.7	**c.** 0.8×0.4
d. 9.7×0.8	**e.** 0.35×0.4	**f.** 0.64×0.3
g. 0.77×0.5	**h.** 0.49×0.9	**i.** 1.63×0.2
j. 5.38×0.8	**k.** 0.347×0.6	**l.** 0.498×0.7
m. 4.832×0.6	**n.** 12.2641×0.4	**o.** $20.032\,79 \times 0.5$

3. Calculate the following.

a. 0.002×0.05	**b.** 0.003×0.004	**c.** 0.7×0.09
d. 0.037×0.006	**e.** $0.000\,061 \times 0.04$	**f.** 0.004×0.09
g. 0.56×0.7	**h.** $0.030\,31 \times 0.02$	**i.** 0.0006×0.007

4. WE20b Calculate the following. (You may use a calculator for parts **g** to **o**.)

a. 0.25×1.2	**b.** 0.37×2.3	**c.** 0.47×5.4
d. 0.79×8.3	**e.** 4.68×3.6	**f.** 8.04×7.5
g. 11.64×4.8	**h.** 15.08×1.9	**i.** 35.17×0.35
j. 67.73×0.28	**k.** 90.65×0.88	**l.** 46.96×0.76
m. 13.56×0.31	**n.** 3.694×0.46	**o.** 12.41×1.2

5. MC **a.** When calculating 8.32×0.64, the number of decimal places in the answer is:

A. 0 **B.** 1 **C.** 2 **D.** 3 **E.** 4

b. 0.2×0.2 equals:

A. 0.004 **B.** 0.04 **C.** 0.4 **D.** 4 **E.** 40

c. 1.4×0.8 equals:

A. 1.12 **B.** 8.2 **C.** 82 **D.** 11.2 **E.** 112

d. 0.0321×0.51 equals:

A. 0.001 5912 **B.** 0.015 912 **C.** 0.156 312 **D.** 0.159 12 **E.** 1.5912

6. WE21, 22 Calculate the following.

a. 0.02^2 **b.** 1.3^2 **c.** 2.05^2

d. $\sqrt{0.09}$ **e.** $\sqrt{0.16}$ **f.** $\sqrt{0.0049}$

7. Calculate the following by moving the position of the decimal point.

a. 6.48×10 **b.** 13.896×100

c. 589.0643×100 **d.** 0.2708×1000

e. $217.148\,96 \times 1000$ **f.** $8.426\,19 \times 100\,000$

g. $0.820\,496\,783 \times 100\,000$ **h.** $32.689\,043\,267 \times 100\,000$

i. $0.984\,326\,641 \times 1\,000\,000$ **j.** $59.027\,683\,017 \times 1\,000\,000$

k. $0.000\,278\,498\,32 \times 1\,000\,000$ **l.** 0.46×1000

m. $529 \times 10\,000$ **n.** $39.486 \times 1\,000\,000$

8. WE23 Calculate the following.

a. 3.64×300 **b.** 7.58×600 **c.** 26.9×500 **d.** 42.6×900

e. 0.127×8000 **f.** $0.543 \times 11\,000$ **g.** $4.6 \times 32\,000$ **h.** $8.1 \times 54\,000$

Understanding

9. Change the following amounts of money to cents.
(*Hint:* There are 100 cents in one dollar.)

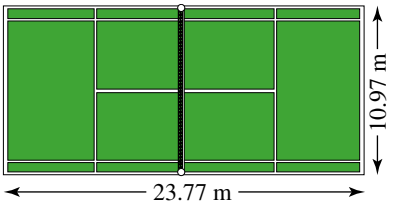
a. \$35 **b.** \$127 **c.** \$11

d. \$25.35 **e.** \$58.20 **f.** \$110.15

10. One thousand Year 7 students contributed 75 cents each to the bushfire appeal. How many dollars did they contribute altogether?

11. Benjamin and Robyn were providing ice-cream for 600 children. How much ice-cream would be needed if each child was expected to eat 0.18 litres?

12. Find the area of the tennis court shown given that area = length × width.



Reasoning

13. Michael bought 0.65 kilograms of cubed steak at a butcher's shop. This meat costs \$8.50 a kilogram. How much did Michael pay for the steak?
Note: Round to the nearest 5c.

14. Judy bought 34.5 litres of petrol at 92.9 cents per litre. How much did she pay for her petrol:

a. in cents

b. in dollars?

15. James is using the recipe for chocolate chip muffins to make 1.5 times the given amount. If the recipe lists 0.25 litres of milk in the ingredients, how much milk should James use for his muffins?

16. A ball is dropped from a height of 1 metre. If it reaches 0.7 times the height of the previous bounce at each bounce, work out how many times the ball bounces until the height is less than 1 centimetre.

Problem solving

17. Sophie, Hamish and Thomas shared a pizza and paid according to what they ate. Sophie ate 4 slices, Hamish ate 3 slices and Thomas ate 5 slices. If Thomas's share of the bill was $4.50, calculate the amount Sophie and Hamish had to pay.

18. A sheet of paper is 0.012 cm thick. Would you notice the difference in the height between a stack of 500 sheets and a stack of 550 sheets?

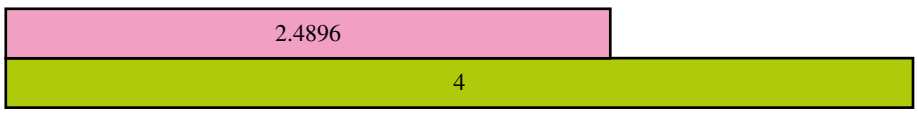
Reflection

What strategy could you use to demonstrate that 0.5 × 0.5 = 0.25?

6.7 Dividing decimals (including by multiples of 10)

6.7.1 Dividing a decimal by a whole number

- The method for dividing a decimal number by a whole number is the same as the method used for dividing whole numbers.
- A visual representation of 2.4896 ÷ 4 is shown below.



2.4896 ÷ 4 can be interpreted as 'how many times does 4 divide into 2.4896?' As shown, 4 divides into 2.4896 less than once.

WORKED EXAMPLE 24

Find the value of 2.4896 ÷ 4.

THINK	WRITE
1 Set out the question as you would for whole numbers.	$4\overline{)2.4896}$
2 Divide 4 into the first digit of 2.4896 (2 ÷ 4 = 0 remainder 2). Write the 0 above the 2, and write the remainder beside the next digit, as shown in black.	$\begin{array}{r} 0 \\ 4\overline{)2.^{2}4896} \end{array}$
3 The second digit in the number being divided is to the right of the decimal point, so write the decimal point in the answer directly above the decimal point in the question, as shown in red.	$\begin{array}{r} 0. \\ 4\overline{)2.^{2}4896} \end{array}$
4 Divide 4 into the second digit, which includes the carried 2 (24 ÷ 4 = 6 remainder 0). Write the 6 above the 4, as shown in green.	$\begin{array}{r} 0.\ 6 \\ 4\overline{)2.^{2}4896} \end{array}$
5 Divide 4 into the third digit (8 ÷ 4 = 2). Write the 2 above the 8, as shown in pink.	$\begin{array}{r} 0.\ 62 \\ 4\overline{)2.^{2}4896} \end{array}$

6 Divide 4 into the fourth digit (9 ÷ 4 = 2 remainder 1). Write the 2 above the 9, and write the remainder beside the next digit, as shown in purple.	$\begin{array}{r} 0.\ 6\ 2\ 2 \\ 4\overline{)2.^{2}4\,8\,9^{1}6} \end{array}$
7 Divide 4 into the fifth digit, which includes the carried 1 (16 ÷ 4 = 4). Write the 4 above the 6, as shown in orange.	$\begin{array}{r} 0.\ 6\ 2\ 2\ 4 \\ 4\overline{)2.^{2}4\,8\,9^{1}6} \end{array}$
8 Write the answer.	2.4896 ÷ 4 = 0.6224

- Sometimes, when you are dividing numbers, you will find that there is a remainder. For example 15.3 ÷ 4:

$$\begin{array}{r} 3.8 \\ 4\overline{)15.3} \end{array} \text{ remainder } 1$$

Instead of leaving a remainder, you can sometimes add zeros to the end of the decimal and keep dividing until there is no remainder.

$$\begin{array}{r} 3.825 \\ 4\overline{)15.300} \end{array}$$

WORKED EXAMPLE 25

Calculate 21.76 ÷ 5. Add zeros and keep dividing until there is no remainder.

THINK	WRITE
1 Set up the division. Write the decimal point in the answer directly above the decimal point in the question and divide as for short division adding zeros as required.	$\begin{array}{r} 4.\ 3\ 5\ 2 \\ 5\overline{)21.^{1}7^{2}6^{1}0} \end{array}$
2 Check the answer by rounding; 20 ÷ 5 = 4 which is close to 4.352.	

6.7.2 Dividing a decimal number by a multiple of 10

- When dividing by a multiple of 10, factorise the multiple to give a power of 10 and its other factor. Divide by the other factor first, and then by the power of 10.

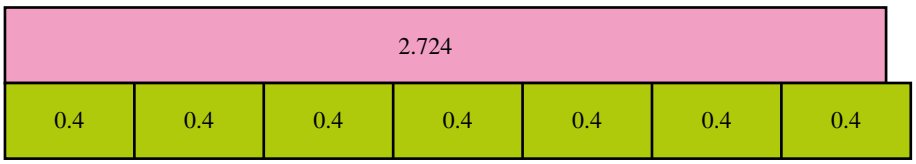
WORKED EXAMPLE 26

Calculate:
a 4.8 ÷ 40 **b 19.2 ÷ 6000.**

THINK	WRITE
a 1 Dividing by 40 is the same as first dividing by 4 then dividing by 10.	**a** $\begin{array}{r} 1.2 \\ 4\overline{)4.8} \end{array}$
2 To divide by 10, move the position of the decimal point 1 place to the left.	1.2 ÷ 10 = 0.12
3 Write your final answer.	4.8 ÷ 40 = 0.12
b 1 Dividing by 6000 is the same as dividing by 6 then dividing by 1000.	**b** $\begin{array}{r} 3.2 \\ 6\overline{)19.2} \end{array}$
2 To divide by 1000, move the position of the decimal point 3 places to the left.	3.2 ÷ 1000 = 0.0032
3 Write your final answer.	19.2 ÷ 6000 = 0.0032

6.7.3 Dividing a decimal number by another decimal number

- A visual representation of 2.724 ÷ 0.4 is shown below.



2.724 ÷ 0.4 can be interpreted as 'how many times does 0.4 divide into 2.724?'
2.724 ÷ 0.4 = 6.81

- When dividing one decimal by another, multiply the decimal you are dividing by (divisor) by a power of 10 to make it a whole number. Multiply the other decimal *by the same power of* 10, as shown at right. This is the same as writing an equivalent fraction with a whole number as the denominator:

First multiply this number by 10 to make a whole number.

2.724 ÷ 0.4

Then multiply this number by 10 also.

$$\frac{2.724}{0.4} \times \frac{10}{10} = \frac{27.24}{4}.$$

WORKED EXAMPLE 27

Calculate:
a 26.724 ÷ 0.4 b 3.0276 ÷ 0.12.

THINK	WRITE
a 1 Rewrite the question as a fraction.	**a** $26.724 \div 0.4 = \frac{26.724}{0.4}$
2 Multiply both the numerator and the denominator by the appropriate multiple of 10.	$= \frac{26.724}{0.4} \times \frac{10}{10}$ $= \frac{267.24}{4}$
3 Divide the decimal by the whole number.	$4\overline{)26^{2}7.^{3}24}$ $6\ 6.\ 81$
4 Write the answer.	$26.724 \div 0.4 = 66.81$
b 1 Rewrite the question as a fraction.	**b** $3.0276 \div 0.12 = \frac{3.0276}{0.12}$
2 Multiply both the numerator and the denominator by the appropriate multiple of 10.	$= \frac{3.0276}{0.12} \times \frac{100}{100}$ $= \frac{302.76}{12}$
3 Divide the decimal by the whole number.	$12\overline{)30^{6}2.^{2}7^{3}6}$ $2\ 5.\ 2\ 3$
4 Write the answer.	$3.0276 \div 0.12 = 25.23$

WORKED EXAMPLE 28

How many litres of petrol could be purchased for $50.88 if 1 litre costs $1.06?

THINK	WRITE
1 Write the problem.	$50.88 \div 1.06$
2 Rewrite the problem as a fraction.	$= \frac{50.88}{1.06}$
3 Multiply the numerator and denominator by the appropriate multiple of 10, in this case 100. Alternatively, the decimal point could be moved twice to the right in both numbers so that the divisor is a whole number (that is, $50.88 \div 1.06 = 5088 \div 106$).	$= \frac{50.88}{1.06} \times \frac{100}{100}$ $= \frac{5088}{106}$
4 Divide the decimal by the whole number. Alternatively, use a calculator.	$106\overline{)508^{84}8}$ $\begin{matrix}4 & 8\end{matrix}$ $50.88 \div 1.06 = 48$
5 Answer the question.	Forty-eight litres of petrol could be purchased for $50.88.

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Complete this digital doc: SkillSHEET: Dividing decimals by a single digit decimal number (doc-6467)

Complete this digital doc: SkillSHEET: Dividing decimals by 10, 100, 1000 (doc-6468)

Complete this digital doc: WorkSHEET: Decimals II (doc-1770)

Exercise 6.7 Dividing decimals (including by multiples of 10)

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–8, 11, 18	Questions: 1–6 (columns 2 and 3), 7–13, 17, 18	Questions: 1–6 (column 3), 7–19

Individual pathway interactivity: int-4342

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Fluency

1. WE24 Calculate:

a. $3.6 \div 6$	**b.** $21.7 \div 7$	**c.** $17.4 \div 6$	**d.** $4.86 \div 9$	**e.** $8.05 \div 5$
f. $14.13 \div 3$	**g.** $9.68 \div 4$	**h.** $1.576 \div 2$	**i.** $17.847 \div 9$	**j.** $8.029 \div 7$
k. $32.5608 \div 8$	**l.** $41.8645 \div 5$	**m.** $20.5782 \div 3$	**n.** $126.4704 \div 4$	**o.** $37.56 \div 12$
p. $46.80 \div 15$	**q.** $24.541 \div 11$	**r.** $17.108 \div 14$	**s.** $77.052 \div 12$	**t.** $121.3421 \div 11$

2. WE25 Calculate the following. In each case, add zeros and keep dividing until there is no remainder.

a. 3.7 ÷ 2 **b.** 9.5 ÷ 2 **c.** 7.3 ÷ 5

d. 9.8 ÷ 4 **e.** 7.5 ÷ 6 **f.** 55.6 ÷ 8

3. Calculate the following by changing the position of the decimal point.

a. 14.07 ÷ 10 **b.** 968.13 ÷ 100 **c.** 985.06 ÷ 100

d. 620.8 ÷ 1000 **e.** 3592.87 ÷ 1000 **f.** 2349.78 ÷ 100 000

g. 5332.0667 ÷ 100 000 **h.** 9.0769 ÷ 100 000 **i.** 103 454.97 ÷ 1000 000

j. 802 405.6 ÷ 1000 000 **k.** 152.70 ÷ 1000 000 **l.** 0.583 ÷ 1000

m. 0.7205 ÷ 10 000 **n.** 0.0032 ÷ 1000 000 **o.** 0.0487 ÷ 1000 000

4. WE26 Calculate the following.

a. 15.9 ÷ 60 **b.** 23.7 ÷ 30 **c.** 164.5 ÷ 700

d. 238 ÷ 400 **e.** 8.79 ÷ 6000 **f.** 5.22 ÷ 3000

5. WE27a Calculate each of the following.

a. 2.5 ÷ 0.5 **b.** 4.2 ÷ 0.6 **c.** 6.4 ÷ 0.8 **d.** 8.1 ÷ 0.9

e. 2.8 ÷ 0.7 **f.** 6.66 ÷ 0.6 **g.** 0.248 ÷ 0.8 **h.** 3.57 ÷ 0.7

i. 1.32 ÷ 0.6 **j.** 39.6 ÷ 0.6 **k.** 57.68 ÷ 0.8 **l.** 4.924 ÷ 0.4

m. 0.2556 ÷ 0.3 **n.** 0.067 354 ÷ 0.2 **o.** 0.5468 ÷ 0.4

6. WE27b Calculate:

a. 172.0488 ÷ 0.11 **b.** 0.510 48 ÷ 0.12 **c.** 6.4032 ÷ 0.32

d. 2.5473 ÷ 0.21 **e.** 21.470 10 ÷ 0.15 **f.** 142.888 ÷ 0.08

g. 0.028 692 ÷ 0.06 **h.** 473.159 61 ÷ 0.03 **i.** 32.619 ÷ 0.02

7. MC **a.** To calculate 9.84 ÷ 0.8, rewrite it as:

A. 9.84 ÷ 8 **B.** 0.984 ÷ 0.8 **C.** 98.4 ÷ 0.8 **D.** 98.4 ÷ 8 **E.** 984 ÷ 8

b. To calculate 151.368 ÷ 1.32, rewrite it as:

A. 151.368 ÷ 132 **B.** 151.368 ÷ 13.2 **C.** 1513.68 ÷ 132

D. 15 136.8 ÷ 132 **E.** 151 368 ÷ 132

c. 0.294 ÷ 0.7 equals:

A. 0.042 **B.** 0.42 **C.** 4.2 **D.** 42 **E.** 420

d. 21.195 ÷ 0.15 equals:

A. 0.1413 **B.** 1.413 **C.** 14.13 **D.** 141.3 **E.** 1413

Understanding

8. Change the following to dollars ($) by dividing by 100.

a. 365 cents **b.** 170 cents **c.** 5685 cents

d. 75 cents **e.** 90 cents **f.** 6350 cents

9. 100 987.5412 ÷ 10^4 = ?
Round the answer to the nearest hundredth.

Reasoning

10. Stephanie spent $6.95 on these chocolates. What was the cost of each chocolate? Give your answer to the nearest 5 cents.

11. If you have $22.50 for bus fares to school for the week, how much would you spend on each of the 5 days?

12. Emily wants to make 10 cushions from 6.75 metres of material that she found on a table of remnants at a fabric shop. How much material would she have for each cushion?

13. WE28 How many 1.25-litre bottles of water could be poured into a 25-litre drink dispenser?

14. The area of an office floor is 85.8 square metres. How many people could she fit in the office if each person takes up 1.2 square metres?

15. How many compact discs can be stacked on a shelf that is 28.6 centimetres high if each compact disc case is 1.1 centimetres high?

16. How many burgers could be bought for $562.80 if each burger costs $2.80?

Problem solving

17. Anna has a wooden rod that is 2.4 metres long. She needs to cut 3 lengths, each of 0.4 metres, and 2 lengths, each of 0.2 metres. What fraction of the original rod will be left over after she has cut the required lengths?

18. The carat is a unit of measure used to weight precious stones. A 2.9 – carat diamond weighs 8.9494 grains. How many grains is 1 carat equal to?

19. A jug is $\frac{1}{5}$ full of water. Jack added more water to the jug until was $\frac{3}{4}$ full. At that stage there was 1.5 L of water in the jug. How much water was originally in the jug?

Reflection

Compare the processes involved when dividing by 0.1 and dividing by $\frac{1}{10}$.

CHALLENGE 6.2

What is my number?

.....

My number contains four different even digits with 3 decimal places. The last digit on the right is half the value of the first digit on the left. The sum of the middle two places is the same as the first digit. Finally, the sum of the first two digits is the same as the sum of the last two digits.

6.8 Review

6.8.1 Review questions

Fluency

1. Give the value of the 7 in each of the following.

a. 1.719 **b.** 3.0726 **c.** 4.7218 **d.** 0.2078 **e.** 23.1487
f. 0.00257 **g.** 17.592 **h.** 50.007

2. Write the following numbers in expanded notation.

a. 2.64 **b.** 0.369 **c.** 18.406 **d.** 96.3428

3. Add 2 tenths to the following.

a. 6.2 **b.** 0.743 **c.** 12.06 **d.** 3.91

4. Add 3 thousandths to the following.
a. 0.456 **b.** 12.803 **c.** 1.6 **d.** 2.79

5. Put < or > between the following.
a. 8.72 _____ 8.27 **b.** 0.35 _____ 0.37
c. 1.06 _____ 1.27 **d.** 10.214 _____ 10.219
e. 0.021 _____ 0.018 **f.** 13.0496 _____ 13.149
g. 0.804 06 _____ 0.804 17 **h.** 0.000 879 _____ 0.000 876

6. Write the following decimals in order from smallest to largest.
a. 0.13, 0.86, 0.34, 0.71, 0.22 **b.** 0.247, 0.274, 0.124, 0.258, 0.285
c. 0.834, 0.826, 0.859, 0.888, 0.891 **d.** 0.356, 0.358, 0.365, 0.385, 0.217

7. Write the following decimals as fractions in simplest form.
a. 0.8 **b.** 0.17 **c.** 0.36 **d.** 0.187 **e.** 0.125 **f.** 0.568 **g.** 0.205 **h.** 0.950

8. Write the following decimals as mixed numerals in simplest form.
a. 1.5 **b.** 4.60 **c.** 3.48 **d.** 5.25 **e.** 2.75 **f.** 2.625 **g.** 1.56 **h.** 8.32

9. Round the following to the number of decimal places shown in the brackets.
a. 1.29 (1) **b.** 2.047 (2) **c.** 13.8649 (2)
d. 0.0482 (3) **e.** 1.925 96 (1) **f.** 17.898 193 (1)

10. Round the following to the nearest unit.
a. 13.6 **b.** 29.02 **c.** 86.99 **d.** 100.09

11. Calculate the following.
a. 1.8 + 7.3 **b.** 4.21 + 5.88 **c.** 6.75 + 0.243
d. 12.047 + 3.6 **e.** 194 + 18.62 + 3.1 **f.** 34.1 + 7.629 + 0.00845

12. Calculate the following.
a. 9.6 − 4.3 **b.** 18.25 − 9.18 **c.** 3.92 − 1.88
d. 100 − 9.341 **e.** 4.876 − 3.927 **f.** 1.6 − 0.025

13. Calculate the following.
a. 6.2 × 3 **b.** 4.67 × 9 **c.** 13.2036 × 5 **d.** 0.7642 × 7

14. Calculate the following.
a. 0.23 × 11 **b.** 16.28 × 41 **c.** 182.94 × 28 **d.** 0.02894 × 32

15. Calculate.
a. 0.26 × 10 **b.** 1.345 × 10 **c.** 0.0645 × 100 **d.** 1.8294 × 100
e. 146.6281 × 100 **f.** 0.048 0643 × 1000 **g.** 0.839204 × 1000 **h.** 0.368 × 1000

16. Calculate.
a. 3.2 × 0.41 **b.** 1.72 × 0.3 **c.** 0.87 × 0.9 **d.** 0.03 × 0.006
e. 0.58 × 1.5 **f.** 2.83 × 0.96 **g.** 11.468 × 1.3 **h.** 1.248 × 0.82

17. Calculate each of the following.
a. 2.5^2 **b.** 0.03^2 **c.** $\sqrt{0.64}$ **d.** $\sqrt{0.0025}$

18. Calculate each of the following.
a. 2.4 ÷ 8 **b.** 1.64 ÷ 4 **c.** 12.48 ÷ 6 **d.** 147.24 ÷ 2 **e.** 1.76 ÷ 11 **f.** 264.88 ÷ 8

19. Calculate each of the following.
a. 14.623 ÷ 10 **b.** 102.36 ÷ 10 **c.** 9612.347 ÷ 1000
d. 20.032 ÷ 100 **e.** 264983.0026 ÷ 1000 **f.** 3462.94 ÷ 100

20. Write as a finite decimal or as an infinite recurring decimal.
a. $\frac{1}{20}$ **b.** $\frac{5}{8}$ **c.** $\frac{5}{16}$ **d.** $\frac{2}{16}$

21. Write these infinite recurring decimals using a short form.
a. 4.555... **b.** 0.8282... **c.** 19.27812781... **d.** 83.01626262...

22. Calculate.
a. 4.8 ÷ 0.6 **b.** 35.7 ÷ 0.7 **c.** 12.1 ÷ 1.1 **d.** 13.72 ÷ 0.4
e. 17.8964 ÷ 0.02 **f.** 372.04572 ÷ 0.06 **g.** 0.28956 ÷ 0.12 **h.** 3214.0170 ÷ 0.15

Problem solving

23. Jim saved the following amounts of pocket money to take away on holidays: $12.50, $15.00, $9.30, $5.70, $10.80. How much money did Jim have to spend on holidays?

24. Mandie poured 0.375 litres from a 1.5-litre bottle of juice. How much juice was left in the bottle?

25. Tara bought 0.350 kilogram of shaved ham at $10.50 per kilogram. How much did Tara pay for the ham?

26. The decimal equivalent of $\frac{3}{13}$ is $0.\overline{230769}$

a. What is the decimal equivalent of $\frac{1}{13}$?

b. Find the decimal equivalent of $\frac{5}{13}$.

27. If a bottle and its cork cost $1.10 and the bottle costs $1.00 more than the cork, how much does the cork cost? (*Hint:* The answer is not 10 cents.)

28. What decimal gives the same result when multiplied by 5 as it does when 5 is added to it?

29. Identify the pattern in this sequence of numbers:
0.1, 0.2, 0.3, 0.6, 1.1, 2.0.
What are the next three numbers in the sequence?

30. Michael added the following on his calculator: 22.5, 0.678, 138.4 and 23.1.

a. He estimated the answer to be about 184 but the calculator display showed 60.118. If his error was putting a decimal point in the wrong place, which number did he enter incorrectly?

b. Michael also performs the following calculation: $5.24 + 23.87 - 2.092$
He compares his answer to the answers of three of his friends and they are all different. The answers are: 5.535, 26.19, 27.018 and 74.178. Determine the correct answer and discuss the key-stroke errors made in the other calculations.

31. Leesa has 3 m of ribbon. It takes 0.15 m of ribbon to make a bow. How many bows can she make with the ribbon?

32. Steven lives $\frac{3}{4}$ km from school. He can walk at 6.25 km/h. How long does it take to walk to school?

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Language

It is important to learn and be able to use correct mathematical language in order to communicate effectively. Create a summary of the topic using the key terms below. You can present your summary in writing or using a concept map, a poster or technology.

decimal digits
decimal place
decimal point
finite decimal
place value
recurring decimal
repeating decimal
rounding
terminating decimal

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STARTERS

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Bruschetta	$4.50
Dips & Bread	$3.50
Potato Skins	$6.50
– served with sour cream	
Basket of fries	$4.25
Nachos with cheese, salsa and sour cream	$6.50
Italian style baby potatoes	$6.50

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All served with salad and fries on your choice of bread

Bacon, Lettuce & Tomato	$5.50
Ham & Cheese	$5.00
Ham, Cheese & Tomato	$5.50
Grilled Chicken & Cheese	$6.00
Egg & Lettuce	$5.50
Top Quality Steak & Onions	$9.95

BURGERS AND HOT DOGS

All served with salad and fries

Jumbo Hot Dog	$5.95
Gourmet Dog	$6.95
– includes smoked sausage	
Double Beef Burger	$5.95
Grilled Chicken Burger	$6.95
Fresh Fish Burger	$7.55
Vegetable Burger	$4.95

Additional items

Bacon	$0.50
Grilled onion	$0.50
Olives	$0.75
Cheddar	$0.75

HOUSE SPECIAL

Today's Special is 250 gram Rib Eye Steak served with fries, vegetables with your choice of sauce	$18.95

PASTA

Your choice of Spaghetti, Fettuccine, Gnocchi, Ravioli or Tagliatelle with the sauces below

Mama's Meaty Bolognaise	$9.95
Creamy Chicken	$9.95
Fresh Seafood Marinara	$10.95
Pesto	$9.95
Tomato and Hot Chilli	$8.95
Vegetable Lasagne	$9.95
Rich and Meaty Lasagne	$9.95

DESSERTS

Homemade Fruit Pie	$3.75
Brownie	$3.50
Cheesecake	$3.95
Frozen Yoghurt	$2.50
– Chocolate or Vanilla	
Jumbo Banana Split	$4.50

When we eat out or buy takeaway food, a number of factors are taken into consideration before we make our purchases. What types of foods are available? What is the size of each serving? What is the cost of the meal? These are examples of things we consider when we try to decide what and where we eat.

In a group of four, you and your friends have decided to eat at a local restaurant. Your parents have given each of you $30.00 to spend on your meal.

1. Using the menu provided, work out how much each of the following meals would cost. On a separate piece of paper, set out each order as it would appear on the final bill. An example is shown at right.

EAT EASY RESTAURANT

Item	Cost
Tossed green salad	$5.50
Homemade fruit pies	$3.75
Total	$9.25

a. Tossed green salad and Homemade fruit pies
b. Chicken Caesar salad, Jumbo hot dog and Vanilla-flavoured yoghurt
c. Garlic bread, Potato skins, House special and Jumbo banana split
d. Vegetable burger with cheddar, Basket of fries and Brownie

Your group has decided to order a three-course meal. The first course consists of items from the Salads and Starters sections and the third course is from the Desserts section. The second course is ordered from the remaining sections.

The group also decides to buy a large jug of orange juice to drink, and plans to share its cost of $4.60 evenly.

2. What is your share of the cost of the drinks?
3. How much does this leave you to spend on your food order?
4. Select your three-course meal, keeping in mind the amount of money you have to spend. Write down your order, and set it out as you did for question **1**.

Obtain the orders from three of your classmates. This will represent your group of four.

5. Write down each group member's order on a separate sheet of paper. Present the information as you would like it to appear on your final bill. Include the total cost of each person's order.
6. How much change did each group member receive from their $30?

Imagine that the restaurant's final bill for your group showed only the total amount owing; that is, there was no information on the breakdown of charges.

7. Comment on how you would feel about this and how you think bills should be presented for group bookings.

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Complete this digital doc: Code puzzle: I came to Australia in the 1860s. Who am I? (doc-14321)

Answers

Topic 6 Decimals

Exercise 6.2 Place value and comparing decimals

1. **a.** $\frac{2}{10}$ **b.** $\frac{2}{100}$ **c.** $\frac{2}{100}$ **d.** $\frac{2}{10}$ **e.** $\frac{2}{1000}$ **f.** $\frac{2}{1000}$
g. $\frac{2}{10000}$ **h.** $\frac{2}{10000}$ **i.** 20 **j.** 2 **k.** $\frac{2}{10000}$ **l.** $\frac{2}{100000}$

2. **a.** $\frac{9}{10}$ **b.** $\frac{9}{10}$ **c.** $\frac{9}{100}$ **d.** $\frac{9}{100}$ **e.** $\frac{9}{100}$ **f.** $\frac{9}{1000}$
g. $\frac{9}{10000}$ **h.** $\frac{9}{100000}$ **i.** 9 **j.** 9 **k.** 900 **l.** 90

3. **a.** Four units, 4; one tenth, $\frac{1}{10}$; $4 + \frac{1}{10}$

b. One unit, 1; eight tenths, $\frac{8}{10}$; five hundredths, $\frac{5}{100}$; $1 + \frac{8}{10} + \frac{5}{100}$

c. Two tenths, $\frac{2}{10}$; seven hundredths, $\frac{7}{100}$; one thousandth, $\frac{1}{1000}$; $\frac{2}{10} + \frac{7}{100} + \frac{1}{1000}$

d. Nine units, 9; two hundredths, $\frac{2}{100}$; $9 + \frac{2}{100}$

e. One ten, 10; six units, 6; one thousandth, $\frac{1}{1000}$; $16 + \frac{1}{1000}$

f. Three units, 3; four tenths, $\frac{4}{10}$; two thousandths, $\frac{2}{1000}$; seven hundred thousandths, $\frac{7}{100\,000}$; $3 + \frac{4}{10} + \frac{2}{1000} + \frac{7}{100\,000}$

4. **a.** $\frac{4}{10}$ **b.** $2 + \frac{7}{10}$ **c.** $6 + \frac{8}{10}$ **d.** $5 + \frac{2}{10} + \frac{3}{100}$
e. $\frac{7}{10} + \frac{6}{100} + \frac{3}{1000}$ **f.** $2 + \frac{1}{10} + \frac{8}{1000}$ **g.** $10 + 9 + \frac{9}{10} + \frac{1}{100}$ **h.** $\frac{1}{10} + \frac{1}{100} + \frac{1}{10000}$
i. $7 + \frac{2}{10} + \frac{9}{100} + \frac{6}{1000} + \frac{4}{10000}$ **j.** $\frac{3}{10} + \frac{3}{100} + \frac{2}{10\,000} + \frac{4}{100\,000}$
k. $300 + \frac{3}{100}$ **l.** $10 + 2 + \frac{2}{10} + \frac{7}{100} + \frac{6}{1000} + \frac{4}{10000} + \frac{3}{1000000}$

5. **a.** $(2 \times 1) + (4 \times \frac{1}{10}) + (7 \times \frac{1}{100})$ **b.** $(3 \times 1) + (6 \times \frac{1}{10}) + (9 \times \frac{1}{100})$
c. $(1 \times 1) + (2 \times \frac{1}{10}) + (5 \times \frac{1}{100})$ **d.** $(5 \times 10) + (6 \times 1) + (1 \times \frac{1}{100})$
e. $(3 \times 10) + (9 \times 1) + (1 \times \frac{1}{100})$ **f.** $(1 \times 10) + (6 \times 1) + (7 \times \frac{1}{100})$
g. $(7 \times 1) + (1 \times \frac{1}{10}) + (2 \times \frac{1}{100}) + (3 \times \frac{1}{1000})$ **h.** $(5 \times 1) + (9 \times \frac{1}{10}) + (8 \times \frac{1}{100}) + (7 \times \frac{1}{1000})$
i. $(1 \times 10) + (3 \times 1) + (4 \times \frac{1}{10}) + (8 \times \frac{1}{100}) + (2 \times \frac{1}{1000})$ **j.** $(3 \times \frac{1}{10}) + (6 \times \frac{1}{1000}) + (2 \times \frac{1}{10000})$
k. $(1 \times \frac{1}{10}) + (3 \times \frac{1}{100}) + (5 \times \frac{1}{10000})$ **l.** $(5 \times \frac{1}{10}) + (2 \times \frac{1}{10000})$

6. **a.** **C** **b.** **D**

7.

		Tens	Units		Tenths	Hundredths	Thousandths
a	0.205	0	0	.	2	0	5
b	1.06	0	1	.	0	6	0
c	74.108	7	4	.	1	0	8
d	0.108	0	0	.	1	0	8
e	50.080	5	0	.	0	8	0

8. **a.** 0.32 **b.** 0.91 **c.** 0.59 **d.** 0.99 **e.** 0.901 **f.** 0.666
g. 0.1440 **h.** 0.039 90 **i.** 2.559 **j.** 10.0233 **k.** 0.110 49 **l.** 0.102 36

9. **a.** > **b.** < **c.** < **d.** < **e.** < **f.** <
g. < **h.** > **i.** > **j.** < **k.** < **l.** >

10. **a.** 0.17, 0.21, 0.33, 0.39, 0.45 **b.** 0.12, 0.16, 0.19, 0.22, 0.27
c. 0.314, 0.391, 0.413, 0.420, 0.502 **d.** 0.613, 0.624, 0.677, 0.690, 0.710
e. 0.803, 0.807, 0.811, 0.821, 0.902 **f.** 0.1002, 0.1033, 0.1164, 0.1196, 0.1245
g. 0.9087, 0.9189, 0.9812, 0.9864, 0.9943 **h.** 0.4004, 0.4076, 0.4100, 0.4139, 0.4826
i. 4.0292, 4.5097, 4.6031, 4.6249, 4.802 **j.** 13.0229, 13.0291, 13.0294, 13.0299, 13.0929
k. 0.003, 0.004 65, 0.005 02, 0.0056, 0.009 **l.** 0.507, 0.5079, 0.595, 0.61, 0.617

11. a. 0.48, 0.39, 0.36, 0.31, 0.19 **b.** 0.38, 0.35, 0.27, 0.16, 0.02
c. 0.99, 0.97, 0.95, 0.91, 0.90 **d.** 0.29, 0.13, 0.09, 0.07, 0.02
e. 1.291, 1.288, 1.279, 1.273, 1.264 **f.** 0.491, 0.442, 0.437, 0.433, 0.406
g. 0.390, 0.372, 0.318, 0.317, 0.309 **h.** 0.602, 0.591, 0.573, 0.556, 0.502
i. 0.8889, 0.8823, 0.8448, 0.8217, 0.8207 **j.** 0.7695, 0.7657, 0.6024, 0.1079, 0.0307
k. 1.843 21, 1.702 96, 1.486 59, 1.486 13, 1.349 54 **l.** 12.9092, 12.392, 12.289 50, 12.208 64, 12.002 36

12. E **13.** D **14.** E

15. a. i. Tenths **ii.** Yes **b. i.** Units **ii.** Yes
c. i. Thousandths **ii.** No **d. i.** Hundredths **ii.** No
e. i. Tenths **ii.** No **f. i.** Tenths **ii.** Yes
g. i. Units **ii.** No **h. i.** Hundredths **ii.** Yes
i. i. Units **ii.** Yes **j. i.** Tens **ii.** Yes

16. a. F **b.** T **c.** F **d.** T **e.** F **f.** T

17. a. Shelley, 0.21 seconds **b.** Carolyn, 0.06 seconds **c.** Mara, 0.11 seconds
d. Shelley, Carolyn, Robyn **e.** Carolyn, Shelley, Kyah **f.** Mara, Jenika, Shelley
g. Yes, Shelley

18. 8.264

19. a. 0.1428571429 0.2857142857 0.42857142857
0.5714285714 0.7142857143 0.8571428571
b. Each number has a repeated group of decimals in the same sequence: 142857.

Exercise 6.3 Converting decimals to fractions and fractions to decimals

1. a. $\frac{3}{10}$ **b.** $\frac{1}{2}$ **c.** $\frac{9}{10}$ **d.** $\frac{21}{100}$ **e.** $\frac{2}{5}$ **f.** $\frac{4}{5}$
g. $\frac{6}{25}$ **h.** $\frac{11}{25}$ **i.** $\frac{49}{100}$ **j.** $\frac{63}{100}$ **k.** $\frac{251}{500}$ **l.** $\frac{617}{1000}$
m. $\frac{3}{25}$ **n.** $\frac{3}{10}$ **o.** $\frac{16}{25}$ **p.** $\frac{7}{25}$ **q.** $\frac{441}{500}$ **r.** $\frac{591}{625}$
s. $\frac{9209}{10000}$ **t.** $\frac{4621}{10000}$ **u.** $\frac{3}{4}$ **v.** $\frac{3}{25}$ **w.** $\frac{143}{500}$ **x.** $\frac{117}{250}$

2. a. $1\frac{3}{10}$ **b.** $1\frac{3}{5}$ **c.** $2\frac{7}{10}$ **d.** $9\frac{2}{5}$ **e.** $1\frac{1}{5}$ **f.** $2\frac{4}{5}$
g. $4\frac{1}{5}$ **h.** $8\frac{1}{2}$ **i.** $2\frac{13}{100}$ **j.** $6\frac{12}{25}$ **k.** $5\frac{27}{100}$ **l.** $19\frac{91}{500}$
m. $12\frac{21}{50}$ **n.** $3\frac{3}{20}$ **o.** $6\frac{1}{4}$ **p.** $9\frac{7}{50}$ **q.** $12\frac{843}{1000}$ **r.** $16\frac{341}{500}$
s. $2\frac{4917}{10000}$ **t.** $4\frac{1693}{5000}$ **u.** $37\frac{41}{200}$ **v.** $18\frac{129}{200}$ **w.** $24\frac{69}{200}$ **x.** $100\frac{3}{625}$

3. a. B **b.** A **c.** C **d.** D **e.** E

4. a. 0.75 **b.** 0.5 **c.** 0.8 **d.** 0.05 **e.** 0.2
f. 0.25 **g.** 0.375 **h.** 0.02 **i.** 0.32

5. a. 0.375 **b.** 0.875 **c.** 0.0625

6. If the denominator only contains factors of 2 or 5, the decimal will terminate.

7. $\frac{3}{7} = 0.\dot{4}2857\dot{1}, \frac{4}{7} = 0.\dot{5}7142\dot{8}, \frac{5}{7} = 0.\dot{7}1428\dot{5}, \frac{6}{7} = 0.\dot{8}5714\dot{2}$

8. No, the second jump was best.

9. 300 m

Exercise 6.4 Rounding and repeating decimals

1. a. 0.32 **b.** 0.86 **c.** 1.25 **d.** 13.05 **e.** 7.13 **f.** 100.81
g. 71.26 **h.** 0.01 **i.** 0.19 **j.** 19.70 **k.** 0.40 **l.** 1.00

2. a. 0.4 **b.** 0.9 **c.** 9.3 **d.** 25.3 **e.** 300.1 **f.** 12.8
g. 99.9 **h.** 8.9 **i.** 17.6 **j.** 0.9 **k.** 94.0 **l.** 1.0

3. a. 2.39 **b.** 14.0 **c.** 0.03 **d.** 0.8765 **e.** 64.2952 **f.** 0.382
g. 96.3 **h.** 3.04 **i.** 8.90 **j.** 47.880 **k.** 0.10 **l.** 0.49

4. a. C **b.** D **c.** D **d.** A

5. a. 11 **b.** 8 **c.** 4 **d.** 93 **e.** 112 **f.** 22
g. 42 **h.** 2138 **i.** 0 **j.** 1 **k.** 1 **l.** 41

6. a. $2.\dot{5}$ **b.** $0.\dot{6}$ **c.** $12.\dot{8}$ **d.** 49.1 **e.** $0.\dot{2}\dot{6}$ **f.** $0.\dot{4}\dot{1}$
g. $0.\dot{9}\dot{1}\dot{3}$ **h.** $8.\dot{6}41\dot{8}$ **i.** $0.040\overline{12}$ **j.** $133.9\overline{462}$ **k.** $1.8\dot{3}$ **l.** $0.12\dot{7}$

7. a. 0.17 **b.** 0.3 **c.** 0.1 **d.** 0.13 **e.** 0.18
f. 0.4 **g.** 0.417 **h.** 0.142 857 **i.** 0.47

8. a. B **b.** A **c.** D **d.** A

9. a. 0.5 **b.** $0.\dot{3}$ **c.** 0.25 **d.** 0.2 **e.** 0.1

10. a. 3.5 **b.** $4.\dot{6}$ **c.** 0.75 **d.** 3.8 **e.** $0.\dot{7}$

11. a. 10 **b.** 80 **c.** 50 **d.** 140 **e.** 260 **f.** 180
g. 310 **h.** 1480 **i.** 10020 **j.** 209720 **k.** 20 **l.** 10

12. a. 300 **b.** 200 **c.** 900 **d.** 100 **e.** 100 **f.** 6800
g. 900 **h.** 9800 **i.** 1700 **j.** 1462800 **k.** 100 **l.** 0

13. a. 3000 **b.** 6000 **c.** 12000 **d.** 19000 **e.** 28000 **f.** 9000
g. 1000 **h.** 497000 **i.** 13000 **j.** 1000 **k.** 130000 **l.** 50000

14. $27.70

15. A more reasonable measurement would be 3.568 m. With the instruments we have available, it would not be possible to cut the timber more accurately.

16. $0.295, 0.\dot{2}9\dot{5}, 0.29\dot{5}, 0.2\dot{9}\dot{5}$

17. Answers will vary. An example is 0.543216.

18. Check with your teacher.

19. 25°C

20. a. i. 0.8 **ii.** 0.84 **iii.** 0.838

b. The rules are the same regardless of the number of decimals.

21. a. 0.166667 1.166667 2.166667 3.166667 4.166667

b. The decimal places are identical.

c. The units are different.

d. 5.167
6.167
7.167

$$\frac{31}{6} = 5\frac{1}{6}$$
$$= 5 + 0.167$$
$$= 5.167$$

22. 50.0000 33.3333 20.0000 14.2857 9.0909
7.6923 5.8824 5.2632 4.3478 3.4483

Exercise 6.5 Adding and subtracting decimals

1. a. 3.5 **b.** 2.69 **c.** 13.239 **d.** 102.1858

2. a. 6.77 **b.** 21.906 **c.** 2012.333 **d.** 261.44906

3. a. 4.6 **b.** 6.9 **c.** 1.59 **d.** 3.471 **e.** 17.280
f. 8.9685 **g.** 17.3271 **h.** 20.4329 **i.** 1056.7863

4. a. 11.69 **b.** 105.22 **c.** 42.031 **d.** 18.951
e. 105.958 **f.** 192.8949 **g.** 20.9916 **h.** 2618.9564

5. a. B **b.** A **c.** A **d.** D

6. a. 0.60 **b.** 4.205 **c.** 0.0806 **d.** 1.3368
e. 5.02 **f.** 89.19 **g.** 87.09 **h.** 2.733

7. a. 3.34 **b.** 5.93 **c.** 3.849 **d.** 0.327 **e.** 4.146
f. 27.83 **g.** 1.934 **h.** 3.289 **i.** 1.2509 **j.** 14.5326
k. 16.766 **l.** 38.238 **m.** 4.1136 **n.** 0.1951 **o.** 0.77216

8. a. C **b.** B **c.** B **d.** B

9. $167.00 **10.** $4.70 **11.** 20.05 kilometres **12.** 8.1 kilometres

13. $6.60 **14.** $10.35 **15.** 50.48 kilograms

16. 11.73 metres **17.** 48.38 seconds **18.** 31.28 metres

19. a.

Team	1	2	3	4	5	6
Time (seconds)	153	159.3	148.9	166.6	155.9	155.8

b. Team 3 **c.** 4.1 seconds

20. 65.53 g, 43.43 g, 32.03 g or 32.04 g **21.** 49 calls

22. Increase 51 c **23.** 0.107

Challenge 6.1

One possible solution is 5.83 + 1.46 = 7.29.

Exercise 6.6 Multiplying decimals (including by multiples of 10)

1. a. 14.0 **b.** 125.6 **c.** 146.7 **d.** 61.2 **e.** 111.70 **f.** 428.67
g. 190.26 **h.** 518.96 **i.** 26.068 **j.** 6.384 **k.** 0.794 43 **l.** 113.541 464

2. a. 0.84 **b.** 2.94 **c.** 0.32 **d.** 7.76 **e.** 0.140
f. 0.192 **g.** 0.385 **h.** 0.441 **i.** 0.326 **j.** 4.304
k. 0.2082 **l.** 0.3486 **m.** 2.8992 **n.** 4.905 64 **o.** 10.016 395

3. a. 0.0001 **b.** 0.000 012 **c.** 0.063 **d.** 0.000 222 **e.** 0.000 002 44
f. 0.000 36 **g.** 0.392 **h.** 0.000 6062 **i.** 0.000 0042

4. a. 0.3 **b.** 0.851 **c.** 2.538 **d.** 6.557 **e.** 16.848
f. 60.300 **g.** 55.872 **h.** 28.652 **i.** 12.3095 **j.** 18.9644
k. 79.7720 **l.** 35.6896 **m.** 4.2036 **n.** 1.699 24 **o.** 14.892

5. a. E **b.** B **c.** A **d.** B

6. a. 0.0004 **b.** 1.69 **c.** 4.2025 **d.** 0.3 **e.** 0.4 **f.** 0.07

7. a. 64.8 **b.** 1389.6 **c.** 58 906.43 **d.** 270.8 **e.** 217 148.96
f. 842 619 **g.** 82 049.6783 **h.** 3 268 904.3267 **i.** 984 326.641 **j.** 59 027 683.017
k. 278.498 32 **l.** 460 **m.** 5 290 000 **n.** 39 486 000

8. a. 1092 **b.** 4548 **c.** 13 450 **d.** 38 340
e. 1016 **f.** 5973 **g.** 147 200 **h.** 437 400

9. a. 3500 **b.** 12 700 **c.** 1100 **d.** 2535 **e.** 5820 **f.** 11 015

10. $750 **11.** 108 L **12.** 260.7569 m^2 **13.** $5.55

14. a. 3205*c* **b.** $32.05 **15.** 0.375 L **16.** 13

17. Together they pay $6.30: Sophie pays $3.60 and Hamish pays $2.70.

18. The 50 extra sheets are 6 mm thick. You probably wouldn't notice the difference.

Exercise 6.7 Dividing decimals (including by multiples of 10)

1. a. 0.6 **b.** 3.1 **c.** 2.9 **d.** 0.54 **e.** 1.61
f. 4.71 **g.** 2.42 **h.** 0.788 **i.** 1.983 **j.** 1.147
k. 4.0701 **l.** 8.3729 **m.** 6.8594 **n.** 31.6176 **o.** 3.13
p. 3.12 **q.** 2.231 **r.** 1.222 **s.** 6.421 **t.** 11.0311

2. a. 1.85 **b.** 4.75 **c.** 1.46 **d.** 2.45 **e.** 1.25 **f.** 6.95

3. a. 1.407 **b.** 9.6813 **c.** 9.8506 **d.** 0.6208 **e.** 3.592 87
f. 0.023 4978 **g.** 0.053 320 667 **h.** 0.000 090 769 **i.** 0.103 454 97 **j.** 0.802 4056
k. 0.0001527 **l.** 0.000 583 **m.** 0.000 072 05 **n.** 0.000 000 0032 **o.** 0.000 000 048 7

4. a. 0.265 **b.** 0.79 **c.** 0.235 **d.** 0.595 **e.** 0.001 465 **f.** 0.001 74

5. a. 5 **b.** 7 **c.** 8 **d.** 9 **e.** 4
f. 11.1 **g.** 0.31 **h.** 5.1 **i.** 2.2 **j.** 66
k. 72.1 **l.** 12.31 **m.** 0.852 **n.** 0.336 77 **o.** 1.367

6. a. 1564.08 **b.** 4.254 **c.** 20.01 **d.** 12.13 **e.** 143.134
f. 1786.1 **g.** 0.4782 **h.** 15 771.987 **i.** 1630.95

7. a. D **b.** D **c.** B **d.** D

8. a. \$3.65 **b.** \$1.70 **c.** \$56.85 **d.** \$0.75 **e.** \$0.90 **f.** \$63.50

9. 10.10 **10.** \$0.60 **11.** \$4.50 **12.** 0.675 m

13. 20 **14.** 71 **15.** 26 **16.** 201

17. $\frac{1}{3}$ **18.** 3.086 grains **19.** 0.4 L

Challenge 6.2

8.264

Exercise 6.8 Review

1. a. 7 tenths **b.** 7 hundredths **c.** 7 tenths **d.** 7 thousandths
e. 7 ten thousandths **f.** 7 hundred thousandths **g.** 7 units **h.** 7 thousandths

2. a. $(2 \times 1) + (6 \times \frac{1}{10}) + (4 \times \frac{1}{100})$
b. $(3 \times \frac{1}{10}) + (6 \times \frac{1}{100}) + (9 \times \frac{1}{1000})$
c. $(1 \times 10) + (8 \times 1) + (4 \times \frac{1}{10}) + (6 \times \frac{1}{1000})$
d. $(9 \times 10) + (6 \times 1) + (3 \times \frac{1}{10}) + (4 \times \frac{1}{100}) + (2 \times \frac{1}{1000}) + (8 \times \frac{1}{10000})$

3. a. 6.4 **b.** 0.943 **c.** 12.26 **d.** 4.11

4. a. 0.459 **b.** 12.806 **c.** 1.603 **d.** 2.793

5. a. $>$ **b.** $<$ **c.** $<$ **d.** $<$
e. $>$ **f.** $<$ **g.** $<$ **h.** $>$

6. a. 0.13, 0.22, 0.34, 0.71, 0.86 **b.** 0.124, 0.247, 0.258, 0.274, 0.285
c. 0.826, 0.834, 0.859, 0.888, 0.891 **d.** 0.217, 0.356, 0.358, 0.365, 0.385

7. a. $\frac{4}{5}$ **b.** $\frac{17}{100}$ **c.** $\frac{9}{25}$ **d.** $\frac{187}{1000}$
e. $\frac{1}{8}$ **f.** $\frac{71}{125}$ **g.** $\frac{41}{200}$ **h.** $\frac{19}{20}$

8. a. $1\frac{1}{2}$ **b.** $4\frac{3}{5}$ **c.** $3\frac{12}{25}$ **d.** $5\frac{1}{4}$
e. $2\frac{3}{4}$ **f.** $2\frac{3}{8}$ **g.** $1\frac{14}{25}$ **h.** $8\frac{8}{25}$

9. a. 1.3 **b.** 2.05 **c.** 13.86 **d.** 0.048 **e.** 1.9260 **f.** 17.90

10. a. 14 **b.** 29 **c.** 87 **d.** 100

11. a. 9.1 **b.** 10.09 **c.** 6.993 **d.** 15.647 **e.** 215.72 **f.** 41.737 45

12. a. 5.3 **b.** 9.07 **c.** 2.04 **d.** 90.659 **e.** 0.949 **f.** 1.575

13. a. 18.6 **b.** 42.03 **c.** 66.0180 **d.** 5.3494

14. a. 2.53 **b.** 667.48 **c.** 5122.32 **d.** 0.926 08

15. a. 2.6 **b.** 13.45 **c.** 6.45 **d.** 182.94
e. 14 662.81 **f.** 48.0643 **g.** 839.204 **h.** 368

16. a. 1.312 **b.** 0.516 **c.** 0.783 **d.** 0.000 18
e. 0.87 **f.** 2.7168 **g.** 14.9084 **h.** 1.023 36

17. a. 6.25 **b.** 0.0009 **c.** 0.8 **d.** 0.05

18. a. 0.3 **b.** 0.41 **c.** 2.08 **d.** 73.62 **e.** 0.16 **f.** 33.11

19. a. 1.4623 **b.** 10.236 **c.** 9.612 347 **d.** 0.200 32 **e.** 264.983 0026 **f.** 34.6294

20. a. 0.05 **b.** 0.625 **c.** 0.3125 **d.** 0.125

21. a. 4.5 **b.** $0.\overline{82}$ **c.** $19.\overline{2781}$ **d.** 83.0162

22. a. 8 **b.** 51 **c.** 11 **d.** 34.3
e. 894.73 **f.** 6200.762 **g.** 2.413 **h.** 21 426.78

23. \$53.30

24. 1.125 L

25. \$3.70 (Rounded to nearest 5c) or \$3.68

26. a. $0.\overline{0.76923}$ **b.** $0.\overline{384615}$

27. 5c

28. 1.25

29. 3.7, 6.8, 12.5

30. a. 13.84 **b.** 17.018

31. 20

32. 7 min 12 s

Investigation — Rich task

1. a. $9.25 **b.** $17.40 **c.** $33.45 **d.** $13.45

2. $1.15

3. $28.85

4. Teacher to check.

5. Teacher to check.

6. Teacher to check.

7. Teacher to check.

TOPIC 7 Percentages

7.1 Overview

Numerous **videos** and **interactivities** are embedded just where you need them, at the point of learning, in your learnON title at www.jacplus.com.au. They will help you to learn the content and concepts covered in this topic.

7.1.1 Why learn this?

Percentages, like fractions and decimals, provide a way of expressing parts of a whole. They are commonly used in advertising, statistics and shopping. Just check out the sales signs when you go shopping and you will see the % sign.

7.1.2 What do you know?

assessON

1. **THINK** List what you know about percentages. Use a thinking tool such as a concept map to show your list.
2. **PAIR** Share what you know with a partner and then with a small group.
3. **SHARE** As a class, create a thinking tool such as a large concept map that shows your class's knowledge of percentages.

LEARNING SEQUENCE

7.1 Overview
7.2 Percentages as fractions
7.3 Percentages as decimals
7.4 Fractions and decimals to percentages
7.5 Finding percentages of an amount
7.6 One amount as a percentage of another
7.7 Common percentages and short cuts
7.8 Review

7.2 Percentages as fractions

7.2.1 Percentages as fractions

- The term **per cent** means 'per hundred'. The symbol for per cent is %. For example, 7% means 7 out of 100.
- Whether a percentage is a whole number, a mixed number, a proper fraction, or a decimal, it can be expressed as a fraction.
- To convert a whole number percentage into a fraction, write it over 100 and simplify.
- To convert a mixed number percentage into a fraction:
  - change the mixed number into an improper fraction
  - multiply the denominator by 100
  - simplify if possible.
- If the percentage is a proper fraction, multiply the denominator by 100 and simplify.
- To convert a percentage containing a decimal number into a fraction:
  - place the decimal number over 100
  - change the numerator to a whole number by multiplying both the numerator and the denominator by a required multiple of 10
  - simplify if possible.

WORKED EXAMPLE 1

Write 47% as a fraction.

THINK	WRITE
Write the percentage and then change it to a fraction with a denominator of 100.	$47\% = \frac{47}{100}$

WORKED EXAMPLE 2

Write 20% as a fraction in simplest form.

THINK	WRITE
1 Write the percentage, change it to a fraction with a denominator of 100 and then cancel by dividing numerator and denominator by the same number.	$20\% = \frac{\cancel{20}^{1}}{\cancel{100}^{5}}$
2 Write the answer and check that the fraction cannot be simplified further.	$= \frac{1}{5}$

WORKED EXAMPLE 3

Write the following percentages as fractions in simplest form.

a $\frac{1}{4}\%$ **b** $15\frac{1}{3}\%$

THINK	WRITE
a 1 Write the percentage and then multiply the denominator by 100. **2** Simplify.	**a** $\frac{1}{4}\% = \frac{1}{4 \times 100}$ $= \frac{1}{400}$
b 1 Write the percentage and then change the mixed number to an improper fraction. **2** Multiply the denominator by 100 and simplify by cancelling. Check to make sure that the fraction cannot be simplified further.	**b** $15\frac{1}{3}\% = \frac{46}{3}\%$ $= \frac{46}{3 \times 100}$ $= \frac{\cancel{46}^{23}}{\cancel{300}^{150}}$ $= \frac{23}{150}$

WORKED EXAMPLE 4

Write 36.4% as a fraction in simplest form.

THINK	WRITE
1 Write the percentage and change it to a fraction out of 100. **2** Change the numerator to a whole number by multiplying it by an appropriate multiple of 10, multiply the denominator by the same multiple of 10 and simplify the fraction. **3** Write the answer.	$36.4\% = \frac{36.4}{100}$ $= \frac{36.4 \times 10}{100 \times 10}$ $= \frac{\cancel{364}^{91}}{\cancel{1000}^{250}}$ $= \frac{91}{250}$

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- **Complete this digital doc:** SkillSHEET: Simplifying fractions with a denominator of 100 (doc-6470)
- **Complete this digital doc:** SkillSHEET: Converting a mixed number into an improper fraction (doc-6471)
- **Complete this digital doc:** SkillSHEET: Changing fractions to equivalent fractions with a denominator of 100 (doc-6472)
- **Complete this digital doc:** Spreadsheet: Percentages as fractions (doc-1909)

Exercise 7.2 Percentages as fractions

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–6, 10, 16	Questions: 1a–h, 2a–k, 3a–k, 4a–i, 5, 6, 9, 12, 14, 16, 17	Questions: 1h–o, 2k–u, 3k–u, 4j–r, 5–17

Individual pathway interactivity: int-4343

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Fluency

1. WE1 Write the following percentages as fractions. (Leave the answer as an improper fraction where appropriate.)

a. 17%	**b.** 29%	**c.** 81%	**d.** 79%	**e.** 99%
f. 43%	**g.** 3%	**h.** 19%	**i.** 67%	**j.** 33%
k. 9%	**l.** 189%	**m.** 243%	**n.** 127%	**o.** 353%

2. WE2 Write the following percentages as fractions in simplest form. (Change the answer to a mixed number where appropriate.)

a. 50%	**b.** 80%	**c.** 25%
d. 35%	**e.** 60%	**f.** 85%
g. 10%	**h.** 45%	**i.** 98%
j. 12%	**k.** 5%	**l.** 56%
m. 74%	**n.** 2%	**o.** 110%
p. 150%	**q.** 90%	**r.** 180%
s. 200%	**t.** 500%	**u.** 112%

3. WE3 Write the following percentages as fractions in simplest form. (Leave answers as mixed numbers where appropriate.)

a. $\frac{1}{2}\%$	**b.** $\frac{1}{5}\%$	**c.** $\frac{3}{4}\%$
d. $\frac{2}{3}\%$	**e.** $\frac{1}{10}\%$	**f.** $\frac{5}{8}\%$
g. $\frac{6}{11}\%$	**h.** $8\frac{1}{4}\%$	**i.** $3\frac{3}{4}\%$
j. $20\frac{2}{3}\%$	**k.** $9\frac{2}{3}\%$	**l.** $14\frac{1}{4}\%$
m. $60\frac{1}{4}\%$	**n.** $15\frac{1}{2}\%$	**o.** $22\frac{1}{2}\%$
p. $11\frac{1}{5}\%$	**q.** $10\frac{3}{8}\%$	**r.** $11\frac{2}{3}\%$
s. $150\frac{1}{2}\%$	**t.** $120\frac{1}{2}\%$	**u.** $33\frac{1}{3}\%$

4. WE4 Write the following percentages as fractions in simplest form.

a. 3.5%	**b.** 7.2%	**c.** 11.8%
d. 19.7%	**e.** 32.4%	**f.** 71.5%
g. 62.9%	**h.** 15.5%	**i.** 8.7%
j. 16.2%	**k.** 28.3%	**l.** 16.25%
m. 41.38%	**n.** 57.99%	**o.** 86.32%
p. 18.14%	**q.** 12.15%	**r.** 0.05%

5. MC **a.** 40% as a fraction in simplest form is:

A. $\frac{1}{4}$ **B.** $\frac{2}{5}$ **C.** $\frac{4}{100}$ **D.** $\frac{4000}{1}$ **E.** $\frac{0.4}{1000}$

b. $10\frac{1}{2}\%$ as a fraction is:

A. $\frac{21}{2}$ **B.** $\frac{10.5}{100}$ **C.** $\frac{2100}{2}$ **D.** $\frac{42}{100}$ **E.** $\frac{21}{200}$

c. 1.5% as a fraction in simplest form is:

A. $\frac{1.5}{100}$ **B.** $\frac{3}{100}$ **C.** $\frac{3}{200}$ **D.** $\frac{15}{100}$ **E.** $\frac{15}{1000}$

d. 138% as an improper fraction in simplest form is:

A. 138 **B.** $\frac{138}{100}$ **C.** $\frac{13.8}{100}$ **D.** $\frac{69}{50}$ **E.** $\frac{138}{1000}$

Understanding

6. Imran saves 20% of his pocket money each week. What fraction does he save?

7. In one game, 35% of a football team is injured. What fraction of the team is injured?

8. Each week Jodie spends 45% of her wages at the supermarket. What fraction of Jodie's wages is spent at the supermarket?

9. What fraction of a class of students are boys if 68% are girls?

10. If the unemployment rate in Australia is 8%:
- **a.** what fraction of the population is unemployed?
- **b.** what fraction of the population is employed?
- **c.** out of 100 people, how many would you expect to be unemployed?

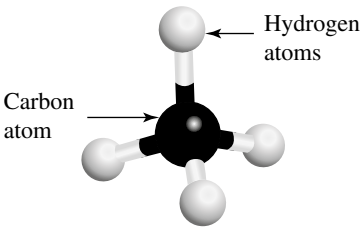
11. Seventeen per cent of visitors to Australia in 2005 were from New Zealand.
- **a.** What fraction of visitors to Australia was from New Zealand?
- **b.** What fraction of visitors to Australia was not from New Zealand?
- **c.** Out of 100 visitors to Australia, how many would you expect to be from New Zealand?

12. Less than 50% of the estimated 200 000 Australian invertebrate species have been described. What fraction is this?

13. What fraction remains if:
- **a.** 65% of the winnings have been spent
- **b.** 19% of the audience hated the movie
- **c.** all stock was discounted by 15%
- **d.** 93.5% of the school population supports the uniform policy?

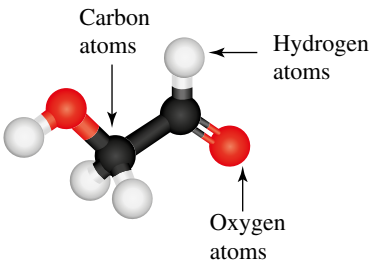
Reasoning

14. a. 80% of the atoms of a molecule of methane (natural gas) are hydrogen atoms. What fraction of the methane molecule is represented by hydrogen atoms?



Methane molecule

- **b.** The diagram below right represents a molecule of glycolaldehyde, a simple form of sugar, found in the gas surrounding a young star called IRAS 16293-2422, located around 400 light-years away from Earth. 50% of the atoms of this molecule are hydrogen atoms. What fraction of the glycolaldehyde molecule is represented by hydrogen atoms?
- **c.** Explain why, although both molecules have the same number of hydrogen atoms, they represent different fractions?



Glycolaldehyde molecule

15. Explain why 33.33% is an approximation and not the identical value of $\frac{1}{3}$.

Problem solving

16. 80% of the students in a Year 7 class have pets and 20% have blue eyes.
- **a.** What fraction of students has pets?
- **b.** What fraction of students has blue eyes?
- **c.** Does this mean that none of the students with blue eyes have pets? Explain your answer.

17. Shirley plays basketball in her school's team. In the last game she scored 50 points out of 75 throws but in the previous game she scored 45 points out of 60 throws.

a. In what game did she score a higher percentage of points relative to the number of throws?

b. Explain how the percentage is affected by:

i. increasing the number of points when the number of throws stay the same

ii. increasing the number of throws when the number of points stay the same

iii. increasing both the number of points and the number of throws at the same percentage rate.

Reflection

Percentages are used everywhere in our daily lives. Why is that so?

7.3 Percentages as decimals

7.3.1 Percentages as decimals

- Percentages can be expressed as decimals.
- To convert a percentage to a decimal, divide the percentage by 100 by moving the decimal point two spaces to the left.

WORKED EXAMPLE 5

Write the following percentages as decimals.

a 81% **b** 16.8%

THINK	WRITE
a 1 Per cent means 'out of 100', so write the percentage as a fraction out of 100.	**a** $81\% = \frac{81}{100}$
2 To write as a decimal, move the position of the decimal point 2 places to the left and place a zero in the units column.	$= 81 \div 100$ $= 0.81$
b 1 Write the percentage as a fraction out of 100.	**b** $16.8\% = \frac{16.8}{100}$
2 To write as a decimal, move the position of the decimal point 2 places to the left and place a zero in the units column.	$= 16.8 \div 100$ $= 0.168$

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Complete this digital doc: Spreadsheet: Percentages as decimals (doc-1910)

Exercise 7.3 Percentages as decimals

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–3, 4, 6, 10, 12	Questions: 1a–i, 2a–l, 3–6, 10, 12, 13	Questions: 1j–r, 2m–x, 3–13

Individual pathway interactivity: int-4344

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To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note*: Question numbers may vary slightly.

Fluency

1. WE5a Write the following percentages as decimals.

a. 36%	**b.** 14%	**c.** 19%	**d.** 28%
e. 73%	**f.** 92%	**g.** 66%	**h.** 59%
i. 11%	**j.** 99%	**k.** 9%	**l.** 7%
m. 4%	**n.** 1%	**o.** 25%	**p.** 200%
q. 150%	**r.** 360%		

2. WE5b Write the following percentages as decimals.

a. 12.3%	**b.** 31.6%	**c.** 59.2%	**d.** 84.9%
e. 37.6%	**f.** 42.1%	**g.** 21.9%	**h.** 16.9%
i. 10.7%	**j.** 11.1%	**k.** 3.1%	**l.** 4.6%
m. 9.2%	**n.** 5.9%	**o.** 6.8%	**p.** 8.8%
q. 14.25%	**r.** 31.75%	**s.** 23.55%	**t.** 45.75%
u. 0.05%	**v.** 1.02%	**w.** 4.01%	**x.** 0.02%

3. MC **a.** 41% as a decimal is:

A. 41 **B.** 0.41 **C.** 4.100 **D.** 4.1 **E.** 0.041

b. 8% as a decimal is:

A. 8 **B.** 0.008 **C.** 0.08 **D.** 0.8 **E.** 800

c. 43.64% as a decimal is:

A. 0.4364 **B.** 4.364 **C.** 43.6400 **D.** 436.4 **E.** 4364.0

d. 110% as a decimal is:

A. $\frac{110}{100}$ **B.** 1100 **C.** 0.11 **D.** 1.10 **E.** 11.0

Understanding

4. Car prices have dropped by 17% over the past 8 years. What is this percentage as a:

a. fraction **b.** decimal?

5. The maximum legal blood alcohol concentration (BAC) for drivers in Victoria is 0.05%. What is the BAC as a:

a. fraction **b.** decimal?

6. In 2013, the number of visitors to Australia during February was 2.85% lower than in January. Write this percentage as a decimal.

7. Over the past decade, insurance premiums have risen by 218%. Write this percentage as a decimal.

8. In the 3 months following a drought, the price of vegetables fell by 13.8%. Write this percentage as a decimal.

9. Government funding to state schools rose by 8.35%. Write this percentage as a decimal.

Reasoning

10. a. Round the following percentages to the nearest whole number: 23.2%, 23.5%, 23.8%, 23.9%.

b. Round the following decimal numbers to two decimal places: 0.232, 0.235, 0.238, 0.239.

c. The percentages in part **a** are the same as the decimal numbers in part **b**. Does it matter whether you round up or down the percentages or the decimal values?

11. Consider the following percentages: 23.6%, 23.6%, 23.66%.

a. Which one of the three percentages is largest?

b. Convert these percentages to decimals. State your answer to four decimal places.

Problem solving

12. Consider the following percentages: 37.8%, 39.6%, 30.9%, 34.5%, 32.8%.

a. Convert these numbers to decimals.

b. Arrange the percentages in order from highest to lowest.

c. Arrange the decimal numbers in order from highest to lowest.

d. Was it easier to arrange the numbers in the order required using decimals or percentages? Explain your answer.

13. Consider the following percentages: 7.291%, 72.91%, 729.1%.

a. Convert these percentages to decimal numbers.

b. What effect does the conversion have on the numbers?

Reflection

When is it more convenient to express a percentage as a decimal, rather than as a fraction?

7.4 Fractions and decimals to percentages

7.4.1 Changing fractions to percentages

- To change a fraction to a percentage, multiply by $\frac{100}{1}$.
- Simplify if possible.
- Add the percentage (%) sign.

WORKED EXAMPLE 6

Change each of the following fractions to a percentage, giving the answer as a mixed number where appropriate.

a $\frac{1}{10}$ **b** $\frac{5}{8}$ **c** $\frac{1}{7}$

THINK	WRITE
a 1 Write the fraction.	**a** $\frac{1}{10} = \frac{1}{\cancel{10}^{1}} \times \frac{\cancel{100}^{10}}{1}\%$
2 Multiply by $\frac{100}{1}$ and include the % sign.	
3 Cancel or simplify as appropriate.	
4 Multiply the numerators and then multiply the denominators.	$= \frac{10}{1}\%$
5 Simplify.	$= 10\%$
b 1 Write the fraction.	**b** $\frac{5}{8} = \frac{5}{\cancel{8}^{2}} \times \frac{\cancel{100}^{25}}{1}\%$
2 Multiply by $\frac{100}{1}$ and include the % sign.	
3 Cancel or simplify as appropriate.	
4 Multiply the numerators and then multiply the denominators.	$= \frac{125}{2}\%$
5 Simplify by writing as a mixed number.	$= 62\frac{1}{2}\%$

c 1 Write the fraction.	**c** $\frac{1}{7}$
2 Multiply by $\frac{100}{1}$ and include the % sign.	$= \frac{1}{7} \times \frac{100}{1}\%$
3 Multiply the numerators and then multiply the denominators.	$= \frac{100}{7}\%$
4 Divide the numerator by the denominator.	$7\overline{)10^{3}0}$ 1 4 rem 2
5 Simplify by writing as a mixed number.	$= 14\frac{2}{7}\%$

WORKED EXAMPLE 7

A survey showed that 3 out of 15 locals rated the Australian Grand Prix in Albert Park as their favourite spectator sport. What percentage is this?

THINK	WRITE
1 Write the information as a fraction by making the numerator the number of people who rated the Grand Prix as their favourite spectator sport and the denominator the total number of people.	$\frac{3}{15} = \frac{3}{\cancel{15}^{3}} \times \frac{\cancel{100}^{20}}{1}\%$
2 Change to a percentage by multiplying by $\frac{100}{1}$ and including the % sign.	
3 Cancel or simplify as appropriate.	$= \frac{\cancel{3}^{1}}{\cancel{3}^{1}} \times \frac{20}{1}\%$
4 Multiply the numerators and then multiply the denominators.	$= \frac{20}{1}\%$
5 Simplify.	$= 20\%$
6 Answer the question by writing a sentence.	20% of the locals surveyed rated the Australian Grand Prix in Albert Park as their favourite spectator sport.

7.4.2 Changing decimals to percentages

- To change a decimal to a percentage, multiply the decimal by 100 by moving the decimal point two spaces to the right, and add a % sign.

WORKED EXAMPLE 8

Change each of the following decimals to percentages.
a 0.38 **b 0.275** **c 4.2**

THINK	WRITE
a 1 Write the decimal, multiply it by 100 (that is, move the decimal point two places to the right) and include the % sign.	**a** $0.38 = 0.38 \times 100\%$
2 Write the answer.	$= 38\%$

b **1** Write the decimal, multiply it by 100 (that is, move the decimal point two places to the right) and include the % sign.

b $0.275 = 0.275 \times 100\%$

2 Write the answer.

$= 27.5\%$

c **1** Write the decimal, multiply it by 100, fill the blanks with zeros and include the % sign.

c $4.2 = 4.2 \times 100\%$

2 Write the answer.

$= 420\%$

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Try out this interactivity: Converting percentages, decimals, fractions and ratios (int-2740)

Complete this digital doc: SkillSHEET: Multiplying fractions by 100 (doc-6473)

Complete this digital doc: SkillSHEET: Multiplying decimals by 100 (doc-6474)

Watch this eLesson: Converting percentages (eles-0005)

Complete this digital doc: Spreadsheet: Fractions/decimals to percentages (doc-1911)

Exercise 7.4 Fractions and decimals to percentages

assessON

Individual pathways

PRACTISE
Questions:
1–8, 10, 14, 19, 24

CONSOLIDATE
Questions:
1–4, 5a–i, 6a–i, 7–11, 15, 19, 22, 24

MASTER
Questions:
1p–x, 2h–l, 3i–o, 5l–r, 6l–r, 7–25

Individual pathway interactivity: int-4345

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To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note*: Question numbers may vary slightly.

Fluency

1. WE6a Change each of the following fractions to a percentage.

a. $\frac{21}{100}$ **b.** $\frac{48}{100}$ **c.** $\frac{9}{100}$ **d.** $\frac{93}{100}$ **e.** $\frac{14}{50}$ **f.** $\frac{36}{50}$

g. $\frac{8}{50}$ **h.** $\frac{40}{50}$ **i.** $\frac{3}{25}$ **j.** $\frac{15}{25}$ **k.** $\frac{10}{25}$ **l.** $\frac{6}{25}$

m. $\frac{2}{20}$ **n.** $\frac{7}{20}$ **o.** $\frac{1}{20}$ **p.** $\frac{2}{10}$ **q.** $\frac{8}{10}$ **r.** $\frac{1}{5}$

s. $\frac{2}{5}$ **t.** $\frac{4}{5}$ **u.** $\frac{7}{10}$ **v.** $\frac{13}{20}$ **w.** $\frac{3}{10}$ **x.** $\frac{3}{5}$

2. WE6b Change each of the following fractions to a percentage.

a. $\frac{1}{2}$ **b.** $\frac{1}{4}$ **c.** $\frac{3}{5}$ **d.** $\frac{3}{12}$ **e.** $\frac{20}{50}$ **f.** $\frac{10}{50}$

g. $\frac{45}{90}$ **h.** $\frac{15}{30}$ **i.** $\frac{3}{4}$ **j.** $\frac{12}{15}$ **k.** $\frac{15}{150}$ **l.** $\frac{4}{20}$

3. WE6c Change each of the following fractions to percentages, giving the answer as a mixed number in simplest form (if appropriate).

a. $\frac{1}{3}$ **b.** $\frac{1}{6}$ **c.** $\frac{2}{9}$ **d.** $\frac{8}{18}$ **e.** $\frac{3}{11}$ **f.** $\frac{7}{15}$

g. $\frac{5}{9}$ **h.** $\frac{9}{16}$ **i.** $\frac{11}{12}$ **j.** $\frac{7}{8}$ **k.** $\frac{1}{13}$ **l.** $\frac{10}{17}$

m. $\frac{1}{7}$ **n.** $\frac{5}{6}$ **o.** $\frac{2}{3}$

4. WE7 MC **a.** To change a fraction to a percentage:

A. divide by 100 **B.** multiply by $\frac{1}{100}$

C. divide by $\frac{1}{100}$ **D.** multiply by 100

E. write a percentage sign (%)

b. Before multiplying, $\frac{3}{10} \times \frac{100}{1}$ would cancel to:

A. $\frac{300}{10}$ **B.** $\frac{3}{1} \times \frac{10}{1}$ **C.** $\frac{3}{1000}$ **D.** $\frac{3}{10} \times \frac{10}{1}$ **E.** $\frac{3}{10} \times \frac{1}{100}$

c. The fraction $\frac{7}{50}$ as a percentage is:

A. 14% **B.** 7% **C.** $3\frac{1}{2}$ **D.** 700% **E.** 70%

d. The fraction $\frac{4}{9}$ as a percentage is:

A. $2\frac{1}{4}$% **B.** 40% **C.** $54\frac{5}{9}$% **D.** 4% **E.** $44\frac{4}{9}$%

5. WE8a Change each of the following decimals to percentages.

a. 0.45	**b.** 0.32	**c.** 0.56	**d.** 0.68	**e.** 0.90	**f.** 0.84
g. 0.12	**h.** 0.08	**i.** 0.02	**j.** 0.10	**k.** 0.99	**l.** 0.05
m. 0.29	**n.** 0.09	**o.** 0.19	**p.** 0.105	**q.** 0.001	**r.** 0.067

6. WE8b, c Write the following decimals as percentages.

a. 0.3	**b.** 0.8	**c.** 0.9	**d.** 0.1	**e.** 0.002	**f.** 0.007
g. 0.005	**h.** 0.009	**i.** 1.32	**j.** 1.50	**k.** 8.65	**l.** 2.05
m. 4.50	**n.** 0.00015	**o.** 2.00	**p.** 10	**q.** 5	**r.** 100

7. MC **a.** 0.14 is the same as:

A. 1.4% **B.** 140% **C.** 0.0014% **D.** 14% **E.** 0.145

b. When multiplying a decimal by 100, move the position of the decimal point:

A. two places right **B.** two places left **C.** one place left

D. one place right **E.** not at all

c. 0.73 as a percentage is:

A. 7.3% **B.** 730% **C.** 0.073% **D.** 0.73% **E.** 73%

d. 6.1 as a percentage is:

A. 6.1% **B.** 61% **C.** 610% **D.** 0.61% **E.** 0.061%

Understanding

8. Put the following in order from smallest to largest by converting the fractions to percentages first.

$$15\%, \frac{1}{4}, \frac{125}{1000}, \frac{85}{100}, \frac{3}{4}\%, \frac{1}{2}, \frac{94}{100}$$

9. One-fifth of the people in the snowfields preferred snowboarding to skiing. What percentage preferred snowboarding?

10. Eight twenty-fifths of people participate in sport or physical activity. What percentage is this?

11. Three-fortieths of the Australian population attended at least one cricket match last year. What percentage attended a cricket match?

12. Twenty-nine out of every 50 kilograms of waste at the tip is household waste. What percentage is household waste?

13. Three out of 10 people surveyed preferred water to soft drink. What percentage preferred water?

14. Sixty students in Year 8 have seen all of the Harry Potter films. If there are 150 students in Year 8, what percentage of them have seen all of the Harry Potter films?

15. Eight out of 22 houses in one street get the newspaper delivered. What percentage get the newspaper delivered?

16. In 2010, 23 out of every 44 Year 12 students were female. What percentage were female?

17. In Australia, 0.7 of all students attend government schools. What percentage of students attend government schools?

18. Sabrine is a secondary-school teacher. She works 0.4 of the week. What percentage of the week does Sabrine work?

19. Sales of notebooks have increased by 0.43 over the past 3 years. By what percentage have notebook sales increased?

20. The last census showed that 0.031 of the Australian population does not speak any English. What percentage does not speak any English?

21. A teacher recorded her class test results as decimals (decimal amount = mark student achieved ÷ total possible mark). The table below shows some of the class marks. Copy the table and convert each mark to a percentage.

	Directed number test	
Student name	**Mark**	**Percentage**
Mandy Adams	0.86	
Sandra Bazumik	0.72	
Malcolm Boncev	0.64	
James Callan	0.91	
Kate D'Arpa	0.79	
Louise Edmonds	0.92	
Chris Edwards	0.95	
Thomas Evancik	0.10	
Jessie Farmer	0.88	
Mia Yazzett	0.46	

Reasoning

22. a. Place the following percentages, decimals and fractions on a number line from lowest to highest.

$$25\%, \frac{3}{4}, 0.39, 78.2\%, \frac{7}{10}$$

b. Give reasons for the placement of each number.

23. a. Convert the following fractions to:

i. decimals

ii. percentages.

$\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}$

b. Add up the following numbers:

i. $\frac{1}{2}, \frac{1}{3}, \frac{1}{6}$

ii. $50\%,\ 33.3\dot{3},\ 16.6\dot{6}$

c. Have you noticed anything about the values in part **b i** and **ii**?

d. Find three other fractions that add up to 1 and check that their percentages add up to 100%.

Problem solving

24. Shirley works for a publishing company for $\frac{2}{5}$ of the week and teaches Mathematics in a school for 0.4 of the week.

a. For what percentage of the week does Shirley work

i. as a publisher

ii. as a teacher?

b. Considering the two jobs, does Shirley work full time? Explain.

c. Shirley would like to volunteer some of her time at the local charity shop for one day during the week.

d. Does she have the time to help at the charity shop?

25. You have a discount voucher for $\frac{1}{3}$ off the price of electrical items at a particular shop. As you walk into the shop, you notice the following sign displayed.

The fine print at the bottom of the sign means that you cannot use your discount voucher to receive a further $\frac{1}{3}$ off the sale price. Will it be cheaper to buy your electrical item today or after the sale using your discount voucher? Explain your answer.

Reflection

Think of a situation related to your school life where it would be convenient to express fractions as percentages.

7.5 Finding percentages of an amount

7.5.1 Finding percentages of an amount using fractions

- To find a percentage of an amount using fractions, follow these steps.
  Step 1: Convert the percentage into a fraction.
  Step 2: Multiply by the amount.
- A percentage of something means a part of it; therefore, the percentage of an amount will be measured in the same units as the amount itself. For example, the percentage of an amount of dollars will be a certain number of dollars; the percentage of an amount of kilograms will be a certain number of kilograms.

WORKED EXAMPLE 9

Find 40% of 135.

THINK	WRITE
1 Write the question.	40% of \$135
2 Write the percentage as a fraction with a denominator of 100, change *of* to × and write the amount as a fraction over 1.	$= \frac{40}{\cancel{100}^{20}} \times \frac{\cancel{137}^{27}}{1}$
3 Cancel and simplify as appropriate.	$= \frac{\cancel{40}^{2}}{\cancel{20}^{1}} \times \frac{27}{1}$
4 Multiply the numerators and then multiply the denominators.	$= \frac{54}{1}$
5 Simplify by dividing the numerator by the denominator.	$=$ \$54

WORKED EXAMPLE 10

Find 26% of 75 and write the answer as a mixed number.

THINK	WRITE
1 Write the question.	26% of 75
2 Write the percentage as a fraction, change *of* to × and write the number as a fraction over 1.	$= \frac{\cancel{26}}{\cancel{100}^{4}} \times \frac{\cancel{75}^{3}}{1}$
3 Cancel and simplify as appropriate.	$= \frac{\cancel{26}^{13}}{\cancel{4}^{2}} \times \frac{3}{1}$
4 Multiply the numerators and then multiply the denominators.	$= \frac{39}{2}$
5 Write the answer as a mixed number by dividing the denominator into the numerator.	$= 19\frac{1}{2}$

WORKED EXAMPLE 11

Of the 250 students selected at random to complete a survey, 18% were in Year 11. How many students were in Year 11?

THINK	WRITE
1 Decide what percentage of the total is required and write an expression to find the percentage of the total.	18% of 250
2 Write the percentage as a fraction, change *of* to × and write the total as a fraction.	$= \frac{18}{\cancel{100}_{2}} \times \frac{\cancel{250}^{5}}{1}$
3 Cancel and simplify as appropriate.	$= \frac{\cancel{18}^{9}}{\cancel{2}^{1}} \times \frac{5}{1}$
4 Multiply the numerators and then multiply the denominators.	$= \frac{45}{1}$
5 Simplify by dividing the numerator by the denominator.	$= 45$
6 Answer the question by writing a sentence.	45 of the 250 students were in Year 11.

7.5.2 Finding percentages of an amount using decimals

- To find the percentage of an amount using decimals, convert the percentage to a decimal and multiply by the amount.
- *Remember:* When you are multiplying decimal numbers, the total number of decimal places in the question gives the number of decimal places in the answer.

WORKED EXAMPLE 12

Evaluate each of the following by converting the percentage to a decimal.
a 20% of 50 **b 34% of 15** **c 4.3% of 12**

THINK	WRITE
a 1 Write the question.	**a** 20% of 50
2 Write the percentage as a number out of 100 and change *of* to ×.	$= \frac{20}{100} \times 50$
3 Write the fraction as a decimal.	$= 0.20 \times 50$
4 Multiply the numbers, inserting the decimal point in the answer. *Note:* There are 2 decimal places in 0.20, so there will be 2 decimal places in the answer.	$= 10.00$ $= 10$
b 1 Write the question.	**b** 34% of 15
2 Write the percentage as a number out of 100 and change *of* to ×.	$= \frac{34}{100} \times 15$
3 Write the fraction as a decimal.	$= 0.34 \times 15$
4 Multiply the numbers, ignoring the decimal point.	$\begin{array}{r} {}^{1}{}^{2}15 \\ \times\ 34 \\ \hline 60 \\ 4_{1}50 \\ \hline 510 \end{array}$

5 Write the answer with the decimal point in the correct position. *Note:* There are 2 decimal places in 0.34, so there will be 2 decimal places in the answer.	34% of 15 = 5.1
c 1 Write the question.	**c** 4.3% of 12
2 Write the percentage as a number out of 100 and change *of* to ×.	$= \frac{4.3}{100} \times 12$
3 Write the fraction as a decimal.	$= 0.043 \times 12$
4 Multiply the numbers, ignoring the decimal point.	$\begin{array}{r} 12 \\ \times\ 43 \\ \hline 36 \\ 408 \\ \hline 516 \end{array}$
5 Write the answer with the decimal point in the correct position. *Note:* There are 3 decimal places in 0.516, so there will be 3 decimal places in the answer.	4.3% of 12 = 0.516

WORKED EXAMPLE 13

Only 2.3% of Zambians have a television set. In a class of 32 Zambian Year 8 students, how many would be expected to have a television set? Write the answer to the nearest whole number.

THINK	WRITE
1 Write the question.	2.3% of 32
2 Write the percentage as a number out of 100 and change *of* to ×.	$= \frac{2.3}{100} \times 32$
3 Write the fraction as a decimal.	$= 0.023 \times 32$
4 Multiply the numbers, ignoring the decimal point.	$\begin{array}{r} 32 \\ \times\ 23 \\ \hline 96 \\ 6_1 40 \\ \hline 736 \end{array}$
5 Write the answer with the decimal point in the correct position.	2.3% of 32 = 0.736
6 Round to the nearest whole number because we are referring to whole students.	≈ 1
7 Write the answer as a sentence.	One student in a class of 32 would be expected to have a television set.

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Complete this digital doc: SkillSHEET: Multiplying fractions by a whole number (doc-6475)
Complete this digital doc: SkillSHEET: Rounding to the nearest whole number (doc-6476)
Complete this digital doc: SkillSHEET: Rounding money to the nearest 5 cents (doc-6477)
Try out this interactivity: Percentages (int-0004)
Complete this digital doc: Spreadsheet: Calculating percentages (doc-1912)
Complete this digital doc: WorkSHEET: Percentages I (doc-6490)

Exercise 7.5 Finding percentages of an amount

assessON

Individual pathways

PRACTISE
Questions:
1–11, 13, 18, 24

CONSOLIDATE
Questions:
1a–e, 2a–k, 3a–j, 4, 5, 6, 7, 8a–j, 9–13, 16, 22, 24, 25

MASTER
Questions:
1f–j, 2l–u, 3i–o, 4, 5, 6k–r, 7i–o, 8i–o, 9–25

Individual pathway interactivity: int-4346

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Fluency

1. Copy each of the following problems and then find the answers by completing the working.

a. 90% of 200 $= \frac{90}{100} \times \frac{200}{1} =$
b. 8% of 50 $= \frac{8}{100} \times \frac{50}{1} =$
c. 50% of 120 $= \frac{50}{100} \times \frac{120}{1} =$
d. 20% of 90 $= \frac{20}{100} \times \frac{90}{1} =$
e. 30% of 150 $= \frac{30}{100} \times \frac{150}{1} =$
f. 75% of 16 $= \frac{75}{100} \times \frac{16}{1} =$
g. 5% of 30 $= \frac{5}{100} \times \frac{30}{1} =$
h. 80% of 55 $= \frac{80}{100} \times \frac{55}{1} =$
i. 15% of 70 $= \frac{15}{100} \times \frac{70}{1} =$
j. 65% of 120 $= \frac{65}{100} \times \frac{120}{1} =$

2. WE9 Find the following.

a. 50% of 20
b. 20% of 80
c. 5% of 60
d. 10% of 30
e. 9% of 200
f. 31% of 300
g. 40% of 15
h. 12% of 50
i. 35% of 80
j. 70% of 110
k. 52% of 75
l. 90% of 70
m. 80% of 5000
n. 44% of 150
o. 68% of 25
p. 24% of 175
q. 38% of 250
r. 95% of 200
s. 110% of 50
t. 150% of 8
u. 125% of 20

3. WE10 Find the following and write the answer as a mixed number.

a. 18% of 20
b. 16% of 30
c. 11% of 70
d. 8% of 120
e. 74% of 25
f. 66% of 20
g. 2% of 95
h. 55% of 45
i. 15% of 74
j. 32% of 220
k. 95% of 62
l. 32% of 65
m. 18% of 80
n. 82% of 120
o. 27% of 60

4. MC **a.** 45% written as a fraction is:

A. $\frac{45}{100}$ **B.** $\frac{45}{1}$ **C.** $\frac{450}{1}$ **D.** $\frac{1}{45}$ **E.** $\frac{90}{45}$

b. When finding 17% of 22, the 'of' will be changed to:

A. ÷ **B.** of **C.** + **D.** × **E.** −

c. Which of the following would find 15% of 33?

A. 15 of 33 **B.** $\frac{15}{1} \times 33$ **C.** $\frac{15}{100} \times \frac{33}{100}$ **D.** $\frac{15}{1} \times \frac{33}{100}$ **E.** $\frac{15}{100} \times \frac{33}{1}$

d. 60% of 30 is:

A. $19\frac{4}{5}$ **B.** $\frac{31}{5}$ **C.** 186 **D.** 19 **E.** 18

5. WE11 Of the 300 Year 7 students selected to complete a questionnaire on ice-cream, 70% said that their favourite flavour was chocolate chip. How many favoured chocolate chip ice-cream?

6. WE12a, b Evaluate each of the following by converting the percentage to a decimal.

a. 10% of 40 **b.** 20% of 90 **c.** 5% of 80 **d.** 10% of 50

e. 20% of 35 **f.** 50% of 82 **g.** 50% of 74 **h.** 5% of 100

i. 80% of 180 **j.** 90% of 45 **k.** 90% of 56 **l.** 60% of 620

m. 30% of 15 **n.** 80% of 318 **o.** 110% of 20 **p.** 70% of 5

q. 40% of 590 **r.** 4% of 312

7. Find the following by converting the percentage to a decimal.

a. 24% of 30 **b.** 12% of 14.5 **c.** 33% of 71.3 **d.** 18% of 12.2

e. 67% of 18.1 **f.** 28% of 39.6 **g.** 19% of 61.7 **h.** 42% of 42.3

i. 8% of 13.4 **j.** 63% of 105.6 **k.** 17% of 192.4 **l.** 33% of 982.3

m. 53% of 507.1 **n.** 75% of 146.85 **o.** $12\frac{1}{2}$% of 9.95

8. WE12c Evaluate each of the following by converting the percentage to a decimal.

a. 3.2% of 14.5 **b.** 12.8% of 22 **c.** 9.4% of 56 **d.** 23.1% of 97

e. 1.7% of 160 **f.** 19.8% of 88 **g.** 14.1% of 27.5 **h.** 17.3% of 176.9

i. 4.6% of 257.25 **j.** 57.2% of 500 **k.** 98.3% of 4500 **l.** 33.3% of 2800

m. 84.1% of 45.50 **n.** 68.5% of 19.95 **o.** 24.5% of 360

9. MC **a.** 7% of 20 equals:

A. 35 **B.** 1.4 **C.** 3.5 **D.** 14 **E.** 2.86

b. 4.2% of 160 equals:

A. 67.2 **B.** 67 200 **C.** 672 **D.** 0.672 **E.** 6.72

c. 63.5% of 12 equals:

A. 76.20 **B.** 1.905 **C.** 7.660 **D.** 7.620 **E.** 19.05

d. 13% of 51.4 equals:

A. 2.056 **B.** 66.82 **C.** 6.682 **D.** 20.56 **E.** 668.2

10. WE13 A thunder day at a given location is a calendar day on which thunder is heard at least once. About 20% of days near Darwin are thunder days. How many days in one year are thunder days?

Understanding

11. Two per cent of Australians play lawn bowls. In a group of 50 people, how many would you expect to play lawn bowls?

12. Two thousand people entered a marathon. Some walked and the rest jogged. If 20% walked:
   a. what percentage jogged?
   b. how many people jogged?

13. William earns $570 per week. He has just received a pay rise of 3%.
   a. How much more will William earn per week?
   b. How much in total does William earn after his pay rise?

14. The Tattslotto jackpot for Saturday night is $20 million. If you win 8% of the jackpot, how much money will you win?

15. The Australian cricket team was fined 15% of their match payments for a slow over rate. If the team was paid $80 000 for the match, how much was the team fined?

16. Australia has 315 species of mammals. Of these, 15% are threatened. How many threatened species of mammals does Australia have? (Round your answer to the nearest whole number.)

17. Two million people attended at least one game of Australian Rules Football last year. Of these people, 30% attended 10 games or more. How many people attended 10 or more games of Australian Rules Football?

18. The water content of a particular brand of shampoo is 35%. After performing a calculation Peter claims that an 800 mL bottle of this particular shampoo contains 28 mL of water.
   a. Why is Peter's calculation incorrect?
   b. How much of the shampoo in the 800 mL bottle is actually water?

19. In Perth, 36% of adults use a telephone to pay bills and 2% use the internet. If 50 adults need to pay bills, how many will use the:
   a. telephone **b.** internet?

20. In Weburbia, 24% of all households have access to the internet. If there are 34 houses in Website Street, how many would you expect to have access to the internet? Write the answer to the nearest whole number.

21. Terry runs a factory that makes parts for trucks. He has increased his staff by 12%. If Terry had 34 workers, how many workers does he have in the factory after the increase? Write the answer to the nearest whole number.

22. In the year 2010, 60% of households in Geelong had a smoke alarm; 94% of these worked. A street in Geelong has 20 houses.
   a. How many houses would you expect to have a smoke alarm?
   b. How many would have a smoke alarm that works? Write the answer to the nearest whole number.

Reasoning

23. During pregnancy or childbirth 6.25% of African women die. Of 30 African women who are pregnant, how many are likely to die during pregnancy or childbirth? Because the answer is people, write it to the nearest whole number. Justify your answer.

24. For radioactive substances a half-life is the time it takes for half of the radioactive atoms in the substance to decay (lose their radioactivity).
400 g of a radioactive substance decays by 50% every 20 days:
a. Calculate the quantity of radioactive substance left after 20 days, 40 days, 60 days, 80 days and 100 days.
b. What percentage of the initial quantity is left after 20 days, 40 days, 60 days, 80 days and 100 days?
c. The quantity of the radioactive substance halves every 20 days. Why is the percentage decrease smaller after every 20 days?

Problem solving

25. *Squeezy fruit drink* is 36% pure orange juice and 64% water. If 1 litre of *Squeezy fruit drink* is mixed with 600 mL of water, what is the concentration of juice in the new mixture?
26. The students in Year 7A were voting for a class captain. There were only two candidates. When the votes were counted, Rachel received 84% of the votes and Susi received 28% of the votes. It was discovered that some students had voted twice. If the class consisted of 25 students, how many of them voted twice?

Reflection

When would it be easier to find a percentage of an amount using fractions, and when using decimals?

CHALLENGE 7.1

A department store stocked a selection of denim jeans. At the start of December, the store had 320 of these jeans. During December, it sold 50% of the jeans. In January, it sold 25% of the remaining denim jeans. How many were left at the end of January?

7.6 One amount as a percentage of another

7.6.1 One amount as a percentage of another

- To express one amount as a percentage of another, write the amounts as a fraction and change to a percentage. (*Remember:* To change a fraction into a percentage, multiply by 100 and add the % sign.)
- When expressing one amount as a percentage of another, make sure that both amounts are in the same units.

WORKED EXAMPLE 14

Express:
a 15 as a percentage of 20
b 9 as a percentage of 33 (write the answer as a mixed number).

THINK	WRITE
a 1 Write the amount as a fraction of the total: $\frac{\text{amount}}{\text{total}}$.	**a** $\frac{15}{20}$
2 Multiply by $\frac{100}{1}$, include the % sign and cancel.	$= \frac{15}{\cancel{20}} \times \frac{\cancel{100}^{5}}{1}\%$
3 Multiply the numerators and then multiply the denominators.	$= \frac{75}{1}\%$
4 Simplify.	$= 75\%$

b 5 Write the amount as a fraction of the total: $\frac{\text{amount}}{\text{total}}$.	**b** $\frac{9}{33}$
6 Multiply by $\frac{100}{1}$ include the % sign and cancel.	$= \frac{\cancel{9}^{3}}{\cancel{33}^{11}} \times \frac{100}{1}\%$
7 Multiply the numerators and then multiply the denominators.	$= \frac{300}{11}\%$
8 Write as a mixed number by dividing the denominator into the numerator.	$= 27\frac{3}{11}\%$

WORKED EXAMPLE 15

Express the number 8 as a percentage of 26. Round the answer to the nearest whole number.

THINK	WRITE
1 Write the amount as a fraction of the total: $\frac{\text{amount}}{\text{total}}$.	$\frac{8}{26}$
2 Multiply by $\frac{100}{1}$, include the % sign and cancel.	$= \frac{\cancel{8}^{4}}{\cancel{26}^{13}} \times \frac{100}{1}\%$
3 Multiply the numerators and then multiply the denominators.	$= \frac{400}{13}\%$
4 Write as a mixed number by dividing the denominator into the numerator.	$= 30\frac{10}{13}\%$
5 Round to the nearest whole number.	$\approx 31\%$

WORKED EXAMPLE 16

Write 45c as a percentage of $2.

THINK	WRITE
1 Write the larger amount using the smaller unit.	$\$2 = 200$ cents
2 Write the first amount as a fraction of the second amount: $\frac{\text{amount}}{\text{total}}$.	$\frac{45}{200}$
3 Multiply by $\frac{100}{1}$ include the % sign and cancel.	$= \frac{45}{\cancel{200}^{2}} \times \frac{\cancel{100}^{1}}{1}\%$
4 Multiply the numerators and then multiply the denominators.	$= \frac{45}{2}\%$
5 Write as a mixed number by dividing the denominator into the numerator.	$= 22\frac{1}{2}\%$ This means that 45c is $22\frac{1}{2}\%$ of $2.

WORKED EXAMPLE 17

Kye obtained 17 out of 30 on his Science test. What percentage did he score? (Round your answer to the nearest whole number.)

THINK	WRITE
1 Write the amount as a fraction of the total: $\frac{\text{amount}}{\text{total}}$.	$\frac{17}{30}$
2 Multiply by $\frac{100}{1}$ include the % sign and cancel.	$= \frac{17}{\cancel{30}} \times \frac{\cancel{100}^{10}}{1}\%$
3 Multiply the numerators and then multiply the denominators.	$= \frac{170}{3}\%$
4 Write as a mixed number by dividing the denominator into the numerator.	$= 56\frac{2}{3}\%$
5 Round to the nearest whole number. (If the fraction is a half or more, add one to the number.)	$\approx 57\%$
6 Write the answer as a sentence.	Kye obtained 57% for his Science test.

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Complete this digital doc: Spreadsheet: One amount as a percentage of another (doc-1913)

Exercise 7.6 One amount as a percentage of another

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–8, 10, 14, 19	Questions: 1a–h, 2a–e, 3a–e, 4a–e, 5–10, 13, 16, 17, 19, 20	Questions: 1b, h, p, 2b, f, j, 3b, f, j, 4, 5d–f, 6–20

Individual pathway interactivity: int-4347

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Fluency

1. WE14a Express:

a. 2 as a percentage of 10
b. 13 as a percentage of 52
c. 6 as a percentage of 12
d. 8 as a percentage of 80
e. 8 as a percentage of 20
f. 35 as a percentage of 700
g. 15 as a percentage of 60
h. 120 as a percentage of 150
i. 9 as a percentage of 15
j. 8 as a percentage of 25
k. 12 as a percentage of 30
l. 30 as a percentage of 150
m. 40 as a percentage of 200
n. 10 as a percentage of 200
o. 9 as a percentage of 45
p. 180 as a percentage of 720.

2. WE14b Express (write the answers as mixed numbers):

a. 63 as a percentage of 200
b. 25 as a percentage of 400
c. 9 as a percentage of 120
d. 32 as a percentage of 500
e. 620 as a percentage of 3000
f. 30 as a percentage of 45
g. 70 as a percentage of 80
h. 12 as a percentage of 42
i. 9 as a percentage of 66
j. 52 as a percentage of 78.

3. Express (write the answers as mixed numbers):

a. 11 as a percentage of 30 **b.** 4 as a percentage of 15 **c.** 8 as a percentage of 60
d. 5 as a percentage of 70 **e.** 4 as a percentage of 18 **f.** 55 as a percentage of 66
g. 3 as a percentage of 7 **h.** 64 as a percentage of 72 **i.** 9 as a percentage of 35
j. 15 as a percentage of 21.

4. WE15 Express (round all answers to the nearest whole number):

a. 7 as a percentage of 42 **b.** 18 as a percentage of 54 **c.** 9 as a percentage of 16
d. 95 as a percentage of 150 **e.** 7 as a percentage of 65 **f.** 2 as a percentage of 22
g. 30 as a percentage of 36 **h.** 14 as a percentage of 18 **i.** 80 as a percentage of 450
j. 8 as a percentage of 68.

5. WE16 Express the first amount as a percentage of the second amount.

a. 30c and $3 **b.** 200 m and 6 km **c.** 5 mm and 4 cm
d. 6 days and 3 weeks **e.** 15 minutes and 1 hour **f.** 25 mL and 2 L

6. MC **a.** 4 as a percentage of 50 is:

A. 2% **B.** 12.5% **C.** 46% **D.** 8% **E.** 4%

b. 9 out of 36 as a percentage is:

A. 25% **B.** 9% **C.** 400% **D.** 27% **E.** 36%

c. 14 as a percentage of 35 is:

A. 14% **B.** 40% **C.** 20% **D.** 35% **E.** $2\frac{1}{2}\%$

d. Alice got 42 out of 60 for her mathematics test. Her mark as a percentage is:

A. 58% **B.** 60% **C.** 30% **D.** 42% **E.** 70%

7. WE17 Alicia Molik has won 7 out of her last 10 singles matches. What percentage of her last 10 matches has Alicia won?

Understanding

8. Three out of five people prefer chocolate ice-cream to vanilla. What percentage prefer chocolate ice-cream to vanilla?

9. Caillan's pocket money increased from $15 per week to $20 per week.

a. By how much has Caillan's pocket money increased?
b. What percentage increase is this?

10. Daniel earns $500 per week. He spends $50 on petrol, $70 on rent and $60 on food.

a. What percentage does Daniel spend on:
i. petrol **ii.** rent **iii.** food?
b. What percentage of his total wage does Daniel spend on petrol, rent and food combined?
c. What percentage does Daniel have left?

11. David shot 40 baskets from the free throw line. Twenty-five of the 40 were goals. What percentage of David's shots were goals?

12. Jordan had a mass of 60 kg. After one month of not eating McDonald's, he lost 4 kg. What percentage of his mass did Jordan lose?

13. After practising for one month, Tayla increased the speed of her tennis serve from 120 km/h to 140 km/h. Give exact answers.

a. By how much has the speed of Tayla's serve increased?
b. By what percentage has the speed of Tayla's serve increased?

14. There are 28 students in a class. A survey showed that 12 walk home, 8 catch a bus and 8 ride in a car. Write the answers as mixed numbers.
- **a.** Calculate the percentage of students who walk.
- **b.** Calculate the percentage of students who catch a bus.
- **c.** Calculate the total percentage who walk or catch a bus.
- **d.** Calculate the percentage of students who ride in a car.

15. During one NRL season, the Melbourne Storm won 16 out of 22 games. What percentage of games did the Storm lose?

16. The teacher's mark book shows Karina has achieved the following test results in Maths.

Topic	Probability	Measurement	Algebra	Equations	Geometry
Score	$\frac{15}{20}$	$\frac{13}{15}$	$\frac{27}{30}$	$\frac{70}{80}$	$\frac{95}{100}$
Percentage					

- **a.** Copy and complete the table to show Karina's percentage for each test (round answers to the nearest whole number).
- **b.** In which test did Karina achieve her best result?
- **c.** Find Karina's average percentage mark by adding up the percentages and dividing by 5, the number of tests.

Reasoning

17. Which of the following quantities represents:
- **a.** the highest percentage
- **b.** the lowest percentage
- **c.** the same percentage?
- **d.** Explain your reasoning.

200 g out of 500 g
300 g out of 450 g
420 g out of 840 g
350 g out of 700 g

18. a. A baker uses 200 g of flour for 500 g of dough in a recipe and 250 g of flour for 500 g of dough in another recipe.
- **i.** Which recipe contains a higher percentage of flour? Explain.
- **ii.** What happens when the quantity of flour is increased but the quantity of dough remains constant?

b. A type of concrete uses 10 kg of cement for a quantity of 65 kg of concrete. Another type of concrete uses 10 kg of cement for a quantity of 85 kg of concrete.
- **i.** Which type of concrete contains a higher percentage of cement? Explain.
- **ii.** What happens when the quantity of cement is constant but the quantity of concrete increases?

Problem solving

19. Roseanne is planning a tour of Australia. She calculates that 70% of the distance will be one sealed roads and the remainder on dirt roads. If there are 1356 km of dirt roads, how long will Roseanne's journey be?

20. a. If you increase \$100 by 25% and then decrease the new amount by 25%, will you end up with more than \$100, less than \$100 or exactly \$100? Give reasons for your answer.

b. Choose another amount and another percentage to test your reasoning.

Reflection

Why do we need to be able to express one quantity as a percentage of another quantity?

CHALLENGE 7.2

My mother is four times older than I am. My sister is 75% of my age. She is also 10% of my grandfather's age. My father is 50, which is 2 years older than my mother. How old are my sister and grandfather?

7.7 Common percentages and short cuts

7.7.1 Common percentages and short cuts

- Common percentages that are often used in everyday life include 10%, 20%, 25%, 50%, 75% and so on.
- It is useful to remember the relationship between some commonly used percentages and their related fractions, as it may help to calculate those percentages mentally. For example, $10\% = \frac{1}{10}$, so to find 10% of any amount we need to find $\frac{1}{10}$ of that amount; this means that we simply need to divide the amount by 10. Likewise, $25\% = \frac{1}{4}$, so to find 25% of any amount we need to find $\frac{1}{4}$ of that amount; this means we simply need to divide the amount by 4.

Percentage	Fraction
1%	$\frac{1}{100}$
5%	$\frac{1}{20}$
10%	$\frac{1}{10}$
$12\frac{1}{2}\%$	$\frac{1}{8}$
20%	$\frac{1}{5}$

Percentage	Fraction
25%	$\frac{1}{4}$
$33\frac{1}{3}\%$	$\frac{1}{3}$
50%	$\frac{1}{2}$
$66\frac{2}{3}\%$	$\frac{2}{3}$
75%	$\frac{3}{4}$

- Finding 10% without a calculator is easy, as it involves dividing the amount by 10; this is done just by moving decimal point 1 space to the left. Once you have found 10% of the amount, it's easy to find some other percentages. For example:
  - to find 5%, halve 10%
  - to find 20%, double 10%
  - to find 15%, add 10% and 5% together.
- $1\% = \frac{1}{100}$, so to find 1% of the amount, divide it by 100 by moving the decimal point two spaces to the left. Once you have found 1% of the amount, it's easy to find some other percentages. For example, to find 2%, double 1%.

WORKED EXAMPLE 18

Find 10% of each of the following, rounding the answer to the nearest 5 cents.
a $37 **b $12.95**

THINK

a Write the question and move the position of the decimal point one place to the left for the answer. Remember that if there is no decimal point, put it at the end of the number. ($17 = $17.00)

b 1 Write the question and move the position of the decimal point one place to the left for the answer.

2 Round to the nearest 5 cents.

WRITE

a 10% of $37 = $3.70

b 10% of $12.95 = $1.295

≈ $1.30

WORKED EXAMPLE 19

Find:
a 5% of $180 **b 20% of $7** **c 15% of $52** **d 25% of 46.**

THINK	WRITE
a 1 Find 10% of the amount.	**a** 10% of \$180 = \$18
2 Find 5% by halving the amount obtained in step 1.	5% of \$180 = \$9
b 1 Find 10% of the amount.	**b** 10% of \$7 = \$0.70
2 Find 20% by doubling the amount obtained in step 1.	20% of \$7 = 2 × \$0.70 = \$1.40
c 1 Find 10% of the amount.	**c** 10% of \$52 = \$5.20
2 Find 5% of the amount by halving 10%, that is, halving \$5.20.	5% of \$52 = \$5.20 ÷ 2 = \$2.60
3 Find 15% of the amount by adding 10% and 5%, that is, \$5.20 and \$2.60.	15% of \$52 = \$5.20 + \$2.60 = \$7.80
d 1 Find 10% of the amount.	**d** 10% of \$46 = \$4.60
2 Find 20% by doubling 10%, that is, doubling \$4.60.	20% = 2 × \$4.60 = \$9.20
3 Find 5% of the original amount by halving 10%, that is, halving \$4.60.	5% = \$4.60 ÷ 2 = \$2.30
4 Find 25% by adding the 20% and the 5%, that is, \$9.20 and \$2.30.	25% = \$9.20 + \$2.30 = \$11.50

WORKED EXAMPLE 20

Find:
a 12% of \$53 **b 43% of \$120.**
Round answers to the nearest 5 cents.

THINK	WRITE
a 1 Break up the percentage into lots of 10% and 1%.	**a** 12% = 10% + 2 × 1%
2 Find 10% of the amount.	10% of \$53 = \$5.30
3 Find 1% of the amount and then double it.	1% of \$53 = 0.53 2% of \$53 = 2 × 0.53 = \$1.06
4 Find 12% of the amount by adding 10% and 2% of the amount.	12% of \$53 = \$5.30 + \$1.06 = \$6.36 Another way to write this solution would be: 12% of \$53 = 10% of \$53 + 2 × 1% of \$53 = \$5.30 + 2 × \$0.53 = \$6.36
5 Round to the nearest 5 cents.	≈ \$6.35
b To find 43%, find 4 × 10% + 3 × 1%.	**b** 43% of \$120 = 4 × 10% of \$120 + 3 × 1% of \$120 = 4 × \$12 + 3 × \$1.20 = \$48 + \$3.60 = \$51.60

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Complete this digital doc: WorkSHEET: Percentages II (doc-6497)

Exercise 7.7 Common percentages and short cuts

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–8, 12, 15, 17, 21	Questions: 1a–l, 2a–j, 3a–h, 4a–f, 5a–f, 6a–i, 7a–i, 8, 9, 12, 15, 18, 21	Questions: 1q–x, 2m–t, 3i–o, 4i–l, 5i–l, 6m–r, 7m–r, 9–21

Individual pathway interactivity: int-4348

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To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note*: Question numbers may vary slightly.

Fluency

1. WE18 Find 10% of each of the following, rounding the answer to the nearest 5 cents.

a. $10.00	**b.** $18.00	**c.** $45.00	**d.** $81.00	**e.** $150.00
f. $112.00	**g.** $93.00	**h.** $79.00	**i.** $47.00	**j.** $22.00
k. $16.50	**l.** $17.20	**m.** $12.60	**n.** $1.50	**o.** $32.90
p. $47.80	**q.** $81.40	**r.** $192.40	**s.** $507.00	**t.** $4620.00
u. $1926.00	**v.** $3041.50	**w.** $7219.60	**x.** $1999.90	

2. Find 10% of the following. Round your answers to the nearest 5 cents.

a. $15	**b.** $51	**c.** $17	**d.** $9	**e.** $137
f. $172	**g.** $4.29	**h.** $6.37	**i.** $8.12	**j.** $39.17
k. $74.81	**l.** $13.95	**m.** $102.75	**n.** $67.87	**o.** $42.96
p. $517.83	**q.** $304.77	**r.** $628.41	**s.** $100.37	**t.** $207.08

3. WE19a Find 5% of each of the following. Round the answers to the nearest 5 cents.

a. $8.20	**b.** $6.40	**c.** $1.60	**d.** $2.20	**e.** $140.20
f. $81.00	**g.** $42.40	**h.** $10.60	**i.** $242.60	**j.** $304.80
k. $1000	**l.** $642.75	**m.** $103.27	**n.** $31.70	**o.** $5.90

4. WE19b Find 20% of the following. Round the answers to the nearest 5 cents.

a. $21.50	**b.** $42.30	**c.** $8.20	**d.** $3.30	**e.** $74.10
f. $0.90	**g.** $0.79	**h.** $16.40	**i.** $135.80	**j.** $261.70
k. $1237	**l.** $5069			

5. WE19c, d Find the following. Round the answers to the nearest 5 cents.

a. 15% of $12	**b.** 15% of $8.00	**c.** 15% of $20.00	**d.** 15% of $60.00
e. 25% of $30.00	**f.** 25% of $45.00	**g.** 25% of $90.00	**h.** 25% of $220.00
i. 30% of $15.00	**j.** 30% of $25.00	**k.** 30% of $47.50	**l.** 30% of $102.20

6. Find 1% of the following. Round the answers to the nearest 5 cents.

a. $268	**b.** $713	**c.** $573	**d.** $604	**e.** $5.60
f. $12	**g.** $13	**h.** $14.80	**i.** $21.70	**j.** $81.75
k. $19.89	**l.** $429.50	**m.** $4.25	**n.** $6.49	**o.** $9.99
p. $0.24	**q.** $0.77	**r.** $1264.37		

7. WE20 Find the following. Round the answers to the nearest 5 cents.

a. 12% of $11	**b.** 21% of $50	**c.** 11% of $30	**d.** 3% of $22
e. 6% of $40	**f.** 22% of $10	**g.** 13% of $14	**h.** 35% of $210
i. 12% of $150	**j.** 9% of $17	**k.** 2% of $53	**l.** 7% of $29
m. 45% of $71.50	**n.** 33% of $14.50	**o.** 42% of $3.80	**p.** 31% of $1.45
q. 64% of $22.50	**r.** 41% of $1200		

8. MC **a.** 10% of $7.25 equals:

A. $725 **B.** $7.30 **C.** $72.50 **D.** $0.73 **E.** $0.72

b. 1% of $31.48 equals:

A. $3.14 **B.** $0.31 **C.** $0.32 **D.** $31.50 **E.** $3.15

c. 15% of $124 equals:

A. $12.40 **B.** $1.24 **C.** $6.20 **D.** $13.64 **E.** $18.60

d. 22% of $5050 equals:

A. $60.60 **B.** $50.50 **C.** $1111 **D.** $43.56 **E.** $55.55

Understanding

9. Maria is buying a new set of golf clubs. The clubs are marked at $950, but if Maria pays cash, the shop will take 10% off the marked price. How much will the clubs cost if Maria pays cash?

10. Thirty per cent of residents in the shire of Sutherland are over the age of 65. If there are 180000 residents, how many are over the age of 65?

11. Jay is buying a new lounge suite worth $2150. Jay has to leave a 15% deposit and then pay the balance in monthly instalments.

a. How much deposit does Jay have to pay?

b. How much will Jay have to pay each month if he plans to pay the balance off in one year?

12. Ninety per cent of students at a school were present for school photographs. If the school has 1100 students, how many were absent on the day the photographs were taken?

13. Jim can swim 50 m in 31 seconds. If he improves his time by 10%, what will Jim's time for 50 m be?

14. In a survey, 40 people were asked if they liked or disliked Vegemite. Of the people surveyed, 5% said they disliked Vegemite. How many people:

a. disliked Vegemite?

b. liked Vegemite?

15. Seventy-two thousand four hundred people went to the MCG to watch a Hawthorn versus Geelong football match. Of the crowd, 42% went to the game by car and 55% caught public transport. How many people:

a. arrived by car

b. caught public transport?

Reasoning

16. When I am 5% older than I am now, I will be 21 years old. How old am I now?

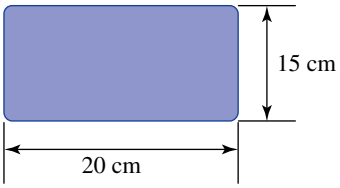
17. The price of bread is now 250% of its previous value, an increase of 150% in the past 20 years. If a loaf of bread costs $2.00 now, how much would it have cost 20 years ago?

18. I am six months old. If I gain 10% of my current mass I will be three times my birth mass. If my birth mass was 3 kg, what is my mass now? Round your answer to 1 decimal place.

19. I am 33 years old. I have lived in England for 8 years. If I stay in England, how old will I be when the number of years I have lived there is 50% of my age? Show how you reached your answer.

Problem solving

20. Kate is a carpenter and made a cutting board as shown.



- **a.** One of her customers asked Kate to make her a similar cutting board but to double its width. What is the percentage increase of the width of the board?
- **b.** A second customer asked Kate to make her a cutting board with dimensions 30 cm by 30 cm.
  - **i.** What is the percentage increase of the width of the board?
  - **ii.** What is the percentage increase of the length of the board?
  - **iii.** What is the percentage increase of the perimeter of the board?
- **c.** Is there a connection between the percentage increase of a side and the percentage increase of the perimeter? Explain.

21. Ten per cent of the messages received by a business were from customers dissatisfied with the product bought. 32% of these customers were males.

- **a.** What is the overall percentage of dissatisfied male customers?
- **b.** What is the overall percentage of dissatisfied female customers?
- **c.** Explain what happens when you apply a percentage to another percentage.

Reflection

Why is it important to be able to quickly evaluate common percentages mentally?

7.8 Review

7.8.1 Review questions

Fluency

1. Write the following percentages as fractions in simplest form.

a. 13%	**b.** 70%	**c.** 26%	**d.** 132%
e. $\frac{1}{4}$%	**f.** $2\frac{2}{5}$%	**g.** 9.7%	**h.** 73.17%

2. Write the following percentages as decimals.

a. 42%	**b.** 5%	**c.** 94%	**d.** 139%
e. 6.7%	**f.** 19.7%	**g.** 58.03%	**h.** 0.8%

3. Change the following fractions to percentages.

a. $\frac{15}{100}$	**b.** $\frac{11}{20}$	**c.** $\frac{3}{5}$	**d.** $\frac{18}{36}$
e. $\frac{30}{80}$	**f.** $\frac{8}{15}$	**g.** $\frac{5}{6}$	**h.** $\frac{4}{11}$

4. Change the following fractions to percentages.

a. 0.71	**b.** 0.84	**c.** 0.03	**d.** 0.2
e. 0.5	**f.** 0.008	**g.** 1.64	**h.** 3.8

5. Change the following fractions to percentages.

a. 5% of 120	**b.** 60% of 20	**c.** 75% of 52	**d.** 130% of 30
e. 14% of 45	**f.** 7% of 530	**g.** 32% of 15	**h.** 17% of 80

6. Express:

a. 30 as a percentage of 50	**b.** 18 as a percentage of 60
c. 32 as a percentage of 80	**d.** 28 as a percentage of 70
e. 44 as a percentage of 132	**f.** 6 as a percentage of 48
g. 7 as a percentage of 15	**h.** 35 as a percentage of 40
i. 4 days as a percentage of 8 weeks	**j.** 20 minutes as a percentage of 3 hours.

7. Find the following percentages by converting the percentage to a decimal.

a. 20% of 25	**b.** 12% of 31	**c.** 4.5% of 50	**d.** 9.2% of 75
e. 21.4% of 90	**f.** 32.3% of 120	**g.** 76.5% of 8	**h.** 42.3% of 96.2

8. Find 10% of each of the following by moving the position of the decimal point. Round your answer to the nearest 5 cents.

a. $63.00	**b.** $42.00	**c.** $105.00	**d.** $216
e. $3.45	**f.** $42.68	**g.** $118.55	**h.** $2125.85

9. Find 5% of the following by finding 10% and halving your answer. Round your answer to the nearest 5 cents.

a. $8.00	**b.** $21.00	**c.** $64.00	**d.** $104.00
e. $35.00	**f.** $52.00	**g.** $205.50	**h.** $77.30

10. Calculate the following using 'shortcuts'. Round your answer to the nearest 5 cents.

a. 1% of $16.00	**b.** 1% of $28.00	**c.** 12% of $42.00	**d.** 30% of $90.00
e. 22% of $220.00	**f.** 43% of $27	**g.** 15% of $19.50	**h.** 8% of $37

Problem solving

11. Ninety-five per cent of Year 8 students participated in the school's athletics day. What fraction of the Year 8 students participated?

12. Twenty-four per cent of visitors to Australia in 2005 were from Germany.

a. What fraction of visitors to Australia was from Germany?

b. What fraction of visitors to Australia was not from Germany?

c. Out of 400 visitors to Australia, how many would you expect to be from Germany?

13. What fraction remains if:

a. 38% of the winnings have been spent? **b.** all stock was discounted 25%?

14. Five-eighths of the plants in my garden are Australian natives. What percentage are Australian natives?

15. Of the Australian population, 0.0041 speak Polish at home. What percentage of Australians speak Polish at home?

16. Eighty per cent of a Year 7 Maths class got question 1 correct on their percentages test. How many got the question correct if there are 30 students in the class?

17. Forty per cent of primary-school children can sing the national anthem. In a group of 675 primary-school children, how many would you expect to be able to sing the national anthem?

18. Nineteen per cent of Australia is forest and woodland. If Australia's area is approximately 7 500 000 km^2, how many km^2 is forest and woodland?

19. Ninety-nine of Canada's 3300 plant species are under threat of extinction. What percentage is threatened?

20. Lake Eyre in South Australia has an annual rainfall of around 100 mm. If 8 mm fell in one week, what percentage of the annual rainfall is this?

21. If a 4-litre can of paint, normally costing $53.50, is discounted by 5%, how much will it cost? Give the answer correct to the nearest 5 cents.

22. A tennis player agreed to pay her coach 8% of her winnings from tournaments. If she wins $145 000 in a tournament, how much does her coach get?

23. A bronze propeller is 75% copper. If there are 300 g of copper in the propeller, what is the mass of the propeller?

24. Alex cuts his fish into 3 pieces — head, body and tail. The head is 30% of the fish and the tail is 10 of the fish. If the body weighs 120 g, how much did the fish weigh?

25. At Queens High School, there are 900 students.
45% are girls.
55% are boys.
12% of students wear glasses.
Which of the following statements is definitely false? (There may be more than one answer.)
a. There are 100 boys with glasses.
b. 10% of the boys and 2% of the girls wear glasses.
c. 20% of girls wear glasses.
d. More than 22% of boys wear glasses.

26. The label on a 500-gram pack of butter states that it contains 81% fat. On a milk bottle it says that the milk contains 3 grams of fat per 100 mL. How many litres of this milk will it take to contain the same amount of fat as there is in the butter?

27. Dragoncity has a population of 85 400. Of these, 74% are sports fans. Of the fans, 90% are followers of some kind of football (soccer, Australian rules and rugby), but only 2.1% of football fans follow all three codes.
a. How many people are interested in soccer, Australian rules and rugby?
b. What percentage of the Dragoncity population are fans of all three codes?

28. After having 24% of her pay taken out as taxes, Maya had $646 left. What was her gross pay?

29. If you travel 50% faster, would you get to your destination 50% sooner? Explain.

30. Two triangles have the same area. Show that if one triangle has a 25% wider base than the other, then it must be 20% shorter too (that is, its height must be 20% less).

31. In a recent survey of house occupancy in a town, it was discovered that 40% of the houses contained 2 or more people. Of those houses that contain only 1 person, 25% of them contained a male. Of all the houses in a town, what percentage of them contains only 1 female and no males?

32. In the Student Council elections at a school, there were 3 candidates — Ann, Bob and Charlie. Ann received 227 votes, Bob 182 and Charlie 239 votes. If 90% of the school population voted, what is the total number of students in the school?

33. For the last two years, the price of entry to a theme park has increased 12.5% each year. The latest price rise increased a ticket to $20.25. What was the entry price 2 years ago?

34. A store prices its T-shirts so that it makes 25% profit. At a sale, the T-shirts were reduced to cost price. By what percentage were the T-shirts reduced from their normal selling price?

35. A television set has a ticketed price of $2500. A 10% discount is allowed on the set.
a. What is the negotiated price of the television set?
b. If a further 10% is negotiated off the price of the set, what is the price paid?
c. Calculate the total discount as a percentage of the original ticketed price.
d. Explain why two successive 10% discounts do not equal a 20% discount.

36. Lena saved $32 when she purchased a pair of jeans at a sale. If the sale had 40% off any item, what was the original price for the pair of jeans?

37. Two different items of clothing costing $45 and $50 are on sale with 10% off. The shop assistant in the store claims that if you buy both items, you will be saving a total of 20%. Is the shop assistant correct?

38. You are given two options at work:
OPTION 1: Your salary can be raised by 10% this month and then reduced by 10% next month, or
OPTION 2: Your salary will be reduced by 10% this month and increased by 10% next month.
At the end of the two months, will your salary be greater than your current salary? Which is the better option?

39. Joseph wants to spend $2500 on a new BBQ. Since he works at Harvey Norman he can get a staff discount of 10%. He can also get a discount of 30% in the stocktaking sale. Joseph wants to know whether he should ask the salesman to apply his staff discount first, or the stock-take sale discount first, to achieve the cheapest price.

a. How much will the BBQ cost if the staff discount is taken off first and then the stock-take sale discount is taken off?

b. How much will it cost if done the other way around?

c. Which way should Joseph select to get the best price?

40. a. When Nathan will be 10% older than he is now, he will be 33 years old. How old is Nathan now?

b. The price of milk has increased by 300% in the past 15 years. If a litre of milk costs $2.50 now, how much would it have cost 15 years ago?

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Try out this interactivity: Crossword: Topic 7 (int-2602)

Complete this digital doc: Sudoku: Topic 7 (int-3168)

Complete this digital doc: Concept map: Topic 7 (doc-10731)

Language

It is important to learn and be able to use correct mathematical language in order to communicate effectively. Create a summary of the topic using the key terms below. You can present your summary in writing or using a concept map, a poster or technology.

deposit
discount
instalments
marked price
original price
per cent
percentage
selling price
survey

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Investigation | Rich task

Sale time — spending time!

Wear what you want		
HALF YEARLY SALE		
Hooped tops	were $39.00	now reduced by 25%
Cargo pants	were $59.00	now reduced by 40%
T-Shirts	were $20.00	now reduced by 10%
Shorts	were $29.00	now reduced by 15%
Jeans	were $79.00	now reduced by 35%
Hurry! Limited stock available! Will not last!		

Use the sale information in the advertisement shown and the skills you have learned in this chapter to answer the following questions.

1. Before the sale, how much would it cost to buy a hooded top and a pair of cargo pants?
2. What items could you buy before the sale if you had $100.00 to spend?
3. Calculate the sale price of each of the 5 items advertised.
4. How much would you pay if you bought a hooded top and a pair of cargo pants at the sale?
5. How much have you saved by buying these 2 items at the sale rather than before the sale?
6. How much change would you receive from $100.00 if you bought a pair of shorts and two T-shirts at the sale?
7. What items could you buy during the sale with the $100.00 you have to spend?
8. Choose one set of items you would prefer to buy. Write a sentence comparing the total cost of the items you have chosen before and during the sale.

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Complete this digital doc: Code puzzle: A comic character (doc-14323)

Answers

Topic 7 Percentages

Exercise 7.2 Percentages as fractions

1. a. $\frac{17}{100}$ **b.** $\frac{29}{100}$ **c.** $\frac{81}{100}$ **d.** $\frac{79}{100}$ **e.** $\frac{99}{100}$ **f.** $\frac{43}{100}$ **g.** $\frac{3}{100}$ **h.** $\frac{19}{100}$
i. $\frac{67}{100}$ **j.** $\frac{33}{100}$ **k.** $\frac{9}{100}$ **l.** $\frac{189}{100}$ **m.** $\frac{243}{100}$ **n.** $\frac{127}{100}$ **o.** $\frac{353}{100}$

2. a. $\frac{1}{2}$ **b.** $\frac{4}{5}$ **c.** $\frac{1}{4}$ **d.** $\frac{7}{20}$ **e.** $\frac{3}{5}$ **f.** $\frac{17}{20}$ **g.** $\frac{1}{10}$ **h.** $\frac{9}{20}$
i. $\frac{49}{50}$ **j.** $\frac{3}{25}$ **k.** $\frac{1}{20}$ **l.** $\frac{14}{25}$ **m.** $\frac{37}{50}$ **n.** $\frac{1}{50}$ **o.** $1\frac{1}{10}$ **p.** $1\frac{1}{2}$
q. $\frac{9}{10}$ **r.** $1\frac{4}{5}$ **s.** $\frac{2}{1} = 2$ **t.** $\frac{5}{1} = 5$ **u.** $1\frac{3}{25}$

3. a. $\frac{1}{200}$ **b.** $\frac{1}{500}$ **c.** $\frac{3}{400}$ **d.** $\frac{1}{150}$ **e.** $\frac{1}{1000}$ **f.** $\frac{1}{160}$ **g.** $\frac{3}{550}$ **h.** $\frac{33}{400}$
i. $\frac{3}{80}$ **j.** $\frac{31}{150}$ **k.** $\frac{29}{300}$ **l.** $\frac{57}{400}$ **m.** $\frac{241}{400}$ **n.** $\frac{31}{200}$ **o.** $\frac{9}{40}$ **p.** $\frac{14}{125}$
q. $\frac{83}{800}$ **r.** $\frac{7}{60}$ **s.** $1\frac{101}{200}$ **t.** $1\frac{41}{200}$ **u.** $\frac{1}{3}$

4. a. $\frac{7}{200}$ **b.** $\frac{9}{125}$ **c.** $\frac{59}{500}$ **d.** $\frac{197}{1000}$ **e.** $\frac{81}{250}$ **f.** $\frac{143}{200}$ **g.** $\frac{629}{1000}$ **h.** $\frac{31}{200}$
i. $\frac{87}{1000}$ **j.** $\frac{81}{500}$ **k.** $\frac{283}{1000}$ **l.** $\frac{13}{80}$ **m.** $\frac{2069}{5000}$ **n.** $\frac{5799}{10000}$ **o.** $\frac{1079}{1250}$ **p.** $\frac{907}{5000}$
q. $\frac{243}{2000}$ **r.** $\frac{1}{2000}$

5. a. B **b.** E **c.** C **d.** D

6. $\frac{20}{100} = \frac{1}{5}$

7. $\frac{35}{100} = \frac{7}{20}$

8. $\frac{45}{100} = \frac{9}{20}$

9. 32% are girls, $\frac{8}{25}$

10. a. $\frac{2}{25}$ **b.** $\frac{23}{25}$ **c.** 8 people

11. a. $\frac{17}{100}$ **b.** $\frac{83}{100}$ **c.** 17

12. Less than $\frac{1}{2}$

13. a. $\frac{7}{20}$ **b.** $\frac{81}{100}$ **c.** $\frac{17}{20}$ **d.** $\frac{13}{200}$

14. a. $\frac{4}{5}$ **b.** $\frac{1}{2}$
c. The percentage is different for the two molecules because the total number of atoms is different in the two molecules.

15. $33.3\dot{3}\%$ is a recurring decimal with no exact value.

16. a. $\frac{4}{5}$ **b.** $\frac{1}{5}$
c. Not necessarily. There could be an overlap of students with blue eyes who have pets.

17. a. 75% in the previous game
b. i. Increased percentage
ii. Decreased percentage
iii. Unchanged

Exercise 7.3 Percentages as decimals

1. a. 0.36 **b.** 0.14 **c.** 0.19 **d.** 0.28 **e.** 0.73 **f.** 0.92 **g.** 0.66 **h.** 0.59
i. 0.11 **j.** 0.99 **k.** 0.09 **l.** 0.07 **m.** 0.04 **n.** 0.01 **o.** 0.25 **p.** 2.00
q. 1.5 **r.** 3.6

2. a. 0.123 **b.** 0.316 **c.** 0.592 **d.** 0.849 **e.** 0.376 **f.** 0.421 **g.** 0.219 **h.** 0.169
i. 0.107 **j.** 0.111 **k.** 0.031 **l.** 0.046 **m.** 0.092 **n.** 0.059 **o.** 0.068 **p.** 0.088
q. 0.1425 **r.** 0.3175 **s.** 0.2355 **t.** 0.4575 **u.** 0.0005 **v.** 0.0102 **w.** 0.0401 **x.** 0.0002

3. a. B **b.** C **c.** A **d.** D

4. a. $\frac{17}{100}$ **b.** 0.17

5. a. $\frac{1}{2000}$ **b.** 0.0005

6. 0.0285

7. 2.18

8. 0.138

9. 0.0835

10. a. 23, 24, 24, 24.

b. 0.23, 0.24, 0.24, 0.24

c. It does not matter as long as both values are rounded up or down at the same digit.

11. a. $23.\dot{6}\%$

b. 0.2360, 0.2367, 0.2366

12. a. 0.378, 0.396, 0.309, 0.345, 0.328

b. 39.6%, 37.8%, 34.5%, 32.8%, 30.9%

c. 0.396, 0.378, 0.345, 0.328, 0.309

d. Various answers are possible.

13. a. 0.07291, 0.7291, 7.291

b. They are 100 times smaller.

Exercise 7.4 Fractions and decimals to percentages

1. a. 21% **b.** 48% **c.** 9% **d.** 93% **e.** 28% **f.** 72% **g.** 16% **h.** 80%
i. 12% **j.** 60% **k.** 40% **l.** 24% **m.** 10% **n.** 35% **o.** 5% **p.** 20%
q. 80% **r.** 20% **s.** 40% **t.** 80% **u.** 70% **v.** 65% **w.** 30% **x.** 60%

2. a. 50% **b.** 25% **c.** 60% **d.** 25% **e.** 40% **f.** 20% **g.** 50% **h.** 50%
i. 75% **j.** 80% **k.** 10% **l.** 20%

3. a. $33\frac{1}{3}\%$ **b.** $16\frac{2}{3}\%$ **c.** $22\frac{2}{9}\%$ **d.** $44\frac{4}{9}\%$ **e.** $27\frac{3}{11}\%$ **f.** $46\frac{2}{3}\%$ **g.** $55\frac{5}{9}\%$ **h.** $56\frac{1}{4}\%$
i. $91\frac{2}{3}\%$ **j.** $87\frac{1}{2}\%$ **k.** $7\frac{9}{13}\%$ **l.** $58\frac{14}{17}\%$ **m.** $14\frac{2}{7}\%$ **n.** $83\frac{1}{3}\%$ **o.** $66\frac{2}{3}\%$

4. a. D **b.** B **c.** A **d.** E

5. a. 45% **b.** 32% **c.** 56% **d.** 68% **e.** 90% **f.** 84% **g.** 12% **h.** 8%
i. 2% **j.** 10% **k.** 99% **l.** 5% **m.** 29% **n.** 9% **o.** 19% **p.** 10.5%
q. 0.1% **r.** 6.7%

6. a. 30% **b.** 80% **c.** 90% **d.** 10% **e.** 0.2% **f.** 0.7% **g.** 0.5% **h.** 0.9%
i. 132% **j.** 150% **k.** 865% **l.** 205% **m.** 450% **n.** 0.015% **o.** 200% **p.** 1000%
q. 500% **r.** 10000%

7. a. D **b.** A **c.** E **d.** C

8. $\frac{3}{4}\%$, 12.5%, 15%, 25%, 50%, 85%, 94%

↓ ↓ ↓ ↓ ↓ ↓ ↓

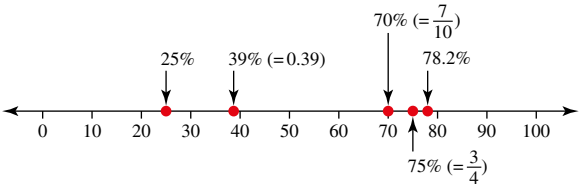
$\frac{3}{4}\%$, $\frac{125}{1000}$, 15%, $\frac{1}{4}$, $\frac{1}{2}$, $\frac{85}{100}$, $\frac{94}{100}$

9. 20% **10.** 32% **11.** 7.5% **12.** 58% **13.** 30% **14.** 40% **15.** $36\frac{4}{11}\%$ **16.** $52\frac{3}{11}\%$
17. 70% **18.** 40% **19.** 43% **20.** 3.1%

21.

Student name	Directed numbers test	
	Mark	Percentage
Mandy Adams	0.86	86
Sandra Bazumik	0.72	72
Malcolm Boncev	0.64	64
James Callan	0.91	91
Kate D'Arpa	0.79	79
Louise Edmonds	0.92	92
Chris Edwards	0.95	95
Thomas Evancik	0.10	10
Jessie Farmer	0.88	88
Mia Yazzett	0.46	46

22. a. 25%, 0.39, $\frac{7}{10}$, $\frac{3}{4}$, 78.2%



a. 25% = 0.25 is less than 0.39
$\frac{7}{10}$ = 0.7 is less than $\frac{3}{4}$ = 0.75 but greater than 0.39.
78.2% is the highest value.

23. a. i. 0.50, 0.3$\dot{3}$%, 0.25, 0.20, 0.1$\dot{6}$ **ii.** 50%, 33.3$\dot{3}$, 25%, 20%, 16.6$\dot{6}$

b. i. 1 **ii.** 100%

c. They are identical: 1 and 100%

d. Various answers possible.

24. a. i. 40% **ii.** 40%

b. No, because the two jobs do not add up to 100%.

c. Yes

25. After the sale with the discount voucher, because $\frac{1}{3} \simeq 33.33\%$

Exercise 7.5 Finding percentages of an amount

1. a. 180 **b.** 4 **c.** 60 **d.** 18 **e.** 45 **f.** 12 **g.** $1\frac{1}{2}$ **h.** 44
i. $10\frac{1}{2}$ **j.** 78

2. a. 10 **b.** 16 **c.** 3 **d.** 3 **e.** 18 **f.** 93 **g.** 6 **h.** 6
i. 28 **j.** 77 **k.** 39 **l.** 63 **m.** 4000 **n.** 66 **o.** 17 **p.** 42
q. 95 **r.** 190 **s.** 55 **t.** 12 **u.** 25

3. a. $3\frac{3}{5}$ **b.** $4\frac{4}{5}$ **c.** $7\frac{7}{10}$ **d.** $9\frac{3}{5}$ **e.** $18\frac{1}{2}$ **f.** $13\frac{1}{5}$ **g.** $1\frac{9}{10}$ **h.** $24\frac{3}{4}$
i. $11\frac{1}{10}$ **j.** $70\frac{2}{5}$ **k.** $58\frac{9}{10}$ **l.** $20\frac{4}{5}$ **m.** $14\frac{2}{5}$ **n.** $98\frac{2}{5}$ **o.** $16\frac{1}{5}$

4. a. A **b.** D **c.** E **d.** E

5. 210 students

6. a. 4 **b.** 18 **c.** 4 **d.** 5 **e.** 7 **f.** 41 **g.** 37 **h.** 5
i. 144 **j.** 40.5 **k.** 50.4 **l.** 372 **m.** 4.5 **n.** 254.4 **o.** 22 **p.** 3.5
q. 236 **r.** 12.48

7. a. 7.2 **b.** 1.74 **c.** 23.529 **d.** 2.196 **e.** 12.127 **f.** 11.088 **g.** 11.723 **h.** 17.766
i. 1.072 **j.** 66.528 **k.** 32.708 **l.** 324.159 **m.** 268.763 **n.** 110.1375 **o.** 1.243 75

8. a. 0.464 **b.** 2.816 **c.** 5.264 **d.** 22.407 **e.** 2.720 **f.** 17.424 **g.** 3.8775 **h.** 30.6037
i. 11.8335 **j.** 286 **k.** 4423.5 **l.** 932.4 **m.** 38.2655 **n.** 13.665 75 **o.** 88.2

9. a. B **b.** E **c.** D **d.** C

10. 73 days

11. 1 person

12. a. 80%

b. 1600 people

13. a. $17.10 **b.** $587.10

14. $1 600 000

15. $12 000

16. 47 species

17. 600 000 people

18. a. Peter calculated 35% of 80 mL rather than 800 mL.

b. 280 mL

19. a. 18 adults **b.** 1 adult

20. 8 houses

21. 38 workers

22. a. 12 houses **b.** 11 houses

23. 2 women

24. a. 200 g, 100 g, 50 g, 25 g and 12.5 g.

b. 50%, 25%, 12.5%, 6.25% and 3.125%.

c. Every 20 days the quantity of the radioactive substance is smaller so there is a 50% reduction of an already reduced quantity.

25. 22.5%

26. 3 students voted twice.

Challenge 7.1

120

Exercise 7.6 One amount as a percentage of another

1. a. 20% **b.** 25% **c.** 50% **d.** 10% **e.** 40% **f.** 5% **g.** 25% **h.** 80%
i. 60% **j.** 32% **k.** 40% **l.** 20% **m.** 20% **n.** 5% **o.** 20% **p.** 25%

2. a. $31\frac{1}{2}\%$ **b.** $6\frac{1}{4}\%$ **c.** $7\frac{1}{2}\%$ **d.** $6\frac{2}{5}\%$ **e.** $20\frac{2}{3}\%$ **f.** $66\frac{2}{3}\%$ **g.** $87\frac{1}{2}\%$ **h.** $28\frac{4}{7}\%$
i. $13\frac{7}{11}\%$ **j.** $66\frac{2}{3}\%$

3. a. $36\frac{2}{3}\%$ **b.** $26\frac{2}{3}\%$ **c.** $13\frac{1}{3}\%$ **d.** $7\frac{1}{7}\%$ **e.** $22\frac{2}{9}\%$ **f.** $83\frac{1}{3}\%$ **g.** $42\frac{6}{7}\%$ **h.** $88\frac{8}{9}\%$
i. $25\frac{5}{7}\%$ **j.** $71\frac{3}{7}\%$

4. a. 17% **b.** 33% **c.** 56% **d.** 63% **e.** 11% **f.** 9% **g.** 83% **h.** 78%
i. 18% **j.** 12%

5. a. 10% **b.** $3\frac{1}{3}\%$ **c.** $12\frac{1}{2}\%$ **d.** $28\frac{4}{7}\%$ **e.** 25% **f.** $1\frac{1}{4}\%$

6. a. D **b.** A **c.** B **d.** E

7. 70%

8. 60%

9. a. \$5 **b.** $33\frac{1}{3}\%$

10. a. i. 10% **ii.** 14% **iii.** 12%

b. 36%

c. 64%

11. $62\frac{1}{2}\%$

12. $6\frac{2}{3}\%$

13. a. 20 km/h **b.** $16\frac{2}{3}\%$

14. a. $42\frac{6}{7}\%$ **b.** $28\frac{4}{7}\%$ **c.** $71\frac{3}{7}\%$ **d.** $28\frac{4}{7}\%$

15. $27\frac{3}{11}\%$

16. a.

Topic	Prob.	M'ment	Algebra	Eqns	Geo.
Score	$\frac{15}{20}$	$\frac{13}{15}$	$\frac{27}{30}$	$\frac{70}{80}$	$\frac{95}{100}$
Percentage	75	87	90	88	95

b. Geometry

c. 87%

17. a. 300 g out of 450 g

b. 200 g out of 500 g

c. 420 g out of 840 g and 350 g out of 700 g

d. 300 g out of 450 g represents 67%

200 g out of 500 g represents 40%

420 g out of 840 g and 350 g out of 700 g both represent 50%

18. a. i. Second recipe. The second recipe contains 50% of flour compared to 40% in the first recipe.

ii. The percentage of flour increases.

b. i. First recipe. The first type of concrete contains 15.4% cement, and the second type of concrete contains 11.8% cement.

ii. The percentage of cement decreases.

19. 4520 km

20. a. Less than $100 because 25% of the new amount is larger than the amount added. So, a higher amount is subtracted.

b. Various answers are possible.

Challenge 7.2

Sister is 9, Grandfather is 90

Exercise 7.7 Common percentages and short cuts

1. a. $1.00 **b.** $1.80 **c.** $4.50 **d.** $8.10 **e.** $15.00 **f.** $11.20 **g.** $9.30 **h.** $7.90
i. $4.70 **j.** $2.20 **k.** $1.65 **l.** $1.70 **m.** $1.25 **n.** $0.15 **o.** $3.30 **p.** $4.80
q. $8.15 **r.** $19.25 **s.** $50.70 **t.** $462.00 **u.** $192.60 **v.** $304.15 **w.** $721.95 **x.** $200.00

2. a. $1.50 **b.** $5.10 **c.** $1.70 **d.** $0.90 **e.** $13.70 **f.** $17.20 **g.** $0.45 **h.** $0.65
i. $0.80 **j.** $3.90 **k.** $7.50 **l.** $1.40 **m.** $10.30 **n.** $6.80 **o.** $4.30 **p.** $51.80
q. $30.50 **r.** $62.85 **s.** $10.05 **t.** $20.70

3. a. $0.40 **b.** $0.30 **c.** $0.10 **d.** $0.10 **e.** $7.00 **f.** $4.05 **g.** $2.10 **h.** $0.55
i. $12.15 **j.** $15.25 **k.** $50 **l.** $32.15 **m.** $5.15 **n.** $1.60 **o.** $0.30

4. a. $4.30 **b.** $8.45 **c.** $1.65 **d.** $0.65 **e.** $14.80 **f.** $0.20 **g.** $0.15 **h.** $3.30
i. $27.15 **j.** $52.35 **k.** $247.40 **l.** $1013.80

5. a. $1.80 **b.** $1.20 **c.** $3.00 **d.** $9.00 **e.** $7.50 **f.** $11.25 **g.** $22.50 **h.** $55.00
i. $4.50 **j.** $7.50 **k.** $14.25 **l.** $30.65

6. a. $2.70 **b.** $7.15 **c.** $5.75 **d.** $6.05 **e.** $0.05 **f.** $0.10 **g.** $0.15 **h.** $0.15
i. $0.20 **j.** $0.80 **k.** $0.20 **l.** $4.30 **m.** $0.05 **n.** $0.05 **o.** $0.10 **p.** $0.00
q. $0.00 **r.** $12.65

7. a. $1.30 **b.** $10.50 **c.** $3.30 **d.** $0.65 **e.** $2.40 **f.** $2.20 **g.** $1.80 **h.** $73.50
i. $18.00 **j.** $1.55 **k.** $1.05 **l.** $2.05 **m.** $32.20 **n.** $4.80 **o.** $1.60 **p.** $0.45
q. $14.40 **r.** $492

8. a. D **b.** B **c.** E **d.** C

9. $855

10. 54 000 residents

11. a. $322.50 **b.** $152.30

12. 110 students

13. 27.9 seconds

14. a. 2 people **b.** 38 people

15. a. 30 240 people **b.** 39 600 people

16. 20 years old

17. $0.80

18. 8.2 kg

19. 50 years old

20. a. 100%

b. i. 100% **ii.** 50% **iii.** 71.4%

c. No, because the two lengths have changed at different percentages.

21. a. 3.2% **b.** 6.8%

c. The overall percentage is a lot smaller.

7.8 Review

1. a. $\frac{13}{100}$ **b.** $\frac{7}{10}$ **c.** $\frac{13}{50}$ **d.** $\frac{33}{25} = 1\frac{8}{25}$ **e.** $\frac{1}{400}$ **f.** $\frac{3}{125}$ **g.** $\frac{97}{1000}$ **h.** $\frac{7317}{10000}$

2. a. 0.42 **b.** 0.05 **c.** 0.94 **d.** 1.39 **e.** 0.067 **f.** 0.197 **g.** 0.5803 **h.** 0.008

3. a. 15% **b.** 55% **c.** 60% **d.** 50% **e.** $37\frac{1}{2}\%$ **f.** $53\frac{1}{3}\%$ **g.** $83\frac{1}{3}\%$ **h.** $36\frac{4}{11}\%$

4. a. 71% **b.** 84% **c.** 3% **d.** 20% **e.** 50% **f.** 0.8% **g.** 164% **h.** 380%

5. a. 6 **b.** 12 **c.** 39 **d.** 39 **e.** $6\frac{3}{10}$ **f.** $37\frac{1}{10}$ **g.** $4\frac{4}{5}$ **h.** $13\frac{3}{5}$

6. a. 60% **b.** 30% **c.** 40% **d.** 40% **e.** $33\frac{1}{3}\%$ **f.** $12\frac{1}{2}\%$ **g.** $46\frac{2}{3}\%$ **h.** $87\frac{1}{2}\%$
i. $7\frac{1}{7}\%$ **j.** $11\frac{1}{9}\%$

7. a. 5 **b.** 3.72 **c.** 2.25 **d.** 6.9 **e.** 19.26 **f.** 38.76 **g.** 6.12 **h.** 40.6926

8. a. \$6.30 **b.** \$4.20 **c.** \$10.50 **d.** \$21.60 **e.** \$0.35 **f.** \$4.25 **g.** \$11.85 **h.** \$212.60

9. a. \$0.40 **b.** \$1.05 **c.** \$3.20 **d.** \$5.20 **e.** \$1.75 **f.** \$2.60 **g.** \$10.30 **h.** \$3.85

10. a. \$0.15 **b.** \$0.30 **c.** \$5.05 **d.** \$27.00 **e.** \$48.40 **f.** \$11.60 **g.** \$2.95 **h.** \$2.95

11. $\frac{19}{20}$

12. a. $\frac{6}{25}$ **b.** $\frac{19}{20}$ **c.** 96

13. a. 62% **b.** 75%

14. $62\frac{1}{2}$ **15.** 0.41% **16.** 24 students **17.** 270 children **18.** 1425000 km^2 **19.** 3%

20. 8% **21.** \$50.80 **22.** \$11600 **23.** 400 g **24.** 200 g **25. b**, **d**

26. 13.5 litres of milk would be needed to make the fat of 500 g of butter.

27. a. 1194 people

b. 1.4%

28. \$850

29. No. (Check with your teacher for explanation.)

30. Check with your teacher.

31. 45% **32.** 720 **33.** \$16 **34.** 20%

35. a. \$2250 **b.** \$2025 **c.** 95%

d. The second 10% discount is not 10% of the original price but 10% of an amount which is only 90% of the original.

36. \$80

37. No. 10% off each item gives a total saving of \$9.50, while 20% of the combined purchase would give a saving of \$19.

38. No. Overall, after two months your salary will decrease by 1% using either option. However, option 1 gives an extra income (10% of the original salary) in the first month.

39. a. \$1575 **b.** \$1575

c. It doesn't matter which discount is applied first; the final price will be the same.

40. a. Nathan is 30 years old now.

a. \$0.63

Investigation — Rich task

1. \$98.00

2. Some examples are:

- a pair of jeans and 1 T-shirt
- a pair each of cargo pants and shorts
- a pair of cargo pants and 2 T-shirts
- 5 T-shirts.

3. Hooded tops = \$29.25; cargo pants = \$35.40; T-shirts = \$18.00; shorts = \$24.65; jeans = \$51.35

4. \$64.65

5. \$33.35

6. \$39.35

7. Some examples are:

- 1 hooded top and a pair each of cargo pants and shorts
- a pair of jeans and 2 T-shirts
- a pair of jeans, 1 hooded top and 1 T-shirt.

8. Answers will vary. Teacher to check.

TOPIC 8 Algebra

8.1 Overview

Numerous **videos** and **interactivities** are embedded just where you need them, at the point of learning, in your learnON title at www.jacplus.com.au. They will help you to learn the content and concepts covered in this topic.

8.1.1 Why learn this?

Algebra is a mathematical language. It simplifies difficult problems by using letters to represent numbers, many of them unknown, in calculations. In learning algebra we learn reasoning skills. The answers to many puzzles and problems can be found by applying algebra.

Sometimes it is hard to see how we would use algebra in everyday life, but the methods used to solve problems will be useful no matter what a person does in life.

Scientists, architects and engineers all use algebra frequently in their work. Without algebra there would be no electrical appliances, no iPod, no iPad, no computers.

8.1.2 What do you know? assessON

1. **THINK** List what you know about algebra. Use a thinking tool such as a concept map to show your list.
2. **PAIR** Share what you know with a partner and then with a small group.
3. **SHARE** As a class, create a thinking tool such as a large concept map that shows your class's knowledge of algebra.

LEARNING SEQUENCE

8.1 Overview
8.2 Using rules
8.3 Finding a formula
8.4 Substitution
8.5 Terms, expressions and equations
8.6 Simplifying and the distributive law
8.7 The associative law
8.8 Review

8.2 Using rules

- Mathematics can be used to describe relationships in the world around us.

8.2.1 Patterns and rules

- A number pattern can be called a **sequence**.
- Each number in the sequence is called a **term**.
- A sequence has a **rule** that describes the pattern.

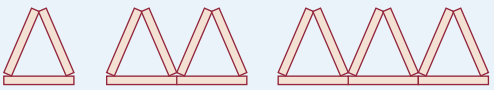
WORKED EXAMPLE 1

Consider the number pattern 1, 4, 7, 10, …
Describe the pattern in words and then write the next three numbers in the pattern.

THINK	WRITE
1 The sequence starts with 1 and increases by 3 to get the second term. Check to see if this works for the remaining terms.	Add 3 to each term to find the next term.
2 We get the next term by adding 3.	The next 3 terms are 13, 16 and 19.

WORKED EXAMPLE 2

Consider the following set of shapes.



a Construct a table to show the relationship between the number of shapes in each figure and the number of matchsticks used to construct it.
b Write a rule in words that describes the pattern relating the number of shapes in each figure and the number of matchsticks used to construct it.
c Use your rule to work out the number of matchsticks required to construct a figure made up of 20 such shapes.

THINK

a Three matches were needed for 1 triangle, 6 matches for 2 triangles and 9 matches for 3 triangles. Construct a table. If there was a fourth triangle, it would need 12 matches.

WRITE

Number of triangles	1	2	3	4
Number of matchsticks	3	6	9	12

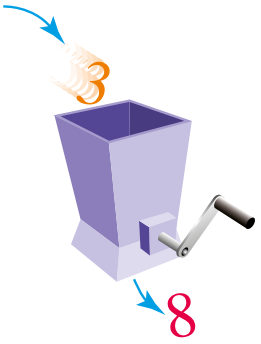
b Look for a pattern between the number of triangles and number of matchsticks.

The number of matchsticks equals 3 times the number of triangles.

c Apply the rule 'the number of matchsticks equals 3 times the number of triangles' for 20 triangles.

$20 \times 3 = 60$
To construct 20 triangles,
60 matchsticks would be required.

8.2.2 Input and output tables



- A rule can be used to produce a set of output numbers from a set of input numbers.
- We can think of this as a machine that takes any input number and changes it according to a given rule. In this case, 5 is added to each input number. Any number can be used as an input number. The machine might print out a table like this:

Input	3	5	89	222
Output	8	10	94	227

- The rule can involve more than one step; for example: 'multiply each input number by 1 and then add 5'.

WORKED EXAMPLE 3

Complete the table using the given rules to work out the correct output numbers.
a Rule: Subtract 3 from each input number.
b Rule: Divide each input number by 2.

Input	4	10	38	144
Output				

THINK

a 1 Take the first input number (4) and apply the rule; that is, subtract 3.

2 Repeat for the other input numbers.

3 Enter these output values in the table.

WRITE

a $4 - 3 = 1$

$10 - 3 = 7$
$38 - 3 = 35$
$144 - 3 = 141$

Input	4	10	38	144
Output	1	7	35	141

b 1 Take the first input number (4) and apply the rule; that is, divide by 2.

b $4 \div 2 = 2$

2 Repeat for the other input numbers.

$10 \div 2 = 5$
$38 \div 2 = 19$
$144 \div 2 = 72$

3 Enter these output values in the table.

Input	4	10	38	144
Output	2	5	19	72

WORKED EXAMPLE 4

Complete the table by using the following rule to work out the correct output numbers in each case.
Rule: Multiply each input number by 8, then subtract 2.

Input	1	3	5	24
Output				

THINK	WRITE
1 Take the first input number (1) and apply the rule. *Note:* First multiply the input number by 8 and then subtract 2 from the answer.	$1 \times 8 = 8$ then $8 - 2 = 6$
2 Repeat for the other input numbers.	$3 \times 8 = 24$ then $24 - 2 = 22$ $5 \times 8 = 40$ then $40 - 2 = 38$ $24 \times 8 = 192$ then $192 - 2 = 190$
3 Enter the output values in the table.	**Input** 1 3 5 24 **Output** 6 22 38 190

WORKED EXAMPLE 5

Use the rule given below to work out the missing input and output numbers.
Rule: Add 4 to each input number.

Input	1	5	25			
Output				6	38	190

THINK	WRITE
1 Consider the input numbers that are given and apply the rule; that is, add 4.	$1 + 4 = 5$ $5 + 4 = 9$ $25 + 4 = 29$
2 Consider the output numbers that are given and perform the opposite operation; that is, subtract 4.	$6 - 4 = 2$ $38 - 4 = 34$ $190 - 4 = 186$
3 Enter the new output and input values in the table.	**Input** 1 5 25 2 34 186 **Output** 5 9 29 6 38 190

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Complete this digital doc: SkillSHEET: Number patterns (doc-6499)
Complete this digital doc: SkillSHEET: Worded operations (doc-6500)
Complete this digital doc: SkillSHEET: Describing a number pattern from a table (doc-6501)

Exercise 8.2 Using rules

assessON

Individual pathways

PRACTISE
Questions:
1a–c, 2a–b, 3a–d, 4a–c, 5a–d, 6a–b, 8, 9

CONSOLIDATE
Questions:
1a–e, 2a–c, 3a–d, 4c–e, 5a–d, 6b–c, 8, 9, 10, 11

MASTER
Questions:
1a–f, 2b–d, 3a–d, 4d–f, 5a–d, 6c–f, 7–12

Individual pathway interactivity: int-4349

learnON ONLINE ONLY

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Fluency

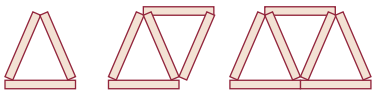
1. WE1 Copy the patterns below, describe the pattern in words and then write the next three numbers in the pattern.

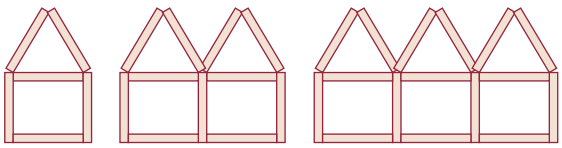
a. 2, 4, 6, 8, . . .
b. 3, 8, 13, 18, . . .
c. 27, 24, 21, 18, . . .
d. 1, 3, 9, 27, . . .
e. 128, 64, 32, 16, . . .
f. 1, 4, 9, 16, . . .

2. WE2 For each of the sets of shapes below:

i. Construct a table to show the relationship between the number of shapes in each figure and the number of matchsticks used to construct it.
ii. Write a rule in words that describes the pattern relating the number of shapes in each figure and the number of matchsticks used to construct it.
iii. Use your rule to work out the number of matchsticks required to construct a figure made up of 20 such shapes.

a.

b.


c.


d.

3. WE3 Copy and complete the following tables. Use the rule given in each case to work out the correct output numbers.

a. Subtract 4 from each input number.

Input	4	5	6	10	14
Output					

b. Add 12 to each input number.

Input	1	2	6	13	107
Output					

c. Multiply each input number by 2.

Input	1	10	51	60	144
Output					

d. Divide each input number by 3.

Input	3	12	21	66	141
Output					

4. WE4 Copy and complete the following tables. Use the rule given in each case to work out the correct output numbers.

a. Multiply each input number by 2, then add 5.

Input	2	3	4	10	17
Output					

b. Multiply each input number by 8, then subtract 4.

Input	2	4	5	20	100
Output					

c. Add 3 to each input number, then multiply by 5.

Input	2	3	5	12	43
Output					

d. Subtract 3 from each input number, then multiply by 11.

Input	3	4	5	8	25
Output					

e. Multiply each input number by itself.

Input	0	1	5	8	10
Output					

f. Multiply each input number by itself, then add 4.

Input	0	1	4	7	12
Output					

5. WE5 Copy and complete the following tables. Use the rule given in each case to work out the missing input and output numbers.

a. Add 2 to each input number.

Input	1	3	27			
				4	55	193

b. Add 20 to each input number.

Input	3	25	56			
Output				20	94	1773

c. Subtract 5 from each input number.

Input	7	15	96			
Output				4	12	104

d. Multiply each input number by 4.

Input	1	6	321			
Output				8	28	412

Understanding

6. a. Multiply each input number by 2, then add 2.

Input	3	13	21			
Output				4	22	102

b. Multiply each input number by 5, then subtract 7.

Input	2	5	16			
Output				8	18	93

c. Divide each input number by 3.

Input	3	15	273			
Output				2	21	54

d. Multiply each input number by itself.

Input	1	5	17			
Output				4	49	144

e. Multiply each input number by 20, then subtract 6.

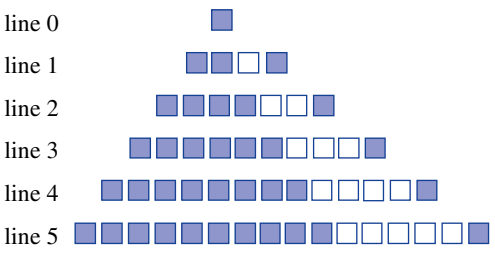
Input	2	7	15			
Output				54	174	214

f. Subtract 3 from each input number, then divide by 4.

Input	7	19	31			
Output				5	12	15

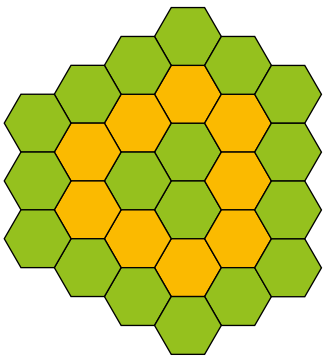
Reasoning

7. The white square represents number 0 and the blue square represents number 1.
A pattern was created using the two types of squares. The number of blue and white squares in each line is determined by a rule that connects the two variables.



a. Explain the pattern for each line.
b. Draw the next two lines in the pattern.

8. Hexagons are used in tessellation to form patterns of colour and shapes.



a. Explain how the pattern is formed in the diagram shown.
b. How many hexagons will be needed for the next layer?

Problem solving

9. With a certain pre-paid mobile phone deal, there is a 20-cent flag-fall that is charged for every call made, then peak hour calls are charged at a rate of 1 cent per second.

a. Set up a table to show the cost of peak calls of 1, 2, 3, …, 8 seconds duration.

b. What is the rule that shows the relationship between the length of the call and its cost?
c. Use the rule to predict the cost of a 2-minute call with this deal.
d. If the cost of a call was $2.00, how long was the call?

10. Triangular numbers are those that can be represented by dots in the shape of an equilateral triangle. The first five triangular numbers are 1, 3, 6, 10 and 15. Investigate these numbers to find a pattern in them. If the 20th triangular number is 210, show how you could determine the 25th triangular number.

11. Using 20 cubes, make four piles so that the first pile contains 4 more cubes than the second pile, the second pile contains 1 cube less than the third pile, and the fourth pile contains twice as many cubes as the second pile. How many cubes are in each pile?

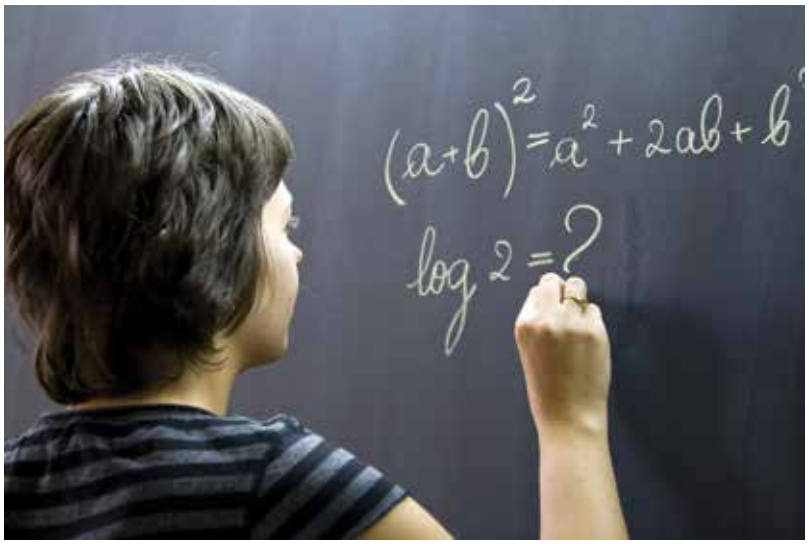
12. A *cryptarithm* is a puzzle in which each digit of a number is replaced by a letter. Using the digits 1, 2, 3, 4, 5 and 6 solve the following product.
A × BC = DEF

Reflection

What strategies can you use to work out the input rule from a table of input and output values?

8.3 Finding a formula

8.3.1 Finding a formula



- **Algebra** can be used to write a rule in a simple and concise manner.
- By using algebra, we can use a letter or symbol in place of a number.
- When a rule is written using a letter or a symbol, it is called a **formula**.
- A letter or symbol in a formula is called a **pronumeral** because its value can change. (It is also called a **variable**.)
- If a letter is used in place of a number, the value is not determined by its position in the alphabet. If we use b for a number, this does not mean that it has to be a smaller number than x or c.
- When multiplying numbers and pronumerals, we do not need to show the multiplication sign, and the number is written in front of the pronumeral, so $h = k \times 5$ is written as $h = 5k$.

WORKED EXAMPLE 6

Rewrite the following formulas, leaving out the multiplication (×) sign.
a $m = q \times 4 + 3$ **b** $b = (m + 2) \times 5$ **c** $g = (2 \times w - 6) \times 3$

THINK	WRITE
a 1 Write the formula.	**a** $m = q \times 4 + 3$
2 Leave out the multiplication sign. (Remember to write the number being multiplied by the pronumeral, in front of the pronumeral.)	$m = 4q + 3$
b 1 Write the formula.	**b** $b = (m + 2) \times 5$

2 Leave out the multiplication sign. (Remember to write the number being multiplied in front of the bracket.)

$b = 5(m + 2)$

c 1 Write the formula.

c $g = (2 \times w - 6) \times 3$

2 Leave out the multiplication signs. (Remember to write the number being multiplied in front of the respective pronumeral and bracket.)

$g = 3(2w - 6)$

WORKED EXAMPLE 7

Look at the table at right and complete the formula by inserting a number in the gap.
$g = f +$ __

f	3	6	8	13	20
g	5	8	10	15	22

THINK

WRITE

1 Look at the first pair of numbers, 3 and 5. Look at the formula and try to guess the number which must be added to 3 to get 5.

$5 = 3 + 2$

2 Look at the next pair of numbers to see if adding 2 works again.

$8 = 6 + 2$

3 Check the other number pairs to see if this works every time.

$10 = 8 + 2$
$15 = 13 + 2$
$22 = 20 + 2$

4 Write the formula.

$g = f + 2$

WORKED EXAMPLE 8

Use the pronumerals given to write a formula for the table at right.

a	4	5	6	7	13
b	3	5	7	9	21

THINK

WRITE

1 Look for two consecutive numbers in the top row, for example, $a = 4$ and $a = 5$, and compare the difference between their corresponding b values.

$5 - 3 = 2$

2 Repeat step **1** for a different pair of consecutive numbers in the top row, for example, $a = 6$ and $a = 7$.

$9 - 7 = 2$

3 If the value obtained in step **1** is the same as the one in step 2, use it along with the basic operations $(+, -, \times, \div)$ or a combination of operations to obtain the correct result.
Note: The b-value increases by 2 for every increase of 1 in the a-value. This means the formula will contain $a \times 2$. Therefore possible rules may be 'Multiply a by 2 and add a number to obtain b' or 'Multiply a by 2 and subtract a number to obtain b'.

Attempt 1

Multiply a by 2 and add a number.

$a = 4$ and $b = 3$

$4 \times 2 = 8$,
$8 +$ any number does not equal 3.

Attempt 2

Multiply a by 2 and subtract a number.

$a = 4$ and $b = 3$

$4 \times 2 = 8$,
$8 - 5 = 3$✓

4 Check that the combination of operations (rule) works for the other number pairs. *Note:* If the rule does not work for all pairs of values in the table further attempts must be used.	$a = 5$ and $b = 5$	$5 \times 2 = 10$, $10 - 5 = 5$
	$a = 6$ and $b = 7$	$6 \times 2 = 12$, $12 - 5 = 7$
	$a = 7$ and $b = 9$	$7 \times 2 = 14$, $14 - 5 = 9$
	$a = 13$ and $b = 21$	$2 \times 13 = 26$, $26 - 5 = 21$
5 Write the formula carefully, being sure to put the correct pronumerals in the right places. Remember when the rule is written using pronumerals, it is called a formula.	$b = 2a - 5$	

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Complete this digital doc: SkillSHEET: Using tables to show number patterns (doc-6503)

Complete this digital doc: WorkSHEET: Algebra I (doc-1819)

Exercise 8.3 Finding a formula

assessON

Individual pathways

PRACTISE
Questions:
1a, d, g, i, m, 2a–d, 3a–f, 4, 5, 6, 7, 8, 13, 14, 16

CONSOLIDATE
Questions:
1b, e, h, k, n, 2c–h, 3c–h, 7, 8, 9, 10, 13, 14, 16

MASTER
Questions:
1g, h, i, k, m, n, 2e–j, 3e–l, 9, 10, 11, 12, 13, 14, 15, 16

Individual pathway interactivity: int-4350

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Fluency

1. WE6 Rewrite the following formulas, leaving out the multiplication (×) sign.

a. $b = 4 \times h$ **b.** $m = f \times 4$ **c.** $r = a \times 5$ **d.** $m = t \times 4$
e. $x = (k + 4) \times 5$ **f.** $k = 6 \times w - 2$ **g.** $t = 4 \times (20 - g)$ **h.** $b = 10 \times a - 5$
i. $d = 6 \times f + 7$ **j.** $h = (x + 5) \times 9$ **k.** $y = (b \times 3 + 6) \times 8$ **l.** $y = 8 \times p - 6$
m. $g = 2 \times (3 \times r + 17)$ **n.** $j = (h \times 5 - 4) \times 18$

2. WE7 Look at the following tables and complete the formula for each table by inserting a number in the gap.

a.

f	0	1	3	6	8
g	9	10	12	15	17

$g = f +$ ____

b.

a	3	4	7	8	11
b	0	1	4	5	8

$b = a -$ ____

c.

k	1	3	5	6	12
t	3	7	11	13	25

$t = 2k +$ ____

d.

x	1	3	5	8	11
y	2	16	30	51	72

$y = 7x -$ ____

e.

a	1	4	6	9	12
g	6	18	26	38	50

$g = 4a +$ ____

f.

m	1	2	3	4	11
t	20	29	38	47	110

$t = 9m +$ ____

g.

p	0	4	5	6	13
w	2	14	17	20	41

$w =$ ____ $\times p + 2$

h.

t	2	3	7	9	12
x	2	7	27	37	52

$x =$ ____ $\times t - 8$

i.

m	1	2	4	7	15
p	2	5	11	20	44

$p = 3m -$ ____

j.

s	1	5	6	7	12
b	3	43	53	63	113

$b = s \times$ ____ $- 7$

3. WE8 Look at the following tables and use the pronumerals given to write a formula for each table.

a.

a	2	4	5	8	12
b	5	7	8	11	15

b.

t	1	2	3	8	15
w	6	7	8	13	20

c.

m	4	5	7	11	12
a	0	1	3	7	8

d.

s	0	1	2	5	12
t	3	5	7	13	27

e.

f	1	3	4	6	11
g	1	5	7	11	21

f.

s	1	3	7	9	12
c	4	10	22	28	37

g.

d	2	4	5	7	12
a	5	13	17	25	45

h.

s	1	2	5	6	10
g	1	6	21	26	46

i.

f	2	3	4	8	11
e	2	13	24	68	101

j.

p	1	2	5	6	12
q	24	44	104	124	244

k.

b	20	55	60	75	100
t	80	220	240	300	400

l.

u	5	10	20	100	7000
p	14	29	59	299	20999

Understanding

For questions **4** to **12**, a formula has been used by an algebra machine to produce each table.

4. MC

a	0	1	3	6	36
b	2	3	5	8	38

The formula used is:

A. $b = a + 5$ **B.** $b = 3a$ **C.** $b = a + 2$

D. $b = a - 2$ **E.** $b = a - 5$

5. MC

g	1	2	4	18	56
k	0	1	3	17	55

The formula used is:
A. $k = g + 1$ **B.** $k = 2g$ **C.** $k = g - 1$
D. $k = 2g - 2$ **E.** $k = 2g + 2$

6. MC

r	5	6	8	9	17
w	0	1	3	4	12

The formula used is:
A. $w = r + 5$ **B.** $w = 5r$ **C.** $w = 2r - 5$
D. $w = r - 5$ **E.** $w = r + 6$

7. MC

p	0	6	7	9	21
m	0	12	14	18	42

The formula used is:
A. $m = p$ **B.** $m = p + 6$ **C.** $m = p + 2$
D. $m = 12p$ **E.** $m = 2p$

8. MC

m	1	2	5	7	98
w	8	9	12	14	105

The formula used is:
A. $w = 8m$ **B.** $w = 2m + 6$ **C.** $w = m + 7$
D. $w = 2m + 5$ **E.** $w = 2m$

9. MC

p	2	3	4	11	78
t	8	11	14	35	236

The formula used is:
A. $t = p + 6$ **B.** $t = 2p + 3$ **C.** $t = 4p$
D. $t = 3p + 2$ **E.** $t = 3p + 1$

10. MC

g	2	3	4	11	78
k	10	15	20	55	390

The formula used is:
A. $k = 5g$ **B.** $k = 4g + 2$ **C.** $k = g + 8$
D. $k = 2g + 6$ **E.** $k = 4g - 2$

11. MC

b	1	2	9	14	65
a	9	11	25	35	137

The formula used is:
A. $a = 8b + 1$ **B.** $a = 3b + 5$ **C.** $a = b + 8$
D. $a = 2b + 7$ **E.** $a = b - 8$

12. MC

m	2	3	5	7	11
e	25	37	61	85	133

The formula used is:
A. $e = m + 78$ **B.** $e = 10m + 11$ **C.** $e = 12m + 1$
D. $e = 6m + 43$ **E.** $e = 10m - 11$

13. Breanna was having some friends over after school. She needed enough biscuits so that they could have 3 biscuits each. Her sisters would also be home. She needed 9 extra biscuits for herself and her sisters. If she wants to have n friends over, write a formula to decide how many biscuits she would need.

Reasoning

14. Look at the following tables and use the pronumerals given to write a formula for each table. Show how you reached your answers.

a.

a	1	2	3	5	10
t	0	3	8	24	99

b.

b	1	3	7	11	15
x	1	2	4	6	8

c.

w	0	1	2	4	10
p	0	2	8	32	200

Problem solving

15. William and Olivia make pool tables with a difference. A diagram of one of their tables is shown. This one is 6 squares long (the base) and 4 squares wide (the height). There are only 4 corner pockets. William and Olivia use their tables to play a game called *Pool multiply*. The rules of the game are simple.

1. Place a ball in front of the corner pocket A (Diagram 1).
2. Shoot the ball through the dotted line shown in the diagram. This will make the ball rebound at 45° when it hits a side cushion (Diagram 2).
3. Count the number of times the ball hits the sides of the table. Include the initial hit of the cue on the ball as one hit and the ball falling into the corner pocket as one hit.
4. Count the number of squares that the ball travels through.

William and Olivia's other pool tables have different numbers of squares along the bases and heights (see some of them in the following table). Explore these pool tables and complete the table.

Diagram 1

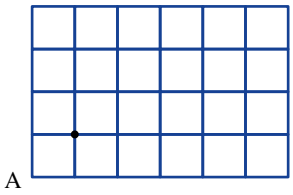
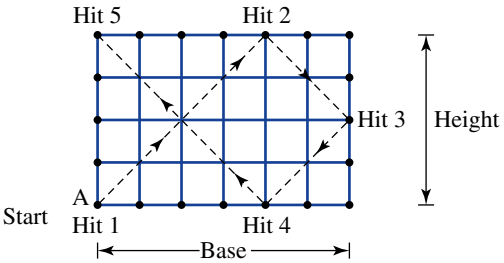


Diagram 2



Base	Height	Number of hits	Number of squares
6	4	5	12
3	4		
3	5		
3	6		
4	8		
b	*h*		

Develop a rule for the number of squares and number of hits if you know the base and height of the table. You may need to try extra tables to help you find the rule.

16. A rectangular chart can be attached to a wall with a drawing pin in each corner. You have 8 rectangular charts all the same size. What is the minimum number of pins you would need to attach all your charts to the wall if you must have a pin in each corner, and if overlapping is allowed at the margins? (Each chart must be visible.)

Reflection

What strategies can you use to help you find a formula?

CHALLENGE 8.1

While on a vacation to the country, a man spent 10 days fishing in the river. On each new day, he caught 4 fish more than he had caught the day before. If he caught a total of 200 fish, how many did he catch on each day?

8.4 Substitution

8.4.1 Substitution

- **Substitution** means that a pronumeral in a formula is replaced by a number.
- If we know the rule or formula for a particular table, we can complete the table by substituting numbers into the formula.

WORKED EXAMPLE 9

If $x = 3t - 6$, substitute the given values of t into the formula to find the value of x in each case.
a $t = 5$ **b** $t = 12$

THINK	WRITE
a 1 Write the formula.	**a** $x = 3t - 6$
2 Substitute 5 for t.	If $t = 5$
3 Perform the multiplication.	$x = 3 \times 5 - 6$
4 Subtract 6 from the above result.	$x = 15 - 6$
5 Write the answer.	$x = 9$
b 1 Write the formula.	**b** $x = 3t - 6$
2 Substitute 12 for t.	If $t = 12$
3 Perform the multiplication.	$x = 3 \times 12 - 6$
4 Subtract 6 from the above result.	$x = 36 - 6$
5 Write the answer.	$x = 30$

WORKED EXAMPLE 10

Find the value of m by substituting the given value of x into the following formula.

$$m = 3(2x + 3), x = 4$$

THINK	WRITE
1 Write the formula.	$m = 3(2x + 3)$
2 Substitute 4 for x.	If $x = 4$ $m = 3(2 \times 4 + 3)$
3 Work inside the brackets first: **a** Multiply 2 by 4 (remember that $2x$ means $2 \times x$). **b** Add 3 to the above result.	$m = 3(8 + 3)$ $m = 3(11)$
4 Multiply by the number outside the brackets.	$m = 3 \times 11$
5 Write the answer.	$m = 33$

Exercise 8.4 Substitution

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1a–d, 3, 4, 5, 10, 11	Questions: 2a–e, 3, 4, 5, 6, 7, 9, 10, 11	Questions: 2d–i, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12

Individual pathway interactivity: int-4351

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Fluency

1. WE9 Substitute the given values into each formula to find the value of m in each case.

a. $m = g - 2$
- **i.** $g = 4$
- **ii.** $g = 5$
- **iii.** $g = 2$
- **iv.** $g = 102$

b. $m = 2t - 3$
- **i.** $t = 7$
- **ii.** $t = 2$
- **iii.** $t = 100$
- **iv.** $t = 8$

c. $m = 12h + 7$
- **i.** $h = 1$
- **ii.** $h = 0$
- **iii.** $h = 5$
- **iv.** $h = 20$

d. $m = 25 - 4w$
- **i.** $w = 1$
- **ii.** $w = 3$
- **iii.** $w = 6$
- **iv.** $w = 0$

2. WE10 Find the value of m by substituting the given value of the pronumeral into the formula.

a. $m = 2(g + 1)$
- **i.** $g = 1$
- **ii.** $g = 0$
- **iii.** $g = 12$
- **iv.** $g = 75$

b. $m = 5(x - 2)$
- **i.** $x = 6$
- **ii.** $x = 10$
- **iii.** $x = 11$
- **iv.** $x = 2$

c. $m = 4(12 - p)$
- **i.** $p = 2$
- **ii.** $p = 3$
- **iii.** $p = 12$
- **iv.** $p = 11$

d. $m = 5(2g - 3)$
- **i.** $g = 2$
- **ii.** $g = 14$
- **iii.** $g = 5$
- **iv.** $g = 9$

e. $m = 2(d + 2) - 3$
- **i.** $d = 3$
- **ii.** $d = 0$
- **iii.** $d = 7$
- **iv.** $d = 31$

f. $m = 3(f - 1) + 17$
- **i.** $f = 1$
- **ii.** $f = 3$
- **iii.** $f = 6$
- **iv.** $f = 21$

g. $m = 4s - s$
- **i.** $s = 3$
- **ii.** $s = 1$
- **iii.** $s = 101$
- **iv.** $s = 72$

h. $m = 3(y + 5) - 2$
- **i.** $y = 1$
- **ii.** $y = 5$
- **iii.** $y = 0$
- **iv.** $y = 12$

i. $m = 50 - 6v$
- **i.** $v = 4$
- **ii.** $v = 7$
- **iii.** $v = 1$
- **iv.** $v = 8$

Understanding

3. Most tinned and packaged foods that we buy have labels which list the nutritional content of the food. Most of the energy (measured in calories or kilojoules) in food comes from fat, protein and carbohydrates.

It is estimated that there are 9 calories in each gram of fat and 4 calories in each gram of protein and each gram of carbohydrate. Hence, if a particular food contained 1.6 grams of fat, 2.1 grams of protein and 16.3 grams of carbohydrate, it would be equal to $1.6 \times 9 = 14.4$ calories of fat, $2.1 \times 4 = 8.4$ calories of protein and $16.3 \times 4 = 65.2$ calories of carbohydrate. The total number of calories would be 88 (that is, $14.4 + 8.4 + 65.2$).

a. From the following table, write the number of grams of fat, protein and carbohydrate in 100 grams of baked beans. Round all values to 1 decimal place.

Nutritional information — 420 g can	
Energy (kJ)	1596
Protein (g)	20.6
Total fat (g)	2.1
Total carbohydrate (g)	57.5

b. Estimate how many calories there would be in 100 grams of baked beans using the information supplied above.

If we wanted to look at a wide variety of foods, it would be more convenient to have a rule or formula into which we could substitute numbers to easily obtain our answer.

c. Clearly explain how you obtained your answer in question **b**.

d. Choose a pronumeral for each of the following quantities:

i. number of grams of fat
ii. number of grams of protein
iii. number of grams of carbohydrate
iv. number of calories.

e. Using the pronumerals you have chosen, write a formula to calculate the number of calories.

f. The table below contains data on the fat, protein and carbohydrate content of selected foods. Copy this table into your workbook and write in your chosen pronumerals in the first row in brackets. Useyour formula to work out the calorie content of each of these foods. (The last column will be completed later on.)

Food	Number of grams of fat	Number of grams of protein	Number of grams of carbohydrate	Number of calories	Number of kilojoules
1 hot cross bun	3	3	19		
100 grams of chocolate cake	16	4	56		
100 grams of roast chicken	14	26	0		
70 grams of bacon	8	21	0		
2 grilled sausages	17	13	15		
1 piece of fish (flake), no batter	1	21	0		
10 grams of sultanas	0	0	6		
25 grams of dried apricots	0	0	17		

Food	Number of grams of fat	Number of grams of protein	Number of grams of carbohydrate	Number of calories	Number of kilojoules
1 banana	0	1	20		
1 apple	0	0	17		
1 serve of carrots	0	0	5		
1 serve of potatoes	0	0	17		
250 millilitres of milk	10	8	12		
210 grams of tinned tomato noodle soup	0.8	2.6	14.9		

g. Which quantity of food in this list has the highest number of calories? Is this what you expected?

h. What types of food in this list generally have a lower number of calories?

A calorie is an energy measurement unit in the imperial system. Nowadays we mostly use the metric system unit of kilojoules to measure energy. It is estimated that the number of kilojoules is equivalent to the number of calories multiplied by 4.2.

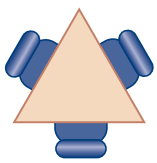
i. Use a pronumeral to represent each of the quantities 'number of calories' and 'number of kilojoules' and write a formula connecting them.

j. Convert the calories you calculated in **b** to kilojoules (rounding to the nearest kilojoule). Compare this to the value calculated in part **a**.

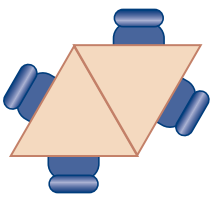
k. Complete the final column of your table by using the formula to find the number of kilojoules for each listed food. (Round your answers to the nearest kilojoule.)

Reasoning

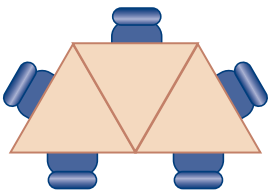
4. The dining tables at a theatre restaurant are triangular in shape. Diners are seated at the tables in the arrangements shown below.



One table, 3 diners



Two tables, 4 diners



Three tables, 5 diners

The manager of the restaurant has received a booking from a large party. She sets out a row of 24 tables. How many diners can be seated at this row of tables?

Follow the steps below to solve this problem.

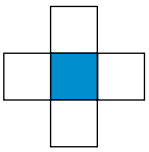
a. Draw a table with a row for the number of tables (T) and another row for the number of diners (D). Using the diagrams above, write in the pairs of numbers.

b. Draw a diagram to work out the number of diners who could sit at a row of 4 tables. Now draw another diagram to show the number of diners who could sit at a row of 5 tables. Write these pairs of values in your table.

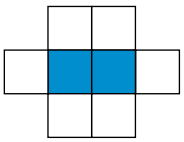
c. Work out the formula which connects the number of diners and the number of tables. Write the formula in the form $D =$ …

d. Substitute $T = 24$ into the formula to find out how many guests could be seated at the row of 24 tables.

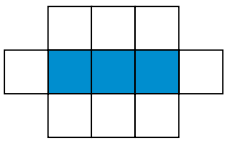
5. Jane is tiling the floor in her bathroom. She has decided to use blue and white tiles in the following pattern:



Number of blue tiles = 1
Number of white tiles = 4



Number of blue tiles = 2
Number of white tiles = 6



Number of blue tiles = 3
Number of white tiles = 8

Find out how many white tiles Jane would need to complete the pattern for a row of 10 blue tiles by following the steps below:

a. Draw a table with two rows; one headed B showing the number of blue tiles and one headed W showing the number of white tiles required. Count the numbers of blue and white tiles from each of the tiling patterns above and put them into the table in the correct row.

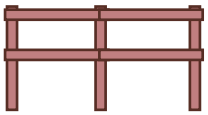
b. Draw a diagram of the tiling pattern for a row of 4 blue tiles and work out the number of white tiles needed to complete the pattern. Then draw another pattern for a row of 5 blue tiles. Add these two sets of values for B and W into the table.

c. Look carefully at the table and work out the formula that connects W and B. Write the formula in the form $W =$ …

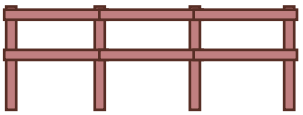
d. Substitute $B = 10$ into the formula to find out how many white tiles would be needed for a row of 10 blue tiles, using Jane's tiling pattern.

6. Michael is constructing a timber fence at his stud farm. The sections of fence are shown below:

1 section, 4 pieces



2 sections, 7 pieces



3 sections, 10 pieces

Michael calculates that he will need 220 sections to fence off his first paddock.
How many pieces of timber will Michael need? Find the cost of the fence if each piece of timber costs $3.85.

Follow the steps below.

a. Draw a table with 2 rows, one headed S showing the number of sections and one headed P showing the number of pieces of timber required. Write in the numbers from the diagrams above.

b. Draw a sketch of the fence with 4 sections, following the same pattern. Count the number of pieces of timber required. Draw a sketch for 5 sections and count the pieces required. Enter both pairs of numbers into your table in the correct columns.

c. Use trial and error to find the formula connecting P and S. Write the formula in the form $P = \ldots$

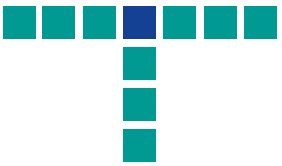
d. Substitute $S = 220$ into the formula to find out how many pieces of timber Michael needs.

e. Find the cost of the fence.

7. A T-shirt shop uses the letter T as its logo. The company wishes to make up a large advertising sign from square lights. Several small T-shirt shop signs are shown below:

Arm length = 1
Number of lights = 4

Arm length = 2
Number of lights = 7



Arm length = 3
Number of lights = 10

Find out how many square light panels would be needed to make up a large sign with an arm length of 75. Follow the steps below.

a. Draw a table with rows headed A (arm length), and L (number of lights needed).

b. Write in the values of A and L from the diagrams above.

c. Complete the diagrams showing the number of lights needed for a sign with an arm length of 4, and a sign with an arm length of 5. Add these values to your table.

d. Look at the table to determine the formula which connects L and A. Write the formula in the form $L = \ldots$

e. Substitute $A = 75$ into the formula to find the number of lights needed for the large display sign.

8. Karl Frederick Gauss, when he was a young man, discovered the sum S of the first n natural numbers is given by the formula $S = \dfrac{n(n+1)}{2}$.

Using this formula, find the answer to

a. $1 + 2 + 3 + 4 + \ldots + 46 + 47 + 48 + 49$

b. $54 + 55 + 56 + 57 + \ldots + 85 + 86 + 87 + 88$.

Problem solving

9. If $a > b$, $b > 2c$, $c > 3d$, $d > 4e$ and $e > 5f$, find the smallest possible value for a if all of the unknowns are positive integers.

10. A *cryptarithm* is a puzzle where each digit of a number has been replaced by a pronumeral or letter. Solve the following cryptarithm by determining the digit that corresponds to each letter to make the calculation true.

$$\begin{array}{r} \text{A B} \\ +\ \text{B A} \\ \hline \text{C A C} \end{array}$$

11. While solving a puzzle, Tiana worked out that $a > m$, $z > a$, $a < g$ and $z < g$. Tiana had to place them in numerical order. What is the numerical order of a, g, m and z?

12. A cryptarithm is a puzzle where each digit of a number has been replaced by a pronumeral or letter. Solve the following cryptarithm by determining the digit that corresponds to each letter to make the calculation true.

$$\begin{array}{r} \text{ONE} \\ +\ \text{TWO} \\ +\ \text{FOUR} \\ \hline \text{SEVEN} \end{array}$$

Reflection

What strategies could you use to check that your answer is reasonable?

8.5 Terms, expressions and equations

8.5.1 Terms and expressions

- A pronumeral or variable is a letter or symbol that is used in place of a number.
- The number in front of a pronumeral is called the **coefficient**. The coefficient of $7x$ is 7. The coefficient of $6xyz$ is 6. The coefficient of p is 1.
- **Expressions** are made up of pronumerals and numbers. Some expressions are $3k - 8$, $3t + 4$, $6(g - 9)$, $9p - 3y + 5$.

WORKED EXAMPLE 11

Write expressions to represent the total number of coins in each of the following situations where **represents a full moneybox and** **represents one coin. Use c to represent the number of coins in a moneybox.**

a Mary fills one moneybox and has two coins left over.

b Valentino and Loris decide to combine their coins. Valentino has three moneyboxes with four coins left over and Loris has two moneyboxes with six coins left over. That is,

Valentino has

Loris has

THINK	WRITE
a 1 The total number of coins will be found by adding the number of coins in one moneybox plus two coins left over.	**a** $1 \times c + 2$
2 Remove the multiplication sign to write the expression.	$= c + 2$
b 1 Add the number of coins Valentino and Loris have.	**b** $(3c + 4) + (2c + 6)$
2 Count the moneyboxes first, and then the coins.	$= (3c + 2c) + (4 + 6)$
3 Simplify.	$= 5c + 10$

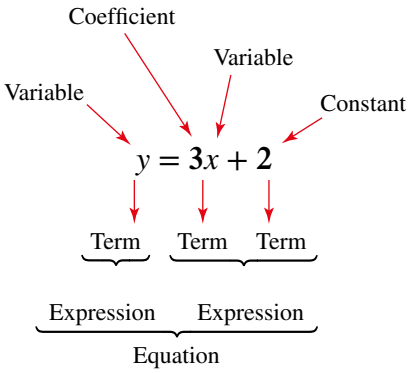
WORKED EXAMPLE 12

Write an expression for the sum of T and G.

THINK	WRITE
The word *sum* means to add together.	The expression is $T + G$ or $G + T$.

- **Equations** always contain an equals sign, whereas expressions do not. Examples of equations are: $y = 4t - 6$, $m + 3 = y - 8$, $2(g + 5) = 5$, $y = 6t$, $w = e - 87$, $9 = 7 + 4r$.
- Expressions and equations are made up of terms. Terms are separated by + or − signs.

- Terms can contain a combination of numbers and one or more pronumerals, or may consist of a number only. Examples of terms are $3t$, $2y$, 7, $5gh$, m, brt.
- The letter, or symbol, is called a variable or pronumeral because its value can change.
- If a term contains only a number and does not have a pronumeral, it is called a **constant**. Examples of constants are 7, 3 and 10.
- The definitions for this section can be summarised as shown below.



WORKED EXAMPLE 13

If Y represents any number, write expressions for:
a 5 times that number
b 2 less than that number
c 8 more than that number
d the number divided by 4 (or the quotient of Y and 4)
e the next consecutive number (that is, the counting number which comes after Y).

THINK	WRITE
In each case think about which operations are being used and the order in which they occur.	
a When multiplying we don't show the multiplication sign. Remember to put the number first.	**a** $5 \times Y$ or $5Y$
b An expression with 2 less means 'subtract 2'.	**b** $Y - 2$
c An expression with 8 more means 'add 8'.	**c** $Y + 8$
d A number divided by 4 means 'write as a quotient (fraction)'.	**d** $\frac{Y}{4}$ or $Y \div 4$
e The next consecutive number means 'add 1 to the number'.	**e** $Y + 1$

WORKED EXAMPLE 14

Write expressions for the following rules.
a Add a number to another number.
b Multiply 2 numbers.
c Add 2 numbers and multiply the answer by 6.
d Multiply a number by 4, then subtract 8 from that answer.

THINK	WRITE
1 In each case choose any pronumeral to represent an unknown number.	

2 In each case think about which operations are being used and the order in which they occur.

a Add two pronumerals.	**a** $B + G$ or $G + B$
b Multiply two pronumerals but don't write a multiplication sign (×).	**b** xy
c Add two pronumerals and place the expression in brackets. Multiply by 6. (Remember to write the numbers first.)	**c** $6(t + f)$
d First multiply a pronumeral by 4, writing the number first ($4b$). Then subtract 8.	**d** $4b - 8$

WORKED EXAMPLE 15

Write equations for the following rules.
a 15 is added to a number and the result is 21.
b The quotient of a number and 7 is equal to 5.

THINK	WRITE
a 1 Choose a pronumeral to represent the number.	**a** Let x = the number
2 Add 15 to the pronumeral then place an equals sign followed by 21.	$x + 15 = 21$
b 1 Choose a pronumeral to represent the number.	**b** Let z = the number
2 Divide the pronumeral by 7 (that is, write it as a fraction) then place an equals sign followed by 5.	$\frac{z}{7} = 5$

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Exercise 8.5 Terms, expressions and equations

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1, 2, 3, 4, 5a–h, 6, 7, 8, 9, 10, 14, 16	Questions: 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 16	Questions: 5, 6, 7, 8, 9, 10, 12, 13, 15, 16, 17

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Fluency

1. WE11a, b Write an expression for the total number of coins in each of the following, using c to represent the number of coins in a moneybox.

a.

b.

c.

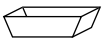
d.

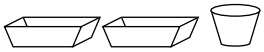
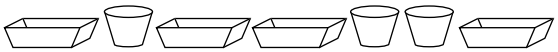
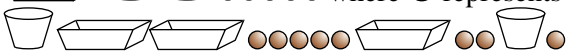
e.

2. Christie and Jane both had two full moneyboxes and seventeen coins each. They combined their money and spent every cent on a day out in town.

$2 \times ($ $+$ $)$

a. Write an expression to represent the number of coins they had in total.
b. Christie opened the first moneybox and counted 52 twenty-cent pieces inside. Use your expression to calculate how many coins the girls had in total, assuming that all the moneyboxes held the same number of coins.
c. What is the total amount that the girls spent on their day out?

3. Luke has three macadamia nut trees in his backyard. He saves takeaway containers to store the nuts in. He has two types of containers, rectangular  and round .
Using m to represent the number of nuts in a rectangular container and n to represent the number of nuts in a round container, write expressions for the following.

a.
b. 
c. 
d. where represents one nut.
e. 

4. Luke found that a rectangular container holds 17 nuts and a round container holds 12 nuts. Calculate how many nuts Luke would have for each part in question 3 by substituting the appropriate values into each expression.

5. WE12 Write an expression for each of the following.

a. The sum of B and 2
b. 3 less than T
c. 6 added to D
d. 5 taken away from K
e. The sum of G, N and W
f. D increased by H
g. N increased by N
h. H added to C
i. G subtracted from 12
j. The product of D and 4
k. 6 added to H
l. The difference between Z and G
m. B multiplied by F
n. Y added to the product of 3 and M

6. WE13 If A, B and C represent any 3 numbers, write an expression for each of the following.

a. The sum of all 3 numbers
b. The difference between A and C
c. The product of A and B
d. The product of all 3 numbers
e. The quotient of B and C (that is, B divided by C)
f. The sum of A and C, divided by B
g. 3 more than A

7. WE14 Write expressions for the following rules.

a. The number of students left in the class if X students leave for the canteen out of a total group of T students
b. The amount of money earned by selling B cakes, where each cake is sold for $4.00
c. The total number of sweets if there are G bags of sweets with 45 sweets in each bag
d. The cost of one concert ticket if 5 tickets cost $\$T$

e. The number of students in a class that contains R boys and M girls
f. The number of insects if there are Y legs altogether
g. The cost of M movie tickets if each ticket costs $11.00
h. The total amount of fruit sold if A apples and H bananas are sold
i. The number of cards left in a pack of 52 cards if j cards are removed
j. The total number of floor tiles if there are B boxes of tiles with 12 tiles in each box
k. The average of B and G
l. The total runs scored by 3 batsmen if they have scores of A, H and K respectively
m. The total number of legs if there are R chairs

8. WE15 Write equations for the following rules.
a. If 8 is added to a number the result is 11.
b. If 11 is subtracted from a number the result is 63.
c. The product of a number and 8 is equal to 128.
d. The quotient of a number and 7 is equal to 5.
e. The total cost of A apples and H bananas is $6.49.
f. When Q cards are removed from a pack of 52 cards only 19 remain.
g. The total cost of M concert tickets at $99 each was $792.
h. The total number of legs if there are R chairs and D tables is 164.

Understanding

9. Answer True or False for each of the statements below.
a. $3x$ is a term.
b. $3mn$ is a term.
c. $g = 23 - t$ is an expression.
d. $g = 5t - 6$ is an equation.
e. $rt = r \times t$
f. $5 + A = A + 5$
g. $3d + 5$ is an expression.
h. $7 - B = B - 7$
i. $3x = 9$ is an expression.
j. The expression $g + 2t$ has two terms.
k. $2f + 4 = 4 + 2f$
l. $5a - 7t + h$ is an equation.

10. MC In each of the following, the letter N has been used to represent a number. For each expression written in words, choose the answer that you think matches:
a. five times the number
A. $N + 5$ **B.** $5N$ **C.** $N - 5$ **D.** $N \div 5$ **E.** $5 - N$
b. the sum of the number and 52
A. $52 - N$ **B.** $52N$ **C.** $N + 52$ **D.** $N - 52$ **E.** $N \div 52$
c. the next consecutive number
A. O **B.** $2N$ **C.** $N - 1$ **D.** $N + 1$ **E.** $12N$
d. the number multiplied by another number
A. $N + T$ **B.** $N \div S$ **C.** $N + 4$ **D.** NS **E.** $N - 5$
e. half the number
A. $2N$ **B.** $N \div 2$ **C.** $N + \frac{1}{2}$ **D.** $N - 2$ **E.** $2 + N$
f. four more than the number
A. $N + 4$ **B.** $4N$ **C.** $N - 4$ **D.** $4 - N$ **E.** $N \div 4$
g. the number plus the product of another number and 2
A. $N + 2D$ **B.** $D(N + 2)$ **C.** $2N + 2B$ **D.** $2N + D$ **E.** $N + T$

11. If x is a whole number, write down the value of:
a. the number 5 more than x
b. the number 8 less than x

c. the number twice as large as x
d. the number one quarter of x
e. the next largest number after x
f. the number of cents in x dollars
g. the number of metres in x centimetres
h. the number of days in x weeks
i. the (mean) average of x and 20
j. how much it costs to rent a tennis court for x hours at \$10 per hour
k. the average of 5, 18 and k.

12. MC **a.** John is x years old. In 5 years his age will be:
A. $5x$ **B.** 17 **C.** $x - 5$ **D.** $x + 5$ **E.** 10

b. Chris is 2 years older than Jenny. If Jenny is x years old then Chris's age is:
A. $x - 2$ **B.** $2x$ **C.** $x + 2$ **D.** 2 **E.** $\frac{x}{2}$

c. Duy earns \$5 per week more than Ben. If Duy earns \$$x$, then Ben earns:
A. $x + 5$ **B.** $5x$ **C.** $x - 5$ **D.** $5 + x$ **E.** $5 - x$

d. Assume x is an even number. The next even number is:
A. $x + 1$ **B.** $2x + 2$ **C.** 4 **D.** $2x$ **E.** $x + 2$

e. Assume x is an odd number. The next odd number is:
A. $x + 1$ **B.** $x + 2$ **C.** 3 **D.** $2x + 1$ **E.** $2x + 2$

f. Gian is half Carla's age. If Gian is x years old, then Carla's age is:
A. $\frac{x}{2}$ **B.** $x + \frac{1}{2}$ **C.** $2x$ **D.** $\frac{1}{2}x$ **E.** $x - \frac{1}{2}$

13. a. Ali earns \$7 more per week than Halit. If Halit earns \$$x$, find their total earnings per week.
b. Sasha is twice as old as Kapila. If Kapila is x years old, find their total age.
c. Frank has had 3 more birthdays than James. If James is $2x$ years old, find their total age.
d. If x is the smaller of 3 consecutive whole numbers, find the sum of the 3 numbers.
e. If x is the smallest of 3 consecutive odd numbers, find the sum of the 3 numbers.
f. If x is the smallest of 3 consecutive even numbers, find the sum of the 3 numbers.

Reasoning

14. a. Write a rule to describe how you would calculate the total cost of two hamburgers and one drink.
b. Let h represent the cost of a hamburger. If the drink costs \$3.75 and the total cost is \$7.25, write an equation which links all this information.

15. To hire a windsurfer, there is a \$50 deposit, and you are charged \$12 per hour.
a. How much will it cost to hire a windsurfer for:
i. 1 hour **ii.** 2 hours **iii.** 3 hours?
b. Write a rule that could be used to calculate the cost of hiring a windsurfer for n hours.
c. Use the rule to calculate the cost of hiring a windsurfer for 8 hours.
d. You have \$100 to spend. Write an equation to help you work out the number of hours for which you could hire a windsurfer.

Problem solving

16. Lisa is 6 years older than Susan, Jessie is 5 years older than Lisa and the total of their ages is 41. How old is Susan?

17. Ten couples go out for dinner. If everyone shakes hands once with everyone else except their spouse, how many handshakes are there?

Reflection

What strategies can you use to check that your formula is correct?

CHALLENGE 8.2

Dad is three times as old as his only son. In 10 years' time, he will be only twice as old. How old are Dad and his son now?

8.6 Simplifying and the distributive law

8.6.1 Simplifying expressions containing like terms

- Terms containing exactly the same pronumerals, regardless of order, are called **like terms**.
- Examples of like and non-like terms are:

Like terms	Non-like terms
$5e$ and $6e$	$5x$ and $5y$
x and $13x$	$12xy$ and $21x$
$2bc$ and $4cb$	$15abc$ and $4bcd$
$3g^2$ and $45g^2$	$16g$ and $45g^2$

- When simplifying expressions, we can add or subtract like terms only.
- Expressions which do not have like terms cannot be added or subtracted.
- Some examples of simplifying are:

$6b + 4b = 10b$
$12y + 3y - 5y = 10y$
$7t - 5t = 2t$
$5xy + 8xy = 13xy$
$4t - 3t = t$

- Some expressions that cannot be simplified are:

$5g + 5c$ $\qquad$ $4t - 4$
$3xy + 4x$ $\qquad$ $14abc - 6bc$
$45b - 43$ $\qquad$ $4g - 4n$.

WORKED EXAMPLE 16

Where possible, simplify the following expressions by adding or subtracting like terms.

a $4g + 6g$ **b** $11ab - ab$ **c** $6ad + 5da$
d $4t + 7t - 5$ **e** $8x + 3y$

THINK	WRITE
a The pronumerals are the same. The terms can be added.	**a** $4g + 6g = 10g$
b The pronumerals are the same. The terms can be subtracted. Note that ab is the same as $1ab$.	**b** $11ab - ab = 10ab$
c Although the order of the pronumerals is different, the terms are like terms and can be added *Note:* Pronumerals may be rearranged in alphabetical order.	**c** $6ad + 5da = 6ad + 5ad$ $= 11ad$

d The pronumerals are the same and the first two terms can be added. Do not add the other term.

d $4t + 7t - 5 = 11t - 5$

e The pronumerals are not the same. These are not like terms and therefore cannot be simplified.

e $8x + 3y$ cannot be simplified.

WORKED EXAMPLE 17

Simplify the following expressions first, then find the value of the expression by substituting a = 4.

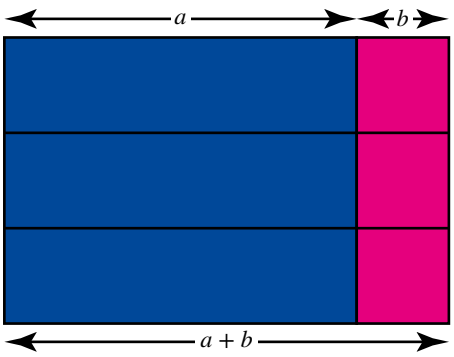
a $5a + 2a$ **b** $7a - a + 5$

THINK	WRITE
a 1 Add the like terms.	**a** $5a + 2a = 7a$
2 Substitute 4 for a in the simplified expression.	If $a = 4$
3 Evaluate.	$7a = 7 \times a$ $= 7 \times 4$ $= 28$
b 1 Subtract the like terms.	**b** $7a - a + 5 = 6a + 5$
2 Substitute 4 for a in the simplified expression.	If $a = 4$
3 Evaluate.	$6a + 5 = 6 \times a + 5$ $= 6 \times 4 + 5$ $= 24 + 5$ $= 29$

8.6.2 The distributive law

- As can be seen in the diagram, $3(a + b) = 3a + 3b$.
- We can think of $3(a + b) = (a + b) + (a + b) + (a + b)$.
- Collecting like terms, $3(a + b) = a + a + a + b + b + b = 3a + 3b$.
- An expression containing a bracket multiplied by a number can be written in expanded or factorised form.

$$\overbrace{3(a + b)}^{\text{factorised form}} = \overbrace{3a + b}^{\text{expanded form}}$$



- The **distributive law** is the name given to this process because the number out in front is distributed to each of the terms in the brackets.

$$\overbrace{a(b + c)} = ab + ac$$

- The distributive law can be used when the terms inside the brackets are either added or subtracted.

$$a(b - c) = ab - ac$$

- The distributive law is not used when the terms inside the brackets are multiplied or divided. You can see this with numbers $2(4 \times 5) = 2 \times 4 \times 5$ not $(2 \times 4) \times (2 \times 5)$.
- When simplifying expressions, you can leave the result in either factorised form or expanded form, but not a combination of both.

WORKED EXAMPLE 18

Write the following expressions in expanded form, then simplify.
a $4(2a + d)$ **b** $a(3x + 2y)$ **c** $3(k + 4a - m)$

THINK	WRITE
a 1 The number 4 is distributed to both the $2a$ and the d.	**a** $4(2a + d) = 4 \times 2a + 4 \times d$
2 Simplify the terms.	$= 8a + 4d$
b 1 The number represented by the pronumeral a is distributed to both the $3x$ and the $2y$.	**b** $a(3x + 2y) = a \times 3x + a \times 2y$
2 Simplify the terms.	$= 3ax + 2ay$
c 1 The number 3 is distributed to k, $4a$ and m.	**c** $3(k + 4a - m) = 3 \times k + 3 \times 4a - 3 \times m$
2 Simplify the terms.	$= 3k + 12a - 3m$

WORKED EXAMPLE 19

Use the distributive law to expand and simplify the following.
a $4(2s + 3k) - 2k$ **b** $a(3 - c) + a$

THINK	WRITE
a 1 Use the distributive law to expand $4(2s + 3k)$.	**a** $4(2s + 3k) - 2k = 4 \times 2s + 4 \times 3k - 2k$
2 Simplify the terms.	$= 8s + 12k - 2k$
3 Collect like terms.	$= 8s + 10k$
b 1 Use the distributive law to expand $a(3 - c)$.	**b** $a(3 - c) + a = a \times 3 - a \times c + a$
2 Simplify the terms.	$= 3a - ac + a$
3 Collect like terms.	$= 4a - ac$

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Complete this digital doc: WorkSHEET: Algebra II (doc-1820)

Exercise 8.6 Simplifying and the distributive law

assessON

Individual pathways

PRACTISE
Questions:
1a–h, 2a, d, g, j, m, p, s, v, 3, 4a–f, 5, 6, 7a–h, 8, 9a–d, 10, 11, 14

CONSOLIDATE
Questions:
1d–n, 2b, e, h, k, n, q, t, w, 4d–g, 5, 6, 7e–l, 8, 9d–g, 10, 11, 12, 14

MASTER
Questions:
1g–o, 2c, f, i, l, o, r, u, x, 4k, l, 5, 6, 7k–r, 8, 9g–j, 10, 11, 12, 13, 14

Individual pathway interactivity: int-4353

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Fluency

1. WE16 Where possible, simplify the following expressions by adding or subtracting like terms.

a. $3a + 2a$ **b.** $9y + 5y$ **c.** $3c + 12c$
d. $4u - 2u$ **e.** $7e + 13e$ **f.** $7t - 2t$
g. $12ab + 2ab$ **h.** $8fg - 2fg$ **i.** $4e - e$
j. $6t + t$ **k.** $f + 4f$ **l.** $6y - 6y$
m. $4x + 14x$ **n.** $3m + 16m - 7m$ **o.** $6a + 4a - 2a$

2. Simplify the following expressions if possible.

a. $24ab + ab - 7$ **b.** $5y + y - 3y$ **c.** $5t + 5s$
d. $18i + 12i - 2$ **e.** $4t + 8t - 3 + 2t$ **f.** $7r + 2r + 5r - r$
g. $2x + 2y$ **h.** $7y + 6$ **i.** $18f - 2f + 5$
j. $4 + 3g - g$ **k.** $6t - 5t$ **l.** $18bg - 18bg$
m. $13mno - 11mno$ **n.** $11pq + 3qp$ **o.** $6pr + 2 + 5rp$
p. $9e - 9t - 1$ **q.** $7t + 4t - 5$ **r.** $32t - 31t$
s. $11aw - 3aw$ **t.** $7xy - 7x$ **u.** $5t + 6t - 8$
v. $5t - t + 3$ **w.** $3g + 7g - 2$ **x.** $5r + 17r + 4 + 2$

3. Simplify each of the following.

a. $3t - 3t$ **b.** $18r - 18r$ **c.** $12ab - 12ab$
d. $5x - 5x + 8$ **e.** $6t + 7 - 6t$ **f.** $9g - 9g + 2$
g. $13xyz - 13xyz$ **h.** $5x + 7 - 5x$ **i.** $5y + 2y - y$

4. WE17 Simplify the following expressions first, then find the value of the expression by substituting $a = 7$.

a. $3a + 2a$ **b.** $7a + 2a$ **c.** $6a - 2a$
d. $9a + a$ **e.** $13a + 2a - 5a$ **f.** $3a + 7a$
g. $17 + 5a + 3a$ **h.** $6a - a + 2$ **i.** $a + a$
j. $7a - 6a$ **k.** $7a - 7a$ **l.** $12a + 5a - 16$

5. WE18 Use the distributive law to expand the following expressions.

a. $3(a + 2b)$ **b.** $5(x - 4z)$ **c.** $10(2g + 3h)$
d. $b(2c - 3a)$ **e.** $2c(8d - q)$ **f.** $3b(3b - 2g)$
g. $x(9a + 7b)$ **h.** $7k(t + 3m)$

6. WE19 Write the following expressions in expanded form, then simplify if possible.

a. $5(a + 2b) - 3a$ **b.** $4(n - 2c) + n$ **c.** $9(b + 2c) - 9b$
d. $x(a - b) + x$ **e.** $y(3 + 2z) + 2y$ **f.** $3x(d - 3) + 4x$
g. $6u(2f - 1) + 20u$ **h.** $7n(10 + 3h) - 50n$

Understanding

7. Answer True or False to each of the following statements.

a. $4t$ and $6t$ are like terms. **b.** $3x$ and x are like terms.
c. $5g$ and $5t$ are like terms. **d.** $7t$ and $5i$ are like terms.
e. $5a$ and $6a$ are like terms. **f.** $4a$ and $4ab$ are like terms.
g. $9fg$ and $4gf$ are like terms **h.** $6gh$ and $7gk$ are like terms.
i. $6r$ and $5r$ are like terms. **j.** yz and $45zy$ are like terms.
k. $3acd$ and $6cda$ are like terms. **l.** $5g$ and $5fg$ are like terms.
m. $8gefh$ and $3efgh$ are like terms. **n.** $6ab$ and $3ba$ are like terms.
o. $8xy$ and $5xy$ are like terms. **p.** $12ep$ and $4pe$ are like terms.
q. $7eg$ and $7g$ are like terms. **r.** $7y$ and $18yz$ are like terms.

8. MC **a.** Which one of the following is a like term for $7t$?

A. $7g$ **B.** $5tu$ **C.** 2 **D.** $6f$ **E.** $5t$

b. Which one of the following is a like term for $3a$?

A. $5ab$ **B.** $6a$ **C.** $3w$ **D.** $a + 2$ **E.** $3 + a$

c. Which one of the following is a like term for $5ab$?

A. $6abe$ **B.** $4b$ **C.** $2ba$ **D.** 5 **E.** $5abe$

d. Which one of the following is a like term for $7bgt$?

A. $7t$ **B.** $4gt$ **C.** $2gb$ **D.** btg **E.** $b + g + t$

e. Which one of the following is a like term for $2fgk$?

A. $fg + k$ **B.** $5fg$ **C.** $2fgk$ **D.** $12f$ **E.** $12p$

f. Which one of the following is a like term for $20m$?

A. $11m$ **B.** $20mn$ **C.** $20f$ **D.** $m + 2$ **E.** 20

g. Which one of the following is a like term for $9xyz$?

A. $9x$ **B.** $2yz$ **C.** $8xz$ **D.** xzy **E.** $9z$

9. Answer True or False to each of the following statements.

a. The equation $y = 4x + 3x$ can be simplified to $y = 7x$.

b. The equation $k = 8y + 4y$ can be simplified to $k = 10y$.

c. The equation $y = 4x + 3$ can be simplified to $y = 7x$.

d. The equation $b = 3a - a$ can be simplified to $b = 2a$.

e. The equation $k = 7a + 4d$ can be simplified to $k = 11ad$.

f. The equation $y = 5x - 3x$ can be simplified to $y = 2x$.

g. The equation $m = 7 + 2x$ can be simplified to $m = 9x$.

h. The equation $p = 3x - 2x$ can be simplified to $p = x$.

i. The equation $t = 3h + 12h + 7$ can be simplified to $t = 15h + 7$.

j. The equation $y = 16g + 6g - 7g$ can be simplified to $y = 15g$.

10. Expand each of the following and then simplify.

a. $3(x - y) + 5(y + 2x)$

b. $5(a - 2y) + a(7 + 6y)$

c. $6c(g + 3d) + g(3c + d)$

d. $y(6d - r) + 2y(3d + 2r)$

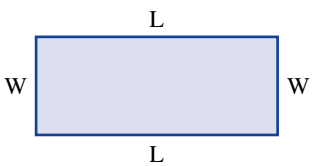
Reasoning

11. a. There are x chocolates in each box. Ahmed buys 7 boxes while Kevin buys 3 boxes. How many chocolates are there altogether?

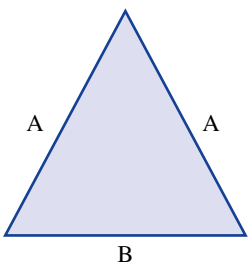
b. There are d mangoes in a kilogram. Nora buys 4 kg but they are so good she buys 2 kg more. How many mangoes did Nora buy altogether?

12. Find the simplest expression for the perimeter of the following shapes:

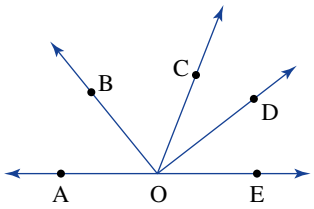
a.



b.



Problem solving



13. Given that OD bisects COE, that $\angle COD = 4x + 16$, and that $\angle DOE = 6x + 4$, explain if there is enough information for you to determine the angles for COD and DOE. If so what are the measure for each?

14. Mrs Makeit sells stuffed teddy bears to a stall holder at the Queen Victoria Market for $10 more than they cost her to make. The stall holder sells the teddy bears to customers for twice what he pays for them.

a. Complete this rule: Price = (Mrs Makeit's costs + __) × ___.

b. Write the rule using c to stand for the price that customers pay in dollars, and n to stand for Mrs Makeit's cost in dollars

c. If a teddy bear costs Mrs Makeit $20 to make, how much will a customer at the market stall pay for it?

Reflection

What strategies will you use to identify like terms?

8.7 The Associative Law

8.7.1 The Associative Law

- The **associative law for addition** is: $(a + b) + c = a + (b + c) = a + b + c$. Using numbers we can see that $(9 + 3) + 7 = 12 + 7 = 19$ and $9 + (3 + 7) = 9 + 10 = 19$.
- The **associative law for multiplication** is: $(a \times b) \times c = a \times (b \times c) = a \times b \times c$. Using numbers we can see that $(2 \times 3) \times 5 = 6 \times 5 = 30$ and $2 \times (3 \times 5) = 2 \times 15 = 30$.
- The associative law does not hold for subtraction and division. Using numbers for subtraction, $(8 - 3) - 2 = 5 - 2 = 3$, but $8 - (3 - 2) = 8 - 1 = 7$. Using numbers for division, $(24 \div 6) \div 2 = 4 \div 2 = 2$, but $24 \div (6 \div 2) = 24 \div 3 = 8$.

WORKED EXAMPLE 20

Use the associative law to complete the following.

a $2w + (8d + \square) = (\square + 8d) + 5h$

b $(x \times \square) \times 4g = \square(d \times 4g)$

THINK	WRITE
a The associative law says that the numbers can be regrouped. $2w$, $8d$ and $5h$ are numbers. $5h$ is missing from the left-hand side and $2w$ is missing from the right-hand side.	**a** $2w + (8d + \boxed{5h}) = (\boxed{2w} + 8d) + 5h$
b The associative law says that the numbers can be regrouped. d is missing from the left-hand side and x is missing from the right-hand side.	**b** $(x \times \boxed{d}) \times 4g = \boxed{x} \times (d \times 4g)$

- The **commutative law for addition** is: $a + b = b + a$. Using numbers we can see that $5 + 3 = 8$ and $3 + 5 = 8$.
- The **commutative law for multiplication** is: $a \times b = b \times a$. Using numbers we can see that $2 \times 3 = 6$ and $3 \times 2 = 6$.
- The commutative law does not hold for subtraction and division. Using numbers, we can see that $5 - 3 \neq 3 - 5$ and $6 \div 2 \neq 2 \div 6$.

WORKED EXAMPLE 21

Use the associative and commutative laws to complete the following.

a $4r + (6t + \square)\ 5g + (\square + \square)$ **b** $a \times (3t \times 2w) = (2w \times \square)\ \square a$

THINK	WRITE
a The associative and commutative laws say that the numbers can be regrouped and rearranged. $5g$ is missing from the left-hand side. Both $4r$ and $6t$ are missing from the right-hand side.	**a** $4r + (6t + \boxed{5g}) = 5g + (\boxed{4r} + \boxed{6t})$
b The associative and commutative laws say that the numbers can be regrouped and rearranged. $3t$ and a multiplication sign are missing from the right-hand side.	**b** $a \times (3t \times 2w) = (2w \times \boxed{3t})\ \boxed{\times} a$

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Try out this interactivity: Associative and commutative laws (int-2349)

Exercise 8.7 The associative law

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1, 2, 3, 4, 5, 8	Questions: 1, 2, 3, 4, 5, 8	Questions: 1, 2, 3, 4, 5, 6, 7

Individual pathway interactivity: int-4354

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Fluency

1. WE20 Use the associative law to complete the following.

a. $w + (r\square 6y) = (\square + \square) + 6y$ **b.** $6t \times (4r \times \square) = (6\square \times 4r)\square 3s$

c. $(9y + \square) + 3w = \square + (2r + 3w)$ **d.** $(z + 2p) + \square = z\square(\square + 6t)$

2. WE21 Use the associative and commutative laws to complete the following.

a. $6t + (3w + \square) = \square + (6t + 7v)$ **b.** $s \times (9r \times \square) = 2c \times (\square \times \square)$

c. $(3c + w) + \square = d + (\square + \square)$ **d.** $(g \times \square) \times 2y = 2y \times (3b \times \square)$

Understanding

3. State whether the following are True or False. (*Hint:* If you're not sure, try substituting values for the pronumerals to help you decide.)

a. $2s + (3w + 5z) = (2s + 3w) + 5z$ **b.** $x \times (d + y) = (x \times d) + y$

c. $g(jk) = (gj)k$ **d.** $4 \div (a \div c) = (4 \div a) \div c$

e. $3g + (k \div m) = (3g + k) \div m$ **f.** $4t - (p + 2b) = (4t - p) + 2b$

g. $3r + (a + 4c) = a + (3r + 4c)$ **h.** $a + (b \times c) = (a \times c) + b$

4. Find the values of the following expressions and comment on the results if $a = 3$, $b = 8$, $c = 2$.

a. **i.** $3a + (2b + 4c)$ **ii.** $(3a + 2b) + 4c$

b. **i.** $9a - (2b - 5c)$ **ii.** $(9a - 2b) - 5c$

c. **i.** $a \times (b \times c)$ **ii.** $(a \times b) \times c$

d. **i.** $4a \div (3b \div 2c)$ **ii.** $(4a \div 3b) \div 2c$

Reasoning

5. Janet and Judy want to buy a set of Blu-rays. Janet has $47 less than the purchase price, and Judy has $2 less. If they pool their money, they still do not have enough to buy the Blu-rays. If the set costs a whole number of dollars, what is its price?

6. The associative law refers to the order in which three numbers may be added, subtracted, multiplied or divided, taking two at a time. It always holds true for addition and multiplication. So:
$12 + (6 + 2) = (12 + 6) + 2$ and $12 \times (6 \times 2) = (12 \times 6) \times 2$.
Generally it does not hold true for subtraction and division. So:
$a - (b - c) \neq (a - b) - c$ and
$a \div (b \div c) \neq (a \div b) \div c$.
Investigate to find values for a, b and c that would make the associative law true for subtraction and division.

Problem solving

7. Andrew thinks of a number, adds 9 and multiplies the result by 3.
Andrea thinks of a number, multiplies it by 3 and adds 9 to the result.
Both Andrew and Andrea get an answer of 60.
Did Andrew and Andrea begin by selecting the same number? Show working out to support your answer.

8. Use the numbers 1, 5, 6 and 7 with any arithmetic operations to result in the number 21.

Reflection

Does the associative law hold when you are adding fractions? How do you know?

8.8 Review

8.8.1 Review questions

Fluency

1. Copy and complete the tables below. For each table, use the rule to work out the correct output numbers.

a. Multiply each input number by 4.

Input	1	3	4	7	13
Output					

b. Add 7 to each input number.

Input	1	4	6	7	15
Output					

c. Multiply each input number by 2, then subtract 3.

Input	3	4	7	10	13
Output					

d. Divide each input number by 4.

Input	0	4	8	16	20
Output					

2. Copy and complete the tables below. Use the rule given in each case to work out the missing input and output numbers.

a. Subtract 7 from each input number.

Input	15	16	33		
Output				3	7

b. Multiply each input number by 3, then add 7.

Input	1	4	15		
Output				22	106

c. Add 5 to each input number, then multiply by 2.

Input	1	3	6		
Output				18	96

d. Multiply each input number by 2, then subtract 5.

Input	8	11	15		
Output				21	71

3. MC Which formula has the algebra machine used in each of the following tables?

a.

a	1	4	5	6	23
g	11	14	15	16	33

A. $g = 5a + 6$ **B.** $g = a - 10$ **C.** $g = a + 10$ **D.** $g = 10a$ **E.** $g = 5a - 10$

b.

m	1	2	3	4	11
t	2	5	8	11	32

A. $t = 3m$ **B.** $t = 3m - 1$ **C.** $t = m + 5$ **D.** $t = 3m + 1$ **E.** $t = 3m - 1$

4. Look carefully at the pattern in each of the following tables. Then complete the formula for each table by inserting a number into the gap.

a.

x	1	4	7	8	23
y	5	32	59	68	203

$y = 9x - ____$

b.

p	0	3	5	6	12
q	6	15	21	24	42

$q = 3p + ____$

5. Use the pronumerals given to write a formula for each table.

a.

x	0	2	3	4	7
y	6	8	9	10	13

b.

c	5	6	8	12	13
d	0	1	3	7	8

c.

g	3	4	6	9	23
h	17	22	32	47	117

d.

m	1	4	5	6	11
n	3	15	19	23	43

6. Complete the tables below by substituting each of the input numbers (x) into the rule.

a. $y = 9x - ____$

x	1	3	4	7	105
y					

b. $d = 3x$

x	1	2	6	7	12
d					

c. $h = 6 - x$

x	0	2	4	5	6
h					

d. $n = 11x + 3$

x	0	1	3	4	7
n					

7. Otto works in a warehouse, packing boxes of CDs for distribution to music stores. An order from Sanity's city store fills 35 boxes with 18 CDs left over.

a. Write an expression to represent the Sanity order, using the pronumeral x to represent a full box of CDs.

b. If each full box contains 30 CDs, use your expression to calculate how many CDs the store ordered.

8. Write expressions for the following:

a. the difference between M and C

b. money earned by selling B cakes for \$3 each

c. the product of X and Y

d. 15 more than G

e. 1 more than D

f. the cost of 12 bananas at H cents each

g. T multiplied by 5.

9. Simplify the following expressions by adding or subtracting like terms.

a. $3g + 4g$	**b.** $8y - 2y$	**c.** $4h + 5h$	**d.** $7\text{ag} - 2\text{ag}$
e. $6\text{gy} - 3\text{yg}$	**f.** $8r - 8r$	**g.** $6y - 2y + y$	**h.** $4t + 6 + 3t$
i. $12\text{gh} + 6\text{hg}$	**j.** $8t - 2m + 3t$	**k.** $3m + m$	**l.** $7g + 8g + 8 + 4$
m. $7h + 4t - 3h$	**n.** $2b + 7c + 8b$	**o.** $11\text{axy} - 3\text{axy}$	

10. Simplify the following expressions first, then find the value of the expression if $x = 5$.

a. $7x + 3x$	**b.** $2x + 3x - 4$	**c.** $11x + 12x$
d. $x + 2x$	**e.** $4x - x$	**f.** $3x - 2x + 16$
g. $21x - 13x + 7$	**h.** $11 + 2x + 5x$	**i.** $7x - 4x + 3x$

11. Use the distributive law to expand the following.

a. $7(m - 3k)$	**b.** $w(g + 9a)$	**c.** $2y(8h - 7)$
d. $10m(7r - 2p)$	**e.** $3g(2a + 2c)$	**f.** $4j(6x - 3y)$

12. Expand the following and simplify if possible.

a. $5(t - 2s) - 3t$	**b.** $4(a + 3g) - 2g$	**c.** $d(3f + 9) - 2d$
d. $4z(2 - a) + 7z$	**e.** $3p(f - 2q) + 7p$	**f.** $10b(a + 6) + 5b$

13. Use the associative law to complete the following.

a. $a + (3b + \square) = (\square + \square) + 5c$

b. $\frac{1}{a} + (2b + \square) = (\square + 2b) \square 7g$

c. $\left(\frac{a}{b} \times c\right) \times \square = \frac{a}{\square} \times (\square \times f)$

d. $(d \times 3y) \times z = \square \times (\square \times \square)$

14. Use the associative and commutative laws to complete the following.

a. $g + (\square + 2k) = (2k + \square) + 3m$

b. $(w + \square) + 4r = \square + (5g + w)$

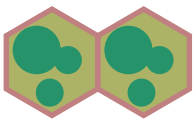
c. $\frac{2}{3} \times (2d \times \square) = (r \times 2d) \times \square$

d. $(3z \times 5b) \times \square = \square \times (6m \times 5b)$

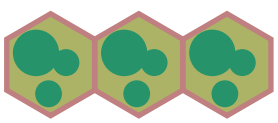
Problem solving

15. Johanna (a landscape architect) is installing hexagonal flowerbeds in a park. The flowerbeds, which are surrounded by wooden sleepers, are arranged in rows as shown below:

1 bed, 6 sleepers required



2 beds, 11 sleepers required



3 beds, 16 sleepers required

How many wooden sleepers will Johanna need to install a row of 12 flowerbeds? Follow the steps below:

a. Draw a table with columns headed B (the number of flowerbeds), and S (the number of sleepers needed). Write in the numbers from the diagrams above.

b. Complete a diagram showing the number of sleepers needed for a row of 4 flowerbeds, and a row of 5 flowerbeds. Add these values to your table.

c. Look at the table to determine the formula which connects S and B. Write the formula in the form $S = \ldots$

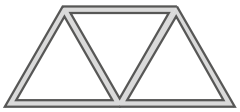
d. Substitute $B = 12$ into the formula to find the number of sleepers needed for the row of flowerbeds.

16. The metal ceiling rafters in a school classroom consist of a series of triangles connected end to end as shown. The number of triangle side lengths required in each rafter depends on the total number of triangles. However, adjacent triangles share a common side length.

How many side lengths will be needed to build a single rafter if each rafter contains 25 triangles? Follow the steps below:

1 triangle, 3 side lengths

2 triangles, 5 side lengths



3 triangles, 7 side lengths

a. Draw up a table with columns headed T (the number of triangles), and S (the number of side lengths needed). Write in the numbers from the diagrams above.

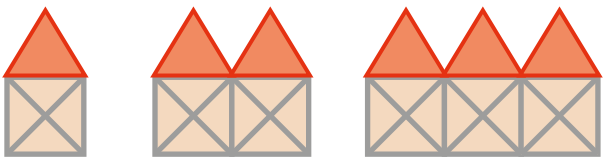
b. Complete a diagram showing the number of side lengths needed for a rafter containing 4 triangles. Add these values to your table.

c. Look at the table to determine the formula which connects S and T. Write the formula in the form $S = \ldots$

d. Substitute $T = 25$ into the formula to find the number of side lengths needed for each ceiling rafter.

17. The diagrams below show the number of struts planned for the back walls of the Surfside beach huts, which are to be built in rows.

Surfside Constructions has a contract to build a row of 34 beach huts at Golden Beach. Based on the plans above, how many struts will be required for the back walls of this row of beach huts? Follow the steps below to solve this problem.



1 hut, 6 struts 2 huts, 11 struts 3 huts, 16 struts

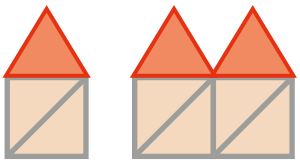
a. Draw a table with rows headed H (the number of huts), and S (the number of struts needed). Write in the numbers from the diagrams above.

b. Following the same pattern, complete a diagram showing the number of struts needed for a row of 4 huts and a row of 5 huts. Add these values to your table.

c. Look at the table to determine the formula that connects S and H. Write the formula in the form $S = \ldots$

d. Substitute $H = 34$ into the formula to find the number of metal struts needed for the row of huts.

The engineers at Surfside Constructions have advised builders that only one of the metal strut cross bracings is required. Only 5 struts are required for the back wall of each hut. The diagram on right is the amended plan for the huts' struts.



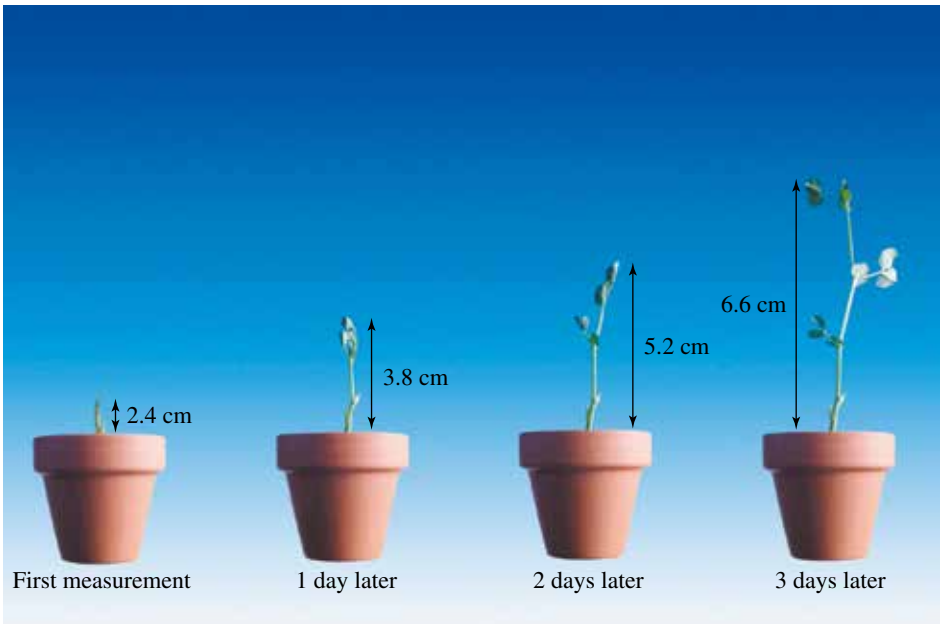
1 hut, 5 struts 2 huts, 9 struts

e. Draw a diagram showing the number of struts needed for a row of 3 huts.

f. Repeat steps **a** to **c** for a beach hut containing 5 struts as illustrated in the diagram above.

g. How many metal struts will now be required for the back wall of a row of 34 beach huts?

18. A seed you have planted in a pot is starting to grow. When you first notice the seedling, you measure it to be 2.4 centimetres high. Each day after this you continue to measure its height.



a. If the seedling continues to grow at the same rate, how tall will it be 4 days later?
b. How tall will it be 5 days later?
c. Write a formula connecting the height of the plant with the number of days after the plant height was first measured. Remember to identify what each pronumeral represents in your formula.
d. Check your formula by calculating the height of the plant after 7 days.
e. What is the height of the plant after 18 days?
f. What would be the height of the plant after 25 days?
g. Challenge: Use your formula to estimate when the plant would be approximately 32 centimetres high.
h. Discuss and give reasons why this formula may not be accurate to predict the plant height after a period of time.

19. It's Bianca's birthday today. She is m years old. She asks the ages of people at her party. Her cousin Paul is 8 years older than Bianca. His age today is $m + 8$ years old.
a. Write the ages of the following using algebra as for Paul's age.
i. Bianca's sister Kate is 5 years younger than Bianca.
ii. Bianca's aunt Theresa is three times Bianca's age.
b. If Bianca is 7 years old today, work out the ages of her relatives and write their names in order from youngest to eldest.
c. If Bianca is 13 years old today, work out the ages of her relatives and write their names in order from youngest to eldest.

20. Use the pronumeral x to make a general statement of the form $\square \times \square = 1$ from the following clues.
$4 \times \frac{1}{4} = 1,\ 30 \times \frac{1}{30} = 1,\ 1.2 \times \frac{1}{1.2} = 1$

21. You work at a restaurant. The manager asks you to push tables together until you have seats for 20 people to sit together. You have tables that seat 4 people. You notice that when you push two tables together you can seat 6 people. Do you see a pattern that will help you to work out how many tables will be needed to seat 20 people?

22. Mary has y books. Tom has 4 more books than Mary has. Cindy has 5 times as many books as Tom has. Write down an expression, in terms of y, for the number of books that Cindy has.

23. If we count by 7s, starting with 3, we get the numbers:
3, 10, 17, 24, 31, 38, …
What is the 50th number in this set?

24. A couple spends $\$x$ on developing eighty small wedding photos and $\$y$ for sixty large wedding photos. How much do they need to spend if they want an extra dozen photos of each size?

25. It costs $\$x$ for four adults' tickets. A child's ticket costs $3 less than an adult's ticket. On family night, all the tickets go for half price. Develop a formula in terms of x to calculate how much it will cost for a family of 2 adults and 3 children on family night.

26. You have four friends to whom you gave some money. To the first friend, you gave $\frac{1}{3}$ of your coins. You then gave $\frac{1}{2}$ of the coins that you had left to your second friend. Then you gave your third friend $\frac{1}{4}$ of the coins that you had left. Finally you gave your fourth friend $\frac{2}{3}$ of your remaining coins. At this point you had 7 coins remaining. How many coins did you begin with?

27. A bird's head is $\frac{1}{3}$ the length of its body. Its tail is as long as its head and body combined. If the total length of the bird is 24 cm, how long is its head?

28. Tom's father is 3 times as old as Tom, and his mother is $\frac{5}{6}$ his father's age. His sister is $\frac{3}{4}$ Tom's age. Tom's father is 6 years older than his mother. How old is Tom's sister?

29. Imagine that a new mathematical operation is being used. Its symbol is #. See the following equations.
$1 \# 1 = 2$ $3 \# 5 = 34$ $6 \# 9 = 117$ $10 \# 14 = 296$
Find the value of $15 \# 19$, and explain your reasoning.

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- **Try out this interactivity:** Crossword: Topic 8 (int-2598)
- **Try out this interactivity:** Sudoku: Topic 8 (int-3169)
- **Complete this digital doc:** Concept map: Topic 8 (doc-10733)

Language

It is important to learn and be able to use correct mathematical language in order to communicate effectively. Create a summary of the topic using the key terms below. You can present your summary in writing or using a concept map, a poster or technology.

algebra	**distributive law**	**input**	**sequence**
associative law	**equate**	**like terms**	**simplify**
coefficient	**equation**	**output**	**substitution**
commutative law	**expression**	**pronumeral**	**term**
constant	**formula**	**rule**	**variable**

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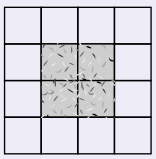
Investigation | Rich task

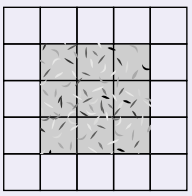
Landscaping

As a part of a town's beautification program, its council wants to create more garden beds in the parklands around the town. All garden beds are to be square, but may vary in size, and will be surrounded by 1-metre-square pavers as shown in the diagrams below.

Garden bed 1



Garden bed 2



Garden bed 3

- **The shaded areas represent the square garden beds.**
- **The unshaded squares represent the 1-metre-square pavers.**

1. Use the 3 diagrams of garden beds shown above to complete the following table.

	Garden bed 1	Garden bed 2	Garden bed 3
Side length of the garden bed (m)			
Area of the garden bed (m^2)			
Number of pavers used			
Total length around the outside of pavers (m)			

Let s represent the side length of the garden bed.

Let a represent the area of the garden bed.

Let *p* represent the number of pavers used.

Let *l* represent the total length around the outside of the pavers.

2. Examine the patterns formed in your table and write formulas linking:

- *a* and *s*
- *p* and *s*
- *l* and *s*
- *p* and *l*.

You have just been appointed the council's new landscape gardener. Your first task is to design a 10 metre × 10 metre square garden bed that is to be constructed in the same way as demonstrated on the previous page.

3. How many pavers will be needed for this design?

The council has decided to put a large, square garden bed on the grounds near the entrance to its offices. The area of ground measures 18 × 20 m and the council wants the largest possible square garden bed to be placed here, and for it to be surrounded by the 1-metre-square pavers.

4. As the landscape gardener, draw a design of the council's plan for this area of ground on a sheet of paper with clear labels showing the measurements involved. Indicate the number of pavers that will be required for the project.
5. Suppose the garden beds are rectangular in size, but are still surrounded by 1-metre-square pavers. On another sheet of paper, draw different rectangular garden beds and investigate the different patterns between the side lengths of your garden beds, their area and the number of pavers used. Report your findings.

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Complete this digital doc: Code puzzle: What was the 'Model T' in 1908? (doc-14325)

Answers

Topic 8 Algebra

Exercise 8.2 Using rules

1. a. Add 2 to each term to get the next term. 10, 12, 14

b. Add 5 to each term to get the next term. 23, 28, 33

c. Subtract 3 from each term to get the next term. 15, 12, 9

d. Multiply each term by 3 to get the next term. 81, 243, 729

e. Divide each term by 2 to get the next term. 8, 4, 2

f. The term number is squared. 25, 36, 49

2. a. i.

Number of squares	1	2	3	4
Number of matchsticks	4	7	10	13

ii. The number of matchsticks equals 3 times the number of squares plus 1. **iii.** 61

b. i.

Number of triangles	1	2	3	4
Number of matchsticks	3	5	7	9

ii. The number of matchsticks equals 2 times the number of triangles plus 1. **iii.** 41

c. i.

Number of houses	1	2	3	4
Number of matchsticks	6	11	16	21

ii. The number of matchsticks equals 5 times the number of squares plus 1. **iii.** 101

d. i.

Number of panels	1	2	3	4
Number of matchsticks	4	7	10	13

ii. The number of matchsticks equals 3 times the number of squares plus 1. **iii.** 61

3. a.

Input	4	5	6	10	14
Output	0	1	2	6	10

b.

Input	1	2	6	13	107
Output	13	14	18	25	119

c.

Input	1	10	51	60	144
Output	2	20	102	120	288

d.

Input	3	12	21	66	141
Output	1	4	7	22	47

4. a.

Input	2	3	4	10	17
Output	9	11	13	25	39

b.

Input	2	4	5	20	100
Output	12	28	36	156	796

c.

Input	2	3	5	12	43
Output	25	30	40	75	230

d.

Input	3	4	5	8	25
Output	0	11	22	55	242

e.

Input	0	1	5	8	10
Output	0	1	25	64	100

f.

Input	0	1	4	7	12
Output	4	5	20	53	148

5. a.

Input	1	3	27	2	53	191
Output	3	5	29	4	55	193

b.

Input	3	25	56	0	74	1753
Output	23	45	76	20	94	1773

c.

Input	7	15	96	9	17	109
Output	2	10	91	4	12	104

d.

Input	1	6	321	2	7	103
Output	4	24	1284	8	28	412

6. a.

Input	3	13	21	1	10	50
Output	8	28	44	4	22	102

b.

Input	2	5	16	3	5	20
Output	3	18	73	8	18	93

c.

Input	3	15	273	6	63	162
Output	1	5	91	2	54	21

d.

Input	1	5	17	2	7	12
Output	1	25	289	4	49	144

e.

Input	2	7	15	3	9	11
Output	34	134	294	54	174	214

f.

Input	7	19	31	23	51	63
Output	1	4	7	5	12	15

7. a. ■ = 2 □ + 1

Every line has an extra white square and two extra blue squares. So, for every white square you have to add two blue squares.

Line 0 has one square so this square has to be added to the pattern.

b.

Line 6

Line 7

8. a. The first inner layer has 3 hexagons.

The second layer is constructed using three hexagons for each original hexagon and hence has 9 hexagons.

The third layer is constructed using five hexagons for each original hexagon and hence has 15 hexagons.

b. 21 hexagons

Seven hexagons for each original hexagon.

9. a.

t	1	2	3	4	5	6	7	8
C	21	22	23	24	25	26	27	28

b. $C = t + 20$ **c.** \$1.40 **d.** 3 minutes

10. 325

11. 7, 3, 4, 6

12. A = 3, B = 5, C = 4, D = 1, E = 6, F = 2

Exercise 8.3 Finding a formula

1. a. $b = 4h$ **b.** $m = 4f$ **c.** $r = 5a$

d. $m = 4t$ **e.** $x = 5(k + 4)$ **f.** $k = 6w - 2$

g. $t = 4(20 - g)$ **h.** $b = 10a - 5$ **i.** $d = 6f + 7$

j. $h = 9(x + 5)$ **k.** $y = 8(3b + 6)$ **l.** $y = 8p - 6$

m. $g = 2(3r + 17)$ **n.** $j = 18(5h - 4)$

2. a. $g = f + 9$ **b.** $b = a - 3$ **c.** $t = 2k + 1$

d. $y = 7x - 5$ **e.** $g = 4a + 2$ **f.** $t = 9m + 11$

g. $w = 3 \times p + 2 (w = 3p + 2)$ **h.** $x = 5 \times t - 8 (x = 5t - 8)$ **i.** $p = 3m - 1$

j. $b = s \times 10 - 7 (b = 10s - 7)$

3. a. $b = a + 3$ **b.** $w = t + 5$ **c.** $a = m - 4$

d. $t = 2s + 3$ **e.** $g = 2f - 1$ **f.** $c = 3s + 1$

g. $a = 4d - 3$ **h.** $g = 5s - 4$ **i.** $e = 11f - 20$

j. $q = 20p + 4$ **k.** $t = 4b$ **l.** $p = 3u - 1$

4. C **5.** C **6.** D

7. E **8.** C **9.** D

10. A **11.** D **12.** C

13. Total biscuits $= 3n + 9$

14. a. $t = a^2 - 1$ **b.** $x = \frac{b + 1}{2}$ **c.** $p = 2w^2$

15.

Base	Height	Number of hits	Number of squares
6	4	5	12
3	4	7	12
3	5	8	15
3	6	3	6
4	8	3	8
b	h	$\frac{(b + h)}{\text{H.C.F.}(b, h)}$	$\frac{(b + h)}{\text{H.C.F.}(b, h)}$

16. 15 pins

Challenge 8.1

2, 6, 10, 14, 18, 22, 26, 30, 34, 38

Exercise 8.4 Substitution

1. a. i. $m = 2$ **ii.** $m = 3$ **iii.** $m = 0$ **iv.** $m = 100$

b. i. $m = 11$ **ii.** $m = 1$ **iii.** $m = 197$ **iv.** $m = 13$

c. i. $m = 19$ **ii.** $m = 7$ **iii.** $m = 67$ **iv.** $m = 247$

d. i. $m = 21$ **ii.** $m = 13$ **iii.** $m = 1$ **iv.** $m = 25$

2. a. i. $m = 4$ **ii.** $m = 2$ **iii.** $m = 26$ **iv.** $m = 152$

b. i. $m = 20$ **ii.** $m = 40$ **iii.** $m = 45$ **iv.** $m = 0$

c. i. $m = 40$ **ii.** $m = 36$ **iii.** $m = 0$ **iv.** $m = 4$

d. i. $m = 5$ **ii.** $m = 125$ **iii.** $m = 35$ **iv.** $m = 75$

e. i. $m = 7$ **ii.** $m = 1$ **iii.** $m = 15$ **iv.** $m = 63$

f. i. $m = 17$ **ii.** $m = 23$ **iii.** $m = 32$ **iv.** $m = 77$

g. i. $m = 9$ **ii.** $m = 3$ **iii.** $m = 303$ **iv.** $m = 216$

h. i. $m = 16$ **ii.** $m = 28$ **iii.** $m = 13$ **iv.** $m = 49$

i. i. $m = 26$ **ii.** $m = 8$ **iii.** $m = 44$ **iv.** $m = 2$

3. a. Fat 0.5 g, Protein 4.9 g, Carbohydrate 13.7 g, Energy 380 kJ

b. 78.9 calories

c. Multiply the number of fat grams by 9 and to this, add the sum of the number of protein and carbohydrate grams multiplied by 4.

d. i. f **ii.** p **iii.** c **iv.** e

e. $e = 9f + 4(p + c)$

f. See the table on the following page.

g. 100 grams of chocolate cake. Yes

h. In general, fruits and vegetables

i. Number of calories: e, number of kilojoules k, $k = 4.2e$

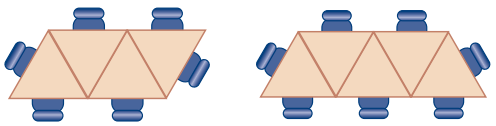
j. 331 kJ

k. See the table on the following page.

4. a.

T	1	2	3	4	5
D	3	4	5	6	7

b.



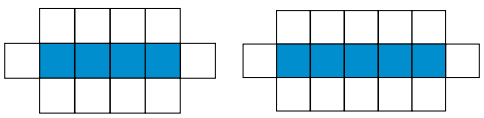
c. $D = T + 2$

d. $D = 26$

5. a.

B	1	2	3	4	5
W	4	6	8	10	12

b.



c. $W = 2B + 2$

d. $W = 22$

3f and **k**

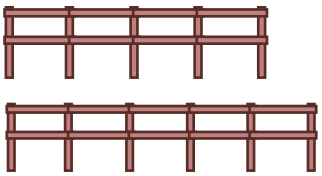
Food	Number of grams of fat	Number of grams of protein	Number of grams of carbohydrate	Number of calories (f)	Number of Kilojoules (k)
1 hot cross bun	3	3	19	115	483
100 grams of chocolate cake	16	4	56	384	1613
100 grams of roast chicken	14	26	0	230	966
70 grams of bacon	8	21	0	156	655
2 grilled sausages	17	13	15	265	1113
1 piece of fish (flake), no batter	1	21	0	93	391
10 grams of sultanas	0	0	6	24	101
25 grams of dried apricots	0	0	17	68	286
1 banana	0	1	20	84	353
1 apple	0	0	17	68	286

Food	Number of grams of fat	Number of grams of protein	Number of grams of carbohydrate	Number of calories (f)	Number of Kilojoules (k)
1 serve of carrots	0	0	5	20	84
1 serve of potatoes	0	0	17	68	286
250 millilitres of milk	10	8	12	170	714
210 grams of tinned tomato noodle soup	0.8	2.6	14.9	77.2	324

6. a.

S	1	2	3	4	5
P	4	7	10	13	16

b.



c. Formula is $P = 3S + 1$ **d.** $P = 661$ pieces of timber **e.** \$2544.85

7. a, b.

A	1	2	3	4	5
L	4	7	10	13	16

c.

d. Formula is $L = 3A + 1$ **e.** $L = 226$ light panels

8. a. 1225 **b.** 2485

9. 154

10.
$$\begin{array}{r} 29 \\ +\ 92 \\ \hline 121 \end{array}$$

11. m, a, z, g

12.
$$\begin{array}{r} 940 \\ 739 \\ +\ 8925 \\ \hline 10604 \end{array}$$

Exercise 8.5 Terms, expressions and equations

1. a. $c + 3$ **b.** $2c$ **c.** $3c + 2$ **d.** $2c + 5$ **e.** $4c + 3$

2. a. $4c + 34$ **b.** 242 **c.** \$48.40

3. a. $2m$ **b.** $2m + n$ **c.** $4m + 3n$ **d.** $m + 2n + 5$ **e.** $3m + 2n + 8$

4. a. 34 **b.** 46 **c.** 104 **d.** 46 **e.** 83

5. a. $B + 2$ **b.** $T - 3$ **c.** $D + 6$ **d.** $K - 5$ **e.** $G + N + W$

f. $D + H$ **g.** $2N$ **h.** $C + H$ **i.** $12 - G$ **j.** $4D$

k. $H + 6$ **l.** $Z - G$ **m.** BF **n.** $3M + Y$

6. a. $A + B + C$ **b.** $A - C$ **c.** AB **d.** ABC

e. $\frac{B}{C}$ **f.** $\frac{A + C}{B}$ **g.** $A + 3$

7. a. $T - X$ **b.** $\$4B$ **c.** $45G$ **d.** $\frac{\$T}{5}$ **e.** $R + M$

f. $\frac{Y}{6}$ **g.** $\$11M$ **h.** $A + H$ **i.** $52 - J$ **j.** $12B$

k. $\frac{B + G}{2}$ **l.** $A + H + K$ **m.** $4R$

8. a. $x + 8 = 11$ **b.** $x - 11 = 63$ **c.** $8x = 128$ **d.** $\frac{x}{7} = 5$

e. $A + H = 6.49$ **f.** $52 - Q = 19$ **g.** $99M = 792$ **h.** $4R + 4D = 164$

9. a. True **b.** True **c.** False **d.** True **e.** True **f.** True

g. True **h.** False **i.** False **j.** True **k.** True **l.** False

10. a. B **b.** C **c.** D **d.** D

e. B **f.** A **g.** A

11. a. $x + 5$ **b.** $x - 8$ **c.** $2x$ **d.** $\frac{x}{4}$

e. $x + 1$ **f.** $100x$ **g.** $\frac{x}{100}$ **h.** $7x$

i. $\frac{x + 20}{2}$ **j.** $10x$ dollars **k.** $\frac{k + 23}{3}$

12. a. D **b.** C **c.** C **d.** E **e.** B **f.** C

13. a. $\$(2x + 7)$ **b.** $3x$ **c.** $4x + 3$ **d.** $3x + 3$ **e.** $3x + 6$ **f.** $3x + 6$

14. a. Cost of a hamburger + cost of a hamburger + cost of a drink = total cost **OR** cost of the drink + twice the cost of a hamburger = total cost

b. $\$3.75 + 2h = \$7.25$

15. a. i. \$62 **ii.** \$74 **iii.** \$86

b. Cost $= 50 + 12n$ **c.** \$146 **d.** $100 = 50 + 12n$

16. 8 years old **17.** 180

Challenge 8.2

Dad is 30, son is 10.

Exercise 8.6 Simplifying and the distributive law

1. a. $5a$ **b.** $14y$ **c.** $15c$ **d.** $2u$

e. $20e$ **f.** $5t$ **g.** $14ab$ **h.** $6fg$

i. $3e$ **j.** $7t$ **k.** $5f$ **l.** 0

m. $18x$ **n.** $12m$ **o.** $8a$

2. a. $25ab - 7$ **b.** $3y$ **c.** Cannot be simplified **d.** $30i - 2$

e. $14t - 3$ **f.** $13r$ **g.** Cannot be simplified **h.** Cannot be simplified

i. $16f + 5$ **j.** $4 + 2g$ **k.** t **l.** 0

m. $2mno$ **n.** $14pq$ **o.** $11pr + 2$ **p.** Cannot be simplified

q. $11t - 5$ **r.** t **s.** $8aw$ **t.** Cannot be simplified

u. $11t - 8$ **v.** $4t + 3$ **w.** $10g - 2$ **x.** $22r + 6$

3. a. 0 **b.** 0 **c.** 0

d. 8 **e.** 7 **f.** 2

g. 0 **h.** 7 **i.** $6y$

4. a. $5a, 35$ **b.** $9a, 63$ **c.** $4a, 28$ **d.** $10a, 70$

e. $10a, 70$ **f.** $10a, 70$ **g.** $17 + 8a, 73$ **h.** $5a + 2, 37$

i. $2a, 14$ **j.** $a, 7$ **k.** $0, 0$ **l.** $17a - 16, 103$

5. a. $3a + 6b$ **b.** $5x - 20z$ **c.** $20g + 30h$ **d.** $2bc - 3ab$

e. $16cd - 2cq$ **f.** $9b^2 - 6bg$ **g.** $9ax + 7bx$ **h.** $7kt + 21km$

6. a. $2a + 10b$ **b.** $5n - 8c$ **c.** $18c$ **d.** $ax - bx + x$

e. $5y + 2yz$ **f.** $3dx - 5x$ **g.** $12fu + 14u$ **h.** $20n + 21hn$

7. a. True **b.** True **c.** False **d.** False

e. True **f.** False **g.** True **h.** False

i. True **j.** True **k.** True **l.** False

m. True **n.** True **o.** True **p.** True

q. False **r.** False

8. a. E **b.** B **c.** C **d.** D

e. C **f.** A **g.** D

9. a. True **b.** False **c.** False **d.** True **e.** False

f. True **g.** False **h.** True **i.** True **j.** True

10. a. $13x + 2y$ **b.** $12a - 10y + 6ay$ **c.** $9cg + 18cd + dg$ **d.** $12dy + 3ry$

11. a. $10x$ **b.** $6d$ **12. a.** 2L + 2W **b.** 2A + B

13. $\angle COD = \angle DOE = 40°$

14. a. (Mrs Makeit's costs + 10) × 2 **b.** $c = 2(n + 10)$ **c.** \$60

Exercise 8.7 The associative law

1. a. $w + (r + 6y) = (w + r) + 6y$ **b.** $6t \times (4r) \times 3s) = (6t \times 4r) \times 3s$

c. $(9y + 2r) + 3w = 9y + (2r + 3w)$ **d.** $(z + 2p) + 6t = z + (2p + 6t)$

2. a. $6t + (3w + 7v) = 3w + (6t + 7v)$ **b.** $s \times (9r \times 2c) = 2c \times (s \times 9r)$

c. $(3c + w) + d = d + (3c + w)$ **d.** $(g \times 3b) \times 2y = 2y \times (3b \times g)$

2. a. T **b.** F **c.** T **d.** F
e. F **f.** F **g.** T **h.** F

3. a. i. 33 **ii.** 33 They are the same.
b. i. 21 **ii.** 1 They are different.
c. i. 48 **ii.** 48 They are the same.
i. i. 2 **ii.** $\frac{1}{8}$ They are different.

4. $48 **5.** $47 or $48

6. Check with your teacher.

7. They did not start with the same number. Andrew started with 11 and Andrea started with 17.

8. $6 \div (1 - 5 \div 7) = 21$

8.8 Review

1. a.

Input	1	3	4	7	13
Output	4	12	16	28	52

b.

Input	1	4	6	7	15
Output	8	11	13	14	22

c.

Input	3	4	7	10	13
Output	3	5	11	17	23

d.

Input	0	4	8	16	20
Output	0	1	2	4	5

2. a.

Input	15	16	33	10	14
Output	8	9	26	3	7

b.

Input	1	4	15	5	33
Output	10	19	52	22	106

c.

Input	1	3	6	4	43
Output	12	16	22	18	96

d.

Input	8	11	15	13	38
Output	11	17	25	21	71

3. a. C **b.** B

4. a. $y = 9x - 4$ **b.** $q = 3p + 6$

5. a. $y = x + 6$ **b.** $d = c - 5$ **c.** $h = 5g + 2$ **d.** $n = 4m - 1$

6. a.

x	1	3	4	7	105
y	0	2	3	6	104

b.

x	1	2	6	7	12
d	3	6	18	21	36

c.

x	0	2	4	5	6
h	6	4	2	1	0

d.

x	0	1	3	4	7
n	3	14	36	47	80

7. a. $35x + 18$ **b.** 1068

8. a. $M - C$ **b.** $\$3B$ **c.** XY **d.** $G + 15$ **e.** $D + 1$ **f.** $12H$ cents **g.** $5T$

9. a. $7g$ **b.** $6y$ **c.** $9h$ **d.** $5ag$ **e.** $3gy$ **f.** 0
g. $5y$ **h.** $7t + 6$ **i.** $18gh$ **j.** $11t - 2m$ **k.** $4m$ **l.** $15g + 12$
m. $4h + 4t$ **n.** $10b + 7c$ **o.** $8axy$

10. a. $10x$, 50 **b.** $5x - 4$, 21 **c.** $23x$, 115 **d.** $3x$, 15 **e.** $3x$, 15 **f.** $x + 16$, 21
g. $8x + 7$, 47 **h.** $11 + 7x$, 46 **i.** $6x$, 30

11. a. $7m - 21k$ **b.** $gw + 9aw$ **c.** $16hy - 14y$ **d.** $70mr - 20mp$ **e.** $6ag + 6cg$ **f.** $24jx - 12jy$

12. a. $2t - 10s$ **b.** $4a + 10g$ **c.** $3df + 7d$ **d.** $15z - 4az$ **e.** $3fp - 6pq + 7p$ **f.** $10ab + 65b$

13. In this question, the order of the terms is important.

a. $a + (3b + \square) = (\square + \square) + 5c$

b. $\frac{1}{a} + (2a + \square) = (\square + 2b)\square 7g$

c. $(d \times 3y) \times z = \square \times (\square \times \square)$

d. $\left(\frac{a}{b} \times c\right) \times \square = \frac{a}{\square} \times (\square \times f)$

14. a. $g + (\square + 2k) = (2k + \square) + 3m$

b. $(w + \square) + 4r = \square + (5g + w)$

c. $\frac{2}{3} \times (2d \times \square) = (r \times 2d) \times \square$

d. $(3z \times 5b) \times = \square \times (6m \times 5b)$

15. a.

B	1	2	3	4	5
S	6	11	16	21	26

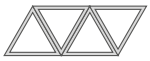
b.

c. Formula is $S = 5B + 1$

d. $S = 5B + 1$ sleepers

16. a.

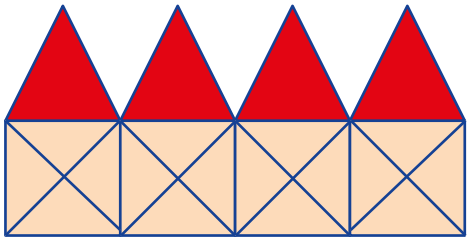
T	1	2	3	4
S	3	5	7	9

b. 

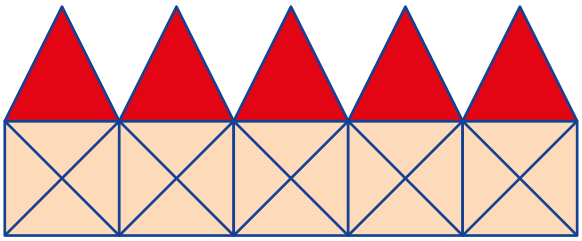
c. $S = 2T + 1$ **d.** $S = 51$

17. a.

H	1	2	3	4	5
S	6	11	16	21	26

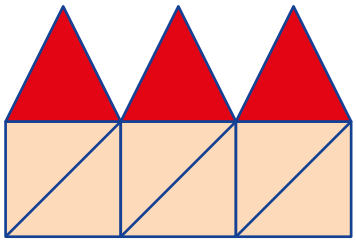
b. 

4 huts, 21 struts



5 huts, 26 huts

c. $S = 5H + 1$ **d.** 171

e. 

3 huts, 13 struts

f.

H	1	2	3
S	5	9	13

$S = 4H + 1$

g. 137

18. a. 8.0 cm **b.** 9.4 cm

c. Let d be the number of days, h be the height, $h = 1.4d + 2.4$.

d. 12.2 cm **e.** 27.6 cm **f.** 37.4 cm

g. Approximately 21 days later

h. The plant will eventually stop growing. According to the formula, it will grow indefinitely.

19. a. i. $m - 5$ **ii.** $3m$

b. Kate 2, Bianca 7, Paul 15, Aunt Theresa 21 **c.** Kate 8, Bianca 13, Paul 21, Aunt Theresa 39

20. $x \times \frac{1}{x} = 1$ **21.** 9 tables **22.** $5y + 20$ **23.** 346 **24.** $\$\frac{3x + 8y}{20} = \frac{3x}{20} + \frac{y}{5}$

25. $\$\frac{5x - 36}{8}$ **26.** 84 coins **27.** 3 cm **28.** 9 years **29.** 586

Investigation — Rich task

1.

	Garden bed 1	Garden bed 2	Garden bed 3
Side length of the garden bed (m)	1	2	3
Area of the garden bed (m^2)	1	4	9
Number of pavers used	8	12	16
Total length around the outside of pavers (m)	12	16	20

2. $a = s \times s, l = 4s + 8, p = 4s + 4, p = l - 4$

3. 44

4. 68

5. Teacher to check.

TOPIC 9 Measurement

9.1 Overview

Numerous **videos** and **interactivities** are embedded just where you need them, at the point of learning, in your learnON title at www.jacplus.com.au. They will help you to learn the content and concepts covered in this topic.

9.1.1 Why learn this?

Measurement is important because it helps us to describe our world using numbers. We use numbers to describe length, perimeter, area, volume and capacity. People use measurements every day. Many types of work involve taking measurements. Geographers, property developers and estate agents all require good knowledge of measurement. Science and engineering would not have progressed so far without an understanding of measurement. Without measurement there would be no roads and freeways, and even no house for you to live in.

9.1.2 What do you know?

assessON

1. **THINK** List what you know about length, area and volume. Use a thinking tool such as a concept map to show your list.
2. **PAIR** Share what you know with a partner and then with a small group.
3. **SHARE** As a class, create a thinking tool such as a large concept map that shows your class's knowledge of length, area and volume.

LEARNING SEQUENCE

9.1 Overview
9.2 Units of measurement and converting units of measurement
9.3 Reading scales and measuring length
9.4 Perimeter
9.5 Area
9.6 Area of composite shapes, using addition and subtraction
9.7 Volume
9.8 Capacity
9.9 Review

9.2 Units of measurement and converting units of measurement

9.2.1 Metric units of length

- The metric system is based on the number 10.
- The base unit of length of the metric system is the metre.
- All other units of length in the metric system are related to the metre by powers of 10.
- The most commonly used units of length together with their abbreviation and approximate examples are shown in the following figures.

1. Kilometre (km)

The distance travelled by a car travelling at the speed limit of 60 kilometres per hour in one minute

2. Metre (m)

The length of an adult's stride

3. Centimetre (cm)

The width of each of your fingers

4. Millimetre (mm)

The width of a wire in this computer chip

WORKED EXAMPLE 1

You have been given the task of measuring the length of a piece of rope to be used around the boundary of a cricket ground. Which metric units of length would you use?

THINK

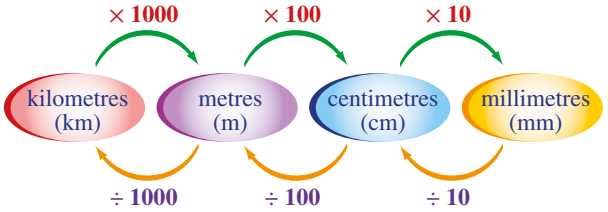
Estimate the length involved. In this case, it is in the order of 100s of metres. Would other units give a more manageable figure? (Would using centimetres, for example, 10 000 cm, be more convenient? No! What about kilometres? No — the length would be less than 1 kilometre.)

WRITE

Metres (or m)

9.2.2 Converting units of length

- The main metric units of length are related as follows:
  1 km = 1000 m 1 m = 100 cm 1 cm = 10 mm
- Units of length can be converted as shown in the following diagram.



- In the preceding diagram, the numbers next to each arrow are called **conversion factors**.
- When converting from a larger unit to a smaller unit, we multiply by the conversion factor; when converting from a smaller unit to a larger unit, we divide by the conversion factor.

WORKED EXAMPLE 2

Complete each of the following metric conversions.

a **0.3285 km = _____ m** **b** **560 m = _____ mm**

c **480 cm = _____ km** **d** **$2\frac{3}{5}$ m = _____ cm**

THINK

a Refer to the metric conversion chart above. To convert kilometres to metres, multiply by 1000. (Move the decimal point 3 places to the right.)

b This conversion involves two steps in the chart. First convert metres to centimetres by multiplying by 100 (since there is no decimal point place two zeros after the final digit). Then convert centimetres to millimetres by multiplying by 10 (place one zero after the final digit).
Note: Overall, we need to multiply by 100 × 10 or 1000.

c To convert 480 centimetres to kilometres involves two division steps. From centimetres to metres, divide by 100 (move the decimal point 2 places to the left). From metres to kilometres, divide by 1000 (move the decimal point 3 places to the left).

d **1** Convert $2\frac{3}{5}$ m to an improper fraction.

2 Convert metres to centimetres by multiplying by 100.
Note: Remember when multiplying fractions by a whole number, the number multiplies the numerator.

3 Simplify the fraction.

WRITE

a 0.3285 km = 0.3285 × 1000 m
= 328.5 m

b 560 m = 560 × 100 cm
= 56 000 cm
= 56 000 × 10 mm
= 56 000 mm

c 480 cm = 480 ÷ 100 m
= 4.8 ÷ 1000 km
= 4.8 ÷ 1000 km
= 0.004 8 km

d $2\frac{3}{5}$ m = $\frac{2 \times 5 + 3}{5}$ m
= $\frac{13}{5}$ m
= $\frac{13}{5}$ × 100 cm
= $\frac{1300}{5}$ cm
= 260 cm

learnON RESOURCES — ONLINE ONLY

Complete this digital doc: SkillSHEET: Metric units of length (doc-6505)

Complete this digital doc: SkillSHEET: Measuring the length of a line (doc-6507)

Complete this digital doc: SkillSHEET: Relationship between unit size and the number of units used (doc-6508)

Complete this digital doc: SkillSHEET: Converting units (metres and kilometres) (doc-6509)

Complete this digital doc: SkillSHEET: Converting units to compare lengths and distances (doc-6510)

Exercise 9.2 Units of measurement and converting units of measurement

assessON

Individual pathways

PRACTISE
Questions:
1, 2a–d, 3, 4, 5, 6, 7, 10, 11, 13, 14, 15, 16, 17, 18, 20, 21, 22, 24, 33, 34

CONSOLIDATE
Questions:
1, 2d–g, 3, 4, 5, 6, 8, 10, 11, 13, 18, 20, 21, 23, 24, 25, 28, 30, 32, 33

MASTER
Questions:
1, 2g–l, 3, 4, 5, 6, 9, 10, 11, 12, 16, 19, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 35

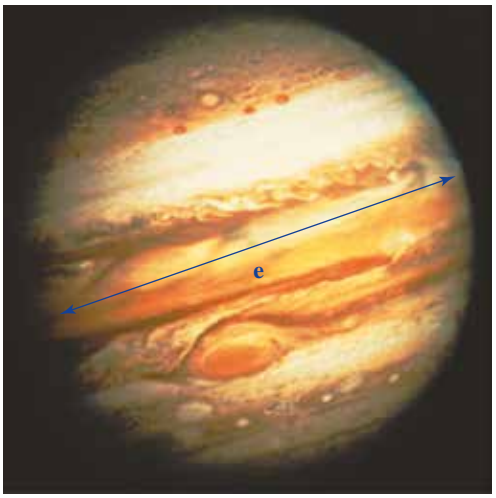
Individual pathway interactivity: int-4355

learnON ONLINE ONLY

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Fluency

1. WE1 Which units (mm, cm, m or km) would be most suitable for measuring the real lengths marked in each photograph? (You may give two different units if you think both would be suitable.)



2. Which metric units of length would you use for measuring the following? (You may give two different units if you think both could be used.)

a. The length of a netball court
b. The diameter of a netball
c. A photo frame's width
d. The thickness of a slice of bread
e. The length of a worm
f. The distance between Melbourne and Sydney
g. The depth of an Olympic swimming pool
h. The depth of a bowl of soup
i. The length of a mathematics textbook
j. The thickness of a magazine
k. The distance around the shores of a reservoir
l. The length of a cut-out shape on a GeoMat mathematics template

3. MC The thicknesses of two different brands of chocolate biscuit would be best compared by measuring in:

A. millimetres
B. kilometres
C. metres
D. centimetres
E. kilometres or metres

4. MC Girouk wishes to measure the length of his mathematics folder to see what size paper he could fit into it. Which of the following would be the best measuring instrument to use?

A. A science department metre ruler
B. A 10-cm mini ruler
C. A 30-cm plastic ruler
D. A 20-m flexible tape measure
E. A one-metre trundle wheel

5. Give three examples (not mentioned earlier in this exercise) of something that would be best measured in:

a. metres
b. millimetres
c. kilometres
d. centimetres.

6. Give an example of a length which each of the following people might measure in their jobs. (For example, a carpet layer would measure the length of a room.)

a. Veterinary surgeon
b. Cartographer (map maker)
c. Costume designer
d. Carpenter
e. Doctor
f. Landscape gardener
g. Plumber
h. Computer technician
i. Track and field athlete
j. Weather reporter

7. WE2a Complete each of the following metric conversions.

a. 2.0 km = _____ m
b. 7.0 km = _____ m
c. 65.0 km = _____ m
d. 5.3 km = _____ m
e. 0.66 km = _____ m
f. 9.0 m = _____ cm
g. 0.25 m = _____ cm
h. 28.0 cm = _____ mm
i. 200.0 cm = _____ mm
j. 700.0 m = _____ cm

8. Convert to the units indicated.

a. 8000 m = _____ km
b. 6500 m = _____ km
c. 700 m = _____ km
d. 50 m = _____ km
e. 10.5 m = _____ km
f. 6000 cm = _____ m
g. 300 cm = _____ m
h. 57 cm = _____ m
i. 9 cm = _____ m
j. 45 mm = _____ cm
k. 835 mm = _____ cm
l. 25 600 mm = _____ cm

9. WE2b, c, d Copy and complete.

a. 8 km = _____ cm
b. 5400 mm = _____ m
c. 101 m = _____ mm
d. 7800 cm = _____ km
e. $60\frac{1}{4}$ km = _____ mm
f. $112\frac{4}{5}$ cm = _____ m
g. 72.33 m = _____ mm
h. 40.5 mm = _____ cm
i. 5.05 km = _____ m
j. $30\frac{7}{20}$ mm = _____ m

10. MC A distance of 6.25 km is the same as:

A. 625 m **B.** 0.006 25 m **C.** 625 000 mm **D.** 625 000 cm **E.** 62.50 cm

11. MC A distance of 7 860 000 cm is equal to:

A. 78.6 km **B.** 786 m **C.** 786 km **D.** 786 000 mm **E.** 7.86 km

12. Convert to the units indicated.

a. $67\frac{7}{10}$ km to m
b. $45\frac{1}{5}$ m to km
c. 560 mm to m
d. 450 000 cm to km
e. $8\frac{3}{4}$ cm to mm
f. 0.0006 km to cm
g. 7.21 m to mm
h. $3\frac{9}{100}$ km to cm
i. 48 mm to cm
j. 11.655 m to mm

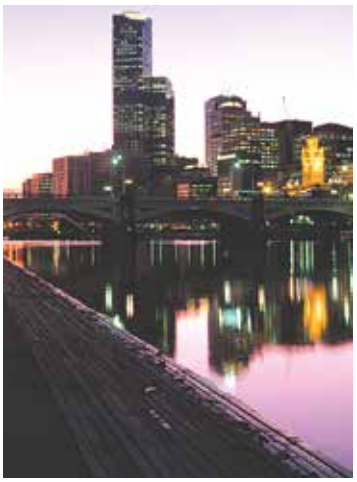
Understanding

13. The longest snake ever held in captivity was a female reticulated python named Colossus. She was measured to be 8.68 m long. Convert this to centimetres and compare it to the adult length of the shortest species of snake, the West Indian *Leptoyphylops bilineata*, which grows to only 108 mm.

14. The world's largest mountains are Everest (8863 m) and K2 in Pakistan (8607 m). Convert these heights to kilometres.

15. The tallest tomato plant on record reached 16.3 m. How many centimetres is this?

16. The tallest Rialto Tower stands 0.448 km high. How many metres is this?



17. Arrange the following in order from smallest to largest.

a. 12.5 m, 150 cm, 0.02 km
b. 350 cm, 0.445 m, 3000 mm
c. 50 km, 500 m, 50 000 mm
d. 1700 cm, 1.7 m, 0.17 km
e. 0.052 cm, 0.0052 mm, 0.000 052 m
f. 990 cm, 0.909 m, 9000 mm

18. Add the following lengths, giving your answer in either unit.

a. 75 cm and 3 m
b. 2700 m and 7.5 km
c. 3800 mm and 52 cm
d. 1.66 m and 58.2 cm
e. 208 cm and 1.83 m
f. 5500 mm and 20.1 m
g. 40 km and 7800 m
h. 0.000 675 km and 87.8 cm

19. Find the difference between each of the following lengths, giving your answer in either unit.

a. 72 km and 5600 m **b.** 418 000 mm and 7.6 m

c. $34\frac{3}{5}$ cm and $\frac{9}{20}$ m **d.** $2\frac{4}{5}$ km and 450 000 cm

20. A particular brand of computer CD is 1.2 mm thick. How high (in centimetres) would a pile of 23 such CDs be?

21. A builder needs to build a wall 3.5 m high. If each layer of bricks adds 8 cm of height, how many layers high will the wall be?

22. Deanne buys a length of rope and cuts it into three smaller sections, each of length 7200 cm. How many metres long was the original piece of rope?

23. Norbert is 1.53 m tall in bare feet. If he wears shoes with 6.3 cm thick soles, how high would he stand then? Give your answer in metres.

24. Adrian is driving a truck with a rooftop 3.4 m above road level, when he approaches an overpass bridge which has a clearance of 376 cm. Will Adrian's truck get under the bridge? If so, how much room does he have to spare? Give your answer in centimetres.

25. A childcare centre has three large cardboard boxes which the children stack up in various combinations. What stack heights are possible for a stack if the boxes' individual heights are 600 mm, 45 cm and 1.1 cm? Give your answer in centimetres.

26. Finita attaches a trellis that is 0.6 m high to the top of her 180 cm high fence. How high is it to the top of the new trellis? Give your answer in metres.

27. Zvenglo is stacking identical boxes of height 330 mm. How high would a stack be if it contained six boxes? Give your answer in centimetres.

28. Waldo's noticeboard is 1.5 m long and 1.2 m wide. If he pins a calendar of length 70 cm and width 60 cm exactly in the middle of the board, what is the width of the space above the calendar? (*Hint*: Draw a diagram of the situation.)

29. A licorice strap machine takes 3.75 m lengths of licorice, and chops them into 10 cm long pieces. How many pieces does each 3.75 m length produce?

Reasoning

30. Astronomers use light-years as a measure of distance in the Universe. A light-year is the distance travelled by light in one year. If light travels approximately 300 000 km in one second, calculate the distance travelled by light in:

a. **i.** one minute
ii. one hour
iii. one day
iv. one year (365 days).

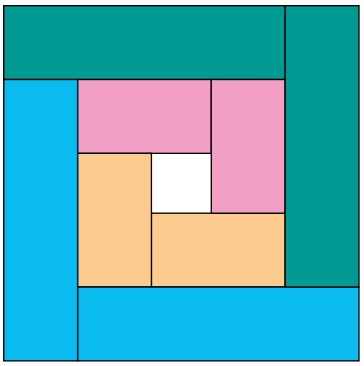
b. Explain why astronomers use this measurement for distance rather than kilometres.

31. Mother and daughter are riding their bikes to the local market. The circumference of the mother's bike wheel is 2 m while the circumference of the daughter's bike wheel is 1 m.

a. How many rotations of the wheel are there in 100 m for the:
i. mother **ii.** daughter?

b. How many rotations of the wheel are there in 1 km for the:
i. mother **ii.** daughter?

Problem solving



32. In a 'log cabin' quilt, a pattern is created from a series of squares and rectangles. From a centre square, 4 congruent rectangles are placed to build a larger square. This process is repeated a number of times to build larger and larger squares. Laurel starts with a centre square of side length 2 cm. If each rectangle has a width of 2 cm and a length that depends on its position in the block, give the dimensions of the rectangles that will need to be cut if the final block is to be 18 cm square. (Don't worry about seam allowance.)

33. It is said that the average person walks the equivalent of four times around the Earth in a lifetime. The circumference of the Earth is about 40000 km. If you lived to age 80, approximately how many kilometres per week would you walk to achieve this distance?

34. A pin, 14 mm long, is made of wire. How many pins could be made from 1 km of wire?

35. Tienna needs to convert her Grandma's favourite cake recipe to metric units. She notes that 1 oz = 30 g and 1 pint = 568 mL. Help her convert the following quantities to metric units.

Water $\frac{3}{4}$ pint
Butter 6 oz
Sugar 5 oz
Flour 10 oz
Eggs 2

Reflection

Why is it important to be able to convert from one metric unit to another?

9.3 Reading scales and measuring length

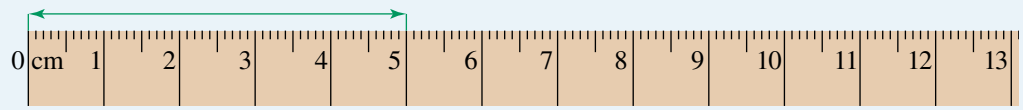
9.3.1 Scales and measurement

- A **scale** is a series of marks used to measure length, mass, temperature or any other quantity.
- When reading scales and measuring lengths:
  - check that the scale starts with zero
  - check the value of each small division by counting along the scale to the next major mark
  - always give units (for example, centimetres) with your answer.

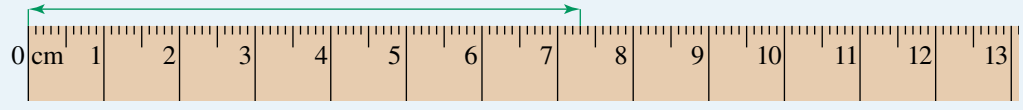
WORKED EXAMPLE 3

State the reading indicated by the arrow, giving answers in:
i decimal form
ii fractional form where appropriate.

a



b



THINK	WRITE
a 1 Check that the line starts at 0. It does.	**a**
2 Note the units printed on the ruler (cm).	
3 Read the last centimetre mark (5).	
4 Does the line go past the last centimetre mark? No.	
5 Write the value with units.	5 cm
Note: There is no need to go any further as the value is a whole number.	
b i 1 Check that the line starts at 0. It does.	**b i**
2 Note the units printed on the ruler (cm).	
3 Read the last centimetre mark (7).	
4 Consider how many smaller intervals there are between the numbered divisions. There are 10, so each smaller division represents 0.1 of a unit.	
5 Count the number of small divisions past the last centimetre mark (3).	
6 Write the value with units.	7.3 cm
ii Express the answer obtained in **b i** as a fraction.	**ii** $7\frac{3}{10}$ cm
Note: The value of the first place to the right of the decimal point is tenths, so the digit 3 is 3 tenths.	

WORKED EXAMPLE 4

Use the given length of the climber's lower leg to estimate the labelled length of the rope on the following diagram.

THINK	WRITE
1 Refer to the 0.5 m lengths indicated on the diagram and determine how many of these lengths correspond to the unknown length of the rope above the climber. *Note:* There are approximately $3\frac{1}{2}$ or 3.5 of these lengths in the rope.	From the above diagram there are approximately $3\frac{1}{2}$ (or 3.5) of these lengths in the rope.
2 Multiply the number of lengths by 0.5 m.	Length $= 3.5 \times 0.5$ $= 1.75$
3 Write the answer.	The length of the rope is approximately 1.75 m.

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Complete this digital doc: SkillSHEET: Reading scales (doc-6506)

Exercise 9.3 Reading scales and measuring length

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1a–f, 2, 3, 4, 5, 8, 9, 11	Questions: 1c–i, 2, 4, 6, 7, 8, 9, 10, 11	Questions: 1e–j, 2, 6, 7, 8, 9, 10, 11, 12

Individual pathway interactivity: int-4356

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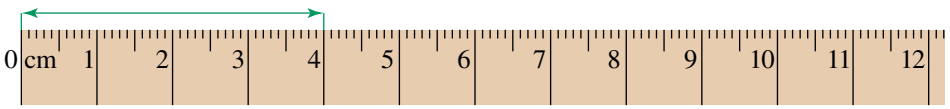
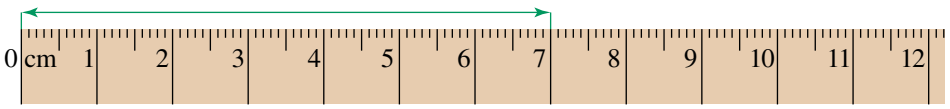
To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note*: Question numbers may vary slightly.

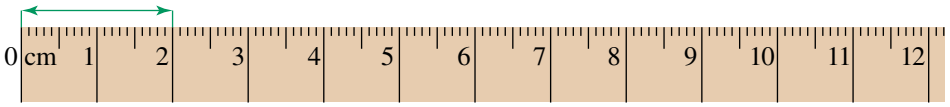
Fluency

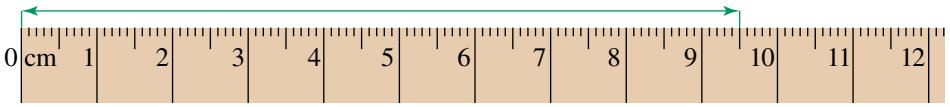
1. WE3a State the reading indicated by the arrow, giving answers in:

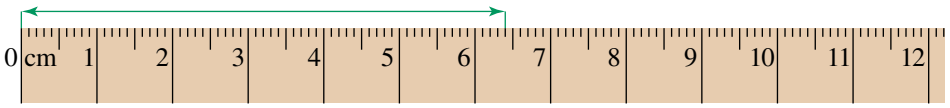
i. decimal form **ii.** fractional form where appropriate.

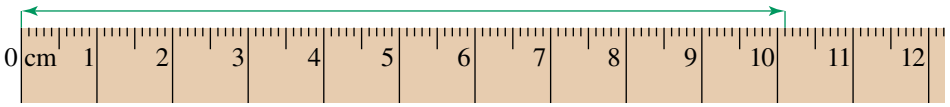
a.


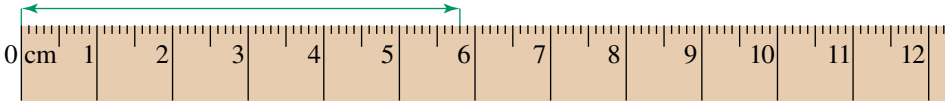
b.


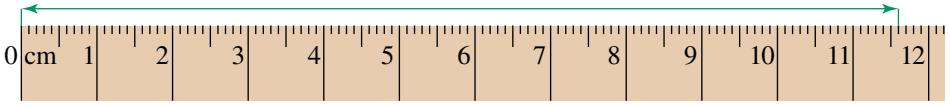
c.


d. WE3b


e.


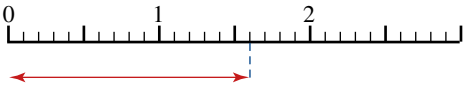
f.


g.


h.


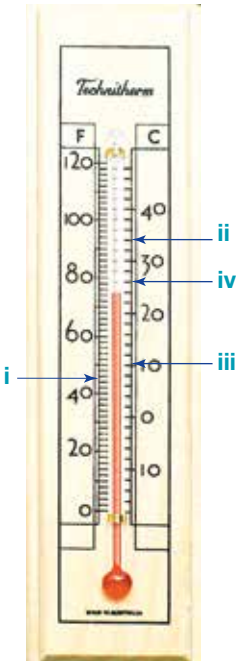
i.

j.



2. Read the values shown on the following scales. For parts **a** to **d** and **g** read from the arrow.

a.



b.

c. i

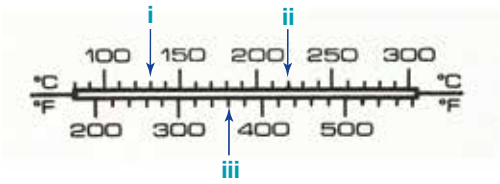
ii

iii

iv

d.

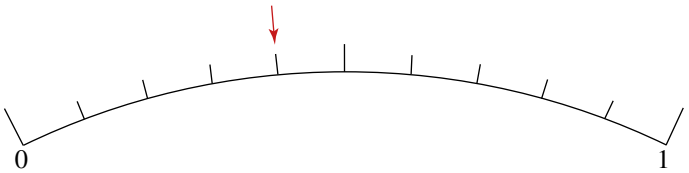
e.



3. MC The reading on the following scale is:

A. 2.2 **B.** 2.4 **C.** 2.6 **D.** 2.8 **E.** 3.3

4. MC The reading on the following scale is:



A. $\frac{4}{5}$ **B.** 4 **C.** $\frac{2}{5}$ **D.** $\frac{2}{10}$ **E.** $\frac{1}{2}$

5. Measure each of the following lines in centimetres.

a. _____

b. _____

c. _____

d. _____

e. _____

f. _____

g. _____

h. _____

6. Measure each of the following lines to the nearest millimetre.

a. _____

b. _____

c. _____

d. _____

e. _____

f. _____

g. _____

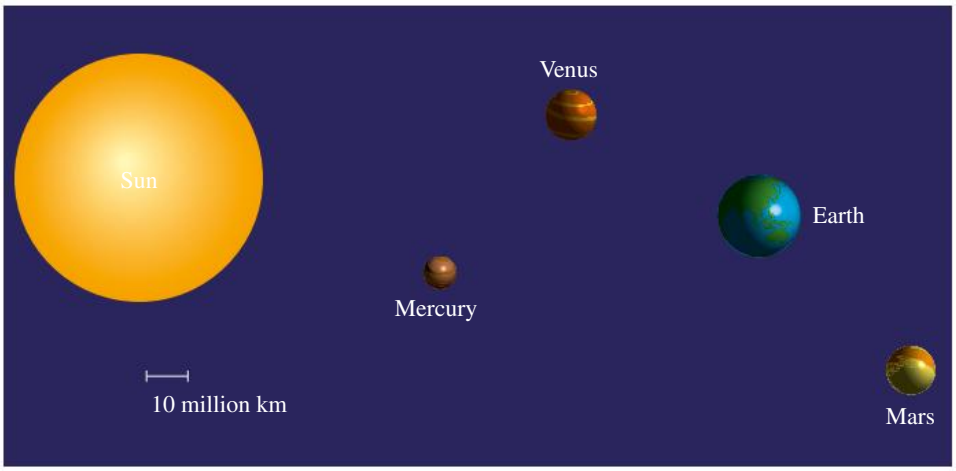
h. _____

7. Convert each of the lengths obtained in question **6** to centimetres in fraction form.

Understanding

8. WE4 In each of the following illustrations, use the given length to estimate the other lengths mentioned.

a. The distance from the centre of the Earth to the centre of the Sun



b. The length of the train if it has four carriages

c. The height of the giraffe

d. The height of the window (to the inside centre of the frame)

e. The distance in a straight line (refer to the following map) from:

i. Cann River to Falls Creek **ii.** Melbourne to Underbool **iii.** Geelong to Canberra

f. The height of the building in the foreground

g. The height (x) of the pole near the utility

Reasoning

9. Explain how you could use a normal ruler to find the thickness of a page in this textbook. Calculate the thickness of the page in your Maths textbook and Science textbook.

10. Devise a way of measuring the length of a student's pace during her walking, jogging and running. (*Hint:* you may use a long tape measure — preferably 30 m — or a trundle wheel.)

Problem Solving

11. Your friend is working on the top floor of a very tall office building. She walks up 726 steps from the ground floor to her office. Each step is 23 cm high. How high does she climb (in km) correct to 1 decimal place?

12. A gift box has dimensions 30 cm by 22 cm by 15 cm. If a ribbon is wrapped around this box as shown using 25 cm for a bow, what is the total length of ribbon needed?

Reflection

Think of a situation from your daily life in which you would need to estimate some lengths.

9.4 Perimeter

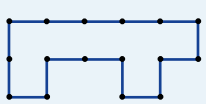
9.4.1 Perimeter

- A **perimeter** is the distance around the outside (border) of a shape.
- To find the perimeter of the shape, change all lengths to the same units and add them up.

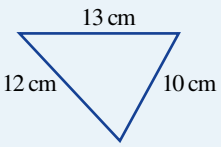
WORKED EXAMPLE 5

Find the perimeter of each shape.

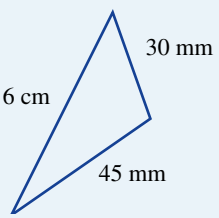
a



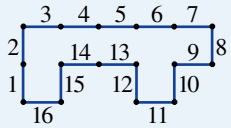
b



c

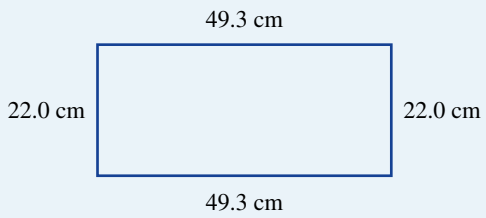
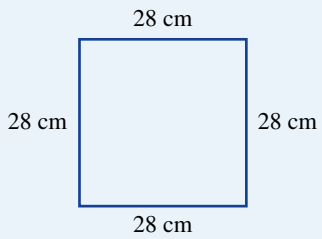


Note: The dots are 1 unit apart.

THINK	WRITE
a 1 Count the number of unit intervals around the outside of the shape (16). 	**a**
2 Write the number.	16 units
b 1 Check that the measurements are in the same units. They are.	**b** 12
2 Add the measurements.	13
3 Write the total, with units.	+10 35 cm
c 1 Notice the measurements are not all the same. Convert to the smaller unit (6 cm = 60 mm).	**c** 6 cm = 60 mm
2 Add the measurements that now have the same unit (mm).	60
3 Write the total using the smaller unit.	30 +45 135 mm

WORKED EXAMPLE 6

Find the perimeter of:
a rectangular print which is 49.3 cm long and 22.0 cm wide
b a square print whose side length is 28 cm.

THINK	WRITE
a 1 Draw a diagram of the print and write in the measurements.	**a** 
2 The perimeter is the distance around the print, so add all the distances together.	$P = 49.3 + 22.0 + 49.3 + 22.0$ $= 142.6$
3 Write the worded answer, with the correct unit.	The perimeter of the print is 142.6 cm.
b 1 Draw a diagram of the print and write in the measurements.	**b** 
2 The perimeter is the distance around the print, so add all the distances together.	$P = 28 + 28 + 28 + 28$ $= 112$
3 Write the worded answer, with the correct unit.	The perimeter of the print is 112 cm.

9.4.2 The perimeter of a rectangle and a square

- The perimeter, P, of a rectangle can be found using the short-cut formula $P = 2(l + w)$, where l is the length of the rectangle and w is its width.
- The perimeter, P, of a square can be found using a short-cut formula $P = 4l$, where l is the side length of the square.

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Complete this digital doc: SkillSHEET: Perimeter (doc-6511)

Complete this digital doc: WorkSHEET: Measurement I (doc-1853)

Exercise 9.4 Perimeter

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1, 2a–f, 3a–f, 4a–e, 5, 6, 7, 8, 10, 14, 18, 20	Questions: 2e–h, 3e–h, 4d–g, 7, 9, 10, 11, 12, 15, 16, 18, 20	Questions: 2g–j, 3g–j, 4f–h, 7, 9, 11, 12, 13, 16, 17, 19, 20

Individual pathway interactivity: int-4357

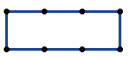
learnON ONLINE ONLY

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note*: Question numbers may vary slightly.

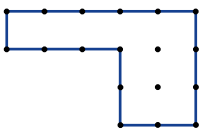
Fluency

1. WE5a Find the perimeter of each shape. The dots are 1unit apart.

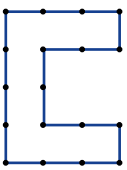
a.



b.



c.



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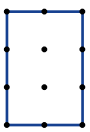

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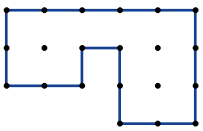

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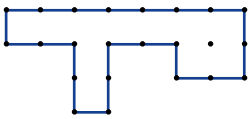

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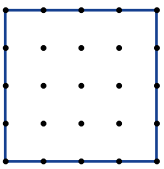

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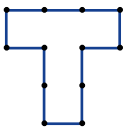

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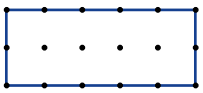

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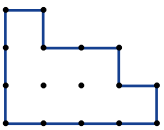
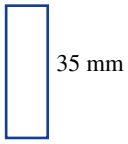


2. WE5b Find the perimeter of each of the following.

a.

b.



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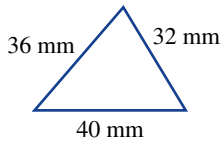

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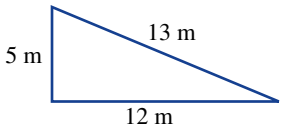
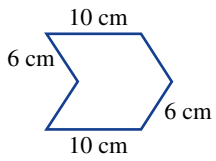


e.



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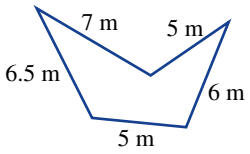

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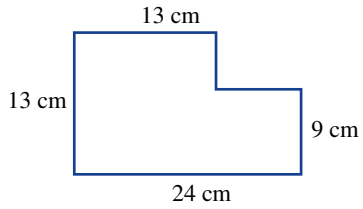

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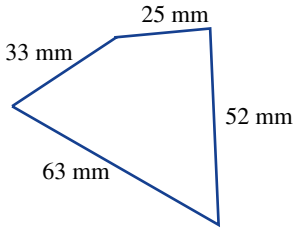

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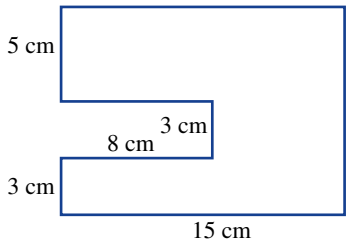
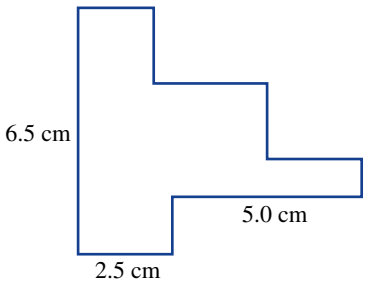


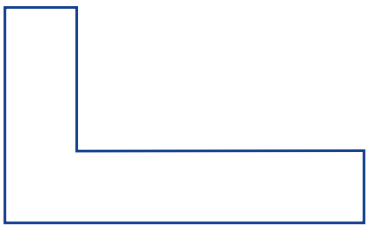
j.



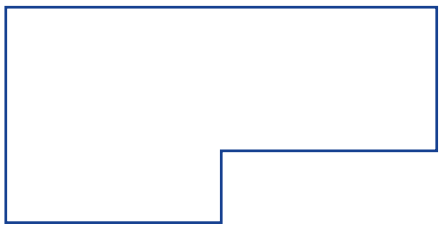
3. Measure the perimeter of each of the following, giving your answers in centimetres.

a.

b.

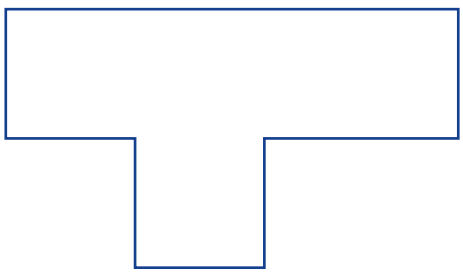


c.

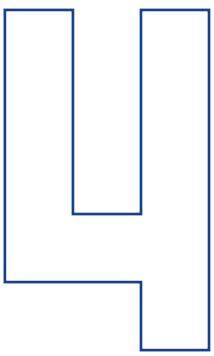


d.

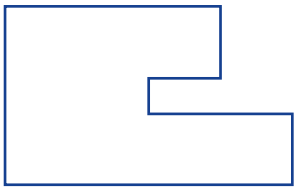
e.



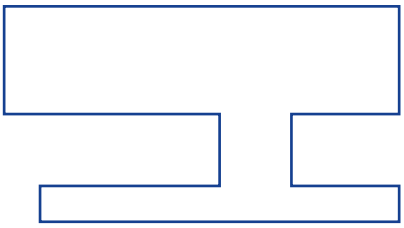
f.



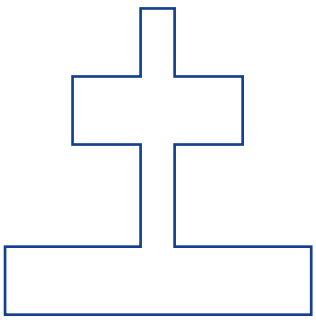
g.



h.



i.



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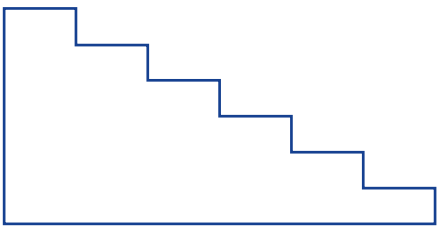
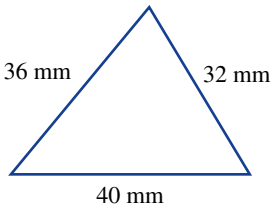
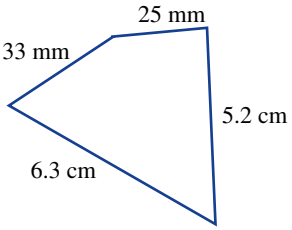


4. WE5c Find the perimeter of each shape, giving answers in the smaller unit in each case.

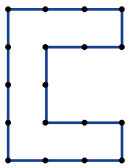
a.



b.

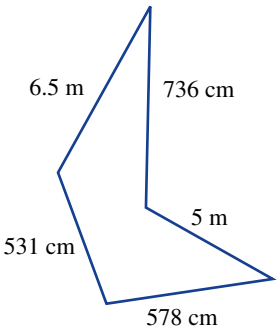


c.

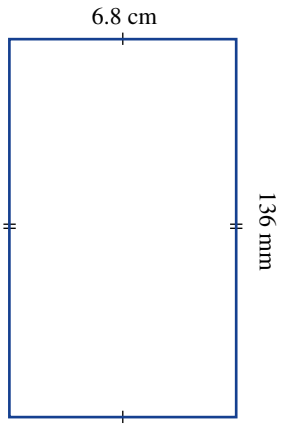


Note: The dots are 1 unit apart.

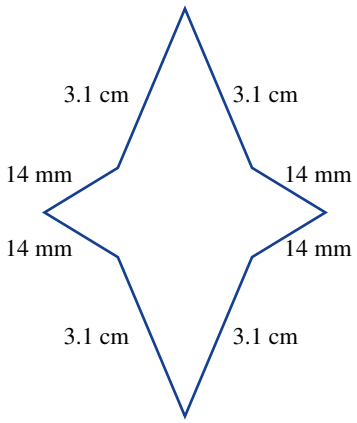
d.



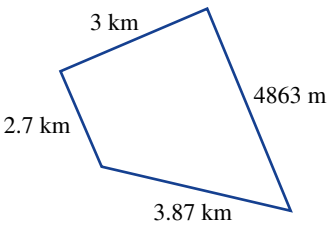
e.



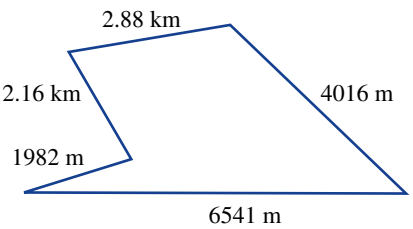
f.



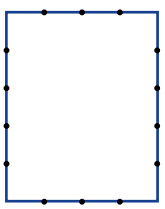
g.



h.



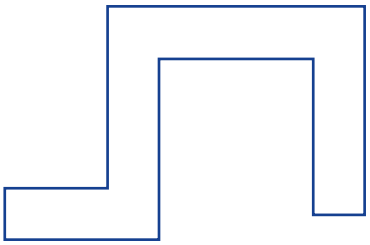
5. MC The perimeter of the shape at right is:



A. 9 units
B. 18 units
C. 16 units
D. 14 units
E. 12 units

Note: The dots are 1 unit apart.

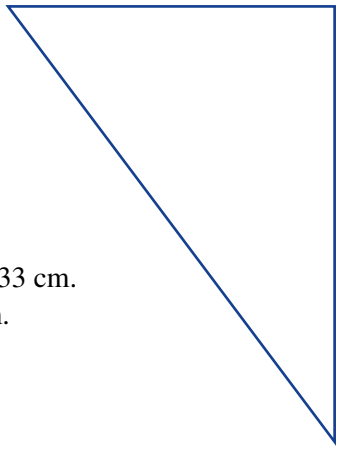
6. MC The shape at right has a perimeter of:



A. 17.6 cm
B. 19.4 cm
C. 20.8 cm
D. 23.2 cm
E. 26.2 cm

Note: Use your ruler to measure.

7. MC Measuring to the nearest half centimetre, the perimeter of the shape at right is:



A. 19.2 cm
B. 20.4 cm
C. 21.2 cm
D. 23 cm
E. 24.6 cm

8. WE6a **a.** Calculate the perimeter of a rectangle of length 45 cm and width 33 cm.
b. Calculate the perimeter of a rectangle of length 2.8 m and width 52.1 cm.
c. Calculate the perimeter of a rectangle of length $4\frac{1}{4}$ m and width $2\frac{1}{5}$ m.

9. a. Calculate the perimeter of a square of side length 3.7 cm.
b. WE6b Calculate the perimeter of a square of side length 2.14 m.
c. Calculate the perimeter of a square of side length $8\frac{1}{5}$ mm.

Understanding

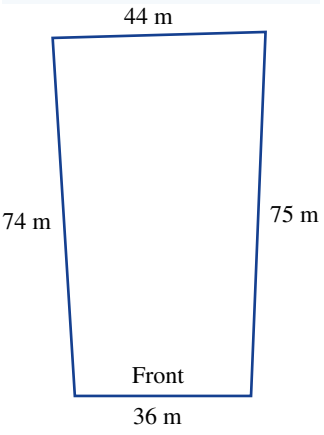
10. What length of party lights is needed to decorate the perimeter of a rectangular tent with dimensions 15.5 m by 8.75 m?

11. Allowing an extra 30 cm for waste, what length of picture frame is required to frame the artwork shown in Figure 1?

FIGURE 1

12. Phang's property boundary dimensions are shown in Figure 2.
a. How many metres of fencing will he need to fence all but the front boundary?
b. If the fencing costs $19 per metre, what will the total cost be?

FIGURE 2



13. Lucille has a quote of $37 per metre for new tennis court fencing. A tennis court is 23.77 m long and 10.97 m wide. There should be a 3.6-m space between the sides of the court and the fence, and a 6.4-m gap between the ends of the court and the fence.

a. Draw a diagram showing all given measurements.

b. How many metres of fencing are needed?

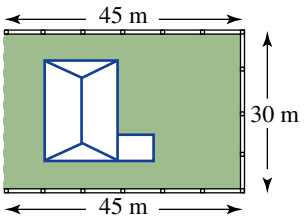
c. What will the total cost of the quote be?

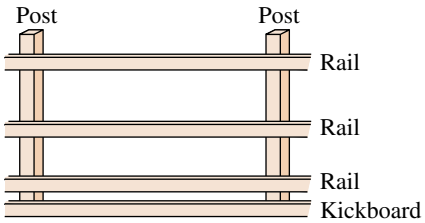
14. Zedken wishes to install three strands of barbed wire at the top of the fences around a rectangular work site. The length of the site is 34.5 m, and its width is 19.8 m. What length of wire will Zedken need?

15. A new game, Bop-ball, is played on a triangular field, where each side of the triangle measures 46.6 m. A greenkeeper is marking the field's perimeter using a chalk dispensing trundle wheel. How far will the greenkeeper walk in marking the field?

Reasoning

16. Marc and Cathy are seeking quotes on the cost of building a fence on three sides of their property. They want to calculate approximate costs for each item to decide whether the quotes supplied sound reasonable.





The new fence requires three rails, a kickboard, posts and palings.

a. What length of timber will be needed for the kickboard?

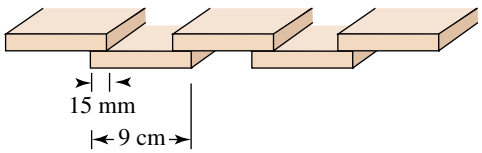
b. Calculate the cost of the kickboard if the timber required for this costs $1.90 per metre.

c. If the timber for the rails costs $2.25 per metre, what will be the total cost of the timber for the railings?

d. How many posts will be needed for the new fence if each post is to be 5 metres apart and there needs to be a post at the end of each straight section of fence?

e. Calculate the cost of the posts if the price of each post is $13.65.

Palings are 9 cm wide and are nailed so they overlap each other by 15 mm on each side.



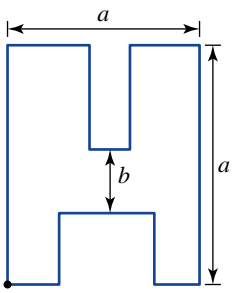
f. Calculate the approximate number of palings needed for the fence.

g. Palings cost $1.05 each. How much money should be allowed for the total cost of the palings?

h. Write an itemised list of all the costs involved. Include an amount to cover labour charges and miscellaneous items like the cost of nails. This amount is around $1000 (two people for two days at approximately $30.00 per hour for an eight-hour day). Estimate the cost of the new fence. This will provide Marc and Cathy with information to use when comparing builders' quotes.

Problem solving

17. Given the following diagram, what is the equation for the perimeter in terms of a and b?



Note: not to scale

18. A piece of paper is 22 cm long, including a 2.2-cm sticky strip at each end. Five strips of paper are stuck together so that the sticky parts overlap exactly to make a loop of paper. What is the circumference of the loop?

19. Kelly has a piece of string 32 cm long. He uses it to form rectangular shapes each with the length and width a whole number of centimetres. How many different rectangular shapes can he form?

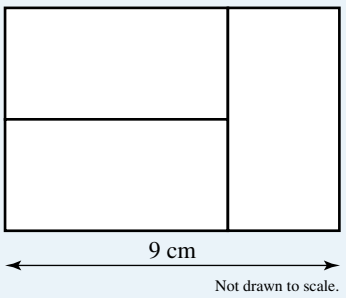
20. The perimeter of a rectangle is 32 cm . Using integers to represent the length and width, predict what the sides of the rectangle will be to give the largest possible area. Check by drawing different rectangles to see if your prediction is correct.

Reflection

In which professions will one need to calculate perimeters on a daily basis?

CHALLENGE 9.1

The following rectangle is made up of three identical, smaller rectangles that fit inside as shown. If its length measures 9 cm, work out its width widthout using a ruler.



Not drawn to scale.

9.5 Area

9.5.1 Metric units of area

- The **area** of a shape is the amount of flat surface enclosed by the shape. For example, if we consider a window, then the frame would represent its perimeter, while the glass inside the frame, its area.
- Area is measured in square units such as square centimetres, square metres and square kilometres. Commonly used metric units of area, with their abbreviations and examples, are shown in the following figures.

  1. Square kilometre (km^2)

The area of a country or a large city like Melbourne is given in square kilometres.

2. Square metre (m^2)

Square metres are used to measure the area of a classroom floor, whiteboard or this window.

3. Square centimetre (cm^2)

Small areas, such as the area of a sheet of A4 paper or a book cover, are measured in square centimetres.

4. Square millimetre (mm^2)

Very small areas, such as the area of a button or a postage stamp, are measured in square millimetres.

- Large areas of land are often measured in hectares (ha). 1 ha = 10 000 m^2.

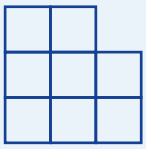
The area of a farm or a large city park is measured in hectares.

- A square grid paper is made up of identical squares with sides that are 1 cm long. The area of each such square is 1 cm^2.
- If a shape is drawn on 1-centimetre grid paper, its area can be found by counting the number of squares that the shape covers.

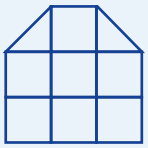
WORKED EXAMPLE 7

The following figures are drawn on centimetre grid paper. Find the area of each one.
(*Note*: The figures are not drawn to scale.)

a



b

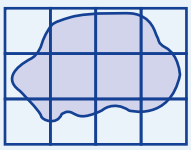


THINK	WRITE
a Count the squares. Remember to include the correct unit (cm^2) in the answer.	**a** 8 cm^2
b Some of the squares are cut in half by the diagonal line. It takes two of these to make up one square centimetre. Count the squares. Remember to include the correct unit (cm^2) in the answer.	**b** 8 cm^2

- If a square of the grid paper is not completely covered by the shape, use the following rule to obtain an estimate of the area:
  - if more than half the square is covered, count it as a full square
  - if less than half the square is covered, do not count it at all.

WORKED EXAMPLE 8

Estimate the shaded area in the following diagram, if the figure is drawn on centimetre grid paper.



THINK

1 Tick the squares that are more than half covered and count them.

2 State the answer.

WRITE

6 cm^2

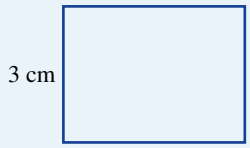
9.5.2 Finding the area of a rectangle

- The area, A, of a rectangle is given by the formula $A = lw$, where l is the rectangle's length and w is its width.
- The area, A, of a square is given by the formula $A = l^2$, where l is the length of the square's side.

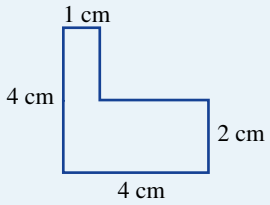
WORKED EXAMPLE 9

Find the area of the following shapes.

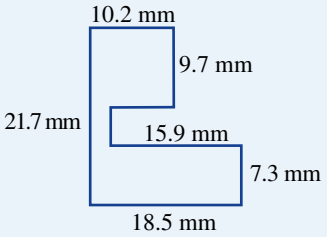
a



b



c



THINK

a 1 Write the formula for the area of a rectangle.

2 Substitute the value 4 for l and 3 for w.

3 Calculate the result. Remember to answer in the correct unit (cm^2).

b 1 Divide the shape into two rectangles.

2 Calculate the area of each rectangle separately by substituting the correct values of l and w into the formula $A = lw$.

3 Add the two areas. Remember to answer in the correct unit (cm^2).

c 1 Divide the shape into three rectangles.

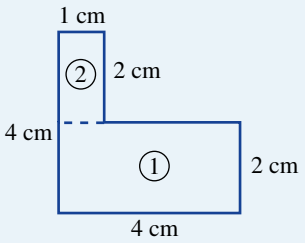
WRITE

a $A = lw$

$A = 4 \times 3$

$= 12 \text{ cm}^2$

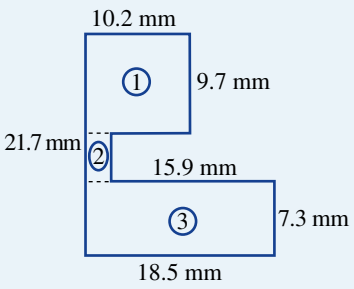
b



Area of rectangle 1 $= l \times w$
$= 4 \times 2$
$= 8 \text{ cm}^2$
Area of rectangle 2 $= l \times w$
$= 2 \times 1$
$= 2 \text{ cm}^2$

Area of shape = area of rectangle 1
+ area of rectangle 2
$= 8 \text{ cm}^2 + 2 \text{ cm}^2$
$= 10 \text{ cm}^2$

c



2 Calculate the area of each rectangle separately by substituting the correct values of l and w into the formula $A = lw$.

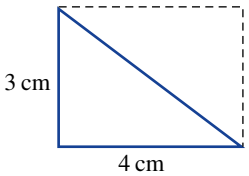
$$\begin{aligned}\text{Area of rectangle 1} &= l \times w \\ &= 10.2 \times 9.7 \\ &= 98.94 \text{ mm}^2 \\ \text{Area of rectangle 2} &= l \times w \\ &= (21.7 - 9.7 - 7.3) \\ &\quad \times (18.5 - 15.9) \\ &= 4.7 \times 2.6 \\ &= 12.22 \text{ mm}^2 \\ \text{Area of rectangle 3} &= l \times w \\ &= 18.5 \times 7.3 \\ &= 135.05 \text{ mm}^2\end{aligned}$$

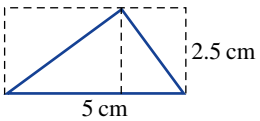
3 Add the three areas. Remember to answer in the correct unit (mm^2).

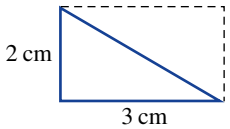
$$\begin{aligned}&\text{Area of shape} \\ &= \text{Area 1} + \text{Area 2} + \text{Area 3} \\ &= 98.94 \text{ mm}^2 + 12.22 \text{ mm}^2 \\ &\quad + 135.05 \text{ mm}^2 \\ &= 246.21 \text{ mm}^2\end{aligned}$$

9.5.3 Finding the area of a triangle

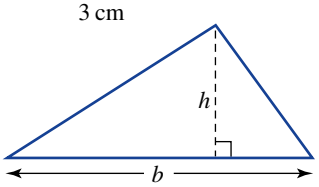
- Triangles can be formed by cutting rectangles in half.







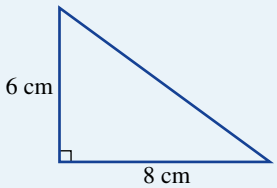
- The area of a triangle is equal to half the area of a rectangle of the same base length and height.
- The area, A, of a triangle is given by the formula $A = \frac{1}{2}bh$, where b is the length of its base and h is its height.



- The base and the height of a triangle are **perpendicular** (at right angles) to each other.
- The height of a triangle could be located inside the triangle, outside the triangle, or as one of the sides of the triangle (if the triangle is right-angled).

WORKED EXAMPLE 10

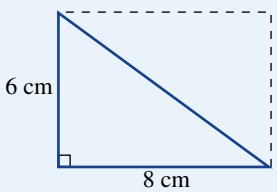
Find the area of the following shape.



THINK

1 Draw an imaginary rectangle that contains the triangle. The rectangle should have the same base length and height as the triangle. Notice that the triangle forms half of the rectangle.

WRITE



2 Use the formula $A = lw$ to find the area of this rectangle.

$A = lw$
$= 8 \text{ cm} \times 6 \text{ cm}$
$= 48 \text{ cm}^2$

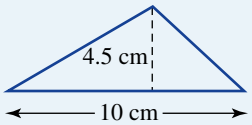
3 Halve the area of the rectangle to get the area of the triangle. Remember to answer in the correct unit (cm^2). *Note*: Formula $A = \frac{1}{2}bh$ could be used.

Area of triangle $= \frac{1}{2}$ of area of rectangle
Area of triangle $= \frac{1}{2} \times 48$
$= 24 \text{ cm}^2$

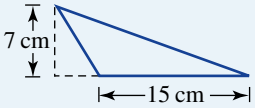
WORKED EXAMPLE 11

Find the area of each of these triangles.

a



b



THINK	WRITE
a 1 Write the formula for the area of a triangle.	**a** $A = \frac{1}{2}bh$
2 Identify the values of b and h.	$b = 10, h = 4.5$
3 Substitute the values of b and h into the formula.	$A = \frac{1}{2} \times 10 \times 4.5$
4 Evaluate. (Since one of the values is even, halve it first to make calculations easier.) Remember to include the correct unit (cm^2).	$= 5 \times 4.5$ $= 22.5 \text{ cm}^2$
b 1 Write the formula for the area of a triangle.	**b** $A = \frac{1}{2}bh$
2 Identify the values of b and h.	$b = 15, h = 7$
3 Substitute the values of b and h into the formula.	$A = \frac{1}{2} \times 15 \times 7$
4 Evaluate. (Since neither value is even, multiply 15 and 7 first, and then divide by 2.) Remember to include the correct unit (cm^2).	$= \frac{1}{2} \times 105$ $= 52.5 \text{ cm}^2$

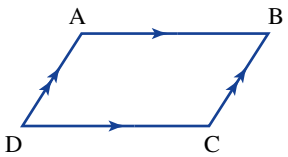
9.5.4 Area of a parallelogram

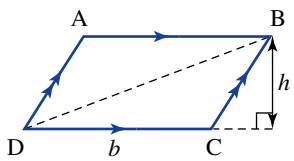
- A **parallelogram** is a quadrilateral such that each pair of opposite sides is parallel.
- To calculate the area of the parallelogram, first draw a diagonal from B to D to form two triangles, ABD and BDC. Label the base b, and the perpendicular height, h.

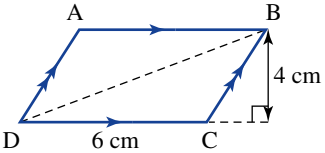
  Then find the area of the two triangles. The area of triangle BCD $= \frac{1}{2}b \times h$ and the area of triangle ABD $= \frac{1}{2}b \times h$.

  So, the area of the parallelogram ABCD $= \frac{1}{2}b \times h + \frac{1}{2}b \times h$
  $= b \times h$

  For example, the area of the following parallelogram is:
  ABCD $= b \times h = 6 \times 4 = 24 \text{cm}^2$.







learnON RESOURCES — ONLINE ONLY

- **Complete this digital doc:** SkillSHEET: Area units (doc-6512)
- **Complete this digital doc:** SkillSHEET: Area of figures drawn in one-centimetre grid paper (doc-6513)
- **Complete this digital doc:** SkillSHEET: Area of rectangles (doc-6514)
- **Complete this digital doc:** SkillSHEET: Area of triangles (doc-6515)
- **Try out this interactivity:** Area (int-0005)
- **Try out this interactivity:** Area of a triangle (int-2350)
- **Complete this digital doc:** Spreadsheet: Area of a rectangle (doc-1876)
- **Complete this digital doc:** Spreadsheet: Area of a triangle (doc-1877)

Exercise 9.5 Area

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1, 2, 3, 4, 5, 6, 8a–f, 9, 10, 11, 12, 13, 14, 15, 16, 17, 24, 25, 28, 29, 33, 36	Questions: 6, 8e–h, 10, 11, 13, 18, 19, 20, 27, 30, 31, 33, 34, 36	Questions: 7, 8e–j, 11, 21, 22, 23, 24, 25, 26, 30, 31, 32, 33, 34, 35, 36, 37

Individual pathway interactivity: int-4358

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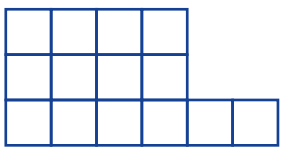
To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note*: Question numbers may vary slightly.

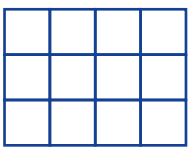
Fluency

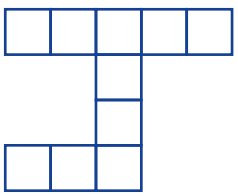
1. Which unit would be most suitable to measure the following areas? Choose from mm^2, cm^2, m^2, ha or km^2.

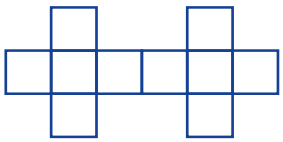
a. A computer screen
b. A sheet of A3 paper
c. Melbourne Cricket Ground
d. A shirt button
e. The Melbourne metropolitan area
f. A compact disc
g. A house block
h. A room
i. Alaska
j. A basketball court
k. A dairy farm
l. The area inside the letter D

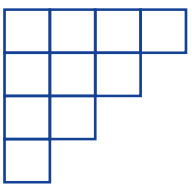
2. WE7a The following figures are drawn on centimetre grid paper. Find the area of each one.

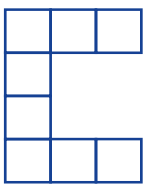
a.


b.


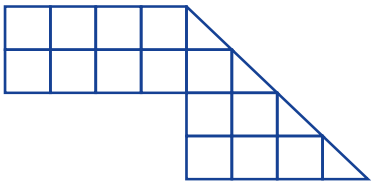
c.


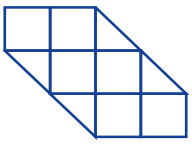
d.


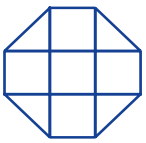
e.


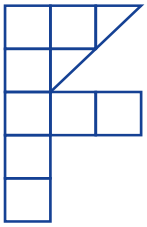
f.


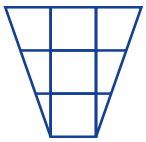
3. WE7b Find the area of the following figures, which are drawn on centimetre grid paper.

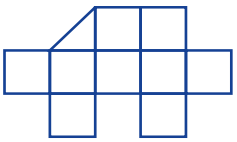
a.


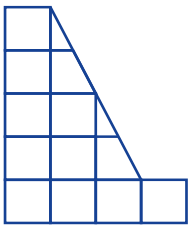
b.


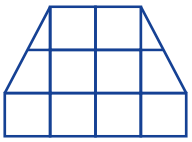
c.


d.


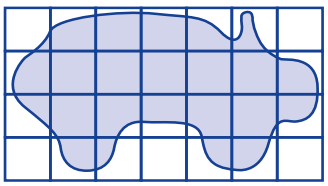
e.


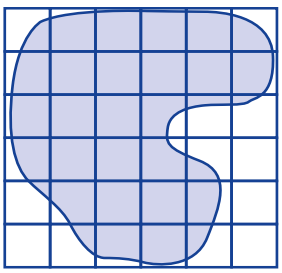
f.


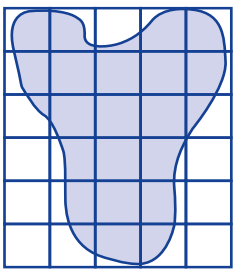
g.


h.


4. WE8 Estimate the shaded area of each of the following figures if each is drawn on centimetre grid paper.

a.


b.


c.


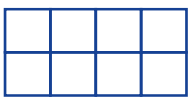
5. Four rectangles are drawn on centimetre grids that follow.

i. State the length and width of each rectangle.

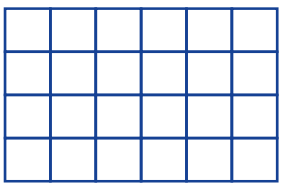
ii. Use your answers to part **i** to calculate the area of each rectangle.

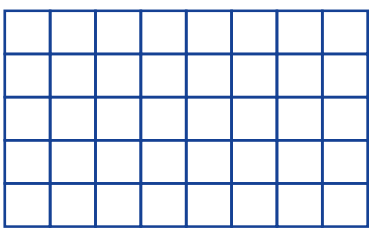
iii. Count the number of 1-cm^2 squares to find the area of each rectangle.

iv. Compare your answers to parts **ii** and **iii**.

a.


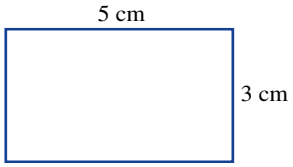
b.

c.


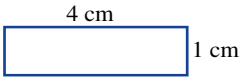
d.


6. WE9a Find the area of the following rectangles. (*Hint:* Use the formula $A = lw$.)

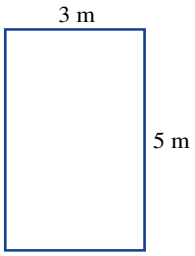
a.



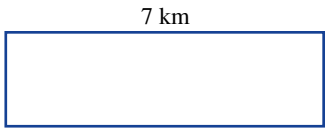
b.



c.



d.



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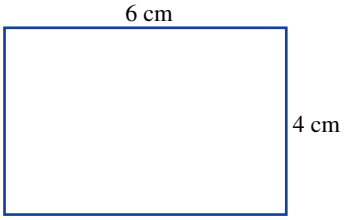

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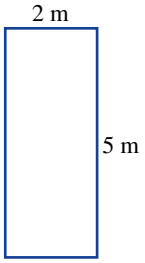
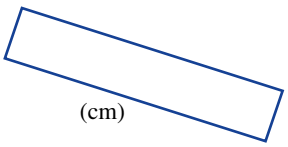


7. Accurately measure the length and width of each rectangle in the units specified in brackets. Hence, calculate the area of the rectangle.

a.

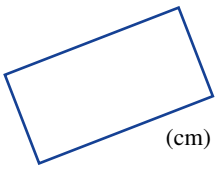
b.



c.

d.

e.



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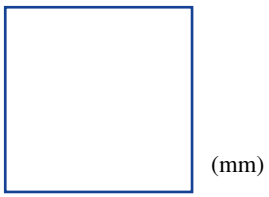
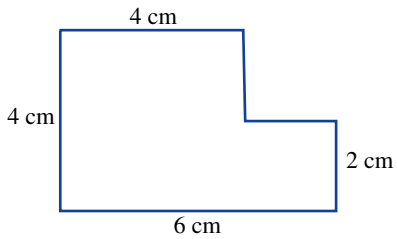
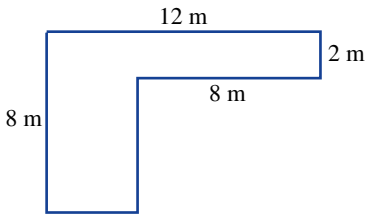


8. WE9b, c Find the area of the following shapes. (*Hint:* Divide the shapes into rectangles and squares before using the formula $A = lw$.)

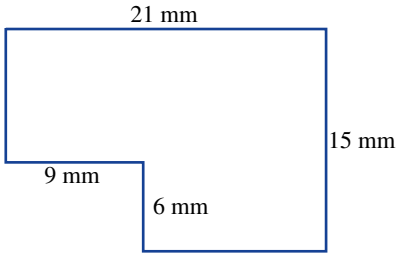
a.



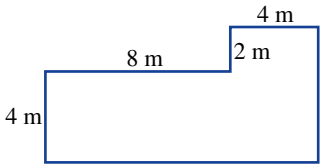
b.



c.



d.



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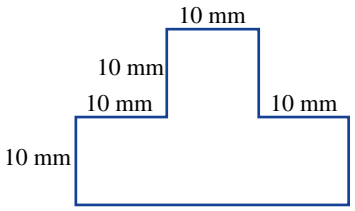

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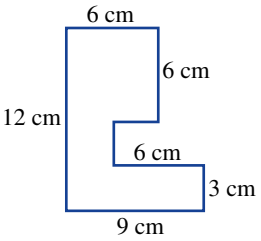
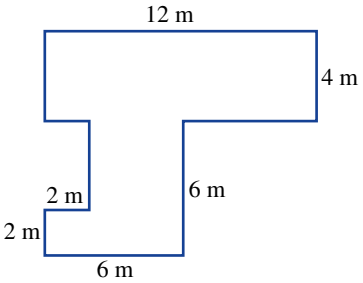


g.



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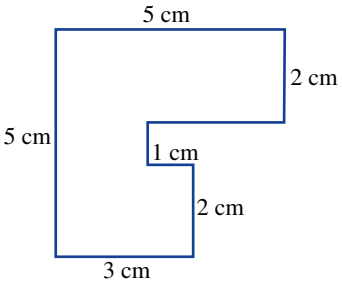

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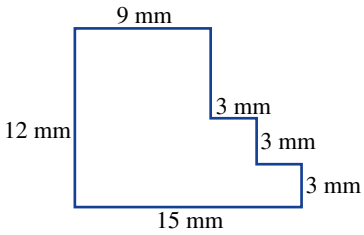
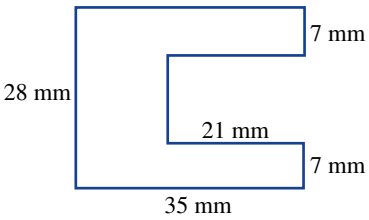
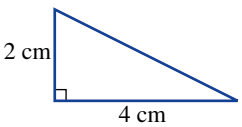


j.

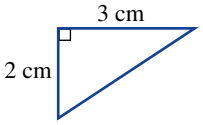


9. WE10 Find the area of the following triangles.

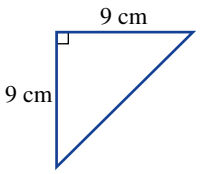
a.



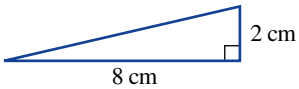
b.



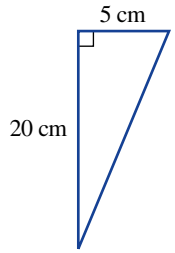
c.



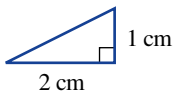
d.



e.

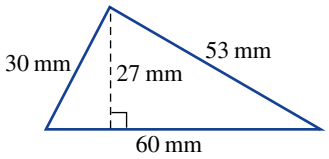


f.



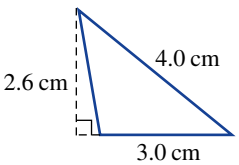
10. MC **a.** The height of the following triangle is:

A. 60 mm **B.** 30 mm **C.** 27 mm
D. 53 mm **E.** 49 mm



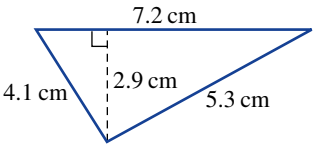
b. The height of the following triangle is:

A. 2.6 cm **B.** 4.0 cm **C.** 3.0 cm
D. 4.2 cm **E.** 3.8 cm



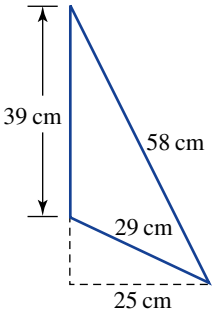
c. The base length of the following triangle is:

A. 4.1 cm **B.** 2.9 cm **C.** 5.3 cm
D. 7.2 cm **E.** 7.0 cm



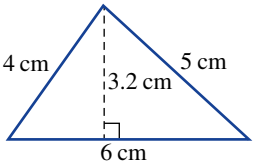
d. The base length of the following triangle is:

A. 39 cm
B. 58 cm
C. 29 cm
D. 25 cm
E. 43 cm

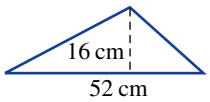


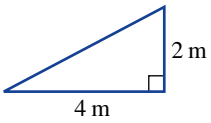
e. The area of the following triangle is:

A. 19.2 cm^2
B. 9.6 cm^2
C. 12 cm^2
D. 15 cm^2
E. 24 cm^2



11. WE11 Find the area of each of these triangles.

a.


b.


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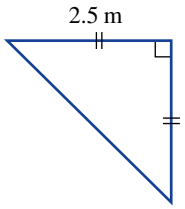

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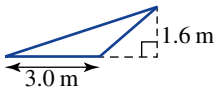

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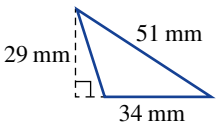
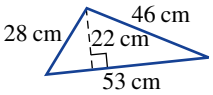
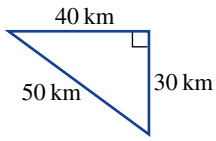


f.


g.


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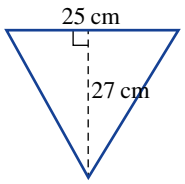

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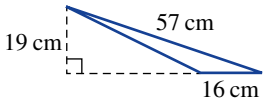
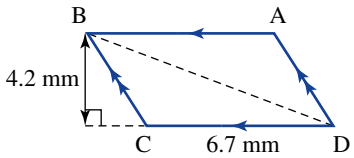


12. Determine the area of the following parallelogram.

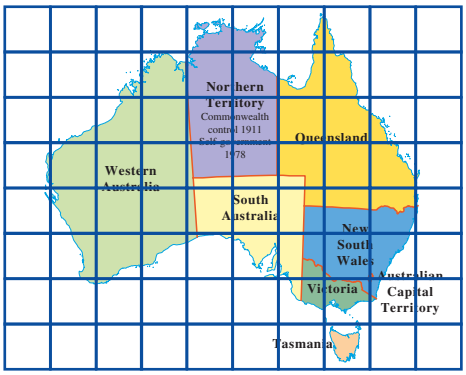


13. The following table gives the bases, heights and areas of various parallelograms. Complete the table.

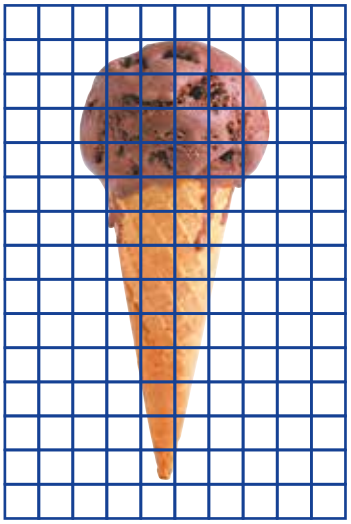
	Base	Height	Area
a	5 mm	3 mm	
b	8 cm		56 cm^2
c		5.2 m	52 m^2
d	6 km	2 km	
e	4 cm		1200 mm^2
f	4.9 m	8.7 m	

Understanding

14. a. Estimate the area of Australia if a 470 km grid is over it as shown.
Note: The area of each square of the grid is 470 km × 470 km or 220 900 km^2.



b. Estimate the area of the chocolate scoop in this picture of an ice-cream if a centimetre grid is over it as shown.



15. One of the largest open-air shopping centres is the Ala Moana centre in Honolulu, Hawaii, USA with more than 200 shops covering an area of 20 hectares. How many square metres is this?

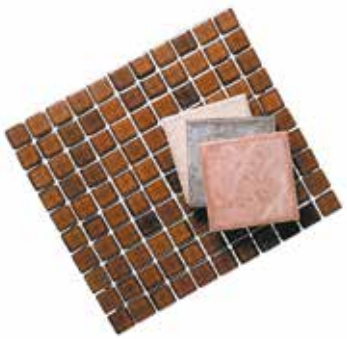
16. The Taupo volcanic eruption, in New Zealand nearly 2000 years ago, is estimated to have flattened an area of 16 000 km^2.
a. How many square metres is this?
b. How many hectares is this?

17. How many square metres of carpet are needed to carpet a rectangular room of length 5 m and width 3.5 m?

18. Find the area of material (cm^2) needed to make a rectangular table napkin 32 cm long and 25 cm wide.

19. Find the area of material needed to make a rectangular rug which is 4.2 m long and 230 cm wide. (Give your answer in square metres.) (*Hint:* Convert the width measurement into metres first.)

20. Toby is tiling his bathroom with ceramic tiles that cost $35.20 per box.
a. How many square metres of tiles will he need, if the rectangular room has a width of 2.5 m and a length of 3 m?
b. How many boxes of tiles should he order if each box contains enough tiles to cover 0.5 m^2?
c. What will be the cost of the tiles?



21. Jane is a landscape gardener who is laying a new lawn. The rectangular lawn is 13 m long and 8 m wide. How many square metres of turf should Jane order? What is the total cost of the turf if it costs $12.50 per square metre?

22. Alana, who works for Fast Glass Replacements, has been asked for a quote to replace three windows. Each window is 1.8 m long and 0.8 m wide. What price should Alana quote if the glass costs $27 per square metre? (Include a delivery cost of $25 in the quote.)

23. Find the total area of the cattle station shown in Figure 1, which has 3 large paddocks. (Give your answer in square kilometres.)

24. Rectangular laminate sheets are 1.8 m long and 0.9 m wide. How many sheets would be needed to cover 4.8 m^2 of bench space in a kitchen?

25. Calculate the total floor area of a concrete slab for a house as shown in Figure 2.

26. How many square centimetres of cardboard are needed to make the cube with an edge length of 12 cm shown in Figure 3?

27. Calculate the approximate area of the triangular mainsail when the sail is flat. Use the dimensions shown in Figure 4.

FIGURE 1

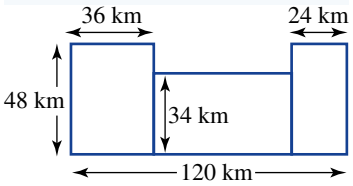


FIGURE 2

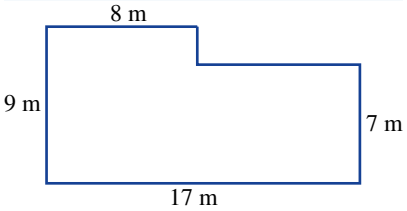


FIGURE 3

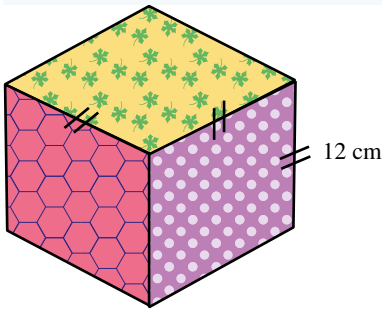
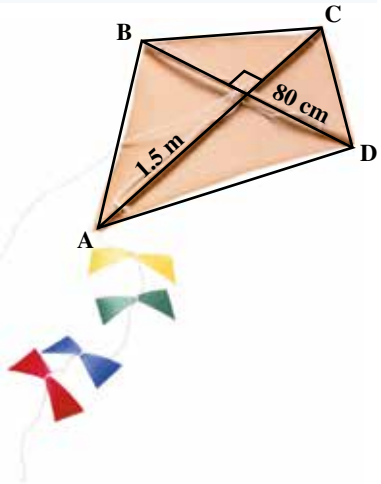


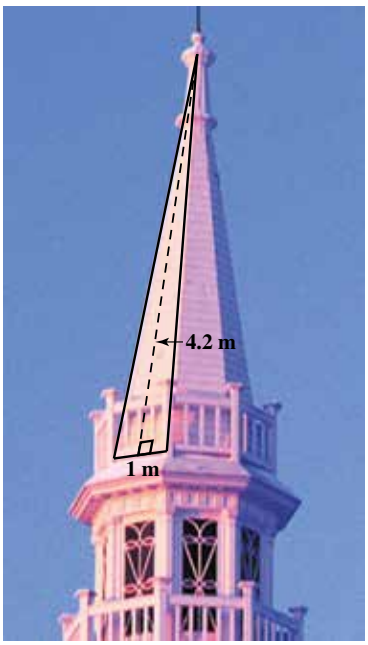
FIGURE 4

28. Find the area of cloth required to make the kite in Figure 5, given that the length BD is 80 cm and the length AC is 1.5 m.

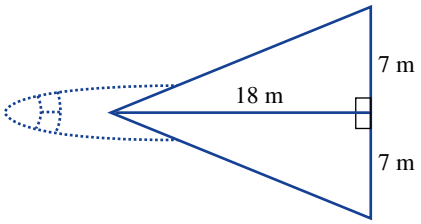
FIGURE 5



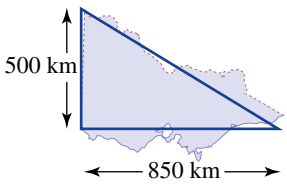
29. A church spire has six identical triangular faces that have the dimensions shown in the following figure. What area of copper roofing would be required to cover all six faces of the spire?



30. Find the total wing area of the following delta-winged jet aircraft.



31. The following triangle can be used to make an estimate of the area of the state of Victoria.



a. Find the area of the triangle shown.
b. Compare your estimate with the correct value from your atlas.

Reasoning

32. A floor tiler charged $640.00 to tile a rectangular room. Her next job is to tile the floor of a rectangular room twice as long and twice as wide. How much should she charge for the larger room? (The answer is not $1280.00.) Justify your answer.

33. A rectangle has an area of 36 cm^2 and a perimeter of 26 cm. Each side of the rectangle is a whole number of centimetres. What is the length and width of this rectangle? Show your working.

34. What is the perimeter of a square that has the same area as a rectangle with length 18 mm and width 8 mm? Show your working.

Problem solving

35. Glen wants to pave his back courtyard. His courtyard is 12 m × 12 m. He has a choice of 10 cm × 25 cm clay pavers that cost $2.30 each, or concrete pavers that are 70 cm × 70 cm and cost $42 each. What would the cheaper option be?

36. Geoff wants to establish a rectangular vegetable garden bed on his farm. His 20-m long work shed will act as one of the boundaries. (*Note:* The garden will not necessarily be this long.) He has 24 metres of fencing which he plans to use to fence the other three sides. How could Geoff use this fencing to enclose the largest possible area?

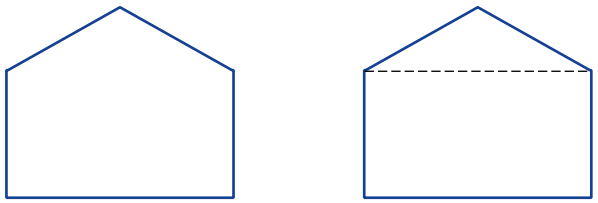
Reflection

Name some objects from your immediate surroundings whose area you can find using the formulae for the areas of rectangles, squares and triangles.

9.6 Area of composite shapes, using addition and subtraction

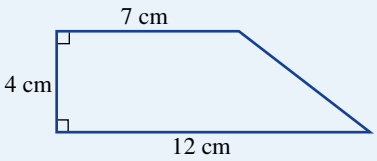
9.6.1 Area of composite shapes, using addition and subtraction

- **Composite shapes** can be divided into parts, each of which is a simple shape (such as, for example, a rectangle or a triangle), whose areas are easy to find using familiar formulae.
- The area of a composite shape can be found by adding the areas of each part of the divided shape. For example, the area of the following pentagon can be found by finding and then adding the areas of the triangle and rectangle.



WORKED EXAMPLE 12

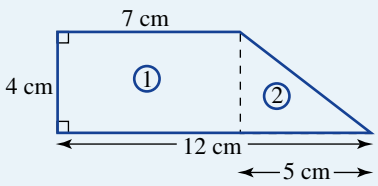
Find the area of the following shape.



THINK

1 Divide the shape into a rectangle and a triangle.

WRITE

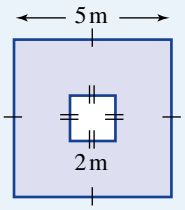


2 Use the formula $A = lw$ to find the area of the rectangle.	Area 1 $= lw$ $= 7 \times 4$ $= 28\ \text{cm}^2$
3 Use the formula $A = \frac{1}{2}bh$ to find the area of the triangle.	Area 2 $= \frac{1}{2}bh$ $= \frac{1}{2} \times 5 \times 4$ $= 10\ \text{cm}^2$
4 Add the two areas to get the total area.	Total area $= A_1 + A_2$ $= 28 + 10$ $= 38\ \text{cm}^2$

- In many situations the area of a given shape can be found by subtracting individual areas from each other.
- If a shaded area is required, the total area is found first and then the area of the 'unshaded' part(s) is subtracted from it.

WORKED EXAMPLE 13

Find the shaded area in the following figure.



THINK	WRITE
1 The figure shown is a large square from which a small square has been removed. Write the formula for the area of the square.	Area of a square $= l^2$
2 Calculate the area of the large square first.	Large square: $A = 5^2$ $= 25\ \text{m}^2$
3 Calculate the area of the small square.	Small square: $A = 2^2$ $= 4\ \text{m}^2$
4 Now find the shaded area by subtracting the area of the small square from that of the large one.	Shaded area $= 25 - 4$ $= 21\ \text{m}^2$

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Complete this digital doc: Spreadsheet: Area (doc-1878)

Exercise 9.6 Area of composite shapes, using addition and subtraction

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1, 3, 4, 5, 7, 10, 14	Questions: 1, 3, 4, 5, 6, 8, 10, 11, 12, 14, 15	Questions: 2, 4, 6, 9, 10, 11, 12, 13, 14, 15

Individual pathway interactivity: int-4359

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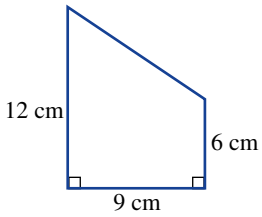
To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note*: Question numbers may vary slightly.

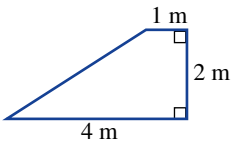
Fluency

1. WE12 Find the area of the following composite shapes.

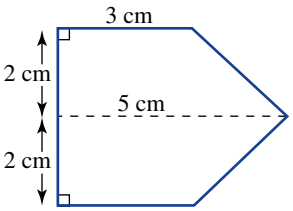
a.



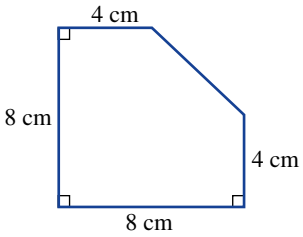
b.



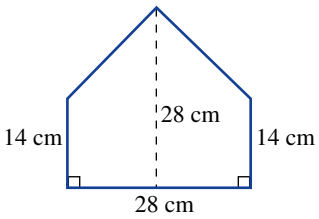
c.



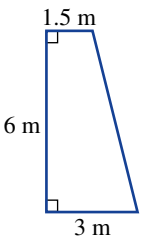
d.



e.

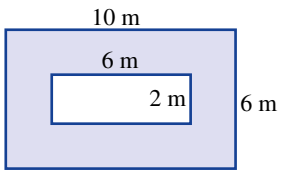


f.

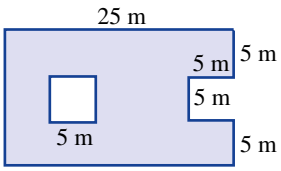


2. WE13 Find the shaded area of the following shapes.

a.

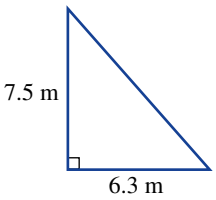


b.



Understanding

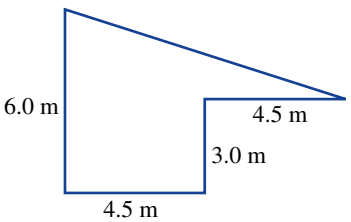
3. The sail shown in the following figure has been constructed for the sailing vessel *Nautilus*.



a. Find the area of cloth required to construct the sail.

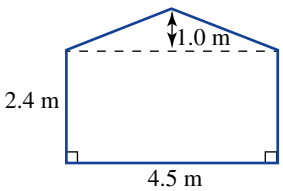
b. What is the cost of the sail if the cloth costs $12.00 per square metre?

4. a. Find the area of carpet needed for the following room.

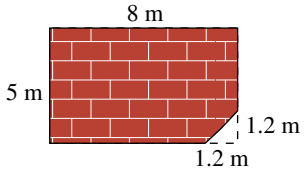


b. What would the carpet for the room cost if it is sold for $25 per square metre?

5. Find the area of the following garage wall.

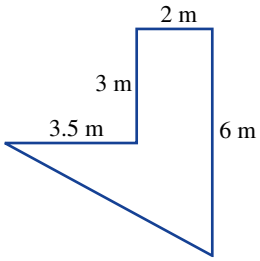


6. Determine the area of bricks needed to cover the following courtyard.



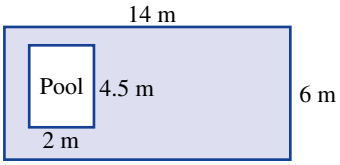
7. A sheet of A4 paper is approximately 30 cm long and 21 cm wide. Find the area of the triangle formed by cutting an A4 sheet in half diagonally.

8. Find the area of the lobby in the following diagram.

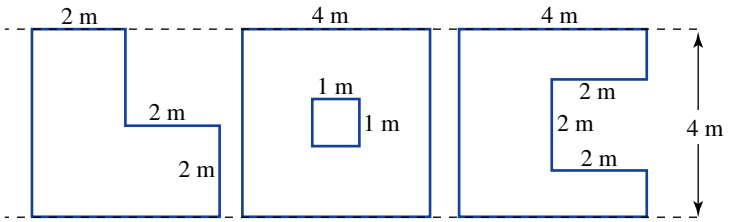


9. Michael is paving a rectangular yard, which is 15.5 m long and 8.7 m wide. A square fishpond with side lengths of 3.4 m is in the centre of the yard. What will be the cost of paving Michael's yard, if the paving material costs $17.50 per square metre?

10. Members of the Lee family want to pave the area around their new swimming pool. The pool is set into the corner of the yard as shown in the following figure. Find the area of paving (in square metres) required to cover the yard around the pool (shaded in the diagram).

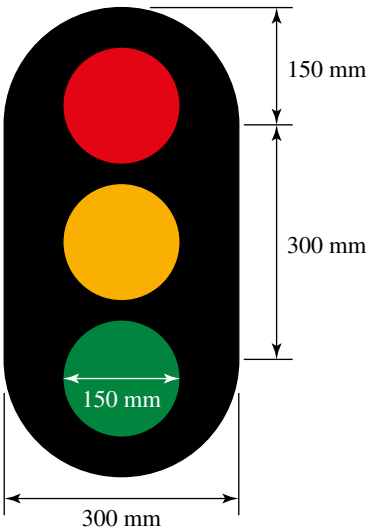


11. In order to construct an advertising sign, the following letters are cut from plastic sheets. Find the total area of plastic required to make all 3 letters.



Reasoning

12. The stylised diagram of a set of traffic lights shown has been constructed using basic shapes.



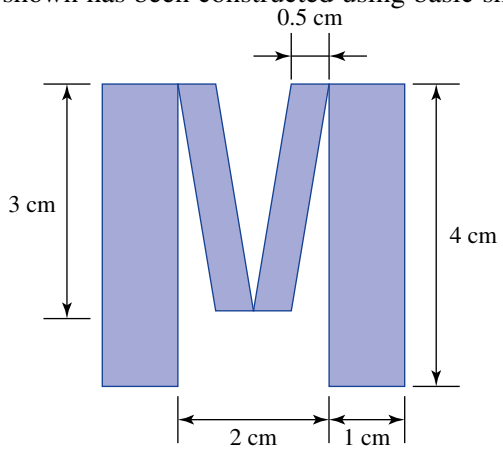
a. State the shapes used to draw the diagram and their dimensions in cm.

b. Explain the steps required in calculating the black area of the traffic lights shown.

c. If the area of one of the large semicircles is closest to 353 cm^2, and the area of one of the small coloured circles is closest to 177 cm^2, calculate the area of the black surface. State the answer in cm^2

d. Explain why it is more appropriate to express the area in cm^2 rather than mm^2.

13. The diagram of the letter M shown has been constructed using basic shapes.

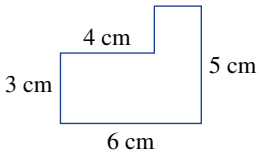


a. Explain two methods of calculating the area of the letter using:
i. addition **ii.** subtraction.

b. Using your preferred method calculate the area of the letter.

Problem solving

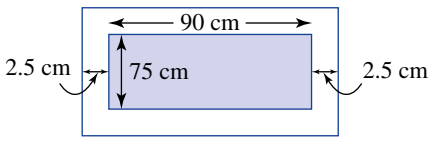
14. a. Calculate the area of the following figure. Show all work.



b. Calculate the area of the figure using an alternative method. Show all work.

c. Explain the differences between the two methods.

15. Ellen wants to frame her cross-stitch work. Her cross-stitch work is 90 cm long and 75 cm wide. The frame's border measures 2.5 cm wide (see the following figure). One company charges $120 for the frame. What is the cost per square centimetre? Round off your answer to 2 decimal places.

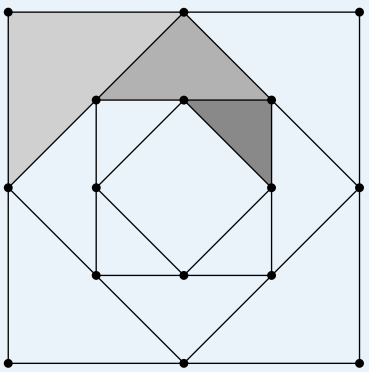


Reflection

How will you decide whether to find the area of a given composite shape by addition or by subtraction?

CHALLENGE 9.2

A square of side length 1 metre is made up of smaller internal squares that are formed by joining the midpoints of the outer square as shown in the following diagram. How much of the square does the shaded area represent?



9.7 Volume

9.7.1 Volume

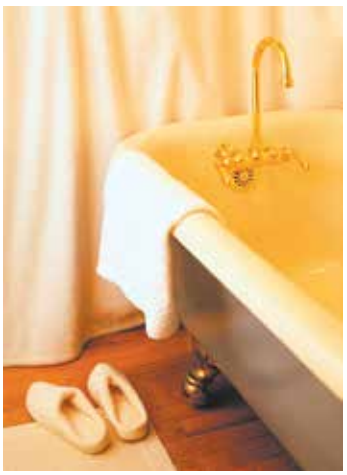
- The **volume** of a 3-dimensional object is the amount of space it occupies.
- Volume is measured in cubic units such as mm^3, cm^3 and m^3.
- A cubic centimetre (cm^3) is the space occupied by a cube with sides of 1cm.

A sugar cube has a volume of about $1\,cm^3$.

The volume of an iPad mini is about $200\,cm^3$.

A cubic metre (m^3) is a space occupied by a cube with sides of 1 m.

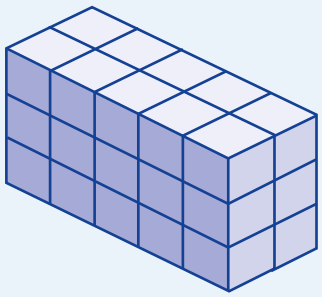
A bathtub can hold about $\frac{1}{2}\,m^3$ of water and each of these recycling bins would hold about $1\,m^3$.



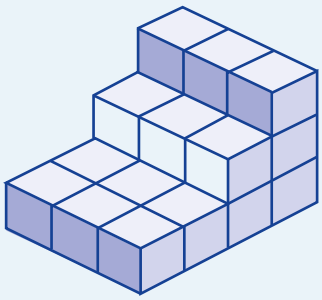
WORKED EXAMPLE 14

How many cubic centimetres are in each solid shape? (Each small cube represents $1\,cm^3$.)

a



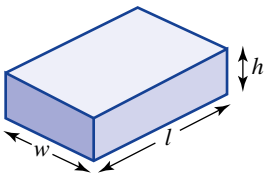
b



THINK	WRITE
a 1 Count the cubes. There are 10 cubes in each layer. There are three layers altogether.	**a** Volume $= 10\ \text{cm}^3 \times 3$
2 Give the answer in cubic centimetres.	$= 30\ \text{cm}^3$
b 1 Count the cubes. There are 12 cubes in the first layer. There are 6 cubes in the second layer. There are 3 cubes in the third layer.	**b** Volume $= 12\ \text{cm}^3 + 6\ \text{cm}^3 + 3\ \text{cm}^3$
2 Give the answer in cubic centimetres.	$= 21\ \text{cm}^3$

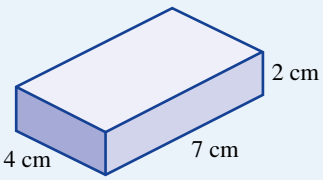
9.7.1 Finding the volume of a rectangular prism

- The volume of a rectangular prism is given by the formula $V = lwh$, where l is the prism's length, w is its width and h is its height.
- The volume of a rectangular prism can also be written as $V = Ah$, where A is the area of the prism's base and h is its height.



WORKED EXAMPLE 15

Use the formula $V = lwh$ to calculate the volume of the following rectangular prism.



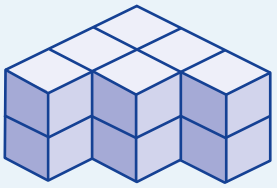
THINK	WRITE
1 Write the formula for volume.	$V = lwh$
2 Identify the length ($l = 7$ cm), width ($w = 4$ cm), and height ($h = 2$ cm) of the prism and substitute the values into the formula.	$= 7\ \text{cm} \times 4\ \text{cm} \times 2\ \text{cm}$
3 Calculate the result. Remember to write the answer using the correct unit (cm^3).	$= 56\ \text{cm}^3$

9.7.2 Finding the volume of other types of prisms and shapes

- The volume of any prism is given by the formula $V = Ah$, where A is the area of its base and h is its height.
- The base of the prism could be of any shape (a rectangle, a square, a triangle etc.).
- The height of the prism is always perpendicular (at right angles) to its base.

WORKED EXAMPLE 16

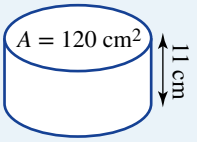
Find the volume of the following prism if each cube has a volume of 1 cm^3.



THINK	WRITE
1 Write the formula for the volume of a prism.	$V = Ah$
2 The area of the base (A) is the same as the area of the top layer which is 6 cm^2. The height is 2 cm.	$A = 6$ cm^2, $h = 2$ cm
3 Substitute the values of A and h into the formula $V = Ah$.	$V = 6$ cm^2 $\times$ 2 cm
4 Calculate the answer and give the units in cubic centimetres.	$= 12$ cm^3

WORKED EXAMPLE 17

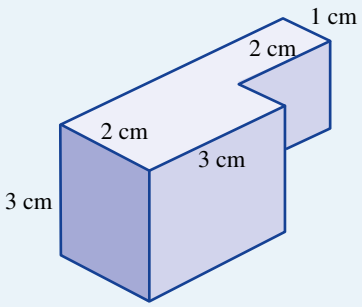
Calculate the volume of this shape.



THINK	WRITE
1 Write the formula for the volume of the shape.	$V = Ah$
2 The area of the base, A, is given (120 cm^2), the height is 11 cm.	$A = 120$ cm^2 and $h = 11$ cm
3 Substitute the values of A and h into the formula $V = Ah$.	$V = 120$ cm^2 $\times$ 11 cm
4 Calculate the answer and give the units in cubic centimetres.	$= 1320$ cm^3

WORKED EXAMPLE 18

Use the information given in the following diagram to calculate the volume of the prism.



THINK	WRITE
1 Write the formula for the volume of a prism.	$V = Ah$
2 The area of the base (A) can be found by adding the areas of the 2 rectangles on the top surface. The height is 3 cm.	Area of large rectangle $= lw$ $= 3$ cm $\times$ 2 cm $= 6$ cm^2 Area of small rectangle $= lw$ $= 2$ cm $\times$ 1 cm $= 2$ cm^2 Total area of base $= 6$ cm^2 + 2 cm^2 $= 8$ cm^2 So A $= 8$ cm^2 and $h = 3$ cm
3 Substitute the values of A and h into the formula $V = Ah$.	$V = 8$ cm^2 $\times$ 3 cm
4 Calculate the answer and give the units in cubic centimetres.	$= 24$ cm^3

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Complete this digital doc: SkillSHEET: Volume units (doc-6516)

Complete this digital doc: SkillSHEET: Volume of a solid constructed from cubic-centimetre blocks (doc-6517)

Exercise 9.7 Volume

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1, 2, 3, 5, 6, 7, 8, 11, 14, 15, 16, 17, 22, 28	Questions: 4, 5, 6, 7, 9, 10, 12, 16, 18, 19, 20, 21, 25, 27, 28	Questions: 5, 6, 8, 9, 10, 12, 13, 15, 19, 21, 22, 23, 24, 25, 26, 27, 29

Individual pathway interactivity: int-4360

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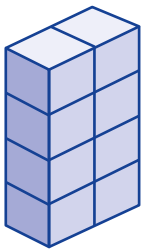
To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note*: Question numbers may vary slightly.

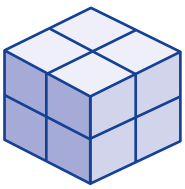
Fluency

1. WE14a How many cubic centimetres are in each solid shape? (Each small cube represents 1 cm^3.)

a.

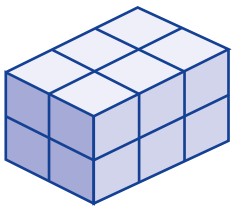


b.

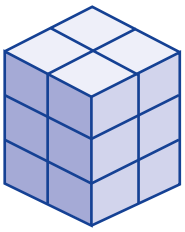


c.

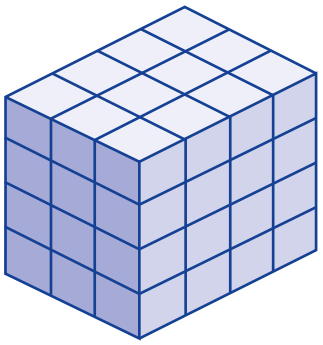
d.



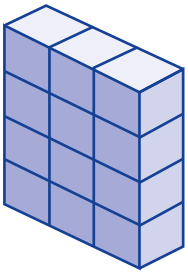
e.



f.



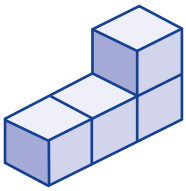
2. MC The volume of the following prism is:



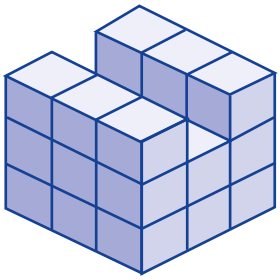
A. 4 cm^3
B. 12 cm^3
C. 1 cm^3
D. 8 cm^3
E. 19 cm^3

3. WE14b Find the volume of the following solids. (Each small cube represents 1 cm^3.)

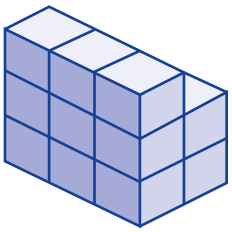
a.



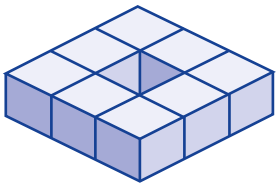
b.



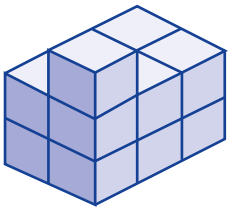
c.



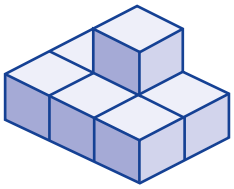
d.



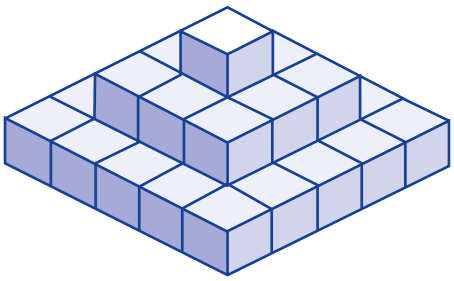
e.



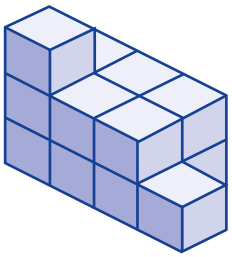
f.



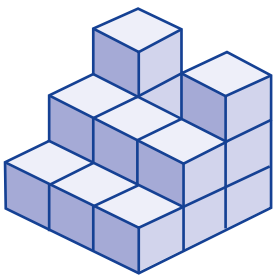
g.



h.



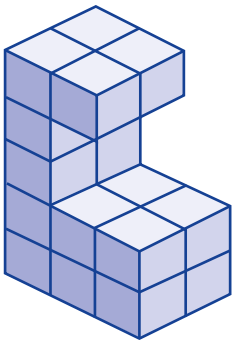
i.



4. MC The volume of the following prism is:

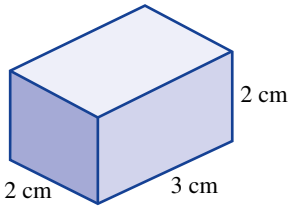
A. 20 cm^3
B. 16 cm^3
C. 10 cm^3
D. 22 cm^3
E. 3 cm^3

Note: Each cube has a volume of 1 cm^3.

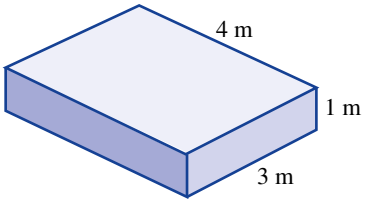


5. WE15 Use the formula $V = lwh$ to calculate the volume of the following rectangular prisms.

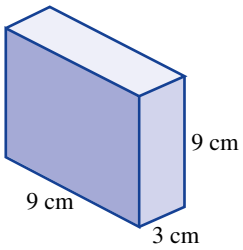
a.



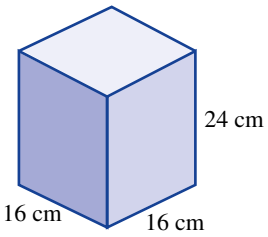
b.



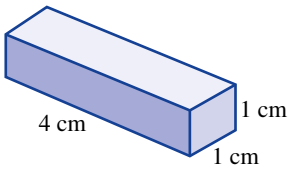
c.



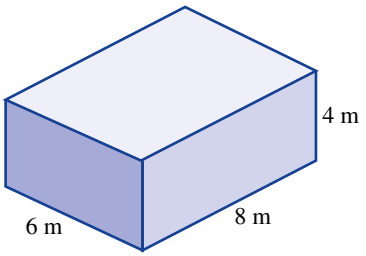
d.



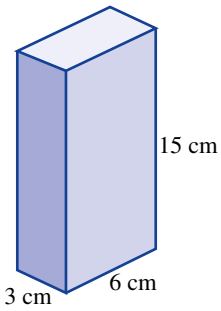
e.



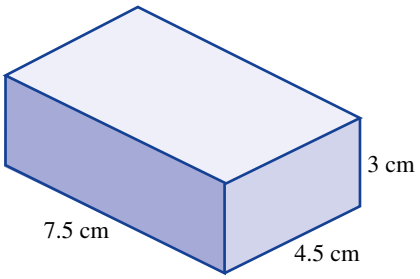
f.



g.

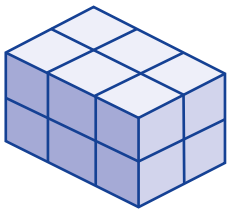


h.

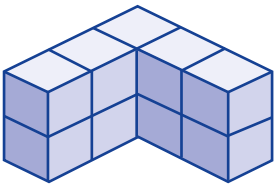


6. WE16 Find the volume of each of the following prisms by copying and completing the following table. (Each cube has a volume of 1 cm^3.)

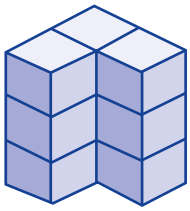
a.



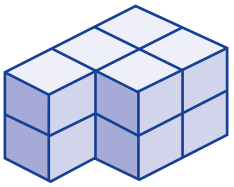
b.



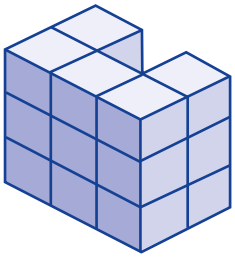
c.



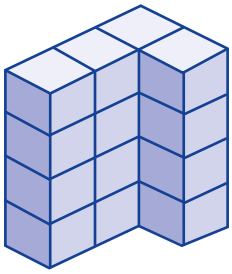
d.



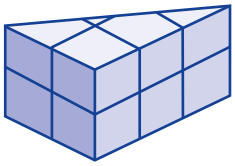
e.



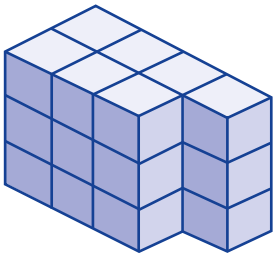
f.



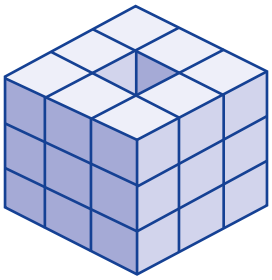
g.



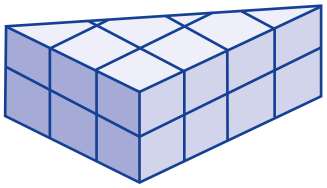
h.



i.



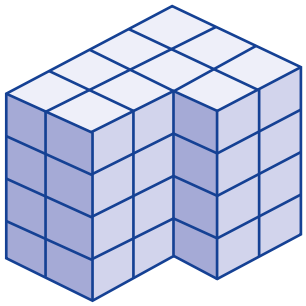
j.



Prism	Area of base (cm^2)	Height (cm)	Volume (cm^3)
a			
b			
c			
d			
e			
f			
g			
h			
i			
j			

7. MC The volume of the following prism (assuming each cube has a volume of 1 cm^3) is:

A. 10 cm^3
B. 20 cm^3
C. 30 cm^3
D. 40 cm^3
E. 80 cm^3



8. MC The volume of the following prism is:

A. 16 cm^3

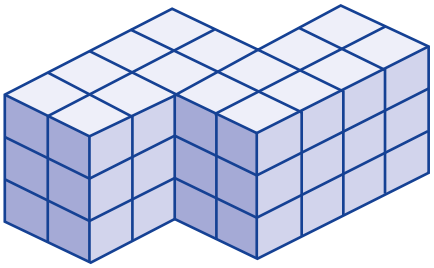
B. 48 cm^3

C. 32 cm^3

D. 40 cm^3

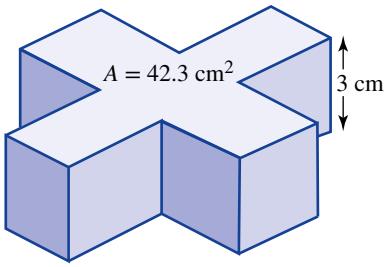
E. 160 cm^3

Each cube has a volume of 1 cm^3.

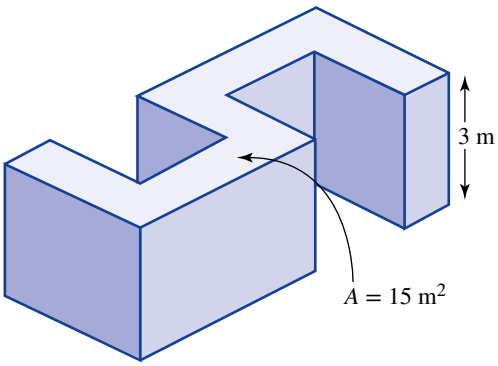


9. WE17 Calculate the volume of each of the following shapes.

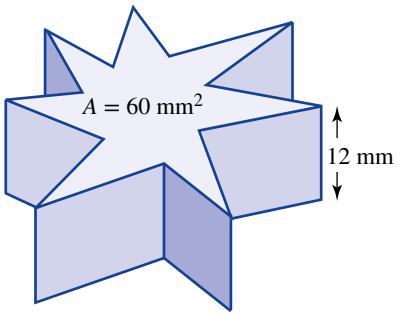
a.



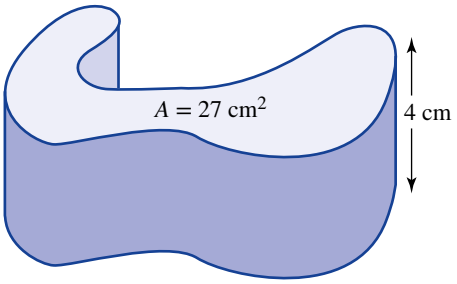
b.



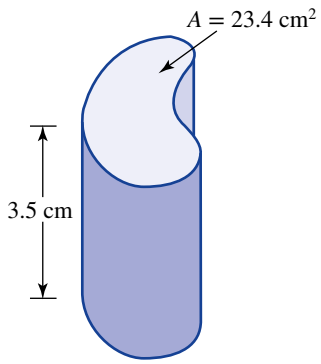
c.



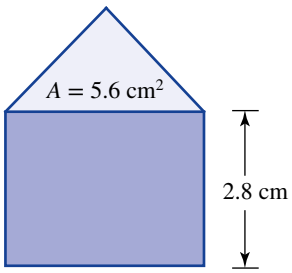
d.



e.

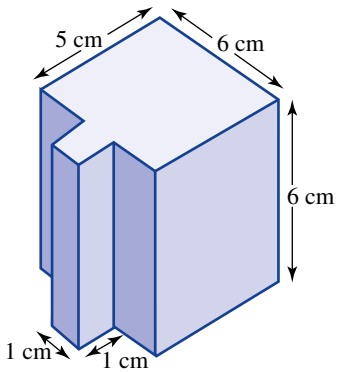


f.

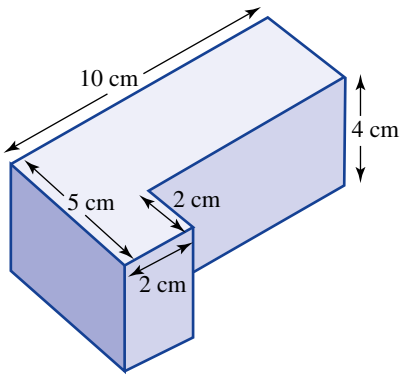


10. WE18 Calculate the volume of the following prisms.

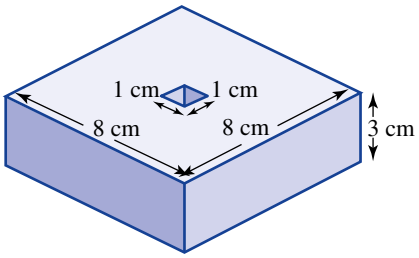
a.



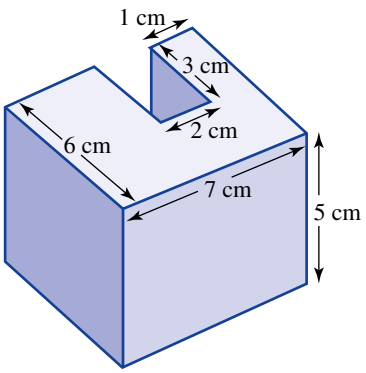
b.



c.

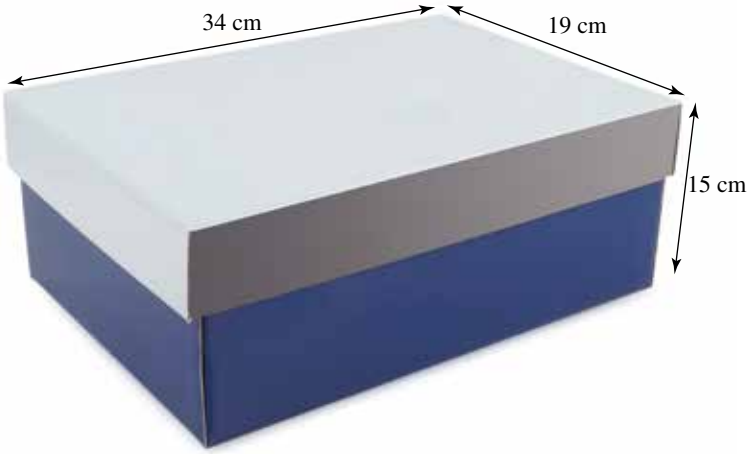


d.



Understanding

11. Find the volume of the following shoe box. (Give your answer in cubic centimetres, cm^3.)



12. The inside dimensions of the following refrigerator are shown. Find the volume available for food storage inside the refrigerator.

13. A rectangular prism has a length of 30 cm and a width of 15 cm. If its volume is 9000 cm^3, find the height of the rectangular prism.

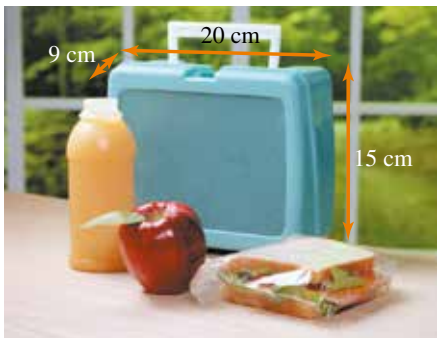
14. A rectangular prism has a length of 13 cm and a width of 17 cm. If its volume is 1989 cm^3, find the height of the rectangular prism.

15. A rectangular prism has a length of 10 cm and a width of 15 cm. If its volume is 1800 cm^3, find the height of the rectangular prism.

16. The lunch box shown is a rectangular prism.

a. Find the volume of the lunch box in cubic centimetres.

b. Change each measurement to millimetres and hence find the volume of the lunch box in cubic millimetres.



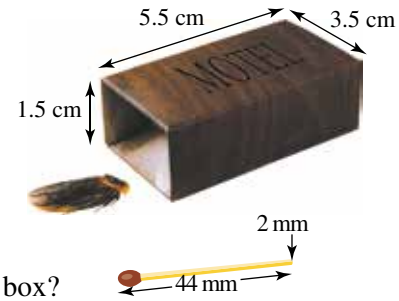
17. Find the volume of concrete (in cubic metres) which would be needed to make the base for a garage that is 6.5 m wide and 3 m long. The concrete base is 0.25 m deep.

18. a. Find the volume (in cubic centimetres) of the following matchbox that has been converted to a cockroach hotel.

b. Change each measurement to millimetres and hence find the volume of the matchbox in cubic millimetres.

c. Matches are rectangular prisms of length 44 mm, width 2 mm and height 2 mm. Find the volume of a match (in cubic millimetres). (Ignore the red substance on the end of each match.)

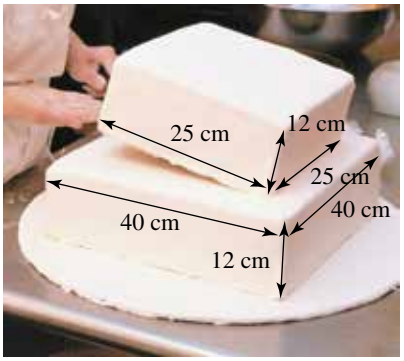
d. If a matchbox contains 50 matches, how much space is left in a full box?



19. How many cubic metres of water would be needed to fill a diving pool which has a length of 16 m, a width of 12 m and a depth of 4 m?

20. Heather wishes to cover a rectangular lawn with topsoil to a depth of 0.1 m. If the lawn is 24 m long and 17 m wide, what volume of soil (in cubic metres) should she order?

21. The fruitcake shown in the following figure is to be divided equally among 100 people at a wedding reception. What volume of cake (in cubic centimetres) will each guest receive?



22. Find the volume of clay in a house brick which has a length of 23 cm, a width of 11 cm and a height of 9 cm.

23. A swimming pool is rectangular and has a width exactly half its length. What volume of water would be needed to fill it if the swimming pool is 50 metres long and has a constant depth of 2 metres?

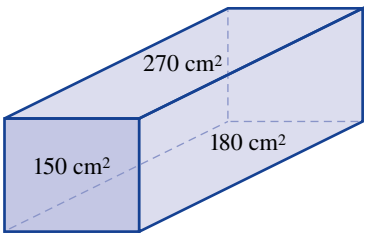
24. The following photograph shows a 'round' of cheese that has a base of area 320 cm^2 when whole. If the height of the cheese is 6 cm, find the following:

a. the volume of the whole cheese round

b. the number of equal segments that could be obtained if the round is cut into segments of 160 cm^3 in volume.

Reasoning

25. The areas of the three sides of a rectangular box are as shown in the figure. What is the volume of the box? Show all of your working.



26. A vase is shaped like a rectangular prism with a square base of length 11 cm. It has 2 litres of water poured into it. To what height (to 1 decimal place) does the water reach in the vase? Show your working. (*Hint*: 1 litre = 1000 cm^3.)

Problem solving

27. A new rectangular patio has been built on the end of the house. It measures 3.8 m by 1.9 m. You want to plant a garden that is 1.5 m wide around the patio on three sides. The garden beds are to be 0.5 metres deep. If you order 7.3 cubic metres, explain with a diagram whether enough soil has been ordered.

28. A big cargo box measures 12 m by 6 m by 5 m. How many small boxes of 40 cm by 25 cm by 10 cm can be put into the cargo box?

29. You have a piece of paper that measures 24 cm by 36 cm. You want to cut out a net that can be folded into a box without a lid. Using whole numbers, find the dimensions of the box which gives the maximum volume.

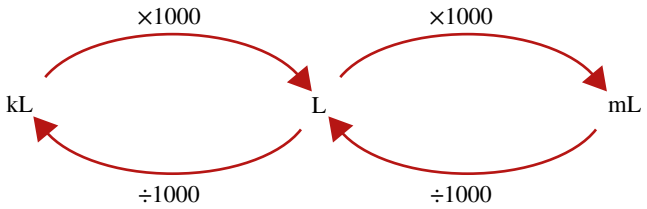
Reflection

What shapes can be classified as prisms?

9.8 Capacity

9.8.1 Capacity

- The **capacity** of a container is the volume of liquid that it can hold.
- Capacity can be measured in the same units as volume (e.g. cm^3 or m^3).
- Capacity is more commonly measured in millilitres (mL), litres (L) and kilolitres (kL).
- Cubic units can be converted to millilitres, litres and kilolitres as follows:
  $1\ cm^3 = 1\ mL$ $1000\ cm^3 = 1\ L$ $1\ m^3 = 1\ kL$
- It may be useful to know that $1\,000\,000\ cm^3 = 1\ m^3$
- The following chart can be used to convert between kilolitres, litres and millilitres.



WORKED EXAMPLE 19

Copy and complete the following unit conversions.
a **6 L =** _____ **mL** **b** **700 mL =** _____ **L** **c** **0.45 L =** _____ **cm^3**

THINK

a Check the conversion chart.
To convert litres to millilitres, multiply by 1000. Since 6 is a whole number, we do this by adding three zeros to the end of the number.

b Check the conversion chart.
To convert millilitres to litres, divide by 1000.
To do this, move the decimal point three places to the left.

c Check the conversion chart.
To convert litres to millilitres, multiply by 1000.
To do this, move the decimal point three places to the right.
Note that $1\ mL = 1\ cm^3$.

WRITE

a $6\ L = 6 \times 1000\ mL$
$= 6000\ mL$

b $700\ mL = 700 \div 1000\ L$
$= 0.7\ L$

c $0.45\ L = 0.45 \times 1000\ mL$
$= 450\ mL$
$= 450\ cm^3$

learnON RESOURCES — ONLINE ONLY

Complete this digital doc: Spreadsheet: Capacity unit conversions (doc-1879)
Complete this digital doc: WorkSHEET: Measurement II (doc-1854)

Exercise 9.8 Capacity

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1a–h, 2, 3, 5, 6, 7, 8, 9, 12, 13, 15, 17, 20, 23	Questions: 1g–n, 2, 4, 5, 8, 9, 10, 11, 16, 17 19, 20, 22, 23	Questions: 1k–r, 2, 4, 5, 10, 11, 12, 13, 14, 15, 16, 19, 20, 21, 22, 23

Individual pathway interactivity: int-4361

learnON ONLINE ONLY

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note*: Question numbers may vary slightly.

Fluency

1. WE19a, b Copy and complete the following unit conversions.

a. 2 L = _____ mL

b. 300 mL = _____ L

c. 13 kL = _____ L

d. 7000 mL = _____ L

e. 5500 mL = _____ L

f. 260 L = _____ kL

g. $2\frac{1}{2}$ L = _____ mL

h. 32 000 mL = _____ L

i. 55 mL = _____ L

j. 0.035 L = _____ mL

k. 420 L = _____ mL

l. 0.99 kL = _____ L

m. $1\frac{87}{100}$ L = _____ mL

n. 22 500 mL = _____ L

o. 350 mL = _____ L

p. $\frac{1}{10}$ L = _____ mL

q. 25 L = _____ kL

r. 0.007 L = _____ mL

2. WE19c Copy and complete the following unit conversions.

a. 750 cm^3 = _____ mL

b. 2500 m^3 = _____ kL

c. 800 cm^3 = _____ mL = _____ L

d. 40 000 cm^3 = _____ mL = _____ L

e. 6 L = _____ mL = _____ cm^3

f. 5200 L = _____ kL = _____ m^3

g. 2.45 L = _____ cm^3

h. 78 000 cm^3 = _____ L

3. MC A capacity of 25 L is equal to:

A. 0.025 mL **B.** 250 mL **C.** 0.25 mL **D.** 25 000 mL **E.** 2500 mL

4. MC A capacity of 35 400 mL is equal to:

A. 35 400 000 L **B.** 0.354 L **C.** 3.5400 L **D.** 35.4 L **E.** 35 L

5. Arrange in order from smallest to largest:

a. $2\frac{1}{2}$ L, 25 000 mL, $\frac{1}{4}$ L, 2.45 L

b. 760 mL, 0.765 L, 7.65 mL, 7.60 L

c. 110 mL, 0.1 L, 0.011 L, 1.1 L

Understanding

6. A water bottle has a capacity of 2 litres. How many 125 mL bottlefuls are required to fill it?

7. A bottle contains 250 mL of orange juice concentrate. How much water should be added to make up 2 L of juice from the concentrate?

8. A scientist dilutes (waters down) an acid solution by adding 120 mL of acid to 1.5 L of water. How much of the diluted solution will this make?

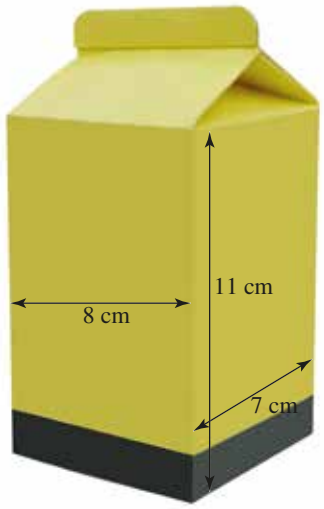
9. Most sports drinks are sold in 500 mL bottles. How many litres of sports drink are there in one dozen such bottles?

10. A medicine bottle contains 125 mL of cough syrup. How many 2.5 mL doses could be administered from this bottle, assuming that none is spilt?

11. Anthea runs a market stall selling detergent. How many 200 mL bottles of detergent could she fill from a 45 L bulk container?

12. A 185 mL container of hair conditioner is sold at the special price of $3.70. A 0.5 L container of the same conditioner costs $11.00. Which is the better buy? (*Hint:* Find the cost of 1 mL of hair conditioner in each case.)

13. A milk bar sells 55 small bottles of lemon drinks in one week. How many litres of drink are sold if each bottle contains 180 mL?



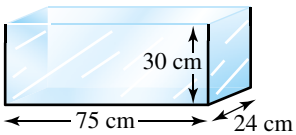
14. Petrov is working as a school laboratory technician. How many litres of salt solution should Petrov prepare for an experiment in which there are 12 groups of students if each group requires 400 mL of solution?

15. How many millilitres of milk does the container above right hold?

16. Claire has made her favourite green cordial with 0.25 L of water and 30 mL of cordial. Will a glass with a capacity of 250 mL hold this volume?

17. Sharmila has made a fresh juice with 0.3 L of apple juice and 40 mL of carrot juice. Will a glass with a capacity of 350 mL hold this volume?

18. Water is to be poured into this fish tank. How many litres of water are needed to fill the tank to a depth of 28 cm?



19. Estimate how many litres of water each sink will hold if filled to the top. Use the measurements shown. (*Hint:* First convert the measurements to centimetres.)

Reasoning

20. To achieve a world record, a 16.4 kL strawberry milkshake was made in the UK in 1996. To understand how large this is, consider the size of a rectangular prism that would have this capacity. Suggest three possible sets of measurements for this container.

21. One litre of paint covers an area of 20 square metres. How many millilitres of paint are required to cover an area of 6 square metres? Show your working.

Problem solving

22. Laurie connected a water tank to his 5 m × 3 m roof in order to collect rain water. In four hours, 2 cm of rain fell. The water tank holds 280 litres. Did the tank overflow?
Hint: 1000 L = 1 m^3.

23. A tap drips water at a rate of one drop every three seconds. If it takes 750 drops of water to fill a 100-ml container, how many litres of water will be lost in one year (365 days)?

Reflection

How will you remember conversion between units for volume and units for capacity?

9.9 Review

9.9.1 Review questions

Fluency

1. Which metric unit would be most suitable for measuring:
- **a.** the distance walked by a teacher at school during a week
- **b.** the length of a piece of spaghetti
- **c.** the width of a pencil?

2. Copy and complete the following conversions.

a. 560 mm = _____ m

b. 2300 cm = _____ km

c. 17 m = _____ cm

d. $\frac{3}{4}$ km = _____ mm

e. 2.09 m = _____ mm

f. $6\frac{4}{5}$ cm = _____ m

g. 22.5 mm = _____ cm

h. $\frac{63}{10000}$ km = _____ m

i. 82 000 000 m = _____ km

j. $5\frac{9}{10}$ mm = _____ cm

3. Arrange from smallest to largest: 44.5 m, 455 cm, 455 000 mm, 0.004 45 km.

4. a. Add 45.6 km to 5600 m.
b. Find the difference between 80 m and 4300 cm.

5. What is the reading in kilograms on these bathroom scales? *Note:* The scale for kilograms is the inner circle.

a.

b.

c.

d.

6. Measure each of the following line segments in centimetres, giving the answers in:

i. decimal form

ii. fractional form.

a.

b.

c.

7. Estimate the height of the following tree, given that the person in the photograph is 1.7 m tall.

8. Which unit of area would be most appropriate for measuring the following? (Choose from mm^2, cm^2, m^2, ha and km^2.)

a. The floor area of your classroom

b. The area of the state of Victoria

c. A pin head

d. A market garden

e. A floor tile

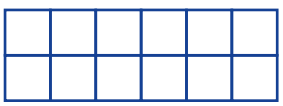
f. A bench top

g. The floor area in a house

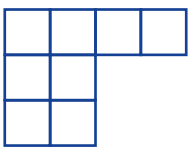
h. The area of the continent of Antarctica

9. Find the areas of the following figures, which are drawn on centimetre grid paper.

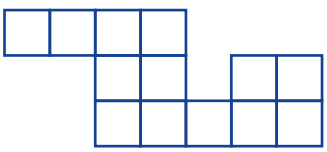
a.



b.

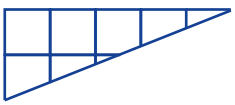


c.

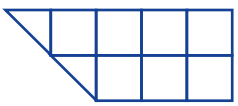


d.

e.

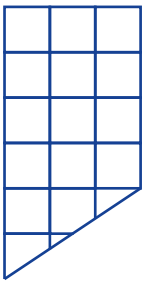


f.

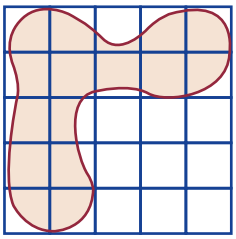


g.

h.



10. Estimate the shaded area if it is drawn on centimetre grid paper. *Note:* Each square on the grid has an area of $1\,cm^2$.

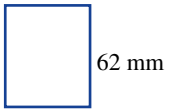


11. Use the formula $A = lw$ to find the area of the following rectangles.

a.

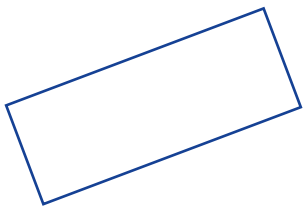
b.

c.



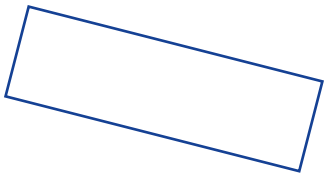
12. Measure the lengths of the sides in centimetres and hence calculate the area of each rectangle.

a.



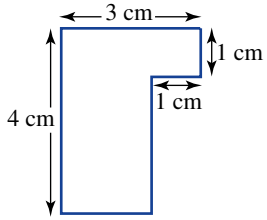
b.

c.

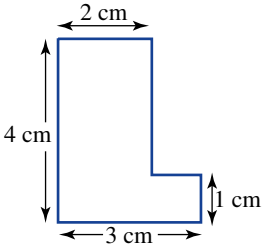


13. Find the area of the following shapes by first dividing them into rectangles.

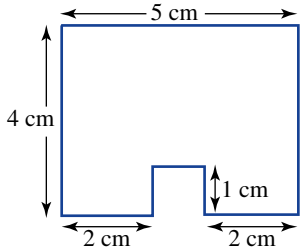
a.



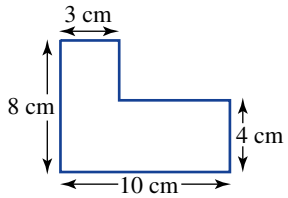
b.



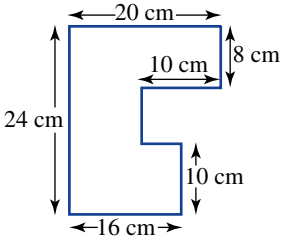
c.



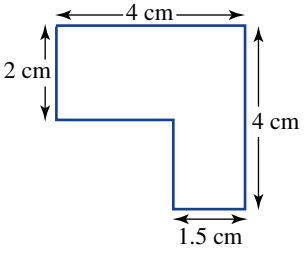
d.



e.

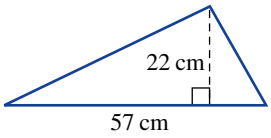


f.

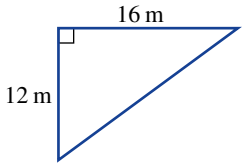


14. Find the area of the following triangles.

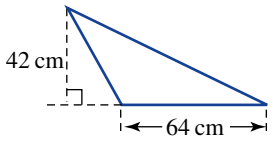
a.



b.

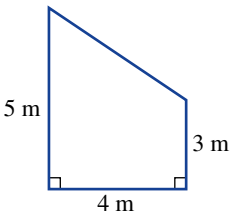


c.

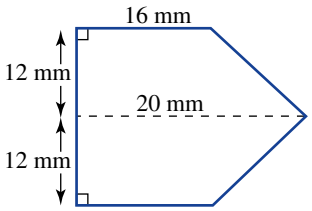


15. Find the area of the following shapes.

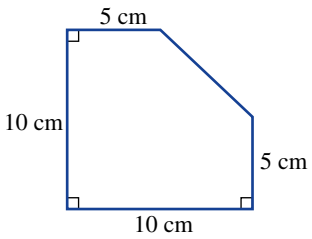
a.



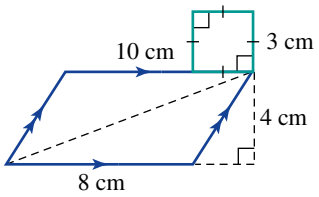
b.



c.

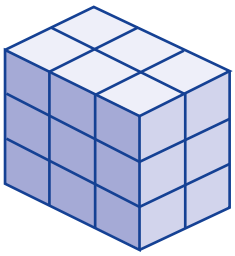


d.

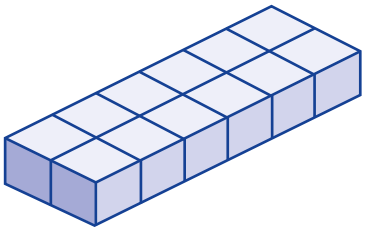


16. How many cubic centimetres (cm^3) are in each shape? Each cube has a volume of $1\,cm^3$.

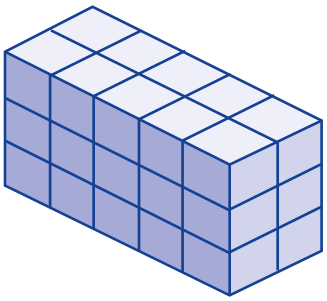
a.



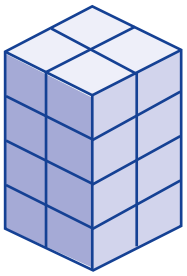
b.



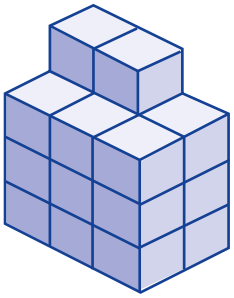
c.



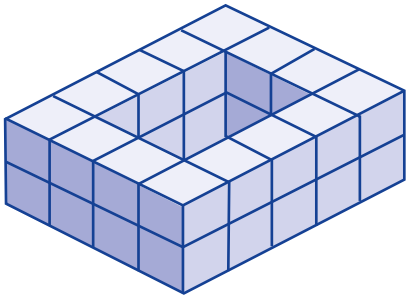
d.



e.

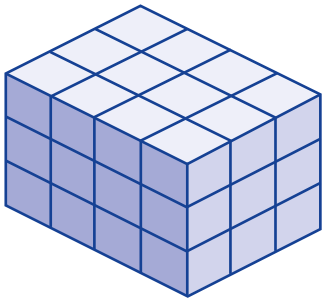


f.

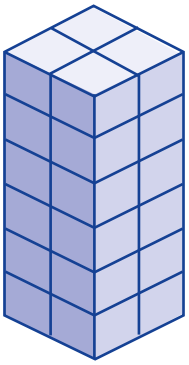


17. Use the rule 'Volume of a prism = area of base × height' to find the volume of the following prisms. Each cube has a volume of $1\,cm^3$.

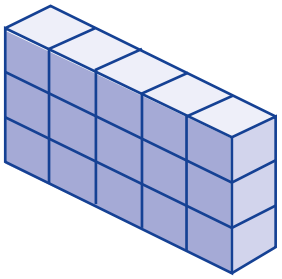
a.



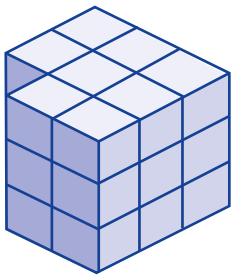
b.



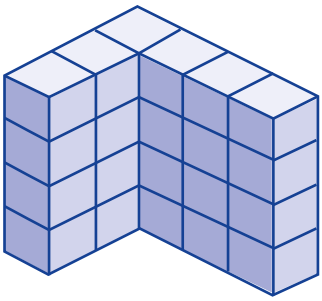
c.



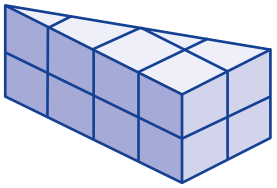
d.



e.

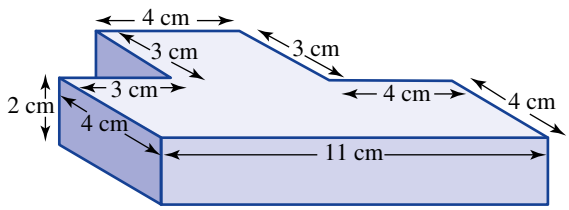


f.

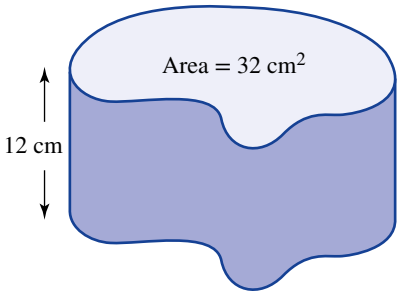


18. Use the information given in the following diagrams to find the volume of each of the following.

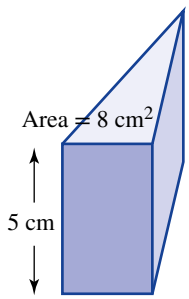
a.



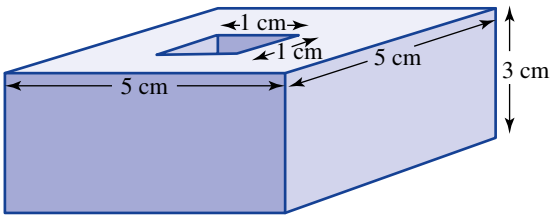
b.



c.



d.



19. Copy and complete the following conversions.

a. 8 L = _____ mL

b. $\frac{21}{50}$ L = _____ mL

c. 3300 mL = _____ L

d. $1\frac{3}{250}$ L = _____ mL

e. $4\frac{3}{10}$ kL = _____ L

f. 0.0034 kL = _____ mL

g. 4755 L = _____ kL

h. 432 mL = _____ L.

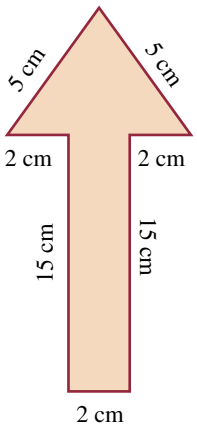
Problem solving

20. During a rescue operation in calm seas, a 16.5 m rope is dangled from a helicopter hovering 20 m above sea level. A 175 cm tall man standing on the deck of a boat reaches 50 cm above his head for the rope. By how much does he fail to reach the rope if the deck is 1 m above sea level?

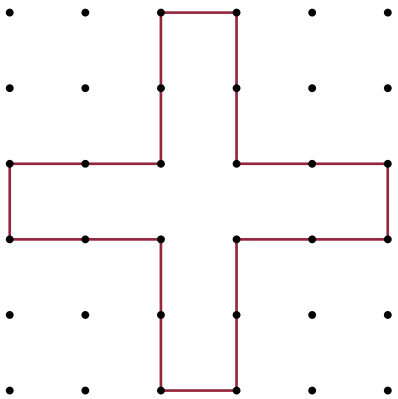
21. The Amazing Ape-Impersonating Gymnastic Troupe consists of two people who are 150 cm tall, and three people who are 1.8 m tall to their shoulders. When these people form a tower five people high by standing on each other's shoulders, how high is the tower? *Note:* The head of the top person is 30 cm high.

22. Find the perimeter of each shape.

a.

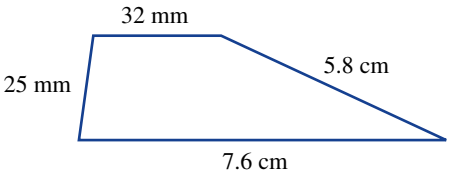


b.

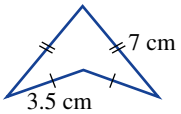


Note: The dots are 1 cm apart.

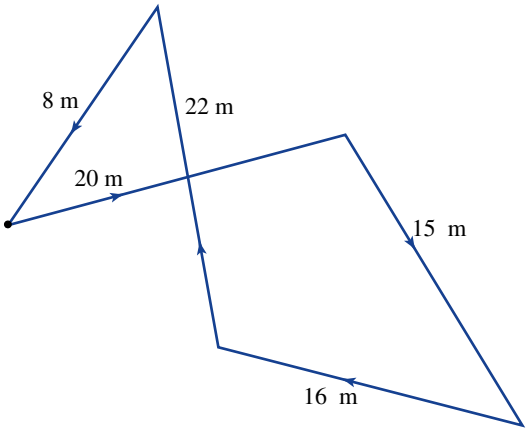
c.



d.



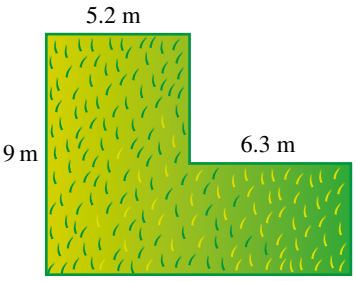
23. Michelle rides three laps of the following dirt-bike track. How far does she ride altogether?



24. A woven rectangular rug is 160 cm wide and 230 cm long. Find the perimeter of the rug.

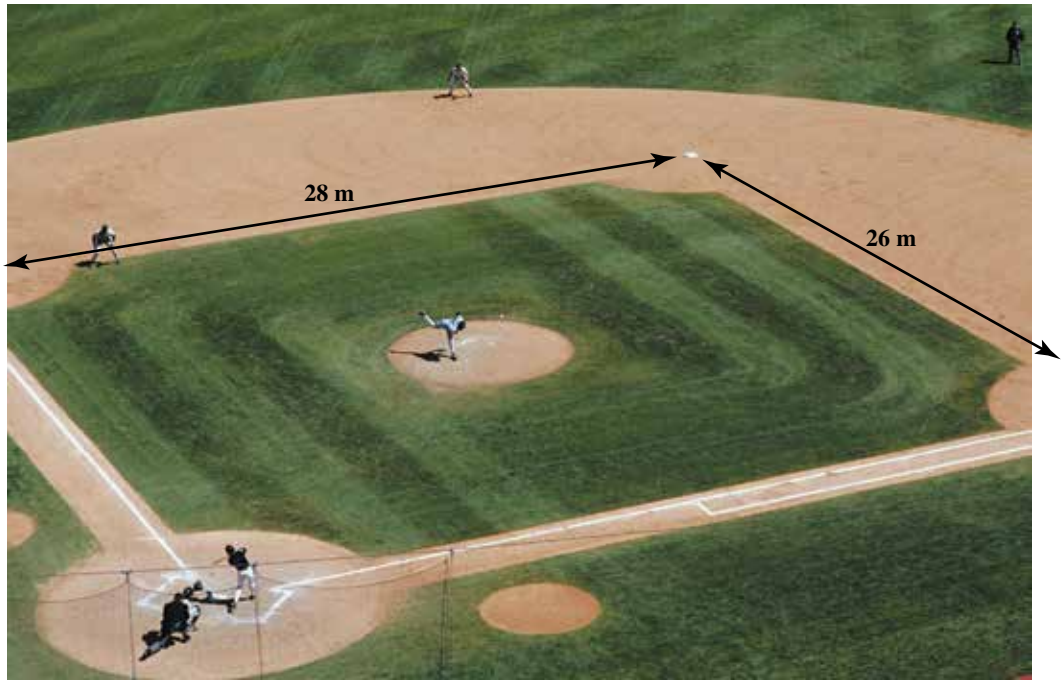
25. a. What length of concrete edging will be required to form an edge around the following lawn area?

b. What will be the cost of the job if edging costs $8.45 per metre?



26. What length of satin ribbon is required to edge a rectangular blanket on all four sides if the blanket is 240 cm long and 195 cm wide? (Assume there is no overlap.)

27. Flavio runs five times around the following baseball field. What distance has he run in total?

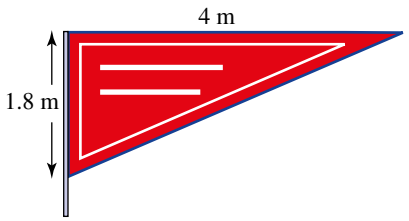


28. On a sheet of graph paper, draw five different figures each having an area of 12 cm.

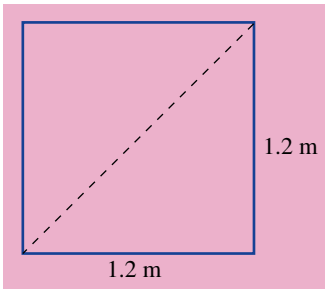
29. Find the area of a rectangular newspaper sheet which is 41 cm long and 30 cm wide.

30. Find the cost of paving a rectangular courtyard which is 6.5 m long and 3.2 m wide. The courtyard is to be paved with concrete paving blocks, which cost $28 per square metre.

31. Find the area of card needed to make the following triangular display sign.



32. When making a box kite, Katie cuts a square piece of material diagonally, as shown by the dotted line. What is the area of each triangular piece of material?



33. The following tip truck is filled with sand, level with the top of the tray. How many cubic metres of sand does the truck hold?

34. Sandro wishes to cover a rectangular lawn with topsoil to a depth of 0.2 m. If the lawn is 30 m long and 24 m wide, what volume of soil (in cubic metres) should he order?

35. A rectangular prism has a length of 40 cm and a width of 26 cm. If its volume is 29 120 cm^3, find the height of the rectangular prism.

36. A rectangular prism has a length of 30 cm and a width of 15 cm. If its volume is 3825 cm^3, find the height of the rectangular prism.

37. If the base area of a compact disc case is 168 cm^2 and its height is 1 cm, find the volume of a stack of 95 compact disc cases.

38. In a food technology class, each of 14 groups of students uses 350 mL of fresh milk to make pancakes. How many litres of milk should be ordered to provide for the whole class?

39. Mario makes up raspberry cordial by pouring 275 mL of concentrate into a 2-litre container and filling the container with cold water.

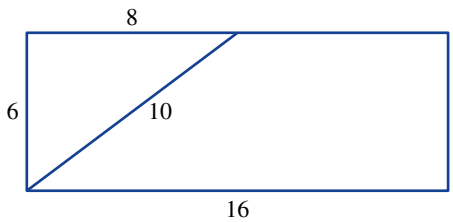
a. How much cold water does Mario add?

b. How many 250-mL glasses of cordial will Mario be able to pour from the large container?

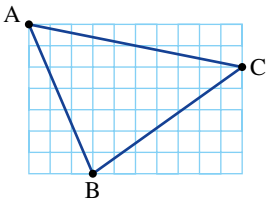
40. Estimate the number of litres of water that a goldfish bowl would hold.

41. Farmer Brown uses his roof to collect rain water, which he stores in a covered rainwater tank. His roof covers a horizontal area of 280 m^2 and the tank has a constant cross-sectional area of 60 m^2. If 11 mm of rain falls overnight, by how much will the depth of water in the tank increase?

42. A 6 × 16 rectangle is cut into two pieces as indicated. The two pieces are rearranged to form a right triangle. What is the perimeter of the resulting triangle?



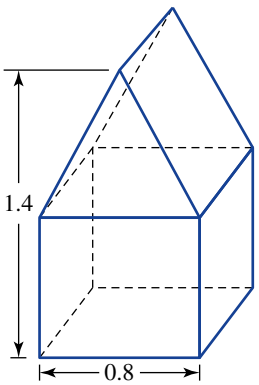
43. Each small square in the following diagram measures 1 cm × 1 cm. What is the area of triangle ABC (in square cm)?



44. Bert the beetle is crawling around the perimeter of a regular pentagonal tile of side length 10 cm. He has a rest when he is $\frac{5}{8}$ of the way around. How much further does he need to travel (to the nearest millimetre) to circumnavigate the tile?

45. Stewart is having a swimming pool built in his back yard. The hole for the pool is 6 metres long, 4 metres wide and 1.5 metres deep. The excavated soil becomes aerated as it is dug out, and its volume is increased by $\frac{1}{10}$. Determine the volume of the excavated soil.

46. A dog kennel is designed in the shape of a cube with a triangular prism as the roof. The width of the kennel is 0.8 metres and the overall height is 1.4 metres. The ceiling is removed to allow more room inside. Determine the volume of space inside the dog kennel.



47. A rectangular cake is three times as long as it is wide. A layer of icing that is 1 cm thick is spread on top of the cake. If the total volume of icing used is 300 cm^3, determine the dimensions of the top of the cake.

48. A gardener is employed to pave a 2-metre-wide path around a 10 m × 5.5 m rectangular swimming pool. Find the area of the paved path.
What is the minimum number of bricks that should be ordered if there are 60 bricks per square metre?

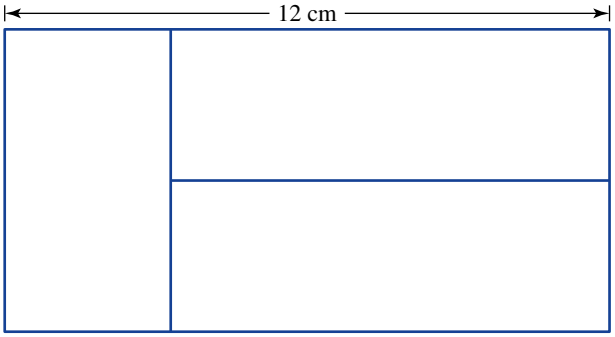
49. The perimeter of a rectangle is 20 cm. Investigate the shape of all such rectangles if the sides are whole numbers. Find the dimensions of the rectangle with the largest area.

50. A display tank at the entrance to Water World is 2.5 m long, 1.4 m wide and 1.3 m high, and has water to a depth of 1 m. A solid cubic decoration with an edge of 70 cm is placed at the bottom of the tank. Explain what will happen to the level of the water in the tank. What is the largest sized cube that can be placed in the tank without the water overflowing?

51. The Reserve Bank of Australia is responsible for printing and issuing Australia's currency notes. The money notes in circulation at the moment ($5, $10, $20, $50 and $100) are all the same width (65 mm), but increase in length as their value increases. The lengths of the five notes (not in order) are: 15.1 cm, 0.158 m, 130 mm, 14.4 cm and 0.137 m. Match each of the notes with its correct length.

52. Lake Eyre is in South Australia, but its basin extends into the Northern Territory and Queensland, and covers one-sixth of our continent. The base of the lake itself varies from 15 m below sea level to 10 m above sea level. The volume of the lake varies, but an average estimate is said to be about 30 million ML. If an Olympic swimming pool holds 2500 kL, how many of these pools could an 'average' Lake Eyre fill?

53. This rectangle is divided into three identical smaller rectangles. The length of the large rectangle is 12 cm.



Determine the width of the large rectangle.

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- **Try out this interactivity:** Crossword: Topic 9 (int-2600)
- **Try out this interactivity:** Sudoku: Topic 9 (int-3170)
- **Complete this digital doc:** Concept map: Topic 9 (int-10735)

Language

It is important to learn and be able to use correct mathematical language in order to communicate effectively. Create a summary of the topic using the key terms below. You can present your summary in writing or using a concept map, a poster or technology.

area
capacity
composite
conversion factors
cubic centimetre
cubic metre
diagonal
hectare
height
length
perimeter
parallelogram
prism
rectangular prism
scale
square centimetre
square metre
triangle
volume
width

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Investigation | Rich task

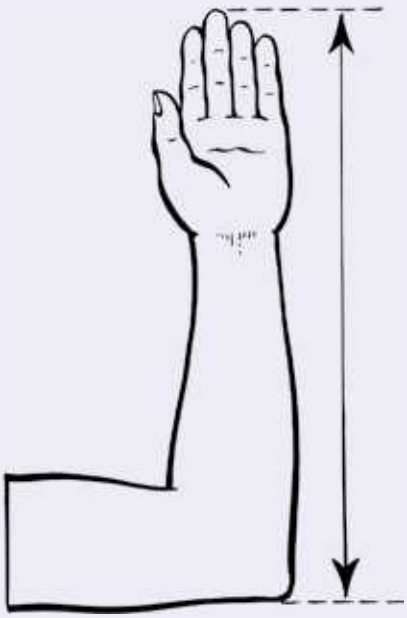
Old units of length

When studying length and perimeter, you were introduced to the common metric units of length: millimetre, centimetre, metre and kilometre. It has been only during the last few decades that these units have been commonly used in Australia. Prior to this, the units of length were based on the *imperial system*. Most countries in the world use units based on the metric system, but a few continue to use imperial units.

The origins of the imperial units are from far and wide. Many have been adapted from ancient times or translated into English from other languages. Other units, as strange as it may seem, are based on parts of the body.

The ancient Egyptians built the Great Pyramid of Giza over four and a half thousand years ago. It was constructed with side lengths of 440 cubits and was 280 cubits high.

The *cubit* was a unit equal to the 'length of the forearm of the bent elbow to the tip of the extended middle finger'.



1. The metric height of the Great Pyramid of Giza is 146.5 metres; its base length is 230 metres. Use these to determine the value of the cubit to 3 decimal places.
2. On the following page are some units of length from the imperial system. Select five units, and find out their equivalence to our better known metric units. Find out where and how they were used (what sorts of objects they measured, in what situations they were used — for example, for small or large distances, for racetracks, for the depth of water or in land surveying).

Although Australians have used units of length from the metric system for over 3 decades, many institutions have stuck with units common to the imperial system. For example, a cricket pitch is still 22 yards long.

league	hand	line	mile
furlong	chain	inch	link
rod	foot	yard	fathom

3. Find another sport that still uses lengths associated with the imperial system and state the unit(s) involved.

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Complete this digital doc: Code puzzle: Find the handy tool (doc-14327)

Answers

Topic 9 Measurement

Exercise 9.2 Units of measurement and converting units of measurement

1. a. m, cm **b.** m, km **c.** m, cm **d.** cm **e.** km

2. a. m **b.** cm **c.** cm **d.** mm **e.** cm, mm **f.** km

g. m, cm **h.** cm, mm **i.** cm, mm **j.** mm **k.** km, m **l.** cm, mm

3. A **4.** C

5. Various answers **6.** Various answers

7. a. 2000 m **b.** 7000 m **c.** 65 000 m **d.** 5300 m **e.** 660 m

f. 900 cm **g.** 25 cm **h.** 280 mm **i.** 2000 mm **j.** 70 000 cm

8. a. 8 km **b.** 6.5 km **c.** 0.7 km **d.** 0.05 km **e.** 0.0105 km **f.** 60 m

g. 3 m **h.** 0.57 m **i.** 0.09 m **j.** 4.5 cm **k.** 83.5 cm **l.** 2560 cm

9. a. 800 000 cm **b.** 5.4 m **c.** 101 000 mm **d.** 0.078 km **e.** 60 250 000 mm

f. 1.128 m **g.** 72 330 mm **h.** 4.05 cm **i.** 5050 m **j.** 0.030 35 mm

10. D **11.** A

12. a. 67 700 m **b.** 0.0452 km **c.** 0.56 m **d.** 4.5 km **e.** 87.5 mm

f. 60 cm **g.** 7210 mm **h.** 309 000 cm **i.** 4.8 cm **j.** 11 655 mm

13. 868 cm, 10.8 cm About 80 times longer **14.** Everest 8.863 km, K2 8.607 km

15. 1630 cm **16.** 448 m

17. a. 150 cm, 12.5 m, 0.02 km **b.** 0.445 m, 3000 mm, 350 cm **c.** 50 000 mm, 500 m, 50 km

d. 1.7 m, 1700 cm, 0.17 km **e.** 0.0052 mm, 0.000 052 m, 0.052 cm **f.** 0.909 m, 9000 mm, 990 cm

18. a. 3.75 m (375 cm) **b.** 10.2 km (10 200 m) **c.** 432 cm (4320 mm) **d.** 2.242 m (224.2 cm)

e. 3.91 m (391 cm) **f.** 25.6 m (25 600 mm) **g.** 47.8 km (47 800 m) **h.** 0.001 553 km (155.3 cm)

19. a. 66.4 km (66 400 m) **b.** 410.4 m (410 400 mm) **c.** 0.104 m (10.4 cm) **d.** 1.7 km (170 000 cm)

20. 2.76 cm **21.** 44 **22.** 216 m **23.** 1.593 m **24.** Yes, 36 cm

25. 215 cm, 105 cm, 155 cm, 170 cm

26. 2.4 m **27.** 198 cm **28.** 40 cm **29.** 38

30. a. i. 18 000 000 km **ii.** 1 080 000 000 km **iii.** 25 920 000 000 km **iv.** 9 460 800 000 000 km

b. Distances in the Universe are very big and it would be inconvenient to use kilometres for measuring distances. The numbers in kilometres are too large and make interpreting and even reading distances difficult.

31. a. i. 50 rotations **ii.** 100 rotations

b. i. 500 rotations **ii.** 1000 rotations

31. 2 cm × 4 cm, 2 cm × 8 cm, 2 cm × 12 cm, 2 cm × 16 cm

32. 38.5 km

33. 71 428 pins

34. Water 426 mL, butter 180 g, sugar 150 g, flour 300 g, eggs 2

Exercise 9.3 Reading scales and measuring length

1. a. i. 4 cm **ii.** N/A **b. i.** 7 cm **ii.** N/A

c. i. 2 cm **ii.** N/A **d. i.** 9.5 cm **ii.** $9\frac{1}{2}$ cm

e. i. 6.4 cm **ii.** $6\frac{2}{5}$ cm **f. i.** 10.1 cm **ii.** $10\frac{1}{10}$ cm

g. i. 5.8 cm **ii.** $5\frac{4}{5}$ cm **h. i.** 11.6 cm **ii.** $11\frac{3}{5}$ cm

i. i. 2.3 units **ii.** $2\frac{3}{10}$ units **j. i.** 1.6 units **ii.** $1\frac{3}{5}$ units

2. a. i. 45° F **ii.** 34° C **iii.** 10° C **iv.** 26° C

b. i. 45 km/ h **ii.** 80 km/ h **iii.** 100 km/ h

c. i. 1250 mL (5 cups) **ii.** 1800 mL (7 cups) **iii.** 500 mL (2 cups) **iv.** 1000 mL (4 cups)

d. 11

e. i. 130° C **ii.** 220° C **iii.** 360° F

3. B

4. C

5. **a.** 5 cm **b.** 7 cm **c.** 3 cm **d.** 8 cm
e. 9 cm **f.** 2.5 cm **g.** 4.5 cm **h.** 6.5 cm

6. **a.** 45 mm **b.** 86 mm **c.** 90 mm **d.** 35 mm
e. 25 mm **f.** 80 mm **g.** 42 mm **h.** 65 mm

7. **a.** $4\frac{1}{2}$ cm **b.** $8\frac{3}{5}$ cm **c.** 9 cm **d.** $3\frac{1}{2}$ cm
e. $2\frac{1}{2}$ cm **f.** 8 cm **g.** $4\frac{1}{5}$ cm **h.** $6\frac{1}{2}$ cm

8. **a.** 150 millionkm **b.** 60 m **c.** 4.9 m **d.** 1.74 m
e. **i.** 187.5 km **ii.** 412.5 km **iii.** 540 km
f. 136 m **g.** 13.32 m

9. Measure the thickness of the book using a ruler, then divide the thickness of the book by the number of pages (two-sided) in the book to find the thickness of each page.

10. Answers will vary. **11.** 0.2 km **12.** 293 cm

Exercise 9.4 Perimeter

1. **a.** 8 units **b.** 16 units **c.** 18 units **d.** 10 units **e.** 18 units
f. 22 units **g.** 16 units **h.** 12 units **i.** 14 units **j.** 14 units

2. **a.** 40 cm **b.** 92 mm **c.** 108 mm **d.** 30 m **e.** 44 cm
f. 29.5 m **g.** 74 cm **h.** 173 mm **i.** 68 cm **j.** 28 cm

3. **a.** 16 cm **b.** 16 cm **c.** 18 cm **d.** 8 cm **e.** 22 cm
f. 22 cm **g.** 15 cm **h.** 25 cm **i.** 22 cm **j.** 18 cm

4. **a.** 108 mm **b.** 173 mm **c.** 18 units **d.** 2995 cm
e. 408 mm **f.** 180 mm **g.** 14 443 m **h.** 17 579 m

5. B **6.** D **7.** A

8. **a.** 156 cm **b.** 664.2 cm **c.** $12\frac{9}{10}$ m or 12.9 m

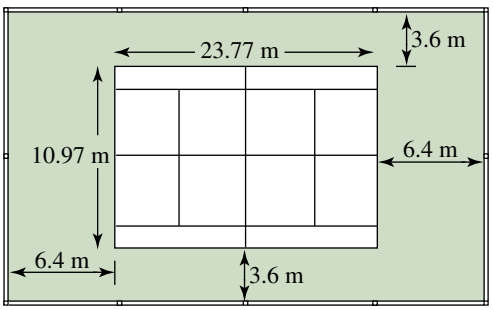
9. **a.** 14.8 cm **b.** 8.56 m **c.** $32\frac{4}{5}$ mm or 32.8 mm

10. 48.5 m

11. 252 cm

12. **a.** 193 m **b.** $3667

13. **a.**



b. 109.48 m **c.** $4050.76

14. 325.8 m **15.** 139.8 m

16. **a.** 120 m **b.** $228 **c.** $810 **d.** 25 posts
e. $341.25 **f.** 1600 **g.** $1680 **h.** $4059.25

17. $P = 6a - 2b$

18. 99 cm

19. 8 different rectangles

20. Square sides 8 cm

Challenge 9.1

6 cm

Exercise 9.5 Area

1. a. cm^2 **b.** cm^2 **c.** m^2 **d.** mm^2
e. km^2 **f.** cm^2 **g.** m^2 **h.** m^2
i. km^2 **j.** m^2 **k.** ha **l.** mm^2

2. a. 14 cm^2 **b.** 12 cm^2 **c.** 10 cm^2
d. 10 cm^2 **e.** 10 cm^2 **f.** 8 cm^2

3. a. 16 cm^2 **b.** 8 cm^2 **c.** 7 cm^2 **d.** 9 cm^2
e. 6 cm^2 **f.** $9\frac{1}{2}$ cm^2 **g.** 12 cm^2 **h.** 10 cm^2

4. Approximate answers — accept answers within 2 cm^2 of those listed here.
a. 19 cm^2 **b.** 24 cm^2 **c.** 19 cm^2

5. a. i. Length 4 cm, width 2 cm
ii. 8 cm^2 **iii.** 8 cm^2 **iv.** same
b. i. Length 6 cm, width 1 cm
ii. 6 cm^2 **iii.** 6 cm^2 **iv.** same
c. i. Length 6 cm, width 4 cm
ii. 24 cm^2 **iii.** 24 cm^2 **iv.** same
d. i. Length 8 cm, width 5 cm
ii. 40 cm^2 **iii.** 40 cm^2 **iv.** same

6. a. 15 cm^2 **b.** 4 cm^2 **c.** 15 m^2
d. 14 km^2 **e.** 24 cm^2 **f.** 10 m^2

7. a. $l = 1.8$ cm $w = 1.2$ cm $A = 2.16$ cm^2 **b.** $l = 3.6$ cm $w = 0.7$ cm $A = 2.52$ cm^2
c. $l = 31$ mm $w = 12$ mm $A = 372$ mm^2 **d.** $l = 25$ mm $w = 4$ mm $A = 100$ cm^2
e. $l = 2.5$ cm $w = 1.3$ cm $A = 3.25$ cm^2 **f.** $l = 25$ mm $w = 25$ mm $A = 625$ mm^2

8. a. 20 cm^2 **b.** 48 m^2 **c.** 261 mm^2 **d.** 56 m^2 **e.** 400 mm^2
f. 72 cm^2 **g.** 76 m^2 **h.** 18 cm^2 **i.** 135 mm^2 **j.** 686 mm^2

9. a. 4 cm^2 **b.** 3 cm^2 **c.** 40.5 cm^2
d. 8 cm^2 **e.** 50 cm^2 **f.** 1 m^2

10. a. C **b.** A **c.** D **d.** A **e.** B

11. a. 416 cm^2 **b.** 4 m^2 **c.** 3.125 m^2 **d.** 2.4 m^2 **e.** 493 mm^2
f. 583 cm^2 **g.** 600 km^2 **h.** 337.5 cm^2 **i.** 152 cm^2

12. 28.14 mm^2

13.

	Base	Height	Area
a	5 mm	3 mm	15 mm^2
b	8 cm	7 cm	56 cm^2
c	10 m	5.2 m	52 m^2
d	6 km	2 km	12 km^2
e	4 cm	30 mm	1200 mm^2
f	4.9 m	8.7 m	42.63 m^2

14. a. 7 952 400 km^2 **b.** 22 cm^2

15. 200 000 m^2

16. a. 16 000 000 000 m^2 **b.** 1 600 000 hectares

17. 17.5 m^2 **18.** 800 cm^2 **19.** 9.66 m^2

20. a. 7.5 m^2 **b.** 15 boxes **c.** $528

21. 104 m^2, $1300 **22.** $141.64 **23.** 4920 km^2 **24.** 3 sheets

25. 135 m^2 **26.** 864 cm^2 **27.** 88.8 m^2

28. 600 cm^2 (0.6 m^2) **29.** 12.6 m^2 **30.** 126 m^2

31. a. 212 500 km^2 **b.** Check with your teacher.

32. $2560 **33.** Length 9 cm, width 4 cm **34.** 48 mm

35. It would be cheaper to use the 10 cm × 25 cm clay pavers.

36. Two sides of the fencing are 6 m long and the other side is 12 m, providing an area of 72 m^2.

Exercise 9.6 Area of composite shapes, using addition and subtraction

1. a. 81 cm^2 **b.** 5 m^2 **c.** 16 cm^2 **d.** 56 cm^2 **e.** 588 cm^2 **f.** 13.5 m^2

2. a. 48 m^2 **b.** 325 m^2

3. a. 23.625 m^2 **b.** $283.50

4. a. 27 m^2 **b.** $675

5. 13.05 m^2 **6.** 39.28 m^2 **7.** 315 cm^2 **8.** 14.25 m^2

9. $2157.60 (rounded to the nearest 5 cents)

10. 75 m^2 **11.** 39 m^2

12. a. Square of side length 30 cm

Three circles of diameter 15 cm

Two semicircles of diameter 30 cm

b. Using addition of areas: square + circle of diameter 30 cm (the two equal semicircles form one circle).

Using subtraction of areas: area calculated in previous step −3 × area of the smaller (coloured) circle.

c. 1075 cm^2

d. The area in mm^2 would be 107 500 mm^2. This is more inconvenient to use because it is a larger number harder to interpret and to use in further calculations if needed.

13. a. i. Two rectangles with length 4 cm and width 3 cm and two parallelograms with length 3 cm and width 0.5 cm

ii. One square of side length 4 cm, one rectangle with length 2 cm and width 1 cm, one triangle with height 3 cm and base 1 cm and two right-angled triangles with height 3 cm and base 0.5 cm. The last two triangles form a triangle with base 1 cm and height 3 cm.

b. 11 cm^2

14. 22 cm^2 **15.** $0.02/$cm^2$

Challenge 9.2

The shaded area is $\frac{7}{32}$ of the square.

Exercise 9.7 Volume

1. a. 8 cm^3 **b.** 8 cm^3 **c.** 6 cm^3 **d.** 12 cm^3 **e.** 12 cm^3 **f.** 48 cm^3

2. B

3. a. 4 cm^3 **b.** 24 cm^3 **c.** 15 cm^3 **d.** 8 cm^3 **e.** 13 cm^3

f. 7 cm^3 **g.** 35 cm^3 **h.** 14 cm^3 **i.** 17 cm^3

4. A

5. a. 12 cm^3 **b.** 12 m^3 **c.** 243 cm^3 **d.** 6144 cm^3

e. 4 cm^3 **f.** 192 m^3 **g.** 270 cm^3 **h.** 101.25 cm^3

6.

Prism	Area of base (cm^2)	Height (cm)	Volume (cm^2)
a	6	2	12
b	5	2	10
c	3	3	9
d	5	2	10
e	5	3	15
f	4	4	16
g	3	2	6
h	7	3	21
i	8	3	24
j	6	2	12

7. D **8.** B

9. a. 126.9 cm^3 **b.** 45 m^3 **c.** 720 mm^3 **d.** 108 cm^3 **e.** 81.9 cm^3 **f.** 15.68 cm^3

10. a. 186 cm^3 **b.** 136 cm^3 **c.** 189 cm^3 **d.** 180 cm^3

11. 9690 cm^3 **12.** 477 750 cm^3 **13.** 20 cm **14.** 9 cm **15.** 12 cm

16. a. 2700 cm^3 **b.** 2 700 000 mm^3

17. 4.875 m^3

18. a. 28.875 cm^3 **b.** 28 875 mm^3 **c.** 176 mm^3 **d.** 20 075 mm^3

19. 768 m^3 **20.** 40.8 m^3

21. Each guest receives 267 cm^3 of cake.

22. 2277 cm^3 **23.** 2500 m^3

24. a. 1920 cm^3 **b.** 12

25. Length 18 cm, width 15 cm and height 10 cm. Volume is 2700 cm^3.

26. 16.5 cm

27. 0.65 m^3 more soil is needed (or 2.075 m^3)

28. 36 000

29. 26 cm × 14 cm × 5 cm

Exercise 9.8 Capacity

1. a. 2000 mL **b.** 3 L **c.** 13 000 L **d.** 7 L **e.** 5.5 L **f.** 0.26 kL

g. 2500 mL **h.** 32 L **i.** 0.055 L **j.** 35 mL **k.** 420 000 mL **l.** 990 L

m. 1870 mL **n.** 22.5 L **o.** 0.35 L **p.** 100 mL **q.** 0.025 kL **r.** 7 mL

2. a. 750 mL **b.** 2500 kL **c.** 800 mL = 0.8 L

d. 40 000 mL = 40 L **e.** 6000 mL = 6000 cm^3 **f.** 5.2 kL = 5.2 m^3

g. 2450 cm^3 **h.** 78 L

3. D **4.** D

5. a. 0.25 L, 2.45 L, 2.5 L, 25 000 mL **b.** 7.65 mL, 760 mL, 0.765 L, 7.60 L **c.** 0.011 L, 0.1 L, 110 mL, 1.1 L

6. 16 bottles **7.** 1750 mL (1.75 L) **8.** 1620 mL (1.62 L)

9. 6 L **10.** 50 doses **11.** 225 bottles

12. The 185 mL container is the better buy.

13. 9.9 L **14.** 4.8 L **15.** 616 mL

16. No **17.** Yes **18.** 50.4 L

19. Approximately 27 L (27.09 L)

20. Answers will vary. 16.4 kL = 16.4 m^3. An example could be length 2 m, width 2 m, height 4.1 m.

21. 300 mL **22.** The tank overflowed. **23.** 1401.6 L

Exercise 9.9 Review

1. a. Kilometres or metres **b.** Centimetres **c.** Millimetres

2. a. 0.56 m **b.** 0.023 km **c.** 1700 cm **d.** 750 000 mm **e.** 2090 mm

f. 0.068 m **g.** 2.25 cm **h.** 6.3 m **i.** 82 000 km **j.** 0.59 cm

3. 0.00445 km, 455 cm, 44.5 m, 455 000 mm

4. a. 51 200 m (51.2 km) **b.** 3700 cm (37 m)

5. a. 114 kg **b.** 42 kg **c.** 80 kg **d.** 95 kg

6. a. i. 4.5 cm **ii.** $4\frac{1}{2}$ cm **b. i.** 3.2 cm **ii.** $3\frac{1}{5}$ cm **c. i.** 2.9 cm **ii.** $2\frac{9}{10}$ cm

7. 5.525 m

8. a. m^2 **b.** km^2 **c.** mm^2 **d.** ha

e. cm^2 **f.** cm^2 or m^2 **g.** m^2 **h.** km^2

9. a. 12 cm^2 **b.** 8 cm^2 **c.** 13 cm^2 **d.** 2 cm^2

e. 5 cm^2 **f.** 8 cm^2 **g.** $6\frac{1}{2}$ cm **h.** 15 cm^2

10. 15 cm^2

11. a. 150 m^2 **b.** 6 cm^2 **c.** 2914 mm^2

12. a. 5.04 cm^2 **b.** 1.98 cm^2 **c.** 4.8 cm^2

13. a. 9 cm^2 **b.** 9 cm^2 **c.** 19 cm^2 **d.** 52 cm^2 **e.** 380 cm^2 **f.** 11 cm^2

14. a. 627 cm^2 **b.** 96 m^2 **c.** 1344 cm^2

15. a. 16 m^2 **b.** 432 mm^2 **c.** 87.5 cm^2 **d.** 41 cm^2

16. a. 18 cm^3 **b.** 12 cm^3 **c.** 30 cm^3 **d.** 16 cm^3 **e.** 20 cm^3 **f.** 28 cm^3

17. a. 36 cm^3 **b.** 24 cm^3 **c.** 15 cm^3 **d.** 24 cm^3 **e.** 24 cm^3 **f.** 8 cm^3

18. a. 112 cm^3 **b.** 384 cm^3 **c.** 40 cm^3 **d.** 72 cm^3

19. a. 8000 mL **b.** 420 mL **c.** 3.3 L **d.** 1012 mL **e.** 4300 L **f.** 3400 mL
g. 4.755 kL **h.** 0.432 L

20. 25 cm

21. 8.7 m (870 cm)

22. a. 46 cm **b.** 20 cm **c.** 191 mm (19.1 cm) **d.** 21 cm (210 mm)

23. 243 m

24. 780 cm

25. a. 41 m **b.** $346.45

26. 870 cm

27. 540 m

28. Answers will vary.

29. 1230 cm^2

30. $582.40

31. 3.6 m^2

32. 0.72 m^2

33. 60 m^3

34. 144 m^3

35. 28 cm

36. 8.5 cm

37. 15960 cm^3

38. 4.9 L

39. a. 1725 mL (1.725 L) **b.** 8 glasses

40. Approximately 3 L

41. 51 mm (approx.)

42. The perimeter is 48.

43. 32 cm^2

44. 188 mm

45. 39.6 cubic metres

46. 0.704 m^3

47. 10 cm by 30 cm

48. Area of paths is 78 m^2; 4680 bricks are needed.

49. Possible sides dimensions: 9×1; 8×2; 7×3; 6×4; 5×5. Largest area is 25 square units.

50. Water level will rise approximately by 0.1 m. The largest cube will have a side length of 1.016 m.

51. $5 is 130 mm long; $10 is 0.137 m long; $20 is 14.4 cm long; $50 is 15.1 cm long; $100 is 0.158 m long

52. 12000000 pools

53. 8 cm

Investigation — Rich task

1. 1 cubit = 0.523 metres

2. Teacher to check.

3. Teacher to check.

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Project: Sustainable by design

Scenario

As part of the EnviroWorks Design architect team, you have been asked to design a house for the McMahon family. The house is to accommodate Mr McMahon, Mrs McMahon and their two children.

As part of their brief to the architect team, the family has stated that the house must meet the following requirements:

- Have at least three bedrooms
- Have a floor plan area of exactly 250 m^2 (square metres)
- Include a minimum of three environmentally sustainable features.

The McMahons have allocated $25 000 to equip their house with environmentally sustainable features. They are interested to see what options are available to them to help make their house more environmentally friendly.

Some of the options they are already aware of are:

- solar hot water system
- water tanks to collect rain water and connected to laundry or toilets
- solar energy
- insulation
- double glazed windows
- recycled materials/furnishing
- aspect (e.g. north/south orientation)

They are also open to other suggestions that your team are able to find, and would like to use the entire $25 000 if possible.

The family has also requested that the team research and price the cost of furnishing the house (not including permanent fixtures such as ovens, bathtubs, taps etc.). They have allowed a budget of $30 000 for furnishings.

The McMahons are very excited about building their dream home and are ready to start construction straight away. They can't wait to see the fantastic ideas your team comes up with for their new home.

Task

You will produce a presentation for the McMahon family showing the floor plan you have designed, the environmentally friendly features included and the furnishings you have selected for their house. Your presentation should explain why you have made the decisions you have in each of the above categories and why you think they are the best choices for the McMahon family. A budget estimate must also be submitted for reference.

Process

Watch the introductory video lesson.

- Research sustainable housing features. Make notes of important facts, and ideas that you discover during your research. Enter your findings as articles under your topics in the Research Forum. You should each find at least three sources of information (including offline resources, such as books, newspapers etc.). You can view and comment on other group members' articles and rate the information they have entered. When your research is complete, print your Research Report to hand in to your teacher.

- Download the PowerPoint sample, the Floor plan template, the Budget template and the PowerPoint planning template to help you prepare your presentation. Your Media Centre also includes images that can help to liven up your presentations.
- Use the Floor plan template to start designing your house layout by printing the template and completing your design by hand, or by using autoshapes in Microsoft Word to complete your design. Make sure you remember the requirements that the McMahon family have requested in their house design. Once you have settled on a design you may want to use Google SketchUp or Floor planner at www.floorplan.com to present your floor plan in more detail.

- Use the PowerPoint template to develop your presentation. Remember that you are trying to convince the McMahon family that your design and proposal will result in their building the house of their dreams. Make sure you include all the information they requested, and that your presentation will grab their attention.

SUGGESTED SOFTWARE

- Microsoft Word
- Microsoft PowerPoint, Keynote or other presentation software
- Microsoft Excel
- Google SketchUp

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Try out this interactivity: A sustainable house (int-2356)

TOPIC 10 Probability

10.1 Overview

Numerous **videos** and **interactivities** are embedded just where you need them, at the point of learning, in your learnON title at www.jacplus.com.au. They will help you to learn the content and concepts covered in this topic.

10.1.1 Why learn this?

Probability allows us to describe how likely an event is to happen. To understand the chances of an event happening it is important to understand the language of probability. Probability is widely used in descriptions of everyday events; for example, the chance that there will be a wet day next week, or the chance of winning lotto.

10.1.2 What do you know?

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1. **THINK** List what you know about probability. Use a thinking tool such as a concept map to show your list.
2. **PAIR** Share what you know with a partner and then with a small group.
3. **SHARE** As a class, create a thinking tool such as a large concept map that shows your class's knowledge of probability.

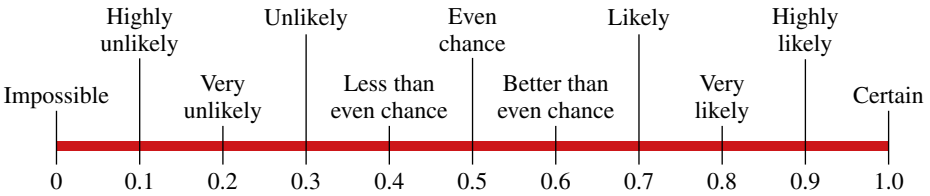
LEARNING SEQUENCE

10.1 Overview
10.2 The language of chance
10.3 The sample space
10.4 Simple probability
10.5 Using a table to show sample spaces
10.6 Experimenting with chance
10.7 Review

10.2 The language of chance

10.2.1 Chance

- An **event** is a result or outcome that may occur.
- When rating the chance of an event occurring, we use words like *certain*, *likely*, *even chance*, *unlikely* and *impossible*.
- The **probability** of an event occurring is given a value between 0 and 1 inclusive.
- If an event is **certain**, like an Australian Cricket team being named this summer, the probability of it occurring is 1.
- If an event is **impossible**, such as Christmas Day falling in June this year, the probability of it occurring is 0.
- If an event has an **even chance** of occurring, such as the probability of tossing a head with a fair coin, the probability of it occurring is 0.5. Probabilities can be written as fractions and percentages. For example, $0.5 = \frac{1}{2} = 50\%$.
- The probability scale, below, associates important words used in describing probability with their approximate corresponding numerical values.



WORKED EXAMPLE 1

For each of the given statements, specify whether the chance of the following events occurring is certain, likely, even chance, unlikely or impossible.

a You will compete in the next Olympics.
b Every student in Year 7 will obtain 100% in their next mathematics test.
c Each person in your class has been to the zoo.
d You flip a coin and Tails comes up.
e March is followed by April.

THINK	WRITE
a 1 Read the given statement and associate the chance of the event occurring with one of the given words from the list. Provide reasons.	**a** The chance of a person competing in the next Olympics is very small; however, it could happen.
2 Answer the question.	It is *highly unlikely* that this event will occur.
b Repeat steps **1** and **2** of part **a**.	**b** Due to each student having different capabilities and the number of students involved, this situation could never happen. It is *impossible* for this event to occur.
c Repeat steps **1** and **2** of part **a**.	**c** The chance that each student in your class has been to the zoo, either with their family or primary school, is very probable. However, there may be a few students who missed out. It is *likely* this event will occur.

d Repeat steps **1** and **2** of part **a**.	**d** When you flip a coin there are only two possibilities, a Head or a Tail. So there is a 50% chance of Tails coming up. There is an *even chance* this event will occur.
e Repeat steps **1** and **2** of part **a**.	**e** This is a true statement. April always follows the month of March. It is *certain* this event will occur.

WORKED EXAMPLE 2

Assign a number between and including 0 and 1 to represent the estimated probability of the following events, depending on how likely they are.

a One of two evenly matched tennis players will win the next game.

b You will guess the correct answer on a multiple choice question with four options.

c Rolling a fair die and obtaining a number less than 6.

THINK	WRITE
a 1 Determine the likelihood of an event occurring, with reasoning.	**a** Since the two players are evenly matched, one does not have an advantage over the other. Therefore, they each have an equal chance of winning the next game.
2 Express the answer as a decimal.	The probability that one player wins the game is $\frac{1}{2}$ or 0.5.
b 1 Determine the likelihood of an event occurring, with reasoning.	**b** When guessing an answer on a multiple choice question with 4 options, 1 out of the 4 possibilities will be correct. One out of 4 may be expressed as a fraction.
2 Express the answer as a decimal.	The probability of guessing the correct answer is $\frac{1}{4}$ or 0.25.
c 1 Determine the likelihood of an event occurring, with reasoning.	**c** When rolling a die there are six possibilities. They are 1, 2, 3, 4, 5, 6. A number less than 6 includes 1, 2, 3, 4, 5. Therefore, five out of the six possibilities may be rolled. Five out of six may be expressed as a fraction.
2 Express the answer as a decimal, correct to 2 decimal places.	The probability of obtaining a number less than six is $\frac{5}{6}$ or approximately 0.83.

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Complete this digital doc: SkillSHEET: Understanding 'chance' words (doc-6522)

Complete this digital doc: SkillSHEET: Comparing the likelihood of different events occurring (doc-6523)

Complete this digital doc: SkillSHEET: Describing the chance of an event occurring as more or less than half (doc-6524)

Exercise 10.2 The language of chance

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Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–7, 10	Questions: 1–8, 10	Questions: 1–11

Individual pathway interactivity: int-4362

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To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note*: Question numbers may vary slightly.

Fluency

1. WE1 For each of the given statements, specify whether the chance of the following events occurring is certain, likely, even chance, unlikely or impossible.
   - **a.** New Year's Day will be on 1 January next year.
   - **b.** Australia will experience at least one earth tremor this year.
   - **c.** Water will boil in the fridge.
   - **d.** There will be at least one day with a maximum temperature under 5 °C in Cairns in January.
   - **e.** A horse will win the Melbourne Cup.
   - **f.** There will be snow at Mt Buller this winter.
   - **g.** You will grow 18 cm taller this year.
   - **h.** You will win first prize in Tattslotto.
   - **i.** You choose a blue ball from a bag which contains only white balls.
   - **j.** You roll a fair die and obtain an odd number.
   - **k.** The year 2020 will be a leap year.
   - **l.** You choose a white ball from a bag which contains only white balls.
   - **m.** You roll a fair die and obtain a number greater than 6.
   - **n.** You choose a yellow ball from a bag containing 4 red balls and 4 yellow balls.
   - **o.** You roll a fair die and obtain a number less than 5.
2. Compare the given events A, B, C, D and order them from least to most likely.
   - **a.** It will be sunny in Queensland most of the time when you visit.
   - **b.** Melbourne Cup Day will be on the first Tuesday in November next year.
   - **c.** You will be the next Australian Idol.
   - **d.** Saturn will be populated next year.
3. WE2 Assign a number between and including 0 and 1 to represent the estimated probability of the following events, depending on how likely they are.
   - **a.** You flip a coin and obtain a Tail.
   - **b.** You choose a red ball from a bag containing only 8 white balls.
   - **c.** You guess the correct answer in a multiple choice question with 5 options.
   - **d.** You roll a die and obtain an even number on a fair die.
   - **e.** You flip a coin and obtain a Head.
   - **f.** Your class elects one class captain from four candidates.
   - **g.** You guess the incorrect answer in a multiple choice question with five options.

h. You choose a green ball from a bag containing only four green balls.
i. You have science classes this year.
j. Australia will win the Boxing Day cricket test.

Understanding

4. List five events that are:
a. impossible **b.** unlikely to happen
c. likely to happen **d.** sure to happen

5. MC The word which has the same meaning as *improbable* is the word:
A. unlikely **B.** impossible **C.** uncertain
D. certain **E.** likely

6. MC The word which has the same meaning as *certain* is the word:
A. definite **B.** possible **C.** likely **D.** probable **E.** unlikely

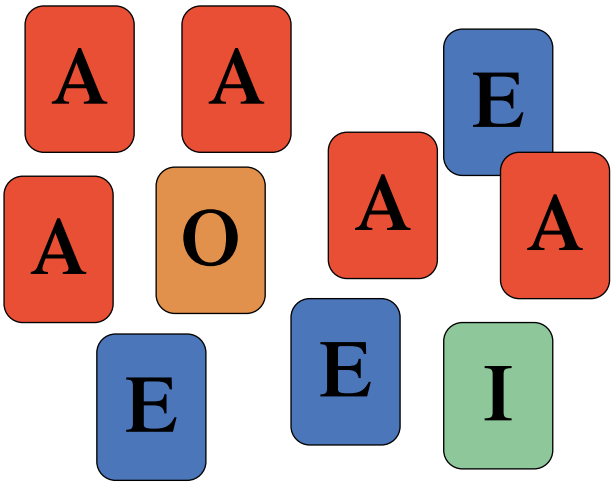
7. Match the words below with one of the numbers between 0 and 1 that are given. Choose the number depending on what sort of chance the word means, between *impossible* and *certain*. You may use a number more than once. If you are unsure, discuss your choice with another class member.
Numbers to choose from: 1, 0.75, 0.25, 0, 0.5.

a. Certain	**b.** Likely	**c.** Unlikely
d. Probable	**e.** Improbable	**f.** Definite
g. Impossible	**h.** Slim chance	**i.** Sure
j. Doubtful	**k.** Not able to occur	**l.** More than likely
m. Fifty-fifty	**n.** Fair chance	

Reasoning

8. Explain your answers to the following questions using the language learned in this chapter.
a. If today is Monday, what is the chance that tomorrow is Thursday?
b. If today is the 28th of the month, what is the chance for tomorrow to be the 29th?
c. If you toss a coin, what is the chance it will land Heads up?

9. Anthony has 10 scrabble pieces as shown. His friend Lian is blindfolded and asked to pick a piece at random. What is the chance she will pick:
a. an I **b.** an A
c. a U **d.** an E?
Explain your answers.



Problem solving

10. 'Fifty-fifty' is an expression commonly used in probability. Explain the meaning of this expression giving its fractional value, its decimal number form and as a percentage.

11. On 1 January two friends, Sharmela and Marcela, were chatting with each other. Sharmela made the comment, 'It is very likely that tomorrow the temperature will be around 35°C.' Marcela replied with the statement, 'It is very likely that tomorrow it is going to snow.' They were both correct.
a. Explain how this is possible.
b. Can you think of other situations like the one described in this question?

Reflection

Give an example of an event that is certain.

10.3 The sample space

10.3.1 The sample space

- The **sample space** refers to the list of all possible outcomes of an experiment. If we think of tossing a coin, the possible outcomes are either Heads or Tails. We write this as $S = \{\text{Heads, Tails}\}$
- Sometimes we don't need to list the elements in the sample space, but the number of elements that are in the sample space. For tossing a coin, we can say $n(S) = 2$.

WORKED EXAMPLE 3

A card is drawn from a standard deck. The suit of the card is then noted. List the sample space for this experiment.

THINK

Although there are 52 cards in the deck we are concerned only with the suit. List each of the four suits as the sample space for this experiment.

WRITE

$S = \{\text{clubs, spades, diamonds, hearts}\}$

WORKED EXAMPLE 4

A die is rolled and the number on the uppermost face is noted. How many elements are in the sample space?

THINK

The die can show the numbers 1, 2, 3, 4, 5 or 6.

WRITE

$n(S) = 6$

Exercise 10.3 The sample space

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Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–8, 11	Questions: 1–8, 11, 12	Questions: 1–12

Individual pathway interactivity: int-4363

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To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note*: Question numbers may vary slightly.

Fluency

1. WE3 A spinner with 10 equal sectors labelled 1 to 10 is spun. List the sample space.

2. For each of the following probability experiments list the sample space.

a. A coin is tossed.

b. A multiple choice question has five alternative answers, A, B, C, D and E.

c. A soccer team plays a match and the result is noted.

d. A card is selected from the four aces in a deck.

e. An exam paper is given the grade A to F.

3. A card is selected from a standard deck. List the sample space if we are interested in:
- **a.** the suit of the card chosen
- **b.** the face of the card chosen
- **c.** the colour of the card chosen.

4. A bag contains 8 red marbles, 9 green marbles and 2 orange marbles. A marble is selected from the bag. List the sample space.

5. WE4 A coin is tossed. How many elements are in the sample space?

6. In each of the following, state the number of elements in the sample space.
- **a.** A card is selected from a standard deck.
- **b.** The first ball drawn in the Tattslotto draw. (Balls are numbered from 1 to 45.)
- **c.** The winner of the AFL premiership. (There are 18 teams.)
- **d.** A day of the year is selected.
- **e.** A letter from the alphabet is selected at random.
- **f.** The first prize in the lottery is chosen from tickets numbered 1 to 180 000.
- **g.** A term is selected from a school year.
- **h.** You win a medal at your chosen event at the world swimming championships.

Understanding

7. MC From the list below select the event that has the most elements in the sample space.
- **A.** Selecting a card from a standard deck
- **B.** Selecting a page at random from this book
- **C.** Selecting an exercise book from your school bag
- **D.** Selecting a student at random from your class
- **E.** Selecting a page at random from the phone directory

Reasoning

8. In how many different ways can change be given for a 50 cent coin using only 20 cent, 10 cent and 5 cent coins? Justify your answer.

9. Create the sample space for each of these experiments.
- **a.** A spinner with four equal sectors (coloured red, green, blue and white) is spun and a coin is flipped.
- **b.** A spinner with three equal sectors (coloured red, green and blue) is spun, a coin is flipped and a standard six-sided die is rolled.

Problem solving

10. Michelle studies elective music. Her assignment this term is to compose a piece of music using as many instruments as she chooses, but only those that she can play. Michelle plays the acoustic guitar, the piano, the double bass and the electric bass. How many choices does Michelle have?

11. Alex has one brother, one sister, a mother and a father. The family owns a 5-seater car. When the family goes out, the parents always sit in the front two seats. There are three seats behind that. How many different seating arrangements are there?

12. If you had any number of ordinary dice, how many different ways could you roll the dice and obtain a total of 6?

CHALLENGE 10.1

Four students, Aimee, Ben, Carla and Donald, are standing in a line waiting to enter the classroom. How many possible ways can these 4 students be arranged if Ben refuses to stand at the front of the line?

Reflection

Are all elements in a sample space equally likely to occur?

10.4 Simple probability

10.4.1 Simple probability

- An **outcome** is a particular result of an experiment.
- A **favourable outcome** is one that we are looking for.
- The **theoretical probability** of a particular result or event is defined as

$$\text{P(event)} = \frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}}.$$

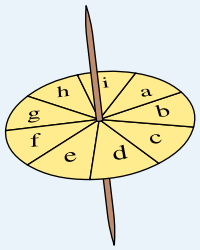
WORKED EXAMPLE 5

State how many possible outcomes there are for each of the following experiments and specify what they are.

a tossing a coin

b spinning a circular spinner with 9 equal sectors labelled from a to i as shown at right

c drawing a picture card (jack, king, queen) from a standard pack of cards



THINK	WRITE
a 1 Make a note of how many sides the coin has and what each side represents.	**a** The coin has 2 sides, a Head and a Tail.
2 Answer the question.	When tossing a coin there are two possible outcomes; they are Head or Tail.
b 1 Make a note of how many sectors the circular spinner has and what each one represents.	**b** The circular spinner has 9 sectors labelled a to i.
2 Answer the question.	When spinning the circular spinner, there are 9 possible outcomes; they are a, b, c, d, e, f, g, h or i.
c 1 Make a note of how many picture cards there are and what they are.	**c** There are 3 picture cards in each of the four suits.
2 Answer the question.	When drawing a picture card there are 12 possible results; they are: the jack, king and queen of clubs, the jack, king and queen of diamonds, the jack, king and queen of hearts and the jack, king and queen of spades.

WORKED EXAMPLE 6

Christopher rolls a fair 6-sided die.

a What are all the possible results that could be obtained?

b What is the probability of obtaining:

i a 4

ii a number greater than 2

iii an odd number?

THINK	WRITE
a Write all the possible outcomes and answer the question.	**a** There are six possible outcomes; they are 1, 2, 3, 4, 5, 6.
b i 1 Write the number of favourable outcomes. A 4 occurs once. Write the number of possible outcomes.	**b i** Number of favourable outcomes = 1 Number of possible outcomes = 6
2 Write the rule for probability.	$\text{P(event)} = \frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}}$
3 Substitute the known values into the rule and evaluate.	$\text{P}(4) = \frac{1}{6}$
4 Answer the question.	The probability of obtaining a 4 is $\frac{1}{6}$.
ii 1 Write the number of favourable outcomes and the number of possible outcomes. *Note:* 'Greater than 2' implies 3, 4, 5, 6.	**ii** Number of favourable outcomes = 4 Number of possible outcomes = 6
2 Substitute the known values into the rule for probability and evaluate.	$\text{P(greater than 2)} = \frac{4}{6}$
3 Simplify the fraction.	$= \frac{2}{3}$
4 Answer the question.	The probability of obtaining a number greater than two is $\frac{2}{3}$.
iii Repeat steps **1** to **4** of part **b ii**. *Note:* 'An odd number' implies 1, 3, 5.	**iii** Number of favourable outcomes = 3 Number of possible outcomes = 6 $\text{P(an odd number)} = \frac{3}{6}$ $= \frac{1}{2}$ The probability of obtaining an odd number is $\frac{1}{2}$ or 50%.

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Complete this digital doc: SkillSHEET: Writing a simple probability as a fraction (doc-6527)

Exercise 10.4 Simple probability

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–8, 12	Questions: 1–9, 11, 12	Questions: 1–13

Individual pathway interactivity: int-4364

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Fluency

1. WE5 State how many possible outcomes there are for each of the following experiments and specify what they are.
- **a.** Rolling a 12-sided die, numbered 1 to 12 inclusive
- **b.** Spinning a spinner for a game that has 5 equal-sized sections, numbered 1 to 5 inclusive
- **c.** Choosing a consonant from the word *cool*
- **d.** Choosing a sock out of a drawer containing 3 different socks coloured red, blue and black
- **e.** Picking a marble out of a bag containing 5 different marbles coloured black, blue, green, red and yellow
- **f.** Rolling an even number on a fair 6-sided die
- **g.** Rolling an even number greater than 2 on a fair 6-sided die
- **h.** Choosing an odd number from the first 20 counting numbers

2. List all the possible results in the following experiments. Comment on whether all results in each case are equally likely. Explain your answer.
- **a.** Rolling a fair 6-sided die
- **b.** Tossing a normal coin
- **c.** Spinning a spinner where half is white and half is black
- **d.** Spinning a spinner where half is white, a quarter is blue and a quarter is red
- **e.** Rolling a 6-sided die that has the numbers 1, 2, 3, 4, 5, 5 on it
- **f.** Shooting at a target where $\frac{1}{3}$ of the area is blue, $\frac{1}{3}$ green and $\frac{1}{3}$ red
- **g.** Choosing a vowel in the word *mathematics*

3. WE6 Christina rolls a fair 10-sided die with faces numbered from 1 to 10.
- **a.** What are all the possible results that could be obtained?
- **b.** What is the probability of obtaining:
  - **i.** a 9
  - **ii.** a number less than 7
  - **iii.** a prime number
  - **iv.** a number greater than 3
  - **v.** a multiple of 3
  - **vi.** a number greater than 10
  - **vii.** an even number greater than 4
  - **viii.** an odd number divisible by 3?

4. Leo has been given a bag of marbles to play with. Inside the bag there are 3 blue, 6 red, 4 green and 7 black marbles.

a. How many marbles are in the bag?

b. If Leo takes out one marble from the bag calculate:

i. P(getting a red marble)

ii. P(getting a green marble)

iii. P(getting a black marble)

iv. P(getting a blue marble)?

c. How many marbles in the bag are either blue or black?

d. Find P(getting a blue or a black marble).

e. Find P(getting a green or red marble).

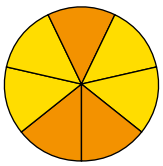
f. Find P(getting a green, red or blue marble).

g. Find P(getting a green, red, blue or black marble).

h. Explain your answer to part **g**.

5. There is a valuable prize behind 2 of the 5 doors in a TV game show. What is the probability that a player choosing any door will win a valuable prize?

6. MC A circular spinner is shown at right. When it is spun, the probability of obtaining an orange sector is:



A. $\frac{4}{7}$ **B.** $\frac{1}{7}$ **C.** 75% **D.** $\frac{1}{2}$ **E.** $\frac{3}{7}$

7. MC For an octagonal spinner with equal sectors numbered 1 to 8, the chance of getting a number *between* 2 and 7 is:

A. $\frac{5}{8}$ **B.** $\frac{3}{8}$ **C.** $\frac{3}{4}$ **D.** 0.5 **E.** 25%

Understanding

8. A pack of playing cards is shuffled and a card is chosen at random (in no particular order or pattern). Find the probability that the card chosen is:

a. a black card (that is, spades or clubs)

b. an ace

c. a diamond

d. a picture card (that is, a jack, king, queen)

e. the queen of hearts

f. a diamond or a black card

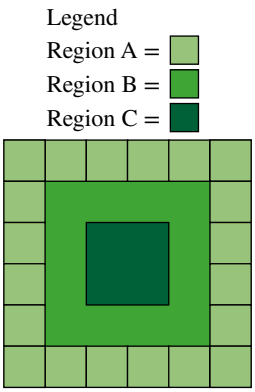
g. not a king

h. a club, diamond, heart or spade

i. not a spade

j. red and a ten.

9. Jim operates a parachute school. Being a man who is interested in statistics, he keeps a record of the landing position of each jump from first-time parachutists. With experience, parachutists are able to land on a particular spot with great accuracy. However, first-time parachutists do not possess this ability. He has marked out his landing field with squares, as shown below.



We are going to look at the areas of each of the regions A, B and C. To do this, we will determine each of the areas in terms of one of the small squares in region C. We will say that each small square has an area of 1 square unit.

a. What is the area of Jim's whole paddock (in square units)?

b. Determine the areas of regions A, B and C (in square units).

c. Assuming that the parachutist lands in the field, calculate the probability that the landing will occur in:
i. region A **ii.** region B **iii.** region C.
These represent theoretical probabilities.

d. Jim's records indicate that, from 5616 jumps of first-time parachutists, the landing positions were:
i. 592 in region C **ii.** 1893 in region B **iii.** 3131 in region A.

Comment on these results in comparison with the probabilities you calculated in question **c**.

Reasoning

10. In a raffle where there is only 1 prize (a car), 100 000 tickets have been sold, at a cost of $5.00 each. What is the chance of winning the prize for a buyer who:

a. purchases only 1 ticket

b. purchases 20 tickets

c. purchases 50 tickets

d. purchases all the tickets?

e. Would the buyer who buys all the tickets have made a wise purchase? Explain.

11. a. If you had only one pair of shoes, what would be the probability that you would wear that pair of shoes on any given day?

b. If you had two pairs of shoes, what would be the probability that you would wear a certain pair of shoes on any given day?

c. If you had seven pairs of shoes, what would be the probability that you would wear a certain pair of shoes on any given day?

d. If you had seven pairs of shoes but two pairs were identical, what would be the probability that you would wear one of the two identical pairs of shoes on any given day?

e. Explain what happens to the probability when the number of pairs of shoes increases.

f. Explain what happens to the probability when the number of identical pairs of shoes increases.

Problem solving

12. A die is rolled 30 times, and gives the following results.

4	3	5	4	3	5	2	1	1	5
3	2	2	4	1	3	1	6	1	3
2	1	6	6	3	5	1	3	5	3

a. Display these results in a frequency table.

b. What is the probability of obtaining a 6 when you roll a die?

c. How many times would you expect to obtain a 6 in 30 rolls of a die?

d. Explain the difference between your expected results and the actual results shown above.

13. Cory records the fact that it has rained on 65 out of 365 days in a year.

a. Write the number of days that it has rained as a simple fraction.

b. Karen says that since any day can be wet or dry, the probability of rain on any day is $\frac{1}{2}$. Is Karen correct?

c. What is the experimental probability of rain on any given day, expressed as a decimal, correct to 2 decimal places?

Reflection

If we know the probability of an event occurring, how can we work out the probability of it not occurring?

CHALLENGE 10.2

An apple, a banana, an orange, a peach and a bunch of grapes are in May's fruit basket. If she chooses two pieces of fruit to take to school with her lunch, what is the probability that one piece of fruit is a banana?

10.5 Using a table to show sample spaces

10.5.1 Tables and sample spaces

- Some experiments have two steps or stages that give a pair of results, such as when we toss 2 coins, or toss a coin and roll a die, or roll 2 dice.
- When the result is a combined one, we usually write the outcome as an **ordered pair**, in brackets, separated by a comma. The ordered pair (H, 6) would correspond to obtaining a Head on the coin and a 6 on the die.
- The sample space of two-step experiments may be displayed in a table.

WORKED EXAMPLE 7

a Draw a two-way table and list the sample space for the experiment 'tossing a coin and rolling a die'.
b State how many different outcomes or results are possible.
c Determine the probability of obtaining:
i a Head **ii a Tail and an even number**
iii a 5 **iv a Tail and a number greater than 2.**

THINK

a 1 Rule a table consisting of 7 rows and 3 columns. Leave the first cell blank.

2 Label the second and third cells of the first row as H and T respectively.

3 Label cells 2 to 7 of the first column as 1, 2, 3, 4, 5, 6 respectively.

4 Combine the outcome pairs in the order in which they occur in each of the remaining cells, that is, the first event result followed by the second event result.

5 Answer the question.

b Count the number of different outcomes and answer the question.

WRITE

a

	H	T
1	H1	T1
2	H2	T2
3	H3	T3
4	H4	T4
5	H5	T5
6	H6	T6

The sample space for the experiment 'tossing a coin and rolling a die' is {(H, 1), (H, 2), (H, 3), (H, 4), (H, 5), (H, 6), (T, 1), (T, 2), (T, 3), (T, 4), (T, 5), (T, 6)}.

b There are 12 different outcomes.

c i 1 Write the number of favourable outcomes.Write the number of possible outcomes. *Note:* The favourable outcomes featuring a Head are (H, 1), (H, 2), (H, 3), (H, 4), (H, 5) and (H, 6).

2 Write the rule for probability.

3 Substitute the known values into the rule and evaluate.

4 Simplify the fraction.

5 Answer the question.

c i Number of favourable outcomes = 6
Number of possible outcomes = 12

$$\text{P(event)} = \frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}}$$

$$\text{P(Head)} = \frac{6}{12}$$

$$= \frac{1}{2}$$

The probability of obtaining a Head is $\frac{1}{2}$ on 50%.

ii Repeat steps **1** to **5** of part **c i**.
Note: The favourable outcomes featuring a Tail and an even number are (T, 2), (T, 4) and (T, 6).

ii Number of favourable outcomes = 3
Number of possible outcomes = 12

$$\text{P(event)} = \frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}}$$

$$\text{P(Tail and even number)} = \frac{3}{12}$$

$$= \frac{1}{4}$$

The probability of obtaining a Tail and an even number is $\frac{1}{4}$ on 25%.

iii Repeat steps **1** to **5** of part **c i**.
Note: The favourable outcomes featuring a five are (H, 5) and (T, 5).

iii Number of favourable outcomes = 2
Number of possible outcomes = 12

$$\text{P(event)} = \frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}}$$

$$\text{P(5)} = \frac{2}{12}$$

$$= \frac{1}{6}$$

The probability of obtaining a five is $\frac{1}{6}$.

iv Repeat steps **1** to **5** of part **c i**.
Note: The favourable outcomes featuring a Tail and a number greater than 2 are (T, 3), (T, 4), (T, 5) and (T, 6).

iv Number of favourable outcomes = 4
Number of possible outcomes = 12

$$\text{P(event)} = \frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}}$$

$$\text{P(Tail and number greater than 2)} = \frac{4}{12}$$

$$= \frac{1}{3}$$

The probability of obtaining a Tail and a number greater than 2 is $\frac{1}{3}$.

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Exercise 10.5 Using a table to show sample spaces

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–8, 12	Questions: 1–9, 12, 13	Questions: 1–13

Individual pathway interactivity: int-4365

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Fluency

1. Write down the sample space as an ordered list for each of the following simple or one-step experiments:
- **a.** rolling a 6-sided die
- **b.** spinning a spinner which can land on any of the numbers from 1 to 10
- **c.** choosing an item from a menu that contains fruit salad, cheesecake, mudcake and cheese platter
- **d.** choosing a number which is a multiple of 5 in the first 50 counting numbers
- **e.** choosing an Australian state or territory for a holiday destination
- **f.** picking the correct answer in a true/false question
- **g.** choosing a king from a pack of standard cards
- **h.** choosing an instrument from the following list: guitar, drum, saxophone, piano and trumpet.

2. WE7 **a.** Draw a two-way table and list the sample space for the experiment 'spinning a circular spinner divided into 3 equal sectors labelled A, B, C and rolling a die'.
- **b.** State the number of different outcomes or results.
- **c.** Determine the probability of obtaining:
  - **i.** the letter A
  - **ii.** the number 4
  - **iii.** a number greater than 2
  - **iv.** a number which is a multiple of 3
  - **v.** an odd number
  - **vi.** the letter C and a prime number
  - **vii.** the letter A, B or C
  - **viii.** any number except the number 6
  - **ix.** the letter B and a number less than 3
  - **x.** a number greater than 6.

3. a. Draw a table to show the sample space for the experiment 'tossing 2 coins at once'. (*Hint:* Call the first coin 'Coin 1' and the other 'Coin 2'.)
- **b.** How many possible results are there for this experiment?
- **c.** How many times does the result (H, H) appear?
- **d.** How many times does the result (T, T) appear?
- **e.** How many times does a result with a Tail and a Head in any order appear?
- **f.** What is P(H, H)?
- **g.** What is P(T, T)?
- **h.** What is P(getting a Tail and a Head in any order)?

4. a. Draw a table to show the sample space for the experiment 'tossing a 5c coin and tossing a 10c coin'.
Note: In this case order will matter.
b. How many possible results are there?
c. What is P(getting a H on the 5c coin and getting a T on the 10c coin)?
d. What is P(getting a H on the 5c coin and a H on the 10c coin)?
e. What is P(getting a T on the 5c coin and a H on the 10c coin)?
f. What is P(getting a T on the 5c coin and a T on the 10c coin)?

5. a. Draw a table to show the sample space for the experiment 'tossing a coin and rolling a 10-sided die'.
b. How many possible results are there?
c. What is P(getting a H on the coin and a 6 on the die)?
d. What is P(getting a H on the coin and an even number on the die)?
e. What is P(getting either a H or a T on the coin and an even number on the die)?
f. What is P(getting a T on the coin and a number divisible by 3 on the die)?
g. What is P(getting a number less than 3)?
h. What is P(getting a number greater than 5)?
i. What is P(getting a H on the coin and a number less than 6 on the die)?
j. What is P(getting either a H or a T on the coin)?

6. a. Draw a table to show the sample space for the experiment 'rolling a red die and a blue die'.
b. How many possible results are there?
(*Note:* There is a difference between a 5 on the red and a 5 on the blue die.)
c. What is P(getting 1 on both dice)?
d. What is P(getting 1 on the red die and 6 on the blue die)?
e. What is P(getting 1 on any die)?
f. What is P(getting an even number on the red die and an odd number on the blue die)?
g. What is P(getting an even number on both dice)?
h. What is P(getting an odd number on both dice)?
i. What is P(getting 2 numbers whose sum is 6)?
j. What is P(getting 2 numbers whose sum is 1)?
k. What is P(getting 2 numbers whose sum is 13)?

7. a. Draw a table to show the sample space for the following experiment 'picking a marble out of a bag containing a red, a blue and a green marble and tossing a coin'.
b. How many possible results are there?
c. What is P(picking out a green marble and getting a Head)?
d. What is P(picking out a red marble and getting a Head)?
e. What is P(picking out a blue marble and getting a Head)?
f. What is P(picking out a blue or green marble and getting a Head)?
g. What is P(picking out a blue or red marble and getting a Head)?
h. What is P(picking out a blue or red or green marble and getting a Head)?
i. How is your answer to part **h** related to the probability of getting a Head if you were only tossing a coin? Can you explain why this is the case?

Understanding

8. MC Two dice are rolled simultaneously. The probability of obtaining the sum of 7 (by adding the results of the two dice being rolled simultaneously) is:

A. $\frac{1}{12}$ **B.** $\frac{7}{36}$ **C.** $\frac{1}{6}$ **D.** $\frac{1}{4}$ **E.** 0

9. For the events given below, determine the following, without listing the sample space.

i. State how many rows and columns would be needed to draw up a table representing the sample space.

ii. State the number of possible outcomes in the sample space.

a. Picking a day in January from a calendar and tossing a coin

b. Tossing a coin and shooting a dart at a board with 3 zones

c. Choosing a pencil from a set of 72 and rolling a 6-sided die

d. Rolling a 10-sided die and rolling a 6-sided die

e. Choosing a member from a class of 30 students and rolling a 6-sided die

f. Choosing a politician from a list of 100 and tossing a coin

Reasoning

10. Within the Australian states, a common number plate system for cars is 3 numbers combined with 3 letters, e.g. ABC 123. How many more number plates does this allow each state to issue than would be the case if the plates were simply 6 numbers (for example, 123456)? Justify your answer.

11. Callum has a regular die and a strange die that has sides numbered 2, 3, 4, 5, 6, 6. If he rolls these two dice together, what is the probability that he gets a double six? Justify your answer.

Problem solving

12. A standard domino set consists of a set of rectangular tiles, each with a line dividing its face into two square ends. Each end is marked with a number of black dots (similar to those on dice), or is blank. A standard domino set has ends ranging from zero dots to six dots. The back side of a domino tile is plain. How many tiles make up a full set of dominoes?

13. Glenn remembered that his mother's car registration plate had 2 letters followed by 3 numbers. He could remember that the letters were R and B and that the numbers were 5, 1 and 4, but he couldn't remember the order. What combination of letters and numbers could his mother's car registration plate have? Make a list of the possibilities.

Reflection

When we toss 2 coins, which result (if any) is more likely: the coins match, or the coins are different?

10.6 Experimenting with chance

10.6.1 Experimenting with chance

- The theoretical probability of a particular result or event is defined as:

$$\text{P(event)} = \frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}}.$$

- In real life, the chance of something occurring may be based on factors other than the number of favourable and possible outcomes. For example, the chances of you beating your friend in a game of tennis could theoretically be $\frac{1}{2}$ as you are one of 2 possible winners. In practice there are other factors (like experience and skill) that would influence your chance of winning.
- A **trial** is one performance of an experiment to collect a result.
- An **experiment** is a process that allows us to collect data by performing trials. In experiments with repeated trials, it is important to keep the conditions for each trial the same.
- A **successful trial** is one that results in the desired outcome.
- The experimental probability of an event is found by conducting an experiment and counting the number of times the event occurs.
- The **experimental probability** of a particular result or event is defined as:

$$\text{P(event)} = \frac{\text{number of successful trials}}{\text{total number of trials}}.$$

WORKED EXAMPLE 8

A coin is flipped 10 times and the results are seven Heads and three Tails. Find the experimental probability of obtaining a Tail.

THINK

1. Obtaining a Tail is considered a success. Each flip of the coin is a trial.
2. Tails was flipped three times, so there were three successful trials out of a total of 10 trials.

WRITE

$$\text{P(sucess)} = \frac{\text{number of successful trials}}{\text{total number of trials}}$$

$$\text{P(Tail)} = \frac{3}{10}$$
$$= 0.3$$

10.6.2 Experimental probability versus actual probability

- The more times an experiment is performed, the closer the average of the measured results should be to the theoretically expected answer for it.
- The **long-term trend** (that is, the trend observed for results from a very large number of trials) shows that results obtained through experimental probability will match those of theoretical probability.

WORKED EXAMPLE 9

a Copy the table below. Toss a coin 10 times and record the results in row 1 of the table.

Experiment number	Heads		Tails	
	Tally	Count	Tally	Count
1 2 3 4 5 6				
	Total		Total	

b What is the probability of obtaining a Head from your experiment?
c What is the probability of obtaining a Tail from your experiment?
d How do these values compare with the theoretical results?
e Repeat step a another 5 times and combine all of your results.
f How does the combined result compare with the theoretical results?

THINK

a Toss a coin 10 times and record the results in the first row of the table. *Notes:* (a) Place a stroke in the appropriate tally column each time an outcome is obtained. Five is denoted by a 'gatepost'; that is, 4 vertical strokes and 1 diagonal stroke (𝍸). (b) The same coin must be used throughout the experiment. The style of the toss and the surface the coin lands on must be the same.

b 1 Calculate the probability of obtaining a Head for this experiment using the rule.

2 Substitute the given values into the rule.

3 Evaluate and simplify.

4 Convert the fraction to a percentage by multiplying by 100%.

5 Answer the question.

WRITE

a

	Heads		Tails	
Exp.No.	Tally	Count	Tally	Count
1	𝍸	4	𝍸	6
2	𝍸	4	𝍸	6
3	𝍸	7	𝍸	3
4	𝍸	3	𝍸	7
5	𝍸	4	𝍸	6
6	𝍸	8	𝍸	2
	Total	30	Total	30

b $\text{P(event)} = \dfrac{\text{number of favourable outcomes}}{\text{number of possible outcomes}}$

$\text{P(Heads)} = \dfrac{\text{number of Heads obtained}}{\text{total number of tosses}}$

$\text{P(Heads)} = \dfrac{4}{10}$

$= \dfrac{2}{5}$

As a percentage $\dfrac{2}{5} = \dfrac{2}{5} \times 100\%$

$= \dfrac{200}{5}\%$

$= 40\%$

The probability of obtaining a Head in this experiment is $\dfrac{2}{5}$ or 40%.

c 1 Calculate the probability of obtaining a Tail for this experiment.

c $P(\text{Tails}) = \dfrac{\text{number of Tails obtained}}{\text{total number of tosses}}$

2 Substitute the given values into the rule and simplify.

$$P(\text{Tails}) = \frac{6}{10}$$
$$= \frac{3}{5}$$

3 Convert the fraction to a percentage by multiplying by 100%.

$$\text{As a percentage } \frac{3}{5} = \frac{3}{5} \times 100\%$$
$$= \frac{300}{5}\%$$
$$= 60\%$$

The probability of obtaining a Tail in this experiment is $\frac{3}{5}$ or 60%.

d Compare the results obtained in parts **b** and **c** with the theoretical results.

d The experimental value obtained for the P(Heads) is $\frac{2}{5}$ (or 40%) and the P(Tails) is $\frac{3}{5}$ (or 60%). The theoretical value of these probabilities is $\frac{1}{2}$ (or 50%). Therefore, the experimental probabilities differ from the theoretical probabilities by 10%.

e 1 Repeat the procedure of part **a** 5 times.

e Refer to the results in the table in part **a**.

2 Calculate the total number of Heads and Tails obtained and enter the results in the table.

f 1 Calculate the probability of obtaining a Head for this experiment.

f $P(\text{Heads}) = \dfrac{\text{number of Heads obtained}}{\text{total number of tosses}}$

2 Substitute the given values into the rule and simplify.

$$P(\text{Heads}) = \frac{30}{60}$$
$$= \frac{1}{2}$$

3 Convert the fraction to a percentage by multiplying by 100%.

$$\text{As a percentage } \frac{1}{2} = \frac{1}{2} \times 100\%$$
$$= \frac{100}{2}\%$$
$$= 50\%$$

4 Answer the question.	The probability of obtaining a Head in this experiment is $\frac{1}{2}$ or 50%. $\text{P(Tails)} = \frac{\text{number of Tails obtained}}{\text{total number of tosses}}$ $\text{P(Tails)} = \frac{30}{60}$ $= \frac{1}{2}$ As a percentage $\frac{1}{2} = \frac{1}{2} \times 100\%$ $= \frac{100}{2}\%$ $= 50\%$ The probability of obtaining a Tail in this experiment is $\frac{1}{2}$ or 50%.
5 Compare the combined result obtained with the theoretical results.	The combined results in this experiment produced probability values that were equal to the theoretical probability values. Therefore, the long-term trend of obtaining a Head or Tail when tossing a coin is equal to $\frac{1}{2}$.

10.6.3 Simulations

- A **simulation** is the process of imitating or modelling a real life situation using simple devices or technology.
- Instead of physically tossing a coin, we can use technology to simulate or copy this process. Because of this, simulations are often quicker, more convenient and safer than carrying out a real life experiment.
- **Random numbers** are sets of numbers that are generated in such a way that each number has an equal chance of occurring each time.
- Random numbers can be used to simulate experiments by assigning a unique number value to each unique outcome in the sample space.

WORKED EXAMPLE 10

Alex wants to know how many packets of cereal she must purchase in order to collect 4 different types of plastic toys during a promotion.

a Design an experiment which will simulate the given situation, providing details of the equipment required and procedure involved.

b Discuss the fairness of the experiment and findings.

THINK	WRITE
a 1 Determine the sample space. *Note:* There are 4 toys, each of which is equally likely to be found in a packet of cereal.	**a** Sample space = { toy 1, toy 2, toy 3, toy 4 } The 4 events are equally likely.
2 Decide on the equipment required for this experiment. *Note:* Choose an item which produces 4 possibilities; that is, a spinner or 4 different coloured balls in a box, etc.	A circular spinner divided into 4 equal sectors and labelled 1, 2, 3, 4 will be used to simulate the outcomes of obtaining a toy from the cereal packet. Sector 1 represents toy 1 (T1). Sector 2 represents toy 2 (T2). Sector 3 represents toy 3 (T3). Sector 4 represents toy 4 (T4).
3 Give details of how the experiment will be conducted.	Spin the pointer on the circular spinner until all 4 toys are represented; that is, until T1, T2, T3 and T4 are obtained. Repeat this procedure another 19 times.
4 Record the results in a table comprising 3 columns, headed: 'Experiment number', 'Results' and 'Number of packets'.	

Experiment number	Results	Number of packets
1	T4 T3 T4 T3 T2 T4 T4 T1	8
2	T3 T4 T4 T3 T4 T1 T2	7
3	T1 T4 T4 T2 T1 T4 T4 T3	8
4	T1 T4 T4 T4 T4 T4 T2 T3	8
5	T1 T4 T3 T4 T1 T2	6
6	T1 T4 T1 T2 T1 T2 T4 T2 T4 T4 T1 T1 T4 T3	14
7	T1 T4 T4 T2 T2 T4 T4 T3	8
8	T4 T2 T2 T3 T2 T4 T3 T2 T4 T4 T1	11
9	T3 T4 T2 T4 T2 T2 T4 T2 T1	9
10	T4 T3 T3 T1 T1 T2	6
11	T3 T1 T4 T3 T1 T1 T1 T3 T1 T4 T1 T3 T1 T1 T3 T2	16
12	T3 T2 T1 T1 T4	5
13	T3 T4 T1 T2	4
14	T4 T2 T3 T3 T4 T1	6
15	T4 T1 T4 T3 T3 T4 T4 T3 T1 T2	10

16	T1 T1 T2 T1 T1 T3 T4	7
17	T4 T2 T3 T2 T4 T2 T4 T4 T4 T4 T3 T4 T4 T3 T4 T3 T4 T4 T3 T4 T2 T1	22
18	T1 T2 T1 T1 T4 T3	6
19	T2 T4 T2 T4 T1 T4 T2 T4 T1 T1 T2 T2 T1 T3	14
20	T1 T1 T1 T1 T4 T3 T2	7
	Total	182

b 1 Comment on the fairness of the experiment.

b In order to ensure fairness throughout the experiment, the pointer of the spinner was spun from the wider end by the same person each time. Twenty experiments were performed in total; however, this amount could be increased.

2 Answer the original question.
Write the total number of packets and total number of experiments.

Total number of packets

$= 8 + 7 + \ \ldots + 14 + 7$
$= 182$

Total number of experiments $= 20$

3 Write the rule for the average number of packets.

$$\text{Average} = \frac{\text{total number of packets}}{\text{total number of experiments}}$$

4 Substitute the known values into the rule.

$$\text{Average number of packets} = \frac{182}{20}$$

5 Evaluate.

$= 9.1$

6 Round the value to the nearest whole number.
Note: Since we are dealing with packets we must work with whole numbers.

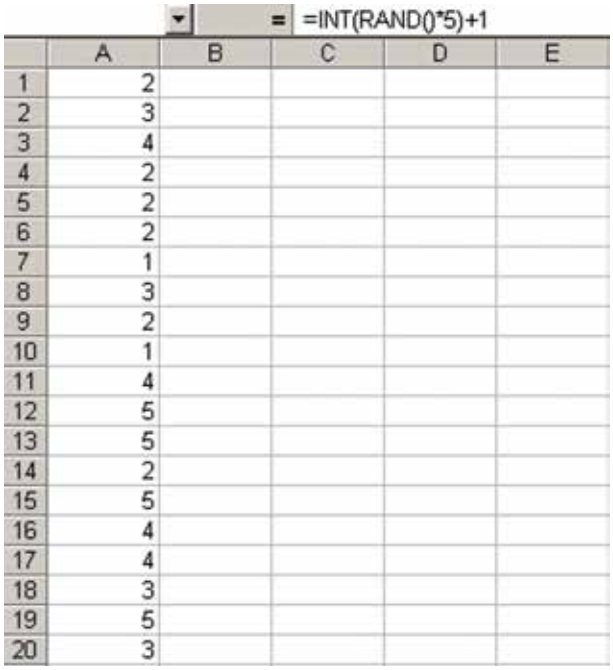
$= 9$

7 Summarise your findings.
Note: Comment on points of interest; that is, the maximum and minimum number of packets that need to be purchased.

The average number of packets of cereal Alex must purchase to obtain each of the 4 plastic toys is 9.
From this experiment, the minimum number of packets needed to obtain the 4 toys is 4 while the largest number of packets needed is 22.

10.6.4 Generating random numbers on an Excel spreadsheet

- The random number generator [=rand()] on the spreadsheet generates a decimal number between 0 and 1. We need to convert this decimal number to a whole number between 1 and 5.



	A	B	C	D	E
1	2				
2	3				
3	4				
4	2				
5	2				
6	2				
7	1				
8	3				
9	2				
10	1				
11	4				
12	5				
13	5				
14	2				
15	5				
16	4				
17	4				
18	3				
19	5				
20	3				

- Enter the formula
  = **INT**(**RAND**()***5**) + **1** into cell A1.
- Multiplying the random decimal number by 5 produces a random decimal number between 0 and 5. The **INT** function will change the decimal to a whole number. We finally add 1 to make it a whole number between 1 and 5.
- Use the **Fill Down** function to fill this formula down to cell A20. You should now have 20 random numbers as shown in this screen dump.
- Note that every time you perform an action on this spreadsheet, the random numbers will change. We can stop this by following these instructions.
  Step 1: Choose **Options** from the **Tools** menu.
  Step 2: Select the calculation tab and click the manual calculation radio button.
  Step 3: You can obtain a recalculation of your random numbers by pressing **F9**.

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Try out this interactivity: Random number generator (int-0089)

Complete this digital doc: WorkSHEET: Probability II (doc-6544)

Exercise 10.6 Experimenting with chance

assesson

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–9, 13	Questions: 1–10, 13	Questions: 1–13

Individual pathway interactivity: int-4366

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To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note*: Question numbers may vary slightly.

Fluency

1. WE8 Teagan was playing Trouble and recorded the number of times she rolled a 6. During the game, she was successful 5 times out of the 25 times she tried. What was the experimental probability of rolling a 6 in the game?

2. WE9 **a.** Copy the table at right. Toss a coin 10 times and record the results in the first row of the table.

b. What is the probability of obtaining a Head from your experiment?

c. What is the probability of obtaining a Tail from your experiment?

d. How do these values compare with the theoretical results?

e. Repeat step **a** another 5 times and combine all your results.

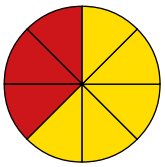
f. How does the combined result compare with the theoretical results?

Experiment number	Heads		Tails	
	Tally	Count	Tally	Count
1				
2				
3				
4				
5				
6				
	Total		Total	

3. If you wanted to create a device that would give a theoretical probability of achieving a particular result as $\frac{1}{4}$, how many sections would a spinner such as this need to be divided into?

4. How would you divide or colour a spinner if you wanted to achieve the probability of a success equal to $\frac{3}{10}$?

5. For the spinner at right, what would be the probability of getting the red section?



6. WE10 Repeat the experiment described in Worked example 10.

Understanding

7. Use your results from question **2** to answer the following.

a. The long-term trend of the probability of obtaining a Head on the toss of a coin is the P(Heads) from your experiment.
What is the long-term trend of the probability after:

i. 10 tosses of the coin **ii.** 20 tosses of the coin

iii. 30 tosses of the coin **iv.** 60 tosses of the coin?

b. Obtain a classmate's 60 results. Combine these with yours. State the long-term trend of P(Heads) obtained.

c. Combine your pair's 120 results with another pair's. State the long-term trend of P(Heads) obtained.

d. Finally, count the results obtained by the whole class for this experiment. (Make sure nobody's results are counted twice.) You should have 60 tosses per person. State the long term trend of P(Heads) obtained.

e. Copy and complete the table below.

Number of tosses	Heads		Tails	
	P(Heads)	P(Heads) as percentage	P(Tails)	P(Tails) as percentage
10				
20				
30				
60				
120				

(*continued*)

Number of tosses	Heads		Tails	
	P(Heads)	P(Heads) as percentage	P(Tails)	P(Tails) as percentage
240				
Whole class (specify number of tosses)				

f. Comment on the changes of the long-term trend value of P(Heads) as you toss the coin more times.

8. Use the **Random number generators** interactivity in your Resources tab to simulate a 5-colour spinner.

a. What is the chance of getting any one of the 5 colours when you spin the spinner (theoretically)?

b. Spin the spinner 10 times and, using a table such as the one below, record your results.

Colour	1	2	3	4	5
Number of times it occurs					

c. From your results, list the probabilities of obtaining each colour. For example, divide the number of times a particular colour was obtained by the total number of spins (that is, 10).

d. Why might these probabilities not be the same as the theoretical probability would suggest?

e. Spin your spinner and record the results for another 10 spins.

f. Spin your spinner so that you have 100 results. Is the experimental probability closer to the pure probability? Why might this be? Discuss.

9. Inside a bag are 36 shapes which are either squares or triangles. One shape is taken out at random, its shape noted and put back in the bag. After this is repeated 72 times, it is found that a triangle was taken out 24 times. Estimate how many triangles and how many squares there are in the bag.

Reasoning

10. You have calculated previously that the chance of getting any particular number on a 6-sided die is $\frac{1}{6}$. You have 2 different coloured dice. Is there any difference in your dice apart from colour? Could one be biased (more likely to give a particular result than theory says it should)?

a. Design a test to determine whether the dice you have are fair. Write down what you are going to do.

b. Perform your test, and record your results.

c. Calculate the probability of getting each of the numbers on each of your dice, based on your tests and on the long-term trend you have observed.

d. What does your test say about your 2 dice? Are there any things that need to be considered before giving your answer? (Perhaps your dice have slightly uneven shapes or something that might cause them to lean towards one result more than others.)

11. Use a box of Smarties, marbles, or technology such as a graphics calculator or an Excel computer program for this question. Count the number of items of each colour in the box before you start.

a. Calculate the theoretical probability of getting a particular colour if you pick 1 out of the box without looking.

b. Design an experiment to determine the probability of getting a particular colour out of the box, using the long-term trend.

c. Why is it important that if you take Smarties (marbles or a colour in the simulation) out of the box for this experiment, you must put them back each time?

d. Could you use something other than your box of Smarties, marbles or technology to determine this probability? What other things could you use to simulate this experiment?

Problem solving

12. In your desk drawer, there are 5 identical red pens and 6 identical black pens. What is the smallest number of pens you have to remove from the drawer in the dark so that you will be absolutely sure of having:

a. 2 black pens
b. 2 red pens
c. 1 black pen and 1 red pen?
d. Explain your answers to parts **a, b** and **c**.

13. a. Conduct the following experiments:

i. Toss a coin 10 times and record the number of heads and tails that occur.
ii. Toss a coin 25 times and record the number of heads and tails that occur.
iii. Toss a coin 50 times and record the number of heads and tails that occur.
iv. Toss a coin 100 times and record the number of heads and tails that occur.

b. Calculate the experimental probability for each experiment.
c. Compare these values and explain your findings.

Reflection

How is the experimental probability of an event related to its theoretical probability?

10.7 Review

10.7.1 Review questions

Fluency

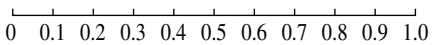
1. For each of the given statements, specify whether the chance of the following events occurring is certain, likely, even chance, unlikely or impossible.

a. Australia is in the southern hemisphere.
b. You will still be alive in the next century.
c. Obtaining an even number on a circular spinner numbered from 1 to 16.
d. England is in the southern hemisphere.
e. You roll a fair die and obtain a number less than or equal to 4.
f. Humans can survive without water indefinitely.

2. List three events where the chance of each occurring would be:

a. impossible
b. certain
c. improbable.

3. Copy the number line below and place the list of words on it to indicate what sort of chance each number would represent.
Words to place: certain, very unlikely, unlikely, likely, highly likely, highly unlikely, even chance, impossible, very likely.



4. List the sample space for the following experiments:

a. tossing a coin
b. drawing a marble out of a bag containing red, green and blue marbles
c. rolling a 6-sided die
d. spinning a circular spinner numbered from 1 to 10
e. drawing a 4 card from a pack of playing cards
f. answering a true/false question in a test
g. competing in a race in which there are 9 runners
h. choosing a year level to represent the secondary school at a public event.

5. For each of the following, state the number of elements in the sample space.
   a. The first prize in a raffle is chosen from tickets numbered from 1 to 5000.
   b. A card selected from the red cards in a standard deck.
   c. A circular spinner numbered 1 to 5 is spun.
   d. A day is selected in the month of July.

6. A six-sided die, labelled 1, 2, 3, 3, 4, 5 is rolled. Find:
   a. P(getting a 5)
   b. P(getting a 3)
   c. P(getting an even number)
   d. P(getting an odd number)
   e. P(getting a number greater than or equal to 3)
   f. P(getting a number less than 3)
   g. P(getting a 0)
   h. P(getting a 6).

7. A person has a normal pack of cards and draws one out. Find:
   a. P(picking a heart)
   b. P(picking a spade)
   c. P(picking a picture card)
   d. P(picking a card with a number less than 5 on it). Do not an count ace as a number.

8. Ten cards are numbered from 1 to 10, shuffled and placed face down on a table. If a card is selected at random, find the probability that the card selected is:
   a. 4
   b. an even number
   c. divisible by 3
   d. an even number and divisible by 3
   e. an even number or divisible by 3
   f. not divisible by 3
   g. greater than 8
   h. 12.

9. Use a table to show the sample space for the experiment 'tossing a coin and rolling a die'. Find:
   a. P(a tail and a number less than 3)
   b. P(a head and a number greater than 3)
   c. P(a tail and an even number)
   d. P(a head and a prime number).

10. Use a table to show the sample space for the experiment 'spinning a spinner with 10 sectors and rolling a die'. Find:
   a. P(the sum totals to an even number)
   b. P(odd number on the spinner and an even number on the die)
   c. P(both digits having a value less than 5)
   d. P(the sum totals to a value between 7 and 10)
   e. P(the sum totals to a value greater than or equal to 13 but less than 16)
   f. P(the sum totals to a prime number).

11. A sample of 250 students at a particular school found that 225 of them had access to the internet at home. Given this sample is a good representation for the entire school, what is the probability that a student selected at random in the school will have internet access at home?

12. Jane and her father have decided to play a game of chess against each other. Neither has played before but each has been given the same explanation of the rules of the game.
   a. What would be the probability of either of them winning a game?
   b. Explain your answer.

Since her first game with her father, Jane has joined a chess club and competes regularly against various players. Her father, on the other hand, plays occasionally. Over a number of years Jane and her father have played 340 games. Jane has won 255 of these games.

c. Given this long-term trend, what would be the probability that the father wins the next game?

d. Does the probability remain the same over a period of time? Explain your answer.

Problem solving

13. In your sock drawer there are only 4 identical blue socks and 5 identical black socks. What is the smallest number of socks you can remove from the drawer in the dark so that you will be absolutely sure of having:

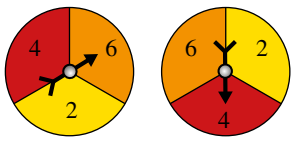
a. a pair of black socks?

b. a pair of blue socks?

c. a black pair and a blue pair of socks?

14. Danielle wants to know how many packets of cereal she must purchase in order to collect 6 different limited edition Disney cards. Design an experiment that will simulate the given situation. Provide clear instructions and details of 2 different devices that may be used for the simulation.

15.



Spinner A Spinner B

The arrows on Spinner A and Spinner B are spun. (If an arrow lands on a line, the spinner is spun again.) The two numbers are added to get a score. For example, in the diagram above, the score is 10.

a. What is the highest total score possible?

b. What are the possible total scores?

c. List all the ways to get a total score of 8.

d. What is the probability of getting a total of 9?

e. What is the probability of getting a total score of 10?

f. Draw a grid showing the probabilities of getting all possible totals.

16. Mark and Cameron play a game in which they toss two coins. Mark wins if two Heads turn up, and Cameron wins if two Tails turn up. If both a Head and a Tail turn up, they throw one of the coins again. If this comes up Heads, Mark wins; if it comes up Tails, Cameron wins. Do you think this is a fair game? Discuss with a classmate and explain your reasoning.

17. Charlotte and Rhianna have a flower garden. They have 17 red flowers, 12 pink flowers and 13 yellow flowers. Charlotte picks some flowers for her vase. She selects 6 red flowers, 3 pink and 3 yellow flowers. If Rhianna picks a flower at random for her hair, what is the probability that it is yellow?

18. A bowl contains blue marbles and white marbles. If there are twice as many blue marbles as white marbles, what is the probability that a blue marble is selected?

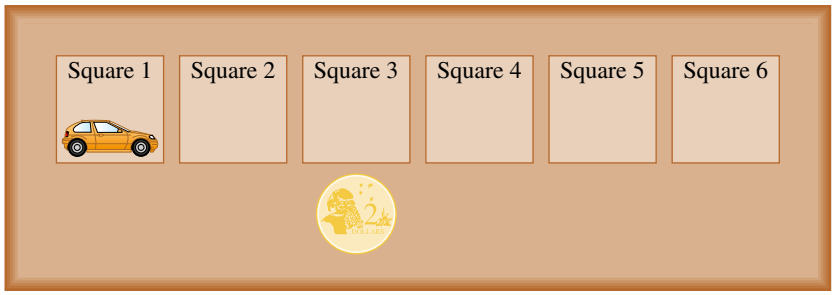
19. Mrs Prince gives Charles 2 containers, 10 green marbles and 10 gold marbles. She tells him to arrange the marbles any way he likes (all of the marbles must go into the containers). She chooses 1 marble from 1 container. If she picks a gold one, she'll give Charles $100. How should Charles arrange the marbles to have the best chance of getting $100?

20. Consider the following game. You have a board (see diagram on the following page), a token to move and a fair coin to flip. Each turn consists of the following two steps.

1. Flip the coin.

2. Move two squares to the right for Heads, and one square to the right for Tails.

What is the probability that you will land on Square 5 in exactly three turns?



21. You have a spinner. Draw a circle graph to represent the following.
Discuss the chances of landing on each of the colours. List the likelihoods from smallest to largest.

Colour	Degree
Red	20°
Orange	40°
Yellow	60°
Blue	100°
Green	20°
Purple	120°

22. Chloë is a contestant on a game show. There are five sealed cases on the podium and each of the cases contains one of the following amounts: $1, $50, $500, $1000 and $5000. The game-show host offers her a deal of $400, or she can choose a case and keep the amount of money in it, instead. What is the probability she will win more than the $400 the host is offering?

23. To 'get out of jail' when playing a board game, the player must throw a double from two dice that are rolled three times. What is the probability of getting at least one double in these three throws?

24. Rebecca plays a sideshow game where she puts four balls into a clown's mouth, and the balls then fall into slots numbered 1 to 6. To win, one of her balls must go into Slot 6. She has noticed that the numbers 2 and 5 come up 4 times as often as 1 and 6, and the numbers 3 and 4 come up 5 times as often as 1 and 6. What is the probability that she will get a 6?

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Complete this digital doc: Concept map: Topic 10 (doc-10737)

Language

It is important to learn and be able to use correct mathematical language in order to communicate effectively. Create a summary of the topic using the key terms below. You can present your summary in writing or using a concept map, a poster or technology.

certain
chance
even chance
event
experiment
experimental probability
favourable outcome
impossible
likely
long-term trend
outcomes
probability
random numbers
sample space
simulation
successful trial
theoretical probability
trial
unlikely

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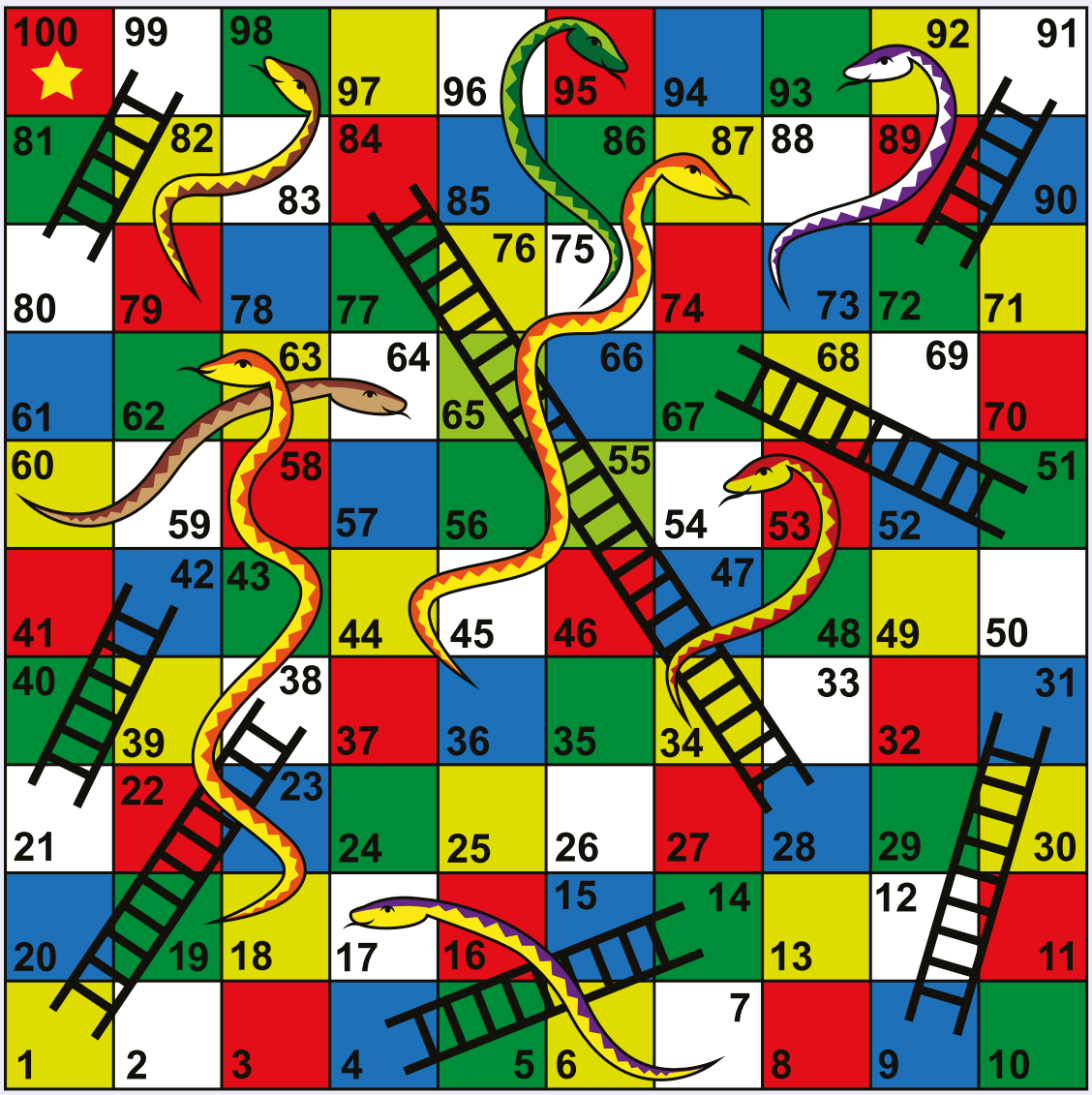
Investigation | Rich task

Snakes, ladders and probability!



In the game of snakes and ladders you roll 2 dice, add the 2 numbers that appear on the uppermost faces and move a marker that many places around a 100-square board. If you are lucky and your marker lands on the base of a ladder, you can advance more quickly in the game by 'climbing' to the top of the ladder. However, if your marker lands on the head of a snake, you must 'slide down' the snake to the end of its tail.

1. Complete the table at right to show the possible totals when 2 dice are rolled. Some of the values have been included for you.

Die 2 \ Die 1	1	2	3	4	5	6
1	2	3	4			
2	3	4	5			
3	4					
4						
5						
6						

2. List the sample space for the possible totals when 2 dice are rolled.
3. Which total appears the most?
4. Which total appears the least?
5. What is the probability of getting a total of 3 when 2 dice are rolled?

Using the table that shows the frequency of the totals, we can investigate the probabilities involved in moving around the snakes and ladders board. The following situations will enable you to investigate some of the possibilities that occur in snakes and ladders.

6. Imagine you landed on square 95 and slid down the snake to square 75. What total would you need to go up the ladder at square 80 on your next move? In how many ways can this total be achieved in one turn?
7. If you slid down the snake at square 87, is it possible to move up the next ladder with your next turn? Explain.
8. Explain what would happen if you were on square 89 and rolled two 1's and rolled two 1's again with your next turn. What would be the likelihood of this happening in a game?
9. Describe how you could get from square 71 to square 78 in one turn. Work out the probability of this happening.
10. Imagine you had a streak of luck and had just climbed a ladder to square 91. Your opponent is on square 89. Explain which player has the greater chance of sliding down the snake at square 95 during the next turn.
11. Investigate the different paths that are possible in getting from start to finish in the fewest turns. For each case, explain the totals required at each turn and discuss the probability of obtaining these totals.

Play a game of snakes and ladders with a partner. Examine your possibilities after each turn, and discuss with each other the likelihood of moving up ladders and keeping away from the snakes' heads as you both move around the board.

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Complete this digital doc: Code puzzle: In 1874 Davis and Strauss made the first ... (doc-14329)

Answers

Topic 10 Probability

Exercise 10.2 The language of chance

1. a. Certain **b.** Likely **c.** Impossible **d.** Impossible **e.** Certain
f. Likely **g.** Unlikely **h.** Unlikely **i.** Impossible **j.** Even chance
k. Certain **l.** Certain **m.** Impossible **n.** Even chance **o.** Likely

2. D, C, A, B

3. a. $\frac{1}{2}$ (or 0.5) **b.** 0 **c.** $\frac{1}{5}$ (or 0.2) **d.** $\frac{1}{2}$ (or 0.5) **e.** $\frac{1}{2}$ (or 0.5)
f. $\frac{1}{4}$ (or 0.25) **g.** $\frac{4}{5}$ (or 0.8) **h.** 1 **i.** 1
j. 0.3–0.7. Any value within this range is reasonable. Discuss.

4. Class discussion or check with your teacher. **5.** A **6.** A

7. a. 1 **b.** 0.75 **c.** 0.25 **d.** 0.75 **e.** 0.25 **f.** 1 **g.** 0
h. 0.25 **i.** 1 **j.** 0.25 **k.** 0 **l.** 0.75 **m.** 0.5 **n.** 0.5

8. a. It is impossible because Tuesday always follows Monday.
b. It is certain because the 29th always follows the 28th day of the month. However, if the month is February, depending on whether the year is a leap year or not, the chance is either impossible or certain.
c. It has an equal chance or a fifty-fifty chance if it is a coin with a Head on one side and a Tail on the other side.

9. a. Highly unlikely because there is only one piece with the letter I. The chance is $\frac{1}{10}$ or 0.1.
b. Equal chance because there are five pieces with the letter A and five pieces with other vowels.
c. Impossible because there are no pieces with the letter U.
d. Unlikely because the chance is $\frac{3}{10}$ or 0.3.

10. Equal chance: 50% or $\frac{1}{2}$ or 0.5.

11. a. The two girls are living in two different countries; one in the northern hemisphere and one in the southern hemisphere.
b. Time zones in different places, night time and day time in different places in the world.

Exercise 10.3 The sample space

1. $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

2. a. $S = \{\text{Heads, Tails}\}$ **b.** $S = \{A, B, C, D, E\}$ **c.** $S = \{\text{win, loss, draw}\}$
d. $S = \{\text{ace of clubs, ace of spades, ace of hearts, ace of diamonds}\}$ **e.** $S = \{A, B, C, D, E, F\}$

3. a. $S = \{\text{clubs, spades, diamonds, hearts}\}$ **b.** $S = (\text{ace, } 2, 3, 4, 5, 6, 7, 8, 9, 10, \text{jack, queen, king}\}$
c. $S = \{\text{red, black}\}$

4. $S = \{\text{red, green, orange}\}$

5. 2

6. a. 52 **b.** 45 **c.** 18 **d.** 365 (or 366 in a leap year)
e. 26 **f.** 180 000 **g.** 4 **h.** 3

7. E

8. 12 different ways:
20, 20, 10; 20, 20, 5, 5; 20, 10, 10, 10;
20, 10, 10, 5, 5; 20, 10, 5, 5, 5, 5; 20, 5, 5, 5, 5, 5, 5;
10, 10, 10, 10, 10; 10, 10, 10, 10, 5, 5;
10, 10, 10, 5, 5, 5, 5; 10, 10, 5, 5, 5, 5, 5, 5; 10, 5, 5, 5, 5, 5, 5, 5, 5; 5, 5, 5, 5, 5, 5, 5, 5, 5, 5

9. a. rH rT bH bT gH gT wH wt

b.

rH1	rH2	rH3	rH4	rH5	rH6
rT1	rT2	rT3	rT4	rT5	rT6
gH1	gH2	gH3	gH4	gH5	gH6
gT1	gT2	gT3	gT4	gT5	gT6
bH1	bH2	bH3	bH4	bH5	bH6
bT1	bT2	bT3	bT4	bT5	bT6

10. 15 choices

11. 12

12. 11

Challenge 10.1

18 ways

Exercise 10.4 Simple probability

1. a. 12: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 **b.** 5: 1, 2, 3, 4, 5 **c.** 2: C, L **d.** 3: red, blue, black

c. 5: black, blue, green, red, yellow **d.** 3: 2, 4, 6 **g.** 2: 4, 6 **h.** 10: 1, 3, 5, 7, 9, 11, 13, 15, 17, 19

2. a. 1, 2, 3, 4, 5, 6. All equally likely.

b. Heads, Tails. Each is equally likely.

c. White, black. Each is equally likely.

d. White, blue, red. It is more likely you will get white rather than red or blue, because it takes up a bigger area.

e. 1, 2, 3, 4, 5. It is more likely that a five will be rolled because it appears twice on the die.

f. Blue, green, red. Each is equally likely.

g. a, e, i. It is more likely that a consonant would be obtained as there are 7 consonants and 4 vowels.

3. a. 1, 2, 3, 4, 5, 6, 7, 8, 9, 10

b. i. $\frac{1}{10}$ **ii.** $\frac{3}{5}$ **iii.** $\frac{2}{5}$ **iv.** $\frac{7}{10}$ **v.** $\frac{3}{10}$ **vi.** 0 **vii.** $\frac{3}{10}$ **viii.** $\frac{1}{5}$

4. a. 20 **b. i.** $\frac{3}{10}$ **ii.** $\frac{1}{5}$ **iii.** $\frac{7}{20}$ **iv.** $\frac{3}{20}$

c. 10 **d.** $\frac{1}{2}$ **e.** $\frac{1}{2}$ **f.** $\frac{13}{20}$ **g.** 1

h. When the probability of a particular result is certain (in this case, that the marble must be either blue, red, green or black), it equals 1.

5. $\frac{2}{5}$

6. E

7. D

8. a. $\frac{1}{2}$ **b.** $\frac{1}{13}$ **c.** $\frac{1}{4}$ **d.** $\frac{3}{13}$ **e.** $\frac{1}{52}$ **f.** $\frac{3}{4}$ **g.** $\frac{12}{13}$ **h.** 1 **i.** $\frac{3}{4}$ **j.** $\frac{1}{26}$

9. a. 36 square units **b.** A 20 square units B 12 square units C 4 square units

c. i. $\frac{5}{9}$ **ii.** $\frac{1}{3}$ **iii.** $\frac{1}{9}$

d. i. $\frac{37}{351}$, which is approx 0.11. This is very close to the theoretical probability.

ii. $\frac{631}{1872}$, which is approx 0.34. This is very close to the theoretical probability.

iii. $\frac{3131}{5616}$, which is approx 0.56. This is very close to the theoretical probability.

10. a. $\frac{1}{100\,000}$ **b.** $\frac{1}{5000}$ **c.** $\frac{1}{2000}$ **d.** 1

e. It would not be a very wise purchase because the total cost would be $500 000 unless of course the car was worth more than $500 000.

11. a. 100% **b.** 50% **c.** $\frac{1}{7}$ **d.** $\frac{2}{7}$ **e.** Decrease **f.** Increase

12. a.

Score	Frequency
1	7
2	4
3	8
4	3
5	5
6	3

b. $P(6) = \frac{1}{6}$

c. 5

d. The result differs by pure chance. The more often we roll the die the closer we can expect the results to match the expected number of sixes.

13. a. $\frac{13}{73}$

b. Karen is not correct, as a wet day (65 out of 365) and a dry day (300 out of 365) are not equally likely.

c. 0.18

Challenge 10.2

The probability of taking a banana is $\frac{2}{5}$.

Exercise 10.5 Using a table to show sample spaces

1. a. {1, 2, 3, 4, 5, 6}

b. {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

c. {fruit salad, cheesecake, mudcake, cheese platter}

d. {5, 10, 15, 20, 25, 30, 35, 40, 45, 50}

e. {Victoria, New South Wales, Queensland, Northern Territory, South Australia, Tasmania, Western Australia, Australian Capital Territory}

f. {true, false}

g. {king of hearts, king of diamonds, king of spades, king of clubs}

h. {guitar, drum, saxophone, piano, trumpet}

2. a.

Die \ Spinner	A	B	C
1	A1	B1	C1
2	A2	B2	C2
3	A3	B3	C3
4	A4	B4	C4
5	A5	B5	C5
6	A6	B6	C6

{(A, 1), (A, 2), (A, 3), (A, 4), (A, 5), (A, 6), (B, 1), (B, 2), (B, 3), (B, 4), (B, 5), (B, 6), (C, 1), (C, 2), (C, 3), (C, 4), (C, 5), (C, 6)}

b. 18

c. i. $\frac{1}{3}$ **ii.** $\frac{1}{6}$ **iii.** $\frac{2}{3}$ **iv.** $\frac{1}{3}$ **v.** $\frac{1}{2}$

vi. $\frac{1}{6}$ **vii.** 1 **viii.** $\frac{5}{6}$ **ix.** $\frac{1}{9}$ **x.** 0

3. a.

Coin 2 \ Coin 1	Head	Tail
Head	H H	T H
Tail	H T	T T

b. 4 **c.** 1 **d.** 1 **e.** 2

f. $\frac{1}{4}$ **g.** $\frac{1}{4}$ **h.** $\frac{1}{2}$

4. a.

10c Coin \ 5c Coin	Head	Tail
Head	H H	T H
Tail	H T	T T

b. 4 **c.** $\frac{1}{4}$ **d.** $\frac{1}{4}$

e. $\frac{1}{4}$ **f.** $\frac{1}{4}$

5. a.

Die \ Coin	Head	Tail
1	H 1	T 1
2	H 2	T 2
3	H 3	T 3
4	H 4	T 4
5	H 5	T 5
6	H 6	T 6
7	H 7	T 7
8	H 8	T 8
9	H 9	T 9
10	H 10	T 10

b. 20 **c.** $\frac{1}{20}$ **d.** $\frac{1}{4}$ **e.** $\frac{1}{2}$ **f.** $\frac{3}{20}$

g. $\frac{1}{5}$ **h.** $\frac{1}{2}$ **i.** $\frac{1}{4}$ **j.** 1

6. a.

Blue die \ Red die	1	2	3	4	5	6
1	11	21	31	41	51	61
2	12	22	32	42	52	62
3	13	23	33	43	53	63
4	14	24	34	44	54	64
5	15	25	35	45	55	65
6	16	26	36	46	56	66

Note: Your table may list the blue die along the top and therefore it would be listed first.

b. 36 **c.** $\frac{1}{36}$ **d.** $\frac{1}{36}$ **e.** $\frac{11}{36}$ **f.** $\frac{1}{4}$ **g.** $\frac{1}{4}$ **h.** $\frac{1}{4}$ **i.** $\frac{5}{36}$ **j.** 0 **k.** 0

7. a.

Coin \ Marble colour	Red	Blue	Green
Head	R H	B H	G H
Tail	R T	B T	G T

b. 6 **c.** $\frac{1}{6}$ **d.** $\frac{1}{6}$ **e.** $\frac{1}{6}$ **f.** $\frac{1}{3}$ **g.** $\frac{1}{3}$ **h.** $\frac{1}{2}$

i. The probability of picking out a red, blue or green marble and getting a Head $= \frac{1}{2}$, which is the same as the result you get if you were just to toss a coin. This is because you must get one of these colour marbles, so the only choice in the experiment here is whether you get a Tail or a Head, so P(Head) $= \frac{1}{2}$.

8. C

9. *Note:* You may have chosen the opposite number for rows and columns. This is also correct.

a. i. 31 rows, 2 columns **ii.** 62

b. i. 2 rows, 3 columns **ii.** 6

c. i. 72 rows, 6 columns **ii.** 432

d. i. 10 rows, 6 columns **ii.** 60

e. i. 30 rows, 6 columns **ii.** 180

f. i. 100 rows, 2 columns **ii.** 200

10. 16 576 000

11. $\frac{1}{18}$

12. 28 dominoes

13. RB145, RB154, RB415, RB451, RB514, RB541, BR145, BR154, BR415, BR451, BR514, BR541

Exercise 10.6 Experimenting with chance

1. $\frac{1}{5}$

2. a.

Exp. No.	Heads		Tails	
	Tally	Count	Tally	Count
1	𝍸	4	𝍸 |	6
2	|	1	𝍸 𝍸	9
3	𝍸 |	6	𝍸	4
4	𝍸 |||	8	||	2
5	𝍸	4	𝍸 |	6
6	𝍸 ||	7	|||	3
	Total	30	Total	30

Note: This is only one possible solution. Answers will differ each time.

b. $\frac{2}{5}(40\%)$

c. $\frac{3}{5}(60\%)$

d. The theoretical value for both results is $\frac{1}{2}$ (50%). They differ by 10%.

e. Refer to table.

f. The combined result equals the theoretical value.

3. 4

4. Divide the spinner into 10 sections; 3 of these sections will be shaded in 1 colour and 7 sections will be shaded in another colour.

5. $\frac{3}{8}$

6. See Worked example 10.

Note: These values will differ each time the experiment is performed.

Spreadsheets could also be used.

7. Values will differ for each group. As you complete more trials, you will probably notice the values you get experimentally are closer to those you would expect from theoretical probability ($\frac{1}{2}$ or 50%). If this is not occurring, though, you will probably need more trials.

8. a. $\frac{1}{5}$

b.

Colour	1	2	3	4	5
Number of times it occurs		||	||	𝍸	||

c. $P(1) = 0$ $P(2) = \frac{1}{5}$ $P(3) = \frac{1}{5}$ $P(4) = \frac{2}{5}$ $P(5) = \frac{1}{5}$

Note: This is the only possible solution. Answers will differ each time.

d. Your spinner may not be evenly balanced and this may lead to an increased likelihood of getting one result or another.

e.–f. Values will differ for each group. As you complete more trials, you will probably notice the values you get experimentally are closer to those you would expect from theoretical probability ($\frac{1}{5}$ or 20%). If this is not occurring, though, you will probably need more trials.

9. 12 triangles, 24 squares

10. a. Each die would need to be thrown about 120 times to get some impression of whether it is biased or not. The same person would need to roll the die each time in the same manner and onto the same surface.

b.

Number on die	Black	White
1	20	19
2	19	21
3	20	20
4	21	20
5	19	20
6	21	20
Total	120	120

Note: These values will differ each time the experiment is conducted.

c.

Number on die	Probability of black	Probability of white
1	$\frac{20}{120} = \frac{1}{6}$	$\frac{19}{120}$
2	$\frac{19}{120}$	$\frac{21}{120} = \frac{7}{40}$
3	$\frac{20}{120} = \frac{1}{6}$	$\frac{20}{120} = \frac{1}{6}$
4	$\frac{21}{120} = \frac{7}{40}$	$\frac{20}{120} = \frac{1}{6}$
5	$\frac{19}{120}$	$\frac{20}{120} = \frac{1}{6}$
6	$\frac{21}{120} = \frac{7}{40}$	$\frac{20}{120} = \frac{1}{6}$
Total	1	1

The long-term trend suggests that the probability of obtaining each value on either dice will be $\frac{1}{6}$.

d. The 2 dice appear to be fair as each value occurred approximately 20 times, which is what we would expect in 120 throws. More trials however could be conducted. It is very important when conducting an experiment such as this that the devices used are even in shape and size and that one doesn't have an advantage over the other.

11. a. Answers will vary for each packet.

b. Check with your teacher.

c. Smarties must be replaced each time they are removed, otherwise the chance of taking out 1 of a certain colour would be changed.

d. You could use a spinner with sections coloured to give the same probability for each colour as you would get for removing the Smarties from the box. (This is only one possible answer.)

12. a. 7 **b.** 8 **c.** 7

d. There are 5 red pens so, to make sure that you pick 2 black pens, you have to pick $5 + 2 = 7$.

e. There are 6 black pens so, to make sure that you pick 2 red pens, you have to pick $6 + 2 = 8$.

f. There are 5 red pens and 6 black pens so, to make sure that you pick 1 black pen and 1 red pen, you have to pick $6 + 1 = 7$.

13. Various answers are possible depending on the results of the experiment.

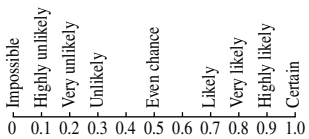
10.7 Review

1. a. Certain **b.** Unlikely **c.** Even chance

d. Impossible **e.** Likely **f.** Impossible

2. Discuss in class.

3.



4. a. {Head, Tail}

b. {red marble, green marble, blue marble}

c. {1, 2, 3, 4, 5, 6}

d. {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

e. {4 of clubs, 4 of diamonds, 4 of hearts, 4 of spades}

f. {true, false}

g. {1, 2, 3, 4, 5, 6, 7, 8, 9}

h. {Year 7, Year 8, Year 9, Year 10, Year 11, Year 12}

5. a. 5000 **b.** 26 **c.** 5 **d.** 31

6. a. $\frac{1}{6}$ **b.** $\frac{1}{3}$ **c.** $\frac{1}{3}$ **d.** $\frac{2}{3}$ **e.** $\frac{2}{3}$ **f.** $\frac{1}{3}$ **g.** 0 **h.** 0

7. a. $\frac{1}{4}$ **b.** $\frac{1}{4}$ **c.** $\frac{3}{13}$ **d.** $\frac{3}{13}$

Note: Do not count an ace as a number.

8. a. $\frac{1}{10}$ **b.** $\frac{1}{2}$ **c.** $\frac{3}{10}$ **d.** $\frac{1}{10}$ **e.** $\frac{7}{10}$ **f.** $\frac{7}{10}$ **g.** $\frac{1}{5}$ **h.** 0

9.

Die \ Coin	Head	Tail
1	H 1	T 1
2	H 2	T 2
3	H 3	T 3
4	H 4	T 4
5	H 5	T 5
6	H 6	T 6

a. $\frac{1}{6}$ **b.** $\frac{1}{4}$ **c.** $\frac{1}{4}$ **d.** $\frac{1}{4}$

10.

Die \ Spinner	1	2	3	4	5	6	7	8	9	10
1	11	21	31	41	51	61	71	81	91	101
2	12	22	32	42	52	62	72	82	92	102
3	13	23	33	43	53	63	73	83	93	103
4	14	24	34	44	54	64	74	84	94	104
5	15	25	35	45	55	65	75	85	95	105
6	16	26	36	46	56	66	76	86	96	106

a. $\frac{1}{2}$ **b.** $\frac{1}{4}$ **c.** $\frac{4}{15}$ **d.** $\frac{1}{5}$ **e.** $\frac{3}{20}$ **f.** $\frac{23}{60}$

11. $\frac{9}{10}$

12. a. $\frac{1}{2}$

b. Since both opponents have not played chess before and are given the same instructions on the rules of the game, it is safe to say they will be evenly matched. Neither has an advantage over the other.

c. $\frac{1}{4}$

d. No, the probabilities are not the same; they have changed because Jane has practised and developed skills while her father has not.

13. a. 6 socks. Worst case scenario is first picking 4 blue socks, then 2 black socks.

b. 7 socks. Worst case scenario is first picking 5 black socks then 2 blue socks.

c. 7 socks. Minimum needed to guarantee a pair of black socks is 6 (from part **a**). Picking one more guarantees a pair of blue socks will be included (from part **b**).

14. Danielle will need to obtain a fair die with 6 sides to represent each card. She will need to roll the die in the same manner and on the same surface each time. Danielle must continue to roll the die until each number 1, 2, 3, 4, 5, 6 has appeared. This will represent one experiment. She will need to repeat this process at least 20 times until she can obtain the average number of cereal packets she must purchase in order to obtain each of the cards. Danielle may also use a circular spinner with 6 equal sectors or 6 tennis balls labelled 1 to 6 in a box and follow the same procedure outlined above. She may also generate random numbers using the calculator or Excel spreadsheet.

15. a. 12 **b.** 4, 6, 8, 10, 12 **c.** 2 + 6, 6 + 2, 4 + 4

d. 0 **e.** $\frac{2}{9}$

f.

	2	4	6
2	4	6	8
4	6	8	10
6	8	10	12

4	6	8	10	12
$\frac{1}{9}$	$\frac{2}{9}$	$\frac{3}{9}$	$\frac{2}{9}$	$\frac{1}{1}$

16. Yes **17.** $\frac{1}{3}$ **18.** $\frac{2}{3}$

19. If one container has one gold marble, if that container is selected, Charles is certain to receive \$100. This leaves 9 gold marbles and 10 green marbles in the second container. The probability that the gold is selected from this container is $\frac{9}{19}$. The overall probability that Charles will receive \$100: $\frac{1}{2} \times 1 + \frac{1}{2} \times \frac{9}{19} = \frac{14}{19}$

20. $\frac{3}{8}$

21. Red, green, orange, yellow, blue, purple

22. $\frac{3}{5}$

23. $\frac{91}{216}$

24. $\frac{1}{20}$

Investigation — Rich task

1.

Die 2 \ Die 1	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

2. $\{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$

3. 7

4. 2, 12

5. $\frac{1}{18}$

6. 5, four different ways.

7. You end up at square 36. The next ladder is at square 51. It is not possible to obtain a 15 when you roll 2 dice.

8. At the end of these two turns you would end up at square 73. P(two ones in a row) $= \frac{1}{1296}$.

9. You need to roll a total of 7. P(7) $= \frac{1}{6}$.

10. Your opponent has a better chance, because the chance of scoring a 6 is higher than that of scoring a 4.

11. There are many different paths that get from start to finish in 5 turns.

TOPIC 11 Numeracy 2

11.1 Overview

Numerous **videos** and **interactivities** are embedded just where you need them, at the point of learning, in your learnON title at www.jacplus.com.au. They will help you to learn the content and concepts covered in this topic.

11.1.1 Why learn this?

Understanding numeracy helps us make sense of mathematical ideas and how they relate to everyday life.

Numeracy includes counting, addition, subtraction, multiplication and division, fractions, decimals, measuring, understanding shapes and graphs.

11.1.2 What do you know?

assessON

1. **THINK** List what you know about numeracy. Use a thinking tool such as a concept map to show your list.
2. **PAIR** Share what you know with a partner and then with a small group.
3. **SHARE** As a class, create a thinking tool such as a large concept map that shows your class's knowledge of numeracy.

LEARNING SEQUENCE

11.1 Overview
11.2 Set C
11.3 Set D

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Watch this eLesson: NAPLAN: strategies and tips (eles-1688)

11.2 Set C

11.2.1 Calculator allowed

1. A school has 434 DVDs in its collection. It has been decided to store them in boxes that hold 32 DVDs each. How many boxes are needed?

A. 13 **B.** 13.5 **C.** 13.6 **D.** 14

2. A cardboard box weighs 244 grams. It contains 48 smaller boxes, each weighing 127 grams. Which of the following is the closest to the total weight of the large box and its contents?

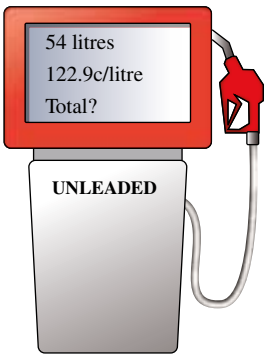
A. 6.3 kg **B.** 6.31 kg **C.** 6.309 kg **D.** 6.4 kg

3. A truck is carrying 12 boxes each weighing 250 kg and 20 pipes each weighing 17 kg. How many excess kilograms is the truck carrying if it has a weight limit of 2.8 tonnes?

A. 2737 **B.** 310
C. 540 **D.** 217

4. The following is the exchange rate given by Travelex.
A$1 = US$0.92
A$1 = €0.58
Jinny has just had a holiday and has brought back US$100 and €150. If she exchanges this into Australian dollars with the above rates, how much will she receive for each?

5. What is the total cost of the petrol shown on the diagram at right?



6. Block chocolate is sold in the following sizes.

A. 100 g for $3.20 **B.** 150 g for $3.95
C. 250 g for $6.90 **D.** 300 g for $8.60

Which is the best buy?

7. The following table shows the number of items sold in one year by a company in each of the locations. In which location was the highest number of items sold per salesperson?

Location	Number of salespeople	Number of items sold
North	52	49 192
Northwest	36	61 164
City	34	23 154
East	20	50 518
South	23	48 048

8. A customer has to wait 8 days for a leaking pipe to be repaired. In 1 hour the pipe loses 4 litres of water. The amount of water that will be lost while the customer is waiting is approximately:

A. 760 L **B.** 770 L **C.** 780 L **D.** 790 L

9. Each number in a sequence is multiplied by 7, and then 1 is added. The first three numbers in the new pattern are 15, 43, 71. What is the fifth number in the original sequence?

A. 99 **B.** 14 **C.** 127 **D.** 18

10. Molecules generally are approximately one billionth of a metre in size. Which one of the following represents that measurement?

A. 1 000 000 000 **B.** 100 000 000 **C.** 0.000 000 000 000 1 **D.** 0.000 000 001

11. The area of a square is 9 cm^2. If the length of each side is doubled, what is the area now?

A. 18 cm^2 **B.** 36 cm^2 **C.** 81 cm^2 **D.** 324 cm^2

12. If each small cake needs 10 g of chocolate and 50 g of rice crispies, how many grams of rice crispies are needed for 50 g of chocolate?

13. A ship's container is loaded with nine boxes each weighing $1\frac{1}{4}$ tonnes. If the container itself weighs 850 kg, what is the total weight of the loaded container?

A. 2100 kg **B.** 10 075 kg

C. 11 250 kg **D.** 12 100 kg

14. Jodie earns $15.00 an hour for 40 hours and $17.50 per hour for any hours of overtime. During one week she earned $810. How many hours did she work?

15. A survey of 300 students in Year 7 looked at the most popular reality TV shows. How many watched *Biggest loser*?

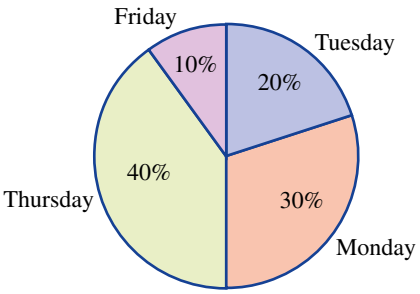
TV show	Number of people
Australian idol	177
Biggest loser	
So you think you can dance	20%
Who do you think you are?	63

A. 40 **B.** 20 **C.** 0 **D.** 54

16. Blueberries cost $14 per kg. You paid $1.75 for a punnet. How many grams of blueberries did you purchase?

A. 125 g **B.** 0.75 g **C.** 200 g **D.** 175 g

17. There are 200 Year 7 students. The diagram at right shows the percentage of students who were late on four days of the week. Which day had the most latecomers? How many students were late on that day?



18. Hayden purchased some content for his MP3 player using the internet. He bought 6 songs at $1.70 each and 4 music videos at $3.20 each. If he used his $20 gift card, how much extra did he need to pay?

19. The hospital has just installed a new scanner to help in the diagnosis of cancer. The state of the art machine detects a tumour as small as 0.5 cm^3. Cancers are made of individual cells, each of which is 0.000 000 1 cm^3. How many cells are needed in a tumour before it is detected by the new scanner?

A. $5 \times 0.000\,001$ **B.** $0.5 \times 100\,000$ **C.** $5 \times 100\,000$ **D.** $50 \times 100\,000$

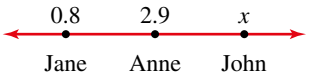
20. A cancer doubles in size every five days. Beginning from one cell, after 50 days how many cells will the cancer contain?

A. 10 **B.** 100 **C.** 512 **D.** 1024

21. A room measures 7 m by 3 m. If 4 pages of *The Australian* newspaper measure one square metre, how many pages are needed to cover the floor area with no overlap?

22. What fraction is halfway between one-quarter and one-half?

23. Anne is standing halfway between her two friends on a number line at 2.9. Jane is at a position 0.8 on the line and John is at x. What is John's position?



24. A square has an area of 36 square cm. If the sides are enlarged by a factor of 5, what is the area of the new square?

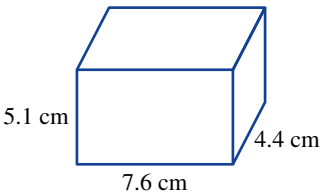
A. 188 cm^2 **B.** 150 cm^2 **C.** 900 cm^2 **D.** 875 cm^2

25. Ten teenagers were weighed in kilograms as follows:
47, 53, 51, 62, 58, 53, 48, 52, 67, 54.
What is the mean of these measurements?

26. A cube is made up of 125 small cubes. How many cubes will make up one face?

A. 5 **B.** 10 **C.** 15 **D.** 25

27. How much wire is needed to create this cuboid?



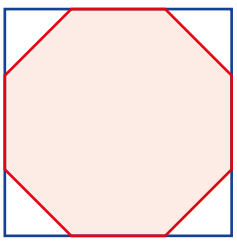
A. 17.1 cm **B.** 34.2 cm
C. 68.4 cm **D.** 94.64 cm

28. Think of two 2-digit numbers, both less than seventy-five. Both numbers have factors of 6, 3 and 5. One number is half the size of the other number. What are the two numbers?

29. Which is the odd one out in the following?

A. 0.8 **B.** $\frac{4}{5}$
C. 80% **D.** $\frac{16}{80}$

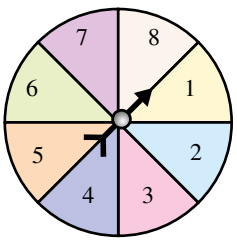
30. The regular octagon shown has an area of 125 cm^2. The square surrounding the octagon has a perimeter of 60 cm. What is the area of one triangle if all the triangles are the same size?



11.3 Set D

11.3.1 Non-calculator

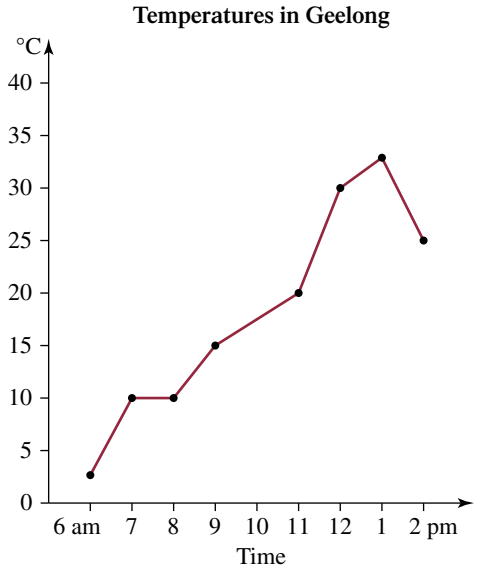
1. You spin the spinner on the right once. What is the probability of selecting a prime number?



A. $\frac{1}{8}$ **B.** $\frac{5}{8}$ **C.** $\frac{4}{8}$ **D.** $\frac{6}{8}$

2. In the graph below, when was the biggest change in temperature?

A. 6 am – 7 am **B.** 1 pm – 2 pm **C.** 11 am – 12 pm **D.** 10 am – 11 am

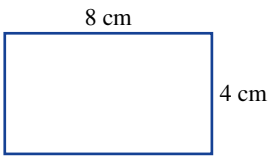


3. Jason packed $5\frac{1}{4}$ shelves in the supermarket and Charlie packed $8\frac{1}{2}$ shelves. How many more shelves did Charlie pack?

A. $3\frac{1}{2}$ **B.** $2\frac{1}{4}$ **C.** $2\frac{1}{2}$ **D.** $3\frac{1}{4}$

4. A class contains 12 boys and 18 girls. Five have red hair, 7 have brown hair and the rest have blond hair. What is the probability that a student chosen has blond hair?

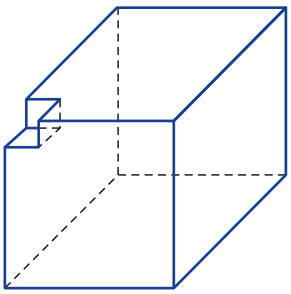
5. A cuboid is made from 1 cm cubes, stacked in layers on the rectangular base shown. If there are 5 layers, how many cubes will be used?



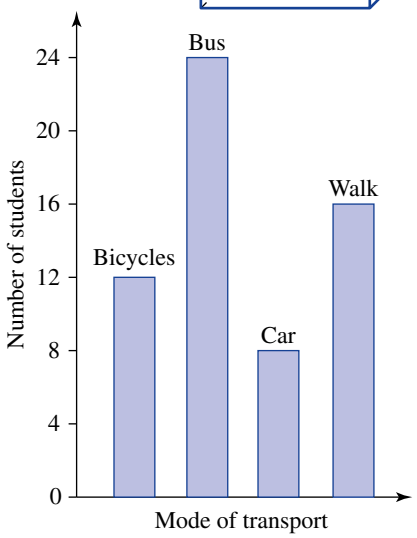
6. The diagram shows a cubic object whose side lengths are 20 cm. A smaller cube with a side length of 4 cm has been cut out of the larger cube. What is the volume of the object that is left?

A. 16 cm^3 **B.** 56 cm^3

C. 736 cm^3 **D.** 7936 cm^3



7. The graph below right shows how 60 students travel to school. How many students travel on wheels?



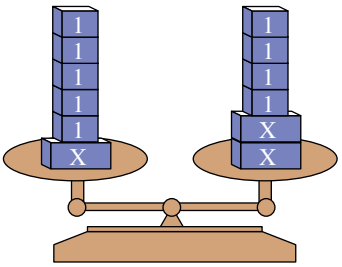
8. A rectangle is twice as long as it is wide. The perimeter of the rectangle is 60 cm. What is the area of the rectangle?

A. 100 cm^2 **B.** 200 cm^2 **C.** 400 cm^2 **D.** 60 cm^2

9. Suem bought a packet of chocolates and shared it with six friends. They each received 4 chocolates and three were left over. If you share the same number of chocolates equally with five friends, how many chocolates will be left over?

A. 2 **B.** 1 **C.** 4 **D.** 0

10. What is the equation represented by the following balance?



11. The Temple of Kukulcan (El Castillo) in Mexico sits on a pyramid-shaped platform. The height of the temple is 6 m and the height to the top of the platform on which it sits is 24 m. What is the ratio of the height of the platform to the height of the temple?

A. 1 : 6 **B.** 6 : 1
C. 6 : 24 **D.** 4 : 1

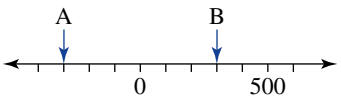
12. Which two numbers will have a difference of 2.5?
−3, −2.5, −2, −1.5, −1, −0.5

13. Julia walked to her friend's house and arrived at 4.10 pm. It took one hour and thirty-five minutes. What time did she begin her walk?

14. What is the value of $3 + 6 \div 3 \times 2$?

A. 6 **B.** 4 **C.** 7 **D.** 1.5

15. What is the range between A and B?



A. 600 **B.** 700
C. 800 **D.** 900

16. To make a batch of chocolate mousse, you need 20 g of chocolate. How many batches can you make from 280 g of chocolate?

17. On a map, a distance of 30 km is represented by 24 cm. What actual distance would be represented by 16 cm on a map?

18. Which of the following is equivalent to $\frac{3}{4}$?

A. $\frac{21}{28}$ **B.** $\frac{8}{40}$ **C.** $\frac{33}{45}$ **D.** $\frac{13}{14}$

19. A plane leaves Brisbane at 0935 and arrives in Perth at 1342 (Brisbane time). How long did the trip take?

A. 3 hours 42 minutes **B.** 3 hours 72 minutes **C.** 4 hours 28 minutes **D.** 4 hours 7 minutes

20. A kindergarten has 120 students. On one day, 42 students were absent. The fraction of students who were present is closest to:

A. one-third **B.** one-quarter **C.** two-thirds **D.** three-quarters

21. In this recipe, how many more teaspoons of sugar than salt are needed?

Marinade for chicken wings

1 cup soy sauce

$\frac{1}{2}$ cup maple syrup

5 cloves of garlic

$1\frac{1}{4}$ teaspoons of salt

$1\frac{2}{3}$ teaspoon brown sugar

2 tablespoons of vinegar

22. The attendance at a Melbourne Storm football match was given as 38 479. Rounded off to the nearest thousand, this number is:

A. 30 000 **B.** 38 500

C. 38 000 **D.** 40 000

23. The temperatures shown at right were measured in cities worldwide. What was the largest difference in temperature between any two cities?

City	Temp.
Athens	37 °C
Cairo	17 °C
Hobart	8 °C
Stockholm	5 °C
Anchorage	−9 °C
New York	7 °C
Helsinki	−5 °C
Paris	18 °C
Rome	35 °C
Perth	23 °C

24. The temperature at midnight was −3 °C. By 8 am it had risen by 6 °C. What was the temperature at 8 am?

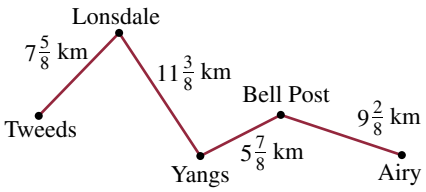
A. 6 °C **B.** 9 °C

C. 3 °C **D.** 8 °C

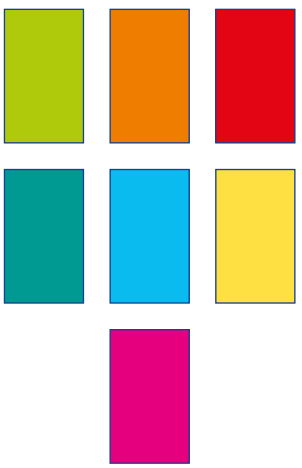
25. At 9 pm, the temperature was 2°C. By midnight, the temperature had dropped to −5 °C. By how many degrees had the temperature fallen?

A. 5 °C **B.** −5 °C **C.** 3 °C **D.** 7 °C

26. From Yangs, how much further is it to Londsdale than to Bell Post? (Give your answer in simplest form.)

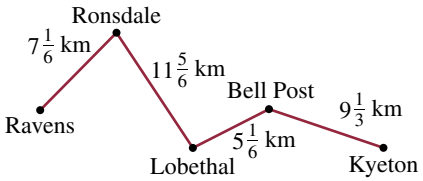


27. A coloured card is chosen and a coin is tossed. The results are recorded. How many different combinations are possible?



A. 3 **B.** 9 **C.** 14 **D.** 21

28. How far is it from Lobethal to Kyeton?



29. Johan Vaaler was the first person to patent the paperclip (1899). If this paperclip is made from 12.5 cm of wire, how much wire do you need to fill a box containing 200 paperclips?

A. 250 cm **B.** 250 m **C.** 25 m **D.** 2.5 m

30. Which one of the following is another way of writing $4 \times 4 \times 4$?

A. 4^3 **B.** $4 + 4 + 4$ **C.** 3×4 **D.** 3^4

Answers

Topic 11 Numeracy 2

11.2 Set C

11.2.1 Calculator allowed

1. D
2. B
3. C
4. \$108.70 and \$258.62
5. \$66.37
6. B
7. East
8. B
9. D
10. D
11. B
12. 250 g
13. D
14. 52
15. C
16. A
17. Thursday 80 students
18. \$3
19. D
20. D
21. 84 pages
22. $\frac{3}{8}$
23. 5
24. C
25. 54.5
26. D
27. C
28. 60 and 30
29. D
30. 25 cm^2

11.3 Set D

11.3.1 Non-calculator

1. C
2. C
3. D
4. $\frac{3}{5}$
5. 160
6. D
7. 44
8. B
9. B
10. $x + 5 = 2x + 4$
11. D
12. -3 and -0.5
13. 2:35 pm
14. C
15. A
16. 14
17. 20 km
18. A
19. D
20. C
21. $\frac{5}{12}$ teaspoon
22. C
23. 46 °C
24. C
25. D
26. $5\frac{1}{2}$ km
27. C
28. $14\frac{1}{2}$ km
29. C
30. A

TOPIC 12
Positive and negative integers

12.1 Overview

Numerous **videos** and **interactivities** are embedded just where you need them, at the point of learning, in your learnON title at www.jacplus.com.au. They will help you to learn the content and concepts covered in this topic.

12.1.1 Why learn this?

There are many everyday situations where a knowledge of positive and negative numbers is useful. Temperatures, money owed and paid back, diving levels and flood measurements are just some examples of where these numbers are used.

12.1.2 What do you know?

assessON

1. **THINK** List what you know about positive and negative integers. Use a thinking tool such as a concept map to show your list.
2. **PAIR** Share what you know with a partner and then with a small group.
3. **SHARE** As a class, create a thinking tool such as a large concept map that shows your class's knowledge of positive and negative integers.

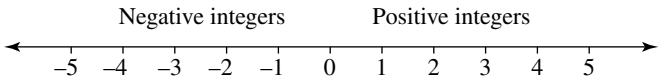
LEARNING SEQUENCE

12.1 Overview
12.2 Integers on the number line
12.3 Integers on the number plane
12.4 Addition and subtraction of integers
12.5 Multiplication and division of integers
12.6 Combined operations
12.7 Review

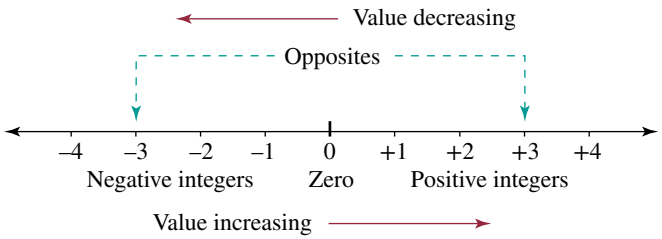
12.2 Integers on the number line

12.2.1 Number line

- **Whole numbers** are the numbers 0, 1, 2, 3, 4, …
- Sometimes it is necessary to use numbers smaller than 0.
- **Integers** are positive and negative whole numbers, including 0.
  The numbers … −3, −2, −1, 0, 1, 2, 3, … are called integers.
- The integers −1, −2, −3, … are called **negative integers**.
- The integers 1, 2, 3, … are called **positive integers** and can be written as +1, +2, +3, …
- 0 is neither positive nor negative.
- Integers can be represented on a number line. The positive integers are written to the right of 0 and the negative integers are written to the left of 0.



- As integers have both size (distance from 0) and direction (above or below 0), they can be called **directed numbers**.
- The further a number is to the right of any number on a number line, the larger it is; +5 is bigger than +2, +2 is greater than 0, +2 is greater than −5.



- The following symbols can be used when comparing integers.
  > means greater than
  < means less than
  ≥ means greater than or equal to
  ≤ means less than or equal to
- **Opposite integers** are equidistant from 0 but on opposite sides; 5 and −5 are opposites.

WORKED EXAMPLE 1

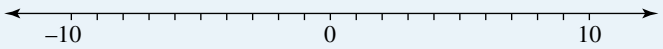
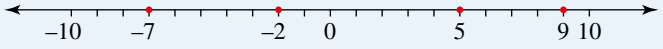
Write an integer suggested by the following descriptions.

a My unit is on the 5th floor. **b A town is 20 m below sea level.**

THINK	WRITE
a Numbers above 0 are positive.	**a** +5 (or 5)
b Numbers below 0 are negative.	**b** −20

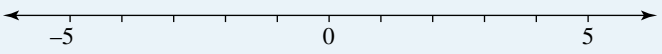
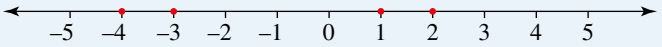
WORKED EXAMPLE 2

Place the numbers 9, −2, −7, 5 on a number line.

THINK	WRITE
1 Draw a number line from −10 to 10 so that you can include all of the numbers.	
2 Mark the values 9, −2, −7, 5 on the number line.	

WORKED EXAMPLE 3

Use a number line to place the numbers −4, 2, −3, 1 in numerical order.

THINK	WRITE
1 A number line from −5 to 5 will be long enough to include all of the numbers. Draw the number line.	
2 Mark the values. As you move from left to right on the number line, the numbers become larger.	 In numerical order: −4, −3, 1, 2

WORKED EXAMPLE 4

Write the opposites of:
a −2 **b** 3 **c** 4.

THINK	WRITE
a An opposite is the same distance from 0, but has the opposite sign.	**a** The opposite of −2 is 2.
b An opposite is the same distance from 0, but has the opposite sign.	**b** The opposite of 3 is −3.
c An opposite is the same distance from 0, but has the opposite sign.	**c** The opposite of 4 is −4.

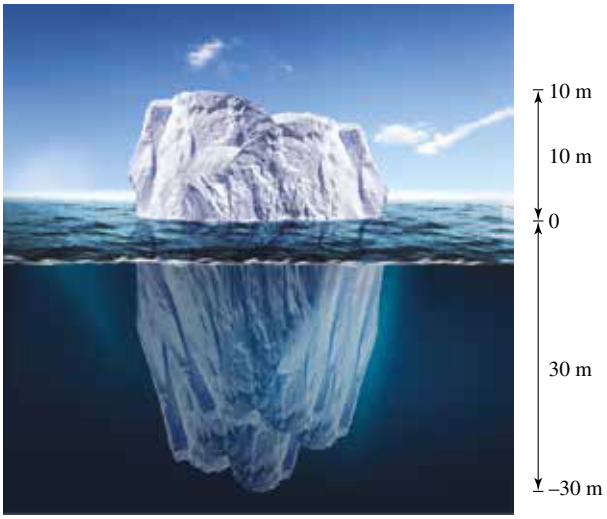
WORKED EXAMPLE 5

Complete each statement by inserting the correct symbol: >, < or =.
a 2 ☐ 5 **b** −4 ☐ −1 **c** 0 ☐ −3 **d** 6 ☐ −2

THINK	WRITE
a 2 is to the left of 5 on the number line, so 2 is smaller.	**a** $2 < 5$
b −4 is to the left of −1 on the number line, so −4 is smaller.	**b** $-4 < -1$
c 0 is to the right of −3 on the number line, so 0 is larger.	**c** $0 > -3$
d 6 is to the right of −2 on the number line, so 6 is larger.	**d** $6 > -2$

12.2.2 Using positive and negative numbers in our daily lives

- Sometimes it is necessary to use numbers smaller than 0 in everyday situations.
- Directed numbers are often used to measure objects from a reference point, for example, temperature above or below zero, or the height above or below sea level, or archaeological measurements below the Earth's surface.
- Directed numbers are used in describing banking transactions. Withdrawals of money from a bank account are given negative signs while deposits are given positive signs.



WORKED EXAMPLE 6

Write an integer suggested by each of the following examples.
a The temperature was 1 degree below zero. **b Kate has $500 in her bank account.**

THINK	WRITE
a Below zero refers to negative.	**a** -1
b She actually has money so the integer is positive.	**b** $+500$

learnON RESOURCES — ONLINE ONLY

- **Watch this eLesson:** Integers on the number line (eles-0040)
- **Complete this digital doc:** SkillSHEET: Using < or > to compare the size of numbers (doc-6563)
- **Complete this digital doc:** SkillSHEET: Ascending and descending order (doc-6564)
- **Complete this digital doc:** SkillSHEET: Marking numbers on a number line (doc-6565)
- **Complete this digital doc:** SkillSHEET: Working with numbers on a number line (doc-6566)
- **Try out this interactivity:** Directed number target (int-0074)

Exercise 12.2 Integers on the number line

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–11	Questions: 1–11	Questions: 1–12

Individual pathway interactivity: int-4367

learnON ONLINE ONLY

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note*: Question numbers may vary slightly.

Fluency

1. WE1 WE6 Write the integer suggested by the following description.

a. The temperature today is 7 degrees below zero.

b. The penthouse is on the 10th floor.

c. The Dead Sea is 422 m below sea level.

d. Mount Kosciuszko is 2228 m above sea level.

e. The Dow Jones dropped 3 points overnight.

f. Chloe borrowed $30 from her mother.

2. WE2 Draw a number line from −10 to 10. Mark the following values on your number line.

a. 2 **b.** −4 **c.** −9 **d.** 7 **e.** 5 **f.** −1 **g.** 0 **h.** −3

3. WE3 Use a number line to place the following numbers in numerical order.

a. 5, −2, 3, −6, 7 **b.** 0, −1, 3, −4, −6 **c.** −5, 7, −2, −3, 1 **d.** −4, 2, 1, −3, 5

4. WE4 Write the opposite of:

a. −6 **b.** 7 **c.** 1 **d.** −8

Understanding

5. WE5 Complete the following by inserting $<$ or $>$ in the box.

a. 2 ☐ −5 **b.** 3 ☐ 7 **c.** −2 ☐ −6

d. 6 ☐ −2 **e.** −1 ☐ 3 **f.** −10 ☐ −6

6. From the following lists, select:

i. the smallest number **ii.** the largest number.

a. −3, 7, −5 **b.** 2, −4, 5, 3, −2 **c.** 7, −10, 5, −2, −4 **d.** −4, −1, 3, 0, −2

Reasoning

7. WE6 Overnight, the temperature in Oslo dropped to −6 °C. During the day, the temperature rose by 4 °C. What temperature was reached?

8. A new apartment building was to include a ground floor reception area. If there were 10 floors of apartments and 3 floors of underground parking, how many buttons were needed in the lift? Explain your answer.

9. a. Historians use BC and AD when discussing dates. Explain what is meant by 2000 AD.

b. If the millennium year AD 2000 (+2000) was considered to be zero:

i. What year would 2010 be?

ii. What year would 1894 be?

c. Captain James Cook first sighted the east coast of Australia in 1770. In 1788 a fleet of British ships with 1530 passengers, including 736 convicts, landed at Botany Bay. If we consider 1788 as being zero, what directed number could be used to describe the following?

i. The year that Captain Cook first sighted the east coast of Australia

ii. The year the following settlements were established:

Hobart (Tasmania) 1803

Brisbane River (Queensland) 1824

Swan River (Western Australia) 1829

Port Phillip Bay (Victoria) 1835

Gulf St Vincent (South Australia) 1836

d. Draw a number line from −20 to 50 to show your answers to part **c**.

10. Lake Mungo in Mungo National Park is a World Heritage–listed area. Archaeologists estimate that the lake dried out 18 000 years ago, and that before that, Aboriginal people lived on the shores of the lake. Lake Mungo became one of the world's most significant archaeological sites following the discovery there of some of the oldest remains of modern man on Earth. Explain what −60 000 could mean in terms of population at Lake Mungo.

Problem solving

11. Raj and Vanessa are going home from school in opposite directions. Raj lives 12 km to the right of the school and Vanessa lives 16 km to the left of the school.

a. Draw a number line to represent the three locations: Raj's house, Vanessa's house and the school. Label the three locations with directed numbers.

b. How far apart do Raj and Vanessa live from each other?

12. An aeroplane is flying at 12 km above sea level and a submarine is 500 m below sea level.

a. Draw a number line to represent the three locations: the aeroplane, the submarine and the ocean level. Label the three locations with directed numbers.

b. What is the distance between the submarine and the aeroplane?

Reflection

Give an example of when a negative integer would be needed.

12.3 Integers on the number plane

12.3.1 Positive integers and zero on the number plane

- The number plane, or Cartesian plane, has two axes: the **horizontal axis** or ***x*-axis** and the **vertical axis** or ***y*-axis**.
- Each point on the number plane is described by its position relative to the x- and y-axes.
- A pair of **coordinates** or an **ordered pair** (x, y) fix the position of the point, where x units is the distance along the x-axis and y units is the distance along the y-axis.

WORKED EXAMPLE 7

In the number plane at right, find the coordinates of:
a A **b** B.

THINK

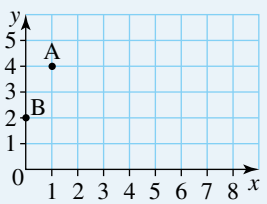
a Point A is 1 unit to the right of zero along the x-axis and 4 units up the y-axis from zero.

b Point B is on the y-axis, so is 0 units to the right of zero along the x-axis and is 2 units up the y-axis from zero.

WRITE

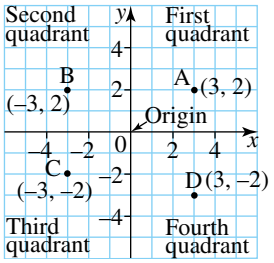
a A (1, 4)

b B (0, 2)



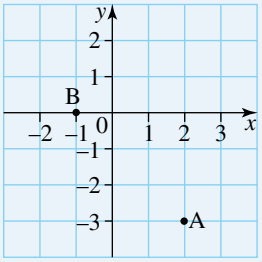
12.3.2 Integers on the number plane

- The axes on the number plane can be extended to include negative integers as shown on the right.
- The **origin** (0, 0) is the point where the two axes intersect, dividing the plane into four **quadrants**.
- The quadrant with positive y and positive x (the quadrant that you have already been using for graphing) is called Quadrant 1 (First quadrant).
- The quadrants are numbered in an anticlockwise direction.



WORKED EXAMPLE 8

Write the coordinates and state the quadrant or axis of each point on this number plane.



THINK	WRITE
a 1 A is 2 units to the right of the origin along the x-axis and is 3 units down from the origin along the y-axis.	**a** A (2, −3)
2 It is in the lower right-hand corner.	A is in the fourth quadrant.
b 1 B is 1 unit to the left of the origin along the x-axis and is 0 units up or down from the origin along the y-axis.	**b** B (−1, 0)
2 It is on the x-axis.	B is on the x-axis.

Exercise 12.3 Integers on the number plane

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–13, 16	Questions: 1–14, 16, 17	Questions: 1–17

Individual pathway interactivity: int-4368

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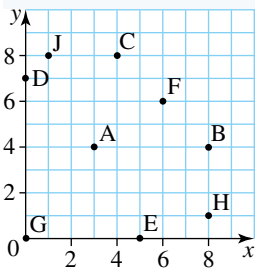
Fluency

Questions **1** and **2** refer to Figure 1.

FIGURE 1



1. WE7 Find the coordinates of the following.

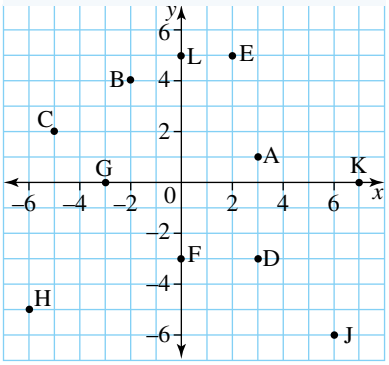
a. A **b.** B **c.** C
d. D **e.** E

2. Write a letter to name each of the points with the following coordinates.

a. (0, 0) **b.** (6, 6) **c.** (8, 1) **d.** (1, 8)

Use Figure 2 to find the answers to questions **3** to **11**.

FIGURE 2



3. WE8 Write the coordinates and state the quadrant or axis of each point.
- **a.** A **b.** B **c.** H
- **d.** F **e.** J

4. Name the point and give its quadrant or axis.
- **a.** $(-5, 2)$ **b.** $(0, 5)$ **c.** $(3, -3)$
- **d.** $(2, 5)$ **e.** $(-3, 0)$

5. Give the x-coordinate of the following.
- **a.** A **b.** D **c.** K **d.** L

6. Give the y-coordinate of the following.
- **a.** C **b.** J **c.** G **d.** F

7. List all points lying in the third quadrant in the diagram.

Understanding

8. What is the x-value of all points on the y-axis?

9. What is the y-value of all points on the x-axis?

10. In which quadrant do all points show the sign pattern $(+, -)$?

11. True or False?
- **a.** F and D have the same y-coordinate.
- **b.** A and D have the same x-coordinate.
- **c.** The origin has coordinates $(0, 0)$.
- **d.** The point at $(3, 5)$ is the same point as $(5, 3)$.
- **e.** The point at $(-5, 4)$ is in the third quadrant.
- **f.** The point at $(0, 2)$ must lie on the y-axis.

12. MC **a.** The point $(5, -2)$ lies:
- **A.** in the first quadrant **B.** in the second quadrant **C.** in the third quadrant
- **D.** in the fourth quadrant **E.** on the x-axis.

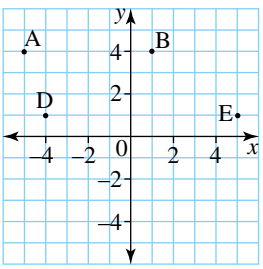
b. The point at $(0, 4)$ lies:
- **A.** in the first quadrant **B.** on the y-axis **C.** in the third quadrant
- **D.** in the fourth quadrant **E.** on the x-axis.

c. The point $(-4, 5)$ lies:
- **A.** in the first quadrant **B.** in the second quadrant **C.** in the third quadrant
- **D.** in the fourth quadrant **E.** on the x-axis.

13. Draw up a number plane with both axes scaled from -6 to 6. Plot the points listed and join them with straight lines in the order given. Name the completed shape.
- **a.** $(5, 5), (3, 2), (-2, 2), (0, 5), (5, 5)$
- **b.** $(4, -1), (4, -5), (-1, -3), (4, -1)$
- **c.** $(-4, 4), (-2, 1), (-4, -5), (-6, 1), (-4, 4)$
- **d.** $(-2, 1), (1, 1), (1, -2), (-2, -2), (-2, 1)$

Reasoning

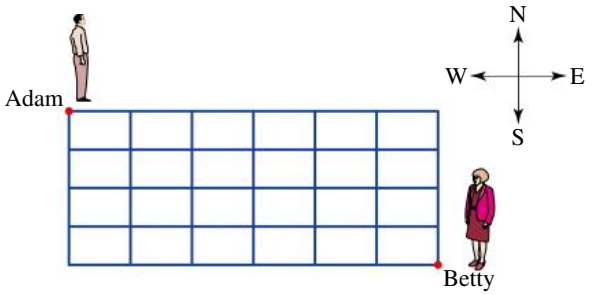
14. **a.** Find the coordinates of a point, C, so that ABCD is a parallelogram.
- **b.** Find the coordinates of a point, F, so that DBEF is a kite shape.
- **c.** Show that the point $(4, -1)$ lies on the line that passes through D and the origin.
- **d.** List 2 points on the line joining D to E.
- **e.** Give the coordinates of a point, T, in the third quadrant which would complete the isosceles triangle ADT.



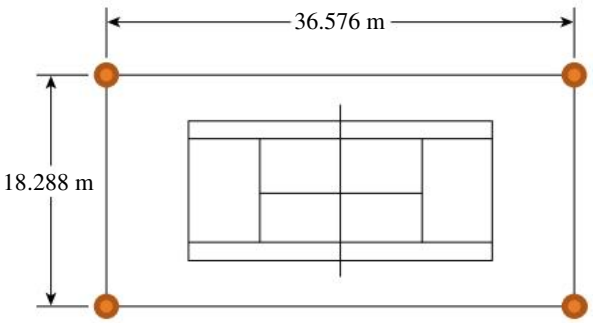
15. a. If you were moving east and then you did a U-turn, what direction would you be moving in now? Assume that a U-turn is the same as putting a negative sign in front of you.
b. What direction would you face if you had two negative signs in front of you?
c. What direction would you face if you had three negative signs in front of you?
d. What direction would you face if you had ten negative signs in front of you?
e. What direction would you face if you had 101 negative signs in front of you? Explain.
f. Explain what would happen if you were moving north and a U-turn would be the same as putting a negative sign in front of you. What direction would you be moving in after doing a U-turn?
g. If you were moving north, explain which direction you would be facing after an odd number of negative signs and after an even number of negative signs.

Problem solving

16. The streets in Metropolis run north–south and east–west (see the following diagram). Adam wants to visit Betty. If he can travel only south or east, how many paths can he follow?



17. The tennis court shown is enclosed in an area which has a post in each corner.
a. Draw a number plane with the origin in the centre of the court.
b. State the coordinates of the four posts on the number plane drawn in part **a**.



Reflection

If you swap the *x*- and *y*-coordinates of a point, it may change to another quadrant. What happens to points from each quadrant if you swap their coordinates?

12.4 Addition and subtraction of integers

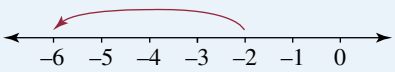
12.4.1 Addition of integers

- A number line can be used to add integers, as demonstrated in Worked example 9. To add a positive integer, move to the right. To add a negative integer, move to the left.
- A sign model can also be used to add integers as demonstrated in Worked example 10. In the sign model, a + and a − cancel each other out.

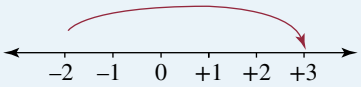
WORKED EXAMPLE 9

Write number sentences to show the addition problems suggested by the following diagrams:

a



b



THINK	WRITE
a Write the addition statement suggested by the diagram. (The first integer is 2 units to the *left* of zero, so start at −2. The second integer is 4 units to the *left* of −2, finishing at −6.) Find the answer.	**a** $-2 + -4 = -6$
b Write the addition statement suggested by the diagram. (The first integer is 2 units to the *left* of zero so start at −2. The second integer is 5 units to the *right* of −2, finishing at +3.) Find the answer.	**b** $-2 + +5 = +3$

WORKED EXAMPLE 10

Draw a sign model for each of the following and state the result.

a $-4 + +1$ **b** $+5 + -3$

THINK	WRITE
a 1 Show −4 as four negative signs.	**a** − − − −
2 Show +1 as one positive sign.	+
3 Cancel out the opposite signs.	~~−~~ − − ~~−~~
4 Write the answer.	$-4 + +1 = -3$
b 1 Show + 5 as 5 positive signs.	**b** + + + + +
2 Show −3 as 3 negative signs.	− − −
3 Cancel out the opposite signs.	~~+~~ ~~+~~ ~~+~~ + + ~~−~~ ~~−~~ ~~−~~
4 Write the answer.	$+5 + -3 = +2$

12.4.2 Subtraction of integers

- Subtracting a number gives the same result as *adding its opposite*.
- By developing and extending a pattern, we can show that subtracting negatives has the same effect as adding a positive; that is, subtracting a negative is the same as adding its inverse. Look at the following pattern.

7 − 4 =	3
7 − 3 =	4
7 − 2 =	5
7 − 1 =	6
7 − 0 =	7
7 − (−1) =	8
7 − (−2) =	9
7 − (−3) =	10
7 − (−4) =	11

It can be seen from the table that subtracting a negative is the same as adding its inverse; for example, $7 - (-2) = 9 = 7 + 2$.

WORKED EXAMPLE 11

Calculate the following.
a $2 - 5$ **b** $-3 - 6$ **c** $5 - -3$ **d** $-5 - -4$

THINK	WRITE
a 1 Write the question.	**a** $2 - 5 = 2 - +5$
2 Rewrite, changing subtraction to addition of the opposite integer.	$= 2 + -5$
3 Add, using the addition rule for addition of integers.	$= -3$
b 1 Write the question.	**b** $-3 - 6 = -3 - +6$
2 Rewrite, changing subtraction to addition of the opposite integer.	$= -3 + -6$
3 Add, using the addition rule for addition of integers.	$= -9$
c 1 Write the question.	**c** $5 - -3$
2 Rewrite, changing subtraction to addition of the opposite integer.	$= 5 + +3$ $= 5 + 3$
3 Add, using the addition rule for addition of integers.	$= 8$
d 1 Write the question.	**d** $-5 - -4 = -5 + +4$
2 Rewrite, changing subtraction to addition of the opposite integer.	$= -5 + 4$
3 Add, using the addition rule for addition of integers.	$= -1$

WORKED EXAMPLE 12

Evaluate the algebraic expression $a + b - c$, if $a = -2, b = 1$ and $c = -5$.

THINK	WRITE
1 Write the expression.	$a + b - c$
2 Replace each pronumeral with the appropriate integer.	$= -2 + 1 - -5$
3 Evaluate the expression; that is:	$= -1 - -5$
a rewrite, changing subtraction to addition of the opposite integer	$= -1 + +5$
b add, using the addition rule for addition of integers.	$= 4$

WORKED EXAMPLE 13

A news flash in Freezonia announced that there had been a record drop in temperature overnight. At 6 pm the temperature was 10 °C and by 4 am it had fallen 25 °C. What was the temperature at 4 am?

THINK	WRITE
1 Write the original temperature. Decide whether the temperature rose (addition) or fell (subtraction). Write the number sentence.	$10 - 25$ $= 10 + -25$
2 Find the answer.	$= -15$
3 Write the answer in a sentence.	The temperature in Freezonia at 4 am was −15 °C.

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Exercise 12.4 Addition and subtraction of integers

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Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–11, 13, 17, 21	Questions: 1–10, 12, 14, 16, 18, 21, 22	Questions: 1–8 columns 2 and 3, 9–22

Individual pathway interactivity: int-4369

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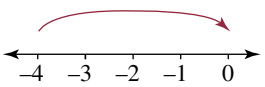
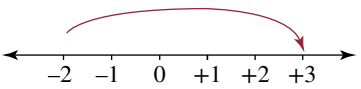
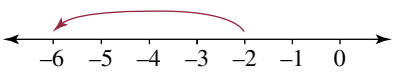
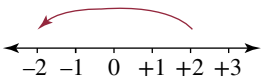
Fluency

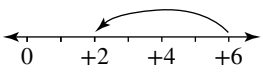
1. WE9 Write number sentences to show the addition problems suggested by the following diagrams.

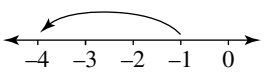
a.


b.


c.


d.


e.


f.


2. WE10 Draw a sign model for each of the following and state the result.

a. +2 + −3 **b.** +3 + −4 **c.** +4 + −4
d. +3 + −2 **e.** −4 + +2 **f.** −5 + +3

3. Copy and complete these number sentences. (Draw a number line or other model if you wish.)

a. 5 + −2 **b.** −3 + −4 **c.** −2 + 2 **d.** 6 + −5
e. −5 + 5 **f.** 4 + −6 **g.** −5 + 7 **h.** 6 + −9
i. −4 + 6 **j.** 3 + −3 **k.** −8 + −2 **l.** 0 + −6

4. Write the answer for each of the following.

a. $-5 + -2$	**b.** $-6 + 4$	**c.** $-8 + 8$	**d.** $3 + -7$
e. $-3 + 7$	**f.** $-3 + -7$	**g.** $-8 + 12$	**h.** $19 + -22$
i. $-64 + -36$	**j.** $-80 + 90$	**k.** $-2 + 4$	**l.** $-15 + -7$

5. Copy and complete this addition table.

+	−13	5
21		
−18		

6. **WE11** Calculate the following.

a. $7 - 5$	**b.** $8 - -2$	**c.** $-4 - 6$	**d.** $-6 - -8$
e. $1 - 10$	**f.** $-5 - 5$	**g.** $-8 - -8$	**h.** $0 - 4$
i. $0 - -3$	**j.** $-10 - -20$	**k.** $-11 - 3$	**l.** $-5 - -5$

7. Calculate the following mentally and write just the answer.

a. $-7 - 3$	**b.** $8 - -5$	**c.** $-6 - -9$	**d.** $0 - -12$
e. $-8 - 8$	**f.** $3 - 20$	**g.** $20 - -3$	**h.** $-4 - 8$

8. Find the missing number in these incomplete number sentences.

a. $8 +$ _____ $= 0$	**b.** $-2 +$ _____ $= -8$	**c.** _____ $+ -6 = 4$
d. _____ $+ 5 = -2$	**e.** _____ $+ -5 = -2$	**f.** _____ $- 7 = -6$
g. $-8 -$ _____ $= -17$	**h.** $-8 -$ _____ $= 17$	**i.** _____ $- -2 = 7$

9. **WE12** Evaluate each algebraic expression, if $a = 5$, $b = -2$, $c = -8$.

a. $a + b$	**b.** $b + c$	**c.** $a - b$	**d.** $b - c$	**e.** $a + b + c$
f. $a - b - c$	**g.** $a - (b + c)$	**h.** $c - b - a$	**i.** $a - b + c$	

Understanding

10. **a.** Copy and complete the addition table shown at right.

+	−2	−1	0	1	2
−2		−3			
−1				0	
0	−2				
1					
2					

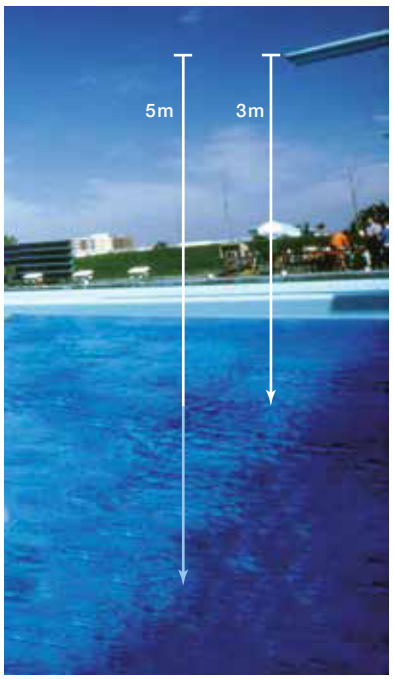
b. What pattern is shown along the leading (dotted) diagonal?
c. What pattern is shown along the other (unmarked) diagonal?
d. Is the chart symmetrical about the leading diagonal?
e. Comment on any other number patterns you can see.

11. Write the number that is:

a. 6 more than −2	**b.** 5 more than −8
c. 8 °C above −1 °C	**d.** 3 °C below 2 °C
e. 3 to the right of −4	**f.** 4 to the left of −3.

12. Model each situation with an integer number sentence that shows the result of the following.

a. From ground level, a lift went down 2 floors, then down another 3 floors.
b. From ground level, a lift went down 3 floors, then up 5 floors.
c. From ground level, a lift went up 5 floors, then down 6 floors.
d. Australia was 50 runs behind, then made another 63 runs.
e. An Olympian dived down 5 metres from a board 3 metres above water level.
f. At 5.00 pm in Falls Creek the temperature was 1°C. It then fell 6 degrees by 11.00 pm.
g. A submarine at sea level dived 50 metres, then rose 26 m.
h. An account with a balance of $200 had $350 withdrawn from it.



13. MC From ground level, a lift went down 2 floors, then down another 3 floors to a level 5 floors below the ground. The number sentence that describes this situation is:

A. $2 + 3 = 5$ **B.** $-2 + -3 = -5$ **C.** $-2 + 3 = 1$
D. $2 + -3 = -1$ **E.** $-3 + -2 = 5$

14. Describe a situation to fit each of the number sentences below.

a. $-3 + -2 = -5$ **b.** $-10 + -40 = -50$ **c.** $2 + -6 = -4$
d. $-20 + 20 = 0$ **e.** $-8 + 10 = 2$

15. Are these number sentences true or false?

a. $7 - 9 = 7 + -9$ **b.** $0 - 8 = -8 - 0$ **c.** $8 - 12 = -12 + 8$ **d.** $0 - p = -p$

16. Show working to calculate the following.

Remember: Order of operations: brackets, × or ÷ moving from left to right, + or − moving from left to right.

a. $6 + -3 - 2$ **b.** $-9 + (5 - 7)$
c. $3 - (8- -2)$ **d.** $6 + -8 + -5 - 2$
e. $4 + (-8 + 10)$ **f.** $16 - (-2 - -6)$

17. MC **a.** $7 + \ - 4 - \ - 2$ is equal to:

A. 9 **B.** 1 **C.** 13
D. 5 **E.** -5

b. $6 - (2 + -3)$ is equal to:

A. 1 **B.** 7 **C.** 11
D. 5 **E.** -7

18. WE13 The temperature in the freezer was −20 °C. Just before he went to bed, Dennis had a spoonful of ice-cream and left the freezer door ajar all night. The temperature in the freezer rose 18 °C and the ice-cream melted. What was the temperature in the freezer when his mother found the ice-cream in the morning?

Reasoning

19. Jill is climbing up a steep and slippery path to fetch a bucket of water. When she is 6 m above her starting point, she slips back 1 m, grasps some bushes by the side of the path and climbs 7 m more to a flat section. How far above her starting point is she when she reaches the resting place? Show your working.

20. a. Write a rule starting with 'When adding two negative numbers, …'

b. Write a rule starting with 'When adding a negative number and a positive number, …'

Problem solving

21. Sharyn opened a bank account when she started working at her local fish and chip shop. Over the four weeks of one month she made deposits of $56.00, $44.00, $52.80 and $39.20 and withdrawals of $45.00, $75.00, $22.00 and $50.00. If her bank account was opened with a $10 deposit, how much was in her account at the end of the month? Was there a time that the balance of her account was negative?

22. A hospital building has 6 above-ground-level floors and 4 floors below ground level. Over a 10-minute period one busy morning, the lift started from ground level (Reception) and made the following moves: up 2 floors, up 1 floor, down 4 floors, down 2 floors, up 3 floors, down 4 floors and up 6 floors. Where did the lift finish and how far did it travel in the 10 minutes?

Reflection

What effect does adding a number to its opposite have? What effect does subtracting the opposite have?

CHALLENGE 12.1

Arrange the numbers in this magic square so that all rows, columns and diagonals add up to the same value. No numbers are used twice.

−3		
4	−2	−6

12.5 Multiplication and division of integers

12.5.1 Multiplication of integers

- Consider the number pattern table below.

×	−3	−2	−1	0	1	2	3
3							
2							
1							
0							
−1							
−2							
−3							

- We already know how to complete part of the table.

×	−3	−2	−1	0	1	2	3
3				0	3	6	9
2				0	2	4	6
1				0	1	2	3
0				0	0	0	0
−1							
−2							
−3							

- Notice that multiplying by 0 always results in 0.
- Notice that multiplying by 1 does not change the value of the number.

×	−3	−2	−1	0	1	2	3
3				0	3	6	9
2				0	2	4	6
1	−3	−2	−1	0	1	2	3
0	0	0	0	0	0	0	0
−1				0	−1		
−2				0	−2		
−3				0	−3		

- Notice that multiplying by −1 changes the sign of the number.

×	−3	−2	−1	0	1	2	3
3	−9	−6	−3	0	3	6	9
2	−6	−4	−2	0	2	4	6
1	−3	−2	−1	0	1	2	3
0	0	0	0	0	0	0	0
−1	3	2	1	0	−1	−2	−3
−2			2	0	−2	−4	−6
−3			3	0	−3	−6	−9

- If we continue the pattern, we get the following.

×	−3	−2	−1	0	1	2	3
3	−9	−6	−3	0	3	6	9
2	−6	−4	−2	0	2	4	6
1	−3	−2	−1	0	1	2	3
0	0	0	0	0	0	0	0
−1				0	−1	−2	−3
−2				0	−2	−4	−6
−3				0	−3	−6	−9

- Completing the pattern results in the following.

×	−3	−2	−1	0	1	2	3
3	−9	−6	−3	0	3	6	9
2	−6	−4	−2	0	2	4	6
1	−3	−2	−1	0	1	2	3
0	0	0	0	0	0	0	0
−1	3	2	1	0	−1	−2	−3
−2	6	4	2	0	−2	−4	−6
−3	9	6	3	0	−3	−6	−9

- From this table, we can see the rules for multiplication:
  - When multiplying two integers with the *same sign*, the answer is *positive*.
  - When multiplying two integers with *different signs*, the answer is *negative*.

WORKED EXAMPLE 14

Evaluate:
a $-5 \times +2$ **b** -4×-6.

THINK	WRITE
a 1 Write the question.	**a** $-5 \times +2$
2 Negative × positive = negative.	$= -10$
b 1 Write the question.	**b** -4×-6
2 Negative × negative = positive.	$= 24$

12.5.2 Division of integers

- The division operation is the *inverse*, or opposite, of multiplication.

Since	$3 \times 2 = 6$	then	$6 \div 3 = 2$	and	$6 \div 2 = 3.$
Since	$3 \times -2 = -6$	then	$-6 \div 3 = -2$	and	$-6 \div -2 = 3.$
Since	$-2 \times -3 = 6$	then	$6 \div -3 = -2$	and	$6 \div -2 = -3.$

- The rules for division are:
  - When dividing two integers with the *same sign*, the answer is *positive*.
  - When dividing two integers with *different signs*, the answer is *negative*.

WORKED EXAMPLE 15

Calculate:
a $10 \div -2$ **b** $-12 \div 4$ **c** $-20 \div -5.$

THINK	WRITE
a 1 Write the question.	**a** $10 \div -2$
2 Look at the directions to decide whether the answer is positive or negative. If they are different, the answer is negative and if they are the same the answer is positive. Divide.	$= -5$
b 1 Write the question.	**b** $-12 \div 4$
2 Different directions give a negative answer. Divide.	$= -3$
c 1 Write the question.	**c** $-20 \div -5$
2 Divide using the kind of directions to indicate the sign.	$= 4$

WORKED EXAMPLE 16

Simplify:
a $\frac{-16}{+2}$ **b** $-5 \times \frac{4}{20}.$

THINK	WRITE
a 1 Write the question. *Note:* $\frac{-16}{+2}$ is the same as $-16 \div +2$.	**a** $\frac{-16}{+2}$
2 Evaluate the expression. *Note:* negative ÷ positive = negative	$= -8$
b 1 Write the question.	**b** $-5 \times \frac{4}{20}$
2 Write the integer as a fraction with a denominator of 1 and simplify by cancelling.	$= \frac{-5}{1} \times \frac{\cancel{4}^{1}}{\cancel{20}_{5}}$
3 Multiply the numerators then multiply the denominators and simplify. *Note:* negative ÷ positive = negative	$= \frac{-5}{5}$
4 Write the answer.	$= -1$

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Complete this digital doc: WorkSHEET: Positive and negative integers II (doc-1923)

Exercise 12.5 Multiplication and division of integers

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–12, 15	Questions: 1–13, 15, 16	Questions: 1–16

Individual pathway interactivity: int-4370

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Fluency

1. WE14 Evaluate the following.

a. 2×-5 **b.** -6×3 **c.** -7×9 **d.** 6×-5 **e.** -2×-3

f. -4×-5 **g.** -5×-5 **h.** 0×-7 **i.** -8×-1 **j.** -24×1

k. 10×-1 **l.** -15×2 **m.** -3×18 **n.** -20×-10 **o.** -6×-6

2. WE15 Calculate the following.

a. $-8 \div -2$ **b.** $8 \div -2$ **c.** $-8 \div 2$ **d.** $12 \div -3$ **e.** $-15 \div 5$

f. $-16 \div -8$ **g.** $-90 \div -10$ **h.** $88 \div -11$ **i.** $-6 \div 1$ **j.** $-6 \div -1$

k. $0 \div -4$ **l.** $-84 \div 4$ **m.** $-184 \div 2$ **n.** $-125 \div -5$ **o.** $-67 \div -1$

p. $129 \div -3$ **q.** $-284 \div 4$ **r.** $336 \div -6$

3. WE16 Simplify the following.

a. $\frac{-6}{2}$ **b.** $\frac{-24}{-8}$ **c.** $\frac{-8}{8}$

d. $3 \times \frac{-2}{-6}$ **e.** $4 \times \frac{-5}{10}$ **f.** $-9 \times \frac{-3}{18}$

4. Simplify the following.

a. $2 \times -3 \times 4$ **b.** $-4 \times -3 \times 3$ **c.** $-8 \times 9 \times -2$

5. Fill in the missing numbers.

a. $-6 \times -3 =$ _____ **b.** $6 \times$ _____ $= -18$ **c.** _____ $\times 3 = -18$

d. $-8 \times$ _____ $= -8$ **e.** $-8 \times$ _____ $= 8$ **f.** $-8 \times$ _____ $= 0$

g. $-1 \times$ _____ $= 1$

6. Fill in the missing numbers in these number sentences.

a. $-21 \div$ _____ $= -7$ **b.** _____ $\div -8 = -4$ **c.** _____ $\times 3 = -8$

d. $-11 \div$ _____ $= 1$ **e.** _____ $\div -7 = 0$ **f.** _____ $\div -4 = 4$

g. $-42 \div$ _____ $= -6$ **h.** $-96 \div$ _____ $= 2$ **i.** $-150 \div -25 =$ _____

7. Evaluate each algebraic expression if $c = -2, d = -5$.

a. $c + d$ **b.** $c \times d$ (or cd) **c.** $d - c$ **d.** dc

e. $3cd$ **f.** $3c + d$ **g.** $\frac{cd}{10}$

Understanding

8. Copy and complete the following.

a. $-4 + -4 =$
$2 \times -4 =$

b. $-2 + -2 + -2 =$
$3 \times -2 =$

c. $-3 + -3 =$
$2 \times -3 =$

d. $-5 + -5 + -5 + -5 =$
$4 \times -5 =$

9. MC The missing numbers in the following number sentences could be:

a. $16 \div _____ =$

A. $2, -8$ **B.** $-2, -8$ **C.** $-4, 4$ **D.** $-2, 8$ **E.** $1, -16$

b. $_____ \div _____ = -5$

A. $-15, 3$ **B.** $15, 5$ **C.** $25, 5$ **D.** $-30, -6$ **E.** $-25, -5$

10. Simplify each algebraic expression.

a. $3 \times 2 \times p$ **b.** $-3 \times 4 \times t$ **c.** $-2 \times -5 \times b$ **d.** $2 \times a \times 4$

e. $-3 \times c \times 5$ **f.** $-2 \times d \times -7$ **g.** $6 \times a \times -2 \times b$ **h.** $-5x \times -2g$

11. If $a = -6$, $b = -3$, $c = 2$, evaluate the following.

a. $a \div b$ **b.** $\frac{a}{c}$ **c.** $a \div b \div c$ **d.** $\frac{bc}{a}$ **e.** $\frac{ab}{c}$

f. $\frac{ab}{bc}$ **g.** $\frac{a}{b} + c$ **h.** $\frac{a}{cb}$ **i.** $\frac{(a + bc)}{b}$

12. MC Six people each owe the bank $50. The combined total of the six accounts is:

A. $300 **B.** $-\$50$ **C.** $50 **D.** $-\$\frac{6}{5}$ **E.** $-\$300$

Reasoning

13. Evaluate the following multiplications.

a. $(-1)(-2)$ **b.** $(-3)(-2)(-1)$ **c.** $(-2)(-3)(-1)(-4)$ **d.** $(-4)(-1)(-5)(-2)(-1)$

e. Write a rule for multiplying more than two negative numbers.

f. Does this rule apply to division of negative numbers also? Explain.

14. A spider is running down the stairs from the first floor of an old lady's house to the basement below. It stops every 5 steps to catch a fly. If there are 26 steps above ground and 14 below, how many flies does the spider catch? Show your working.

Problem solving

15. Dawn was taking her younger brother and sister to the local pool for a swim. She knew it cost $2 entry for each person and as she had spent all her money she borrowed from her parents. How much money did Dawn have after paying the entry cost?

16. Mengtong is jumping down the stairs stepping on every second step only.

a. If she jumps ten times:

i. how many steps has she skipped

ii. how many steps are there altogether?

b. If each step is 25 cm high, how far has she travelled in total?

c. Going up the stairs is considered going in a positive direction and going down the stairs is considered going in a negative direction. Rewrite your answer to part **b** with this information in mind.

Reflection

When you multiply two integers, is the result an integer? What about when you divide two integers?

CHALLENGE 12.2

On a test, each correct answer scores 5 points, each incorrect answer scores −2 points, and each question left unanswered scores 0 points. A student answers 16 questions on the test correctly, 3 incorrectly and does not answer 1 question. Write an expression for the student's score and find the score.

12.6 Combined operations

12.6.1 Combined operations

- The mathematical rules about order of operations apply when we work with directed numbers.
- When we are simplifying numbers, the order of operations, BIDMAS, helps us to remember the correct order in which we should perform the various operations. This means we do brackets first; then powers or indices; then × or ÷ (working from left to right); and finally, + and − (working from left to right).

WORKED EXAMPLE 17

Calculate $58 - (2 \times -8 + 3^2)$ using the correct order of operations.

THINK	WRITE
1 Write the question.	$58 - (2 \times -8 + 3^2)$
2 Working inside grouping symbols (Brackets), simplify the squared term (Indices).	$= 58 - (2 \times -8 + 9)$*
3 Multiplication within the brackets. (Multiplication)	$= 58 - (-16 + 9)$
4 Addition within the brackets. (Addition)	$= 58 - -7$
5 When brackets have been removed work the subtraction outside the brackets. (Subtraction)	$= 58 + +7$* $= 65$ ***These steps could be omitted with practice.**

WORKED EXAMPLE 18

Evaluate 5a ÷ b, if $a = -20$ and $b = 4$.

THINK	WRITE
1 Write the expression as given.	$5a \div b$
2 Substitute the given value for each pronumeral, inserting operation signs as required.	$= 5 \times -20 \div 4$
3 Perform the operations as they occur from left to right.	$= -100 \div 4$ $= -25$

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Complete this digital doc: SkillSHEET: Evaluating squares, cubes and cube roots (doc-6567)

Complete this digital doc: SkillSHEET: Order of operations II (doc-6568)

Exercise 12.6 Combined operations

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–5, 8	Questions: 1, 2, 3 1st columns, 4–6, 8, 9	Questions: 1, 2, 3 2nd columns, 4–9

Individual pathway interactivity: int-4371

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Fluency

1. WE17 Calculate the following, using the correct order of operations.

a. $6 + 3 \times -4$
b. $18 - 12 \div -3$
c. $8 + -4 - 10$
d. $17 - 3 + -8$
e. $6 \times -3 \div 9$
f. $72 \div 8 \times -3$
g. $7 + (-3 - 4)$
h. $(6 + 3) \div -9$
i. $-3 \times -2 + 3 \times -1$
j. $-6 \times 5 - 2 \times -6$
k. $-4 \times (6 - -4)$
l. $(-8 + 3) \times -7$

2. Evaluate each of the following.

a. $4 + 7 \times -3 - 2$
b. $-6 - 4 + (-3)^2$
c. $(-2)^3 - 3 \times -2$
d. $3 + (2 - 8) + -6$
e. $-8 \div 2 + (-2)^2$
f. $-4 \times -8 - [\, 2 + (-3)^2 \,]$
g. $(-7 + 5) - -24 \div 6$
h. $-15 \div (2 - 5) - 10$
i. $54 \div -6 + 8 \times -9 \div -4$
j. $(9 - -6) \div -5 + -8 \times 0$
k. $-7 + -7 \div -7 \times -7 - -7$
l. $-9 \times -5 - (3 - -2) + -48 \div 6$

3. WE18 Evaluate each of the following.

a. $2x + 3x$, if $x = -4$
b. $5 + 3d$, if $d = -2$
c. $-5b - 3$, if $b = -7$
d. $a(b + c)$, if $a = 6$, $b = -2$, $c = -4$
e. $x^3 - y$, if $x = -4$, $y = 4$

Understanding

4. MC The expression $6 + 2 \times -5 - -10 \div 2$ is equal to:

A. -15 **B.** -35 **C.** -60 **D.** 1 **E.** 3

5. Model each situation with integers, then find the result.

a. A submarine dives 100 m from sea level, rises 60 m then dives 25 m. What is its final position?
b. Jemma has $274 in the bank, then she makes the following transactions: 2 withdrawals of $68 each, and then 3 deposits of $50 each. How much money does Jemma now have in the bank?
c. If 200 boxes of apples were each 3 short of the stated number of 40 apples, what was the overall shortfall in the number of apples?
d. A person with a mass of 108 kg wants to reduce his mass to 84 kg in 3 months. What average mass reduction is needed per month?

Reasoning

6. Local time in Melbourne is 3 hours ahead of Singapore time, which is 5 hours behind Auckland (NZ) time. Auckland is 11 hours ahead of Berlin (Germany) time. What is the time difference between:

a. Melbourne and Berlin
b. Singapore and Berlin?

7. Merlin is riding his bike east at a steady 10 km/h, while Morgan is riding her bike west at a steady 8 km/h. They pass each other on Backpedal Bridge at 12 noon.
(Assume that east is the positive direction and west is negative and that time *before* noon is negative and *after* noon is positive.)
a. What is the location of each person with respect to the bridge at 9 am?
b. What are their locations with respect to the bridge at 2 pm?
c. How far apart were they at 10 am?
d. How far apart will they be at 4 pm?

Problem solving

8. Emilia bakes and sells cupcakes. A customer ordered 3 lemon cupcakes at $4.25 each, 3 chocolate cupcakes at $4.75 each and 2 original cupcakes at $3.75 each.
a. Write an expression to help Emilia calculate the amount the client has to pay.
b. Calculate the final amount.

9. In a Maths competition marks are awarded as follows: 3 marks for a hard question (H), 2 marks for a medium question (M) and 1 mark for an easy question (E). If the answer is incorrect (I), 1 mark is deducted.
The top three students had the following scores:
Student 1: 10H, 3M, 5E, 2I
Student 2: 7H, 7M, 3E, 3I
Student 3: 9H, 6M, 5E, 0I
a. How many marks did each of them receive?
b. Who won the competition?

Reflection

What do you need to be careful about when you are applying operation order to positive and negative integers?

12.7 Review

12.7.1 Review questions

Fluency

1. Which of the following are integers?

a. $-2\frac{1}{2}$ **b.** 0.45 **c.** 0 **d.** −201

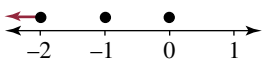
2. Complete each statement by inserting the correct symbol: >, < or =.

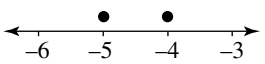
a. −6 ☐ −2 **b.** −7 ☐ 7 **c.** 0 ☐ −5 **d.** −100 ☐ 9

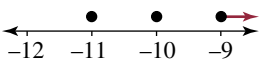
3. List the integers between −21 and −15.

4. Arrange in *descending* order: −3, 2, 0, −15.

5. Describe the integers graphed on each number line.

a. 

b. 

c. 

6. Graph each set of integers on a number line:

a. integers between −7 and −2 **b.** integers > -3 **c.** integers ≤ -4.

7. State whether the following points are on the *x*-axis, the *y*-axis, both axes or in the first quadrant.

a. (0, 0) **b.** (0, 5) **c.** (3, 0) **d.** (3, 2)

8. Draw and label appropriately a set of axes. Plot the following points in the order given, joining each point to the next one. Name the shape that has been drawn.
(−2, 3), (1, 3), (2, −2), (−1, −2), (−2, 3)

9. In which quadrant or on which axes do the following points lie?

a. (−2, 3) **b.** (3, −1) **c.** (−4, −1) **d.** (0, 2) **e.** (−1, 0) **f.** (7, 9)

10. Calculate the following.

a. −12 + 7 **b.** −9 + −8 **c.** 18 + −10 **d.** 5 + 1

11. Write the number that is 2 more than each of the following integers.

a. −4 **b.** 5 **c.** −1 **d.** 0 **e.** −2

12. A snail begins to climb up the side of a bucket. It climbs 3 cm and slips back 2 cm, then climbs a further 4 cm and slips back 1 cm. Write a number sentence to help you find how far the snail is from the bottom of the bucket.

13. Calculate the following.

a. −5 − 3 **b.** 17 − −9 **c.** −6 − −9 **d.** 6 − 8 **e.** 12 − 20 **f.** −10 − −12

14. Evaluate the following.

a. -6×7 **b.** 4×-8 **c.** -2×-5 **d.** $(-8)^2$ **e.** -8^2 **f.** $-2 \times (-8)^2$

15. Calculate the following.

a. −36 ÷ 3 **b.** −21 ÷ −7 **c.** 45 ÷ −9 **d.** $\frac{-18}{-2}$ **e.** 64 ÷ −4 **f.** −100 ÷ 25

16. Calculate the following.

a. $10 - 6 \times 2$ **b.** $-7 - -8 \div 2$ **c.** $-3 \times -5 - -6 \times 2$
d. $(-2)^3$ **e.** $(-3 - 12) \div (-10 + 7)$ **f.** $2c + 3c$, if $c = -4$
g. $-2x(x + 5)$, if $x = -2$ **h.** $2a^2 + a$ if $a = -3$ **i.** b^3 if $b = -5$

17. Replace the box with =, ≤ or > to make the statement correct.

a. −5 ☐ −3 **b.** -22×-2 ☐ 44 **c.** 4 ☐ 2
d. -5×-3 ☐ −15 **e.** 0 ☐ −7 **f.** $-2(-4 + 7)$ ☐ 6
g. -2×5 ☐ −9 **h.** $5 \times (-2 - 18) \div -4$ ☐ 25 **i.** $12 \div -4$ ☐ 3
j. $5 \times (-2 - 18) \div 4$ ☐ −25 **k.** -10×-5 ☐ 50 **l.** $5 \times -2 - 18 \div 4$ ☐ 2.5

18. Model this situation with integers, then find the result: A scuba diver at 52 metres below sea level made his ascent in 3 stages of 15 metres each. At what level was he then?

19. Some historians believe that the Roman era began in 146 BC and ended in 455 AD. For how many years did the Roman era last?

20. The lowest temperature recorded on Earth was −90 °C in Antarctica and the highest was 58 °C in Africa. What is the difference between these two temperatures?

21. Rachel's bank account has a balance of −$43. She pays a bill using direct debit of $51. What is Rachel's bank balance now?

Problem solving

22. a. Complete the addition table below.

+	6		3
	11	15	
12			
	15		

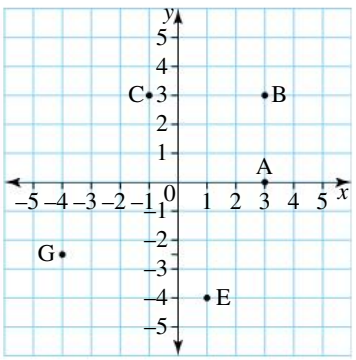
b. Complete the multiplication table below.

×	4		−6
	−12	24	18
5			
	8		

23. For the following expressions determine if the answer will be negative or positive. Explain your answers using mathematical reasoning.

a. −17489 − 25636 **b.** −65234 + 123468 **c.** −44322 + 31212

24. Examine the graph at right.



a. Find the coordinates of the point D, so that ABCD is a square.

b. State the points that:

i. make a right-angled triangle

ii. make an isosceles triangle that doesn't include a right angle.

(There may be more than one solution.)

c. Write down the coordinates of the point F, so that ACFE is a parallelogram. Mark this on the grid and draw the parallelogram.

d. State the coordinates of the midpoint of CA.

e. State the coordinates of the midpoint of AE.

f. Estimate the coordinates of G.

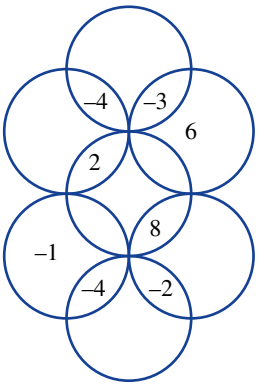
25. At many locations around the world, minimum and maximum temperatures are recorded.

a. On one particular day, the minimum temperature in the Arctic circle was recorded as −24 °C and the maximum temperature was recorded as −7 °C. In London, UK, the maximum temperature was 32 °C and the minimum temperature was −2 °C. In Mexico, the maximum temperature was 38 °C and the minimum temperature was 16 °C. What was the difference in maximum and minimum temperatures at each of these three places?

b. In Melbourne, the temperature often changes rapidly during the day. On one very hot day the temperature reached 42 °C. A cool change arrived during the afternoon and the temperature dropped by 18 °C. What was the temperature after the cool change?

c. In Alaska, the weather often changes suddenly as storms sweep across the frozen plains. On one day, the temperature was 3 °C, but a storm caused the temperature to drop by 24 °C. What was the temperature during the storm?

26. a. Fill the gaps in this diagram so that the numbers in each circle add to zero.



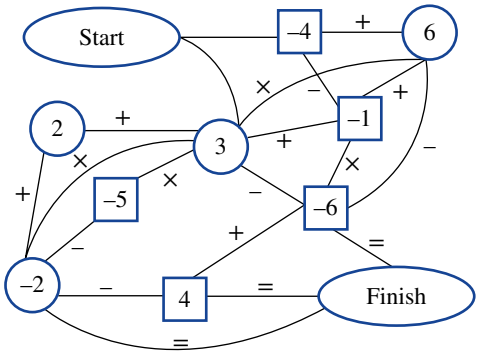
b. Now fill in the gaps so that the numbers in each circle sum to –2.

27. There are many ways to go through the following number network. Follow the addition, subtraction or multiplication signs as you progress from the Start to the Finish.

a. Find a path (using each number only once) that gives a final answer of 0.

b. Find a path (using each number only once in the order that they come) that gives the highest final answer.

c. Find a path (using each number only once in the order that they come) that gives the lowest final answer.



28. You have 30 questions to answer on a multiple-choice test. Each correct answer scores one mark while each incorrect answer scores –2 marks. Any unmarked questions will be counted as incorrect. What is the least number of questions that you can answer correctly and still obtain a score greater than 0? How many marks will you earn? Explain your reasoning.

29. If $X - Y > X$ and $Y - X < Y$ and Y are integers, can you determine whether X and Y are positive or negative values?

30. Place the digits 1 – 9 (no repeats) in the grid below so that the equations reading across and down are true.

	–		×		=	–16
–		×		–		
	–		+		=	7
×		÷		÷		
	–		–		=	4
=		=		=		
–13		10		–3		

31. The negative integers −2 to −1000 are arranged in five columns as shown.

Column 1	Column 2	Column 3	Column 4	Column 5
	−2	−3	−4	−5
−9	−8	−7	−6	
	−10	−11	−12	−13
−17	−16	−15	−14	

In which column will −1000 lie?

11. The following table lists the profit and loss that has occurred for the past four years. Which two consecutive years had the maximum difference?

Year	2006	2007	2008	2009
$Profit/loss	−6420	3808	−2651	8222

12. a. Copy and use your calculator to complete the following table.

Integer (x)	x^2	x^3	x^4	x^5
2				
−2				
3				
−3				
4				
−4				

b. Look at your results in the x^2 column. What is the sign of all the numbers?

c. Consider the sign of the numbers in the x^3 column. What do you notice?

d. Describe the resulting sign of the numbers in the x^4 and x^5 columns.

e. Is the resulting sign in the x^2 column the same as that in the x^4 column? What about the signs of the numbers in the x^3 and x^5 columns?

f. Copy and complete the following statements.
When a positive number is raised to any power, the sign of the answer is _____.
When a negative number is raised to an even power, the sign of the answer is _____.
When a negative number is raised to an odd power, the sign of the answer is _____.

g. Let us now look at the reverse of raising a number to a power — taking the root of a number. You will notice that $2^2 = 4$ and $(-2)^2 = 4$. It follows that, if we take the square root of 4, we can get +2 or −2. (Your calculator will only give you the positive answer.) This has a shorthand way of being written as $\sqrt{4} = \pm 2$. Similarly, you will notice that $\sqrt[4]{16} = \pm 2$. This only applies to even roots. Write a statement showing the square root of 100.

h. It is not possible to take the even root of a negative number (because no number raised to an even power will produce a negative number). What happens when you try to evaluate $\sqrt{-144}$ on the calculator?

i. Notice that with odd-numbered roots the sign of the answer is the same as the sign of the original number; $\sqrt[3]{8} = 2$ but $\sqrt[3]{-8} = -2$. Calculate $\sqrt[3]{-125}$.

j. Copy and complete the following table. (Fill in the blocks that are not blacked out.) Consider each answer carefully, as some are not possible.

Integer (x)	$\sqrt{x}$	$\sqrt[3]{x}$	$\sqrt[4]{x}$	$\sqrt[5]{x}$
16	± 4			
−16				
27				
−27				
32				
−32				
81				
−81				
64				
−64				

k. In your own words, describe the sign resulting after taking odd and even roots of positive and negative numbers.

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Try out this interactivity: Word search: Topic 12 (int-2603)

Try out this interactivity: Crossword: Topic 12 (int-2604)

Try out this interactivity: Sudoku: Topic 12 (int-3172)

Complete this digital doc: Concept map: Topic 12 (doc-10739)

Language

It is important to learn and be able to use correct mathematical language in order to communicate effectively. Create a summary of the topic using the key terms below. You can present your summary in writing or using a concept map, a poster or technology.

ascending
descending
directed number
horizontal
integer
negative integers
opposite integer
origin
positive integers
quadrants
vertical
***x*-axis**
***y*-axis**

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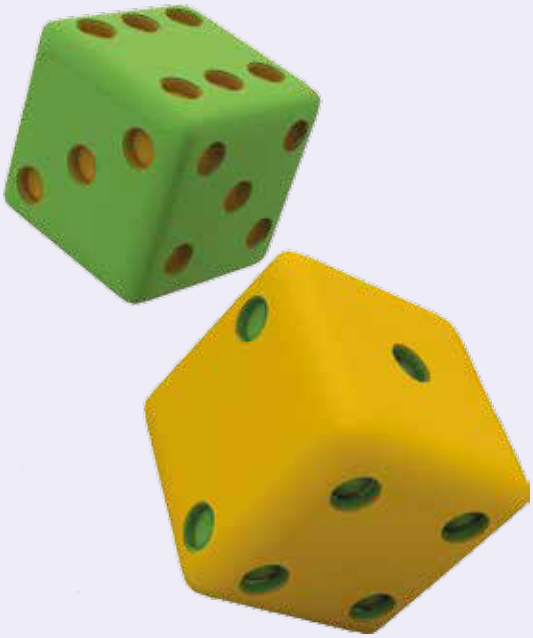
Investigation | Rich task

Directed numbers dice game

In this game, the winner is the first to complete a row of three, horizontally, vertically or diagonally.

Equipment: 2 players, 2 standard dice, Connect-3 board, dice placement sheet, 9 blue counters, 9 red counters.



Instructions

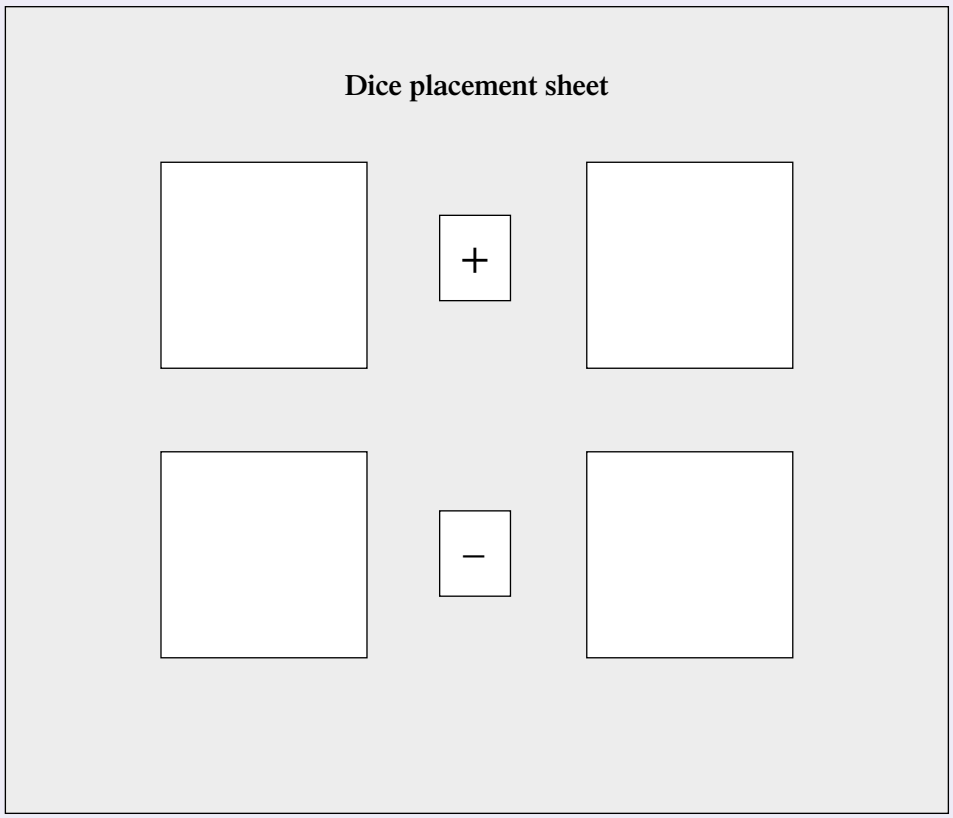
Roll the two dice and decide whether to add or subtract the numbers on the dice. Put the dice on one of the squares on the dice placement sheet to produce a total shown on the Connect-3 board.

Once you have worked out the answer, place your counter on the appropriate number on the Connect-3 board. (You should check each other's answers.) You cannot cover a number that has already been covered. If you are unable to find a total that has not been covered, you must pass. The winner is the first to complete a row of three numbers: horizontally, vertically or diagonally.

Make your own board game using dice with negative numbers on each face and put different numbers on the board. The dice placement sheet could also be changed to include × or ÷ symbols.

Connect-3 board

	−5	−4	−3	−2
−1	0	1	2	3
4	5	6	7	8
9	10	11	12	



learnon RESOURCES — ONLINE ONLY

Complete this digital doc: Code puzzle: What has six eyes but cannot see? (doc-14331)

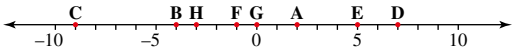
Answers

Topic 12 Positive and negative integers

Exercise 12.2 Integers on the number line

1. a. −7 **b.** 10 or + 10 **c.** −422
d. 2228 or + 2228 **e.** −3 **f.** −30

2.



3. a. −6, −2, 3, 5, 7 **b.** −6, −4, −1, 0, 3 **c.** −5, −3, −2, 1, 7 **d.** −4, −3, 1, 2, 5

4. a. 6 **b.** −7 **c.** −1 **d.** 8

5. a. > **b.** < **c.** > **d.** > **e.** < **f.** <

6. a. i. −5 **ii.** 7 **b. i.** −4 **ii.** 5
c. i. −10 **ii.** 7 **d. i.** −4 **ii.** 3

7. −2 °C **8.** 14

9. a. 2000 years after the birth of Christ.
b. i. 10 **ii.** −106
c. i. −18 **ii.** +15, +36, +41, +47, +48
d. Check with your teacher.

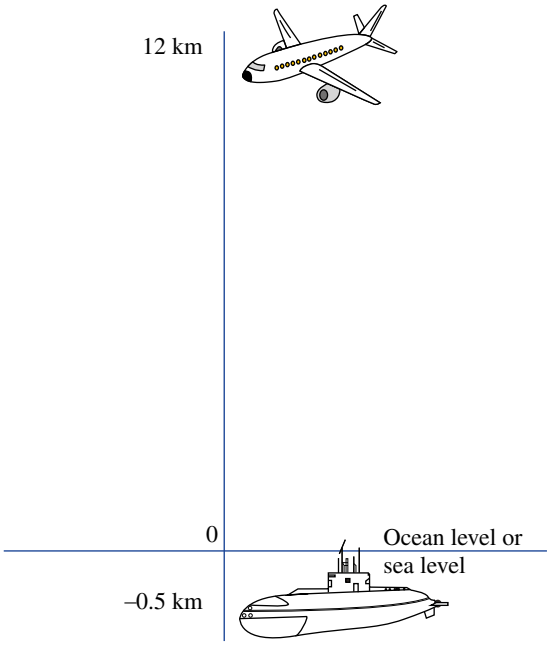
10. Answers will vary.

11. a.

Vanessa −16 | School 0 | Raj 12

b. 2.8 km

12. a.



b. 12.5 km

Exercise 12.3 Integers on the number plane

1. a. (3, 4) **b.** (8, 4) **c.** (4, 8) **d.** (0, 7) **e.** (5, 0)

2. a. G **b.** F **c.** H **d.** J

3. a. (3, 1); 1st quadrant **b.** (−2, 4); 2nd quadrant **c.** (−6, −5); 3rd quadrant
d. (0, − 3); y-axis **e.** (6, − 6); 4th quadrant

4. a. C; 2nd quadrant **b.** L; y-axis **c.** D; 4th quadrant
d. E; 1st quadrant **e.** G; x-axis

5. a. 3 **b.** 3 **c.** 7 **d.** 0

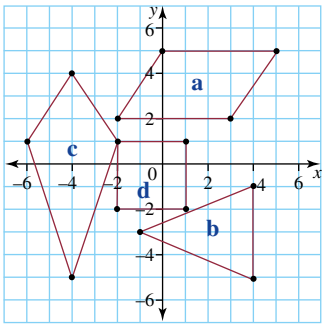
6. a. 2 **b.** −6 **c.** 0 **d.** −3

7. H **8.** 0 **9.** 0 **10.** 4

11. a. T **b.** T **c.** T **d.** F **e.** F **f.** T

12. a. D **b.** B **c.** B

13.



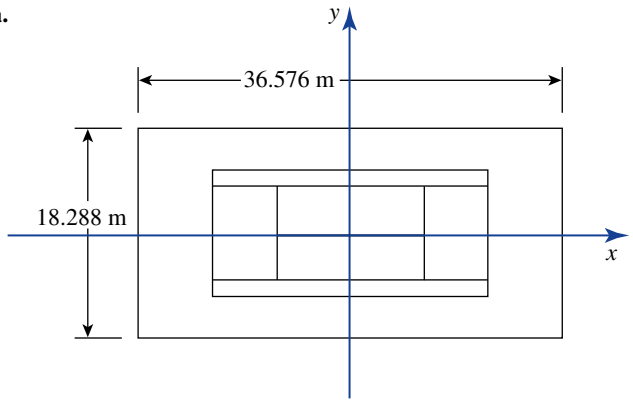
a. Parallelogram **b.** Isosceles triangle **c.** Kite **d.** Square

14. a. (2, 1) **b.** (1, −2) **c.** Draw line DO
d. (−3, 1), (−2, 1), (−1, 1) etc. **e.** (−5, −2)

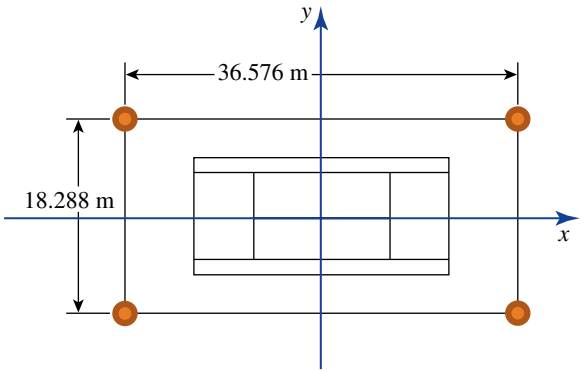
15. a. West **b.** East **c.** West **d.** East
e. An odd number of negative signs turns you west, and an even number of negative signs turns you east.
f. South
g. An odd number of negative signs turns you south, and an even number of negative signs turns you north.

16. 210

17. a.



b.



Top left post (−18.288, 9.144) Top right post (18.288, 9.144)
Bottom right post (18.288, −9.144) Bottom left post (−18.288, −9.144)

Exercise 12.4 Addition and subtraction of integers

1. a. $-4 + +4 = 0$ **b.** $-1 + +3 = +2$ **c.** $-2 + -4 = -6$
d. $+2 + -4 = -2$ **e.** $+6 + -4 = +2$ **f.** $-1 + -3 = -4$

2. a. -1 **b.** -1 **c.** 0 **d.** $+1$ **e.** -2 **f.** -2

3. a. $5 + -2 = 3$ **b.** $-3 + -4 = -7$ **c.** $-2 + 2 = 0$ **d.** $6 + -5 = 1$
e. $-5 + 5 = 0$ **f.** $4 + -6 = -2$ **g.** $-5 + 7 = 2$ **h.** $6 + -9 = -3$
i. $-4 + 6 = 2$ **j.** $3 + -3 = 0$ **k.** $-8 + -2 = -10$ **l.** $0 + -6 = -6$

4. a. -7 **b.** -2 **c.** 0 **d.** -4 **e.** 4 **f.** -10
g. 4 **h.** -3 **i.** -100 **j.** 10 **k.** 2 **l.** -22

5.

+	−13	5
21	8	26
−18	−31	−13

6. a. 2 **b.** 10 **c.** -10 **d.** 2 **e.** -9 **f.** -10
g. 0 **h.** -4 **i.** 3 **j.** 10 **k.** -14 **l.** 0

7. a. -10 **b.** 13 **c.** 3 **d.** 12
e. -16 **f.** -17 **g.** 23 **h.** -12

8. a. -8 **b.** -6 **c.** 10 **d.** -7 **e.** 3
f. 1 **g.** 9 **h.** -25 **i.** 5

9. a. 3 **b.** -10 **c.** 7 **d.** 6 **e.** -5
f. 15 **g.** 15 **h.** -11 **i.** -1

10. a.

+	−2	−1	0	1	2
−2	−4	−3	−2	−1	0
−1	−3	−2	−1	0	1
0	−2	−1	0	1	2
1	−1	0	1	2	3
2	0	1	2	3	4

b. Even numbers; doubles **c.** Zeroes (addition of opposites)
d. Yes **e.** Discuss

11. a. 4 **b.** -3 **c.** $7\ ^\circ C$ **d.** $-1\ ^\circ C$ **e.** -1 **f.** -7

12. a. $-2 + -3 = -5$ **b.** $-3 + 5 = 2$ **c.** $5 + -6 = -1$ **d.** $-50 + 63 = 13$
e. $3 + -5 = -2$ **f.** $1 + -6 = -5$ **g.** $-50 + 26 = -24$ **h.** $200 + -350 = -150$

13. B **14. a** to **e** Discuss

15. a. T **b.** T **c.** T **d.** T

16. a. 1 **b.** -11 **c.** -7 **d.** -9 **e.** 6 **f.** 12

17. a. D **b.** B

18. $-2\ ^\circ C$ **19.** 12 m

20. a. 'When adding two negative numbers, add the two numbers as if they were positive numbers and then write a negative sign in front of the answer.'

b. 'When adding a negative number and a positive number, ignore the signs, subtract the smaller number from the larger number and then write the sign the larger number has.'

21. Balance at end of month $10. During the second week her balance was negative.

22. 2nd floor. It travelled 22 floors in 10 minutes.

Challenge 12.1

−3	−4	3
4	−2	−6
−5	2	−1

Exercise 12.5 Multiplication and division of integers

1. a. −10 **b.** −18 **c.** −63 **d.** −30 **e.** 6 **f.** 20 **g.** 25 **h.** 0
i. 8 **j.** −24 **k.** −10 **l.** −30 **m.** −54 **n.** 200 **o.** 36

2. a. 4 **b.** −4 **c.** −4 **d.** −4 **e.** −3 **f.** 2 **g.** 9 **h.** −8 **i.** −6
j. 6 **k.** 0 **l.** −21 **m.** −92 **n.** 25 **o.** 67 **p.** −43 **q.** −71 **r.** −56

3. a. −3 **b.** 3 **c.** −1 **d.** 1 **e.** −2 **f.** $1\frac{1}{2}$

4. a. −24 **b.** 36 **c.** 144

5. a. 18 **b.** −3 **c.** −6 **d.** 1 **e.** −1 **f.** 0 **g.** −1

6. a. 3 **b.** 32 **c.** −72 **d.** −11 **e.** 0
f. −16 **g.** 7 **h.** −48 **i.** 6

7. a. −7 **b.** 10 **c.** −3 **d.** 10 **e.** 30 **f.** −11 **g.** 1

8. a. −8, −8 **b.** −6, −6 **c.** −6, −6 **d.** −20, −20

9. a. B **b.** A

10. a. $6p$ **b.** $-12t$ **c.** $10b$ **d.** $8a$
e. $-15c$ **f.** $14d$ **g.** $-12ab$ **h.** $10xg$

11. a. 2 **b.** −3 **c.** 1 **d.** 1 **e.** 9 **f.** −3 **g.** 4 **h.** 1 **i.** 4

12. E

13. a. 2 **b.** −6 **c.** 24 **d.** −40
e. When multiplying an even number of negative numbers, the answer is always positive. When multiplying an odd number of negative numbers, the answer is always negative.
f. Yes, because multiplication and division have the same rules when working with negative numbers.

14. 8 flies **15.** −$6

16. a. i. 10 **ii.** 20 **b.** 5 metres **c.** −5 metres

Challenge 12.2

16 × 5 + 3 × −2 + 0 = 80 − 6 = 74 marks

Exercise 12.6 Combined operations

1. a. −6 **b.** 22 **c.** −6 **d.** 6 **e.** −2 **f.** −27
g. 0 **h.** −1 **i.** 3 **j.** −18 **k.** −40 **l.** 35

2. a. −19 **b.** −1 **c.** −2 **d.** −9 **e.** 0 **f.** 21
g. 2 **h.** −5 **i.** 9 **j.** −3 **k.** −7 **l.** 32

3. a. −20 **b.** −1 **c.** 32 **d.** −36 **e.** −68

4. D

5. a. −100 + 60 + −25 = −65 **b.** 274 + 2 × −68 + 3 × 50 = 288
c. 600 **d.** 8 kg

6. a. Sydney 9 h ahead **b.** Singapore 6 h ahead

7. a. Merlin: −3 × 10 = −30, 30 km west of bridge
Morgan: −3 × − 8 = 24, 24 km east of bridge
b. Merlin: 2 × 10 = 20, 20 km east of bridge
Morgan: 2 × − 8 = −16, 16 km west of bridge
c. 36 km **d.** 72 km

8. a. 3 × (4.25 + 4.75) + 2 × 3.75 **b.** $34.50

9. a. 39 marks, 35 marks, 44 marks **b.** Student 3 with 44 marks

Exercise 12.7 Review

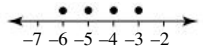
1. a. No **b.** No **c.** Yes **d.** Yes

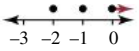
2. a. < **b.** > **c.** < **d.** >

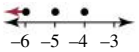
3. −20, −19, −18, −17, −16

4. 2, 0, −3, −15

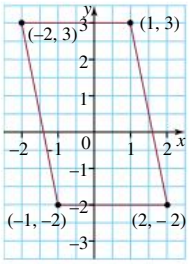
5. a. < 1 or ≤ 0 **b.** Between -6 and -3 **c.** > -12 or ≥ -11

6. a.


b.


c.


7. a. Both axes **b.** y-axis **c.** x-axis **d.** First quadrant

8.


Parallelogram

9. a. Second **b.** Fourth **c.** Third **d.** y-axis **e.** x-axis **f.** First

10. a. -5 **b.** -17 **c.** 8 **d.** 6

11. a. -2 **b.** 7 **c.** 1 **d.** 2 **e.** 0

12. $3 + -2 + 4 + -1 = 4$

snail is 4 cm from the bottom of the bucket.

13. a. -8 **b.** 26 **c.** 3 **d.** -2 **e.** -8 **f.** 2

14. a. -42 **b.** -32 **c.** 10 **d.** 64 **e.** -64 **f.** -128

15. a. -12 **b.** 3 **c.** -5 **d.** 9 **e.** -16 **f.** -4

16. a. -2 **b.** -3 **c.** 27 **d.** -8 **e.** 5
f. -20 **g.** 12 **h.** 15 **i.** -125

17. a. $-5 < -3$ **b.** $-22 \times -2 = 44$ **c.** $4 > 2$
d. $-5 \times 3 > -15$ **e.** $0 > -7$ **f.** $-2(-4 + 7) < 6$
g. $-2 \times 5 < -9$ **h.** $5 \times (-2 - 18) \div 4 < 25$ **i.** $12 \div -4 < 3$
j. $5 \times (-2 - 19) \div 4 = -25$ **k.** $-10 \times -5 = 50$ **l.** $5 \times -2 - 18 \div 4 < 2.5$

18. $-52 + 3 \times 15 = -7$ **19.** 601 years **20.** 148 °C **21.** −$94

22. a.

+	6	10	3
5	11	15	8
12	18	22	15
9	15	19	12

b.

×	4	−8	−6
−3	−12	24	18
5	20	−40	−30
2	8	−16	−12

23. a. Negative **b.** Positive **c.** Negative

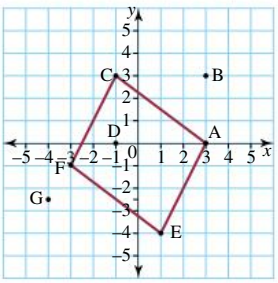
Two negative numbers added together will always be negative. When a positive and a negative number are added together if the larger number is positive the result will be positive, if the larger number is negative the result will be negative.

24. a. D$(-1, 0)$

b. i. CBA, BAD others are possible; check with your teacher.

ii. ADE or BCE

c. F$(-3, -1)$

d.


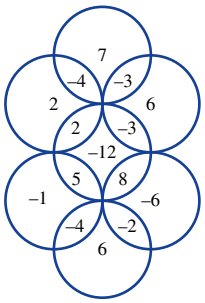
e. (1, 1.5) **f.** (2, −2) **g.** G$(-4, -2.5)$

25. a. Arctic Circle 17 °C, London 34 °C, Mexico 22 °C

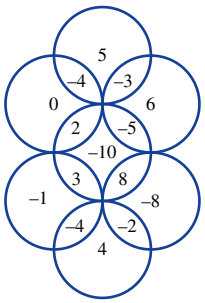
b. 24 °C

c. −21 °C

26. a.



b.



27. a. Start −4 + 6 + −1 + 3 + 2 + −2 − 4 = 0. There are more possibilities — check with your teacher.

b. There are several possibilities — check with your teacher.

c. There are several possibilities — check with your teacher.

28. 21 questions for 3 marks

29. *X* is positive and *Y* is negative.

30.

	−		×		=	−16
−		×		−		
	−		+		=	7
×		÷		÷		
	−		−		=	4
=		=		=		
−13		10		−3		

31. Column 2

32. 2009 and 2008

33. a.

Integer (x)	x^2	x^3	x^4	x^5
2	4	8	16	32
−2	4	−8	16	−32
3	9	27	81	243
−3	9	−27	81	−243
4	16	64	256	1024
−4	16	−64	256	−1024

b. Positive

c. If x was negative then x^3 is negative.

d. For x^4, all the results are positive, for x^5, if x is negative then x^5 is negative.

e. The sign in the x^2 is the same as the sign in the x^4 column. The sign in the x^3 column is the same as the matching term in the x^5 column.

f. When a positive number is raised to any power, the sign of the answer is positive.
When a negative number is raised to an even power, the sign of the answer is positive.
When a negative number is raised to an odd power the sign of the answer is negative.

g. The square root of 100 is 10 or −10.

h. It will give an error message.

i. -5

j.

Integer (x)	$\sqrt{x}$	$\sqrt[3]{x}$	$\sqrt[4]{x}$	$\sqrt[5]{x}$
16	4		2	
-16	Not possible		Not possible	
27		3		
-27		-3		
32				2
-32				-2
81	9		3	
-81	Not possible		Not possible	
64	8	4		
-64	Not possible	-4		

k. When taking an even root of a positive number the answer can be positive or negative.

It is not possible to take the even root of a negative number.

When taking the odd root of a positive number the answer is always positive.

When taking the odd root of a negative number the answer is always negative.

Investigation — Rich task

Teacher to check.

TOPIC 13
Linear equations

13.1 Overview

Numerous **videos** and **interactivities** are embedded just where you need them, at the point of learning, in your learnON title at www.jacplus.com.au. They will help you to learn the content and concepts covered in this topic.

13.1.1 Why learn this?

Many everyday situations can be represented by equations. You will almost certainly have been solving equations without even realising it. If you pay for an item which costs $4.25 and you hand $5.00 to the shop assistant, you will expect 75c in change. This answer is obtained using an equation. To work out which mobile phone plan is better value requires solving equations. If you are travelling to a country which uses different currency, you can use the exchange rate and equations to work out the equivalent cost of an item in Australian dollars.

13.1.2 What do you know?

assessON

1. **THINK** List what you know about equations. Use a thinking tool such as a concept map to show your list.
2. **PAIR** Share what you know with a partner and then with a small group.
3. **SHARE** As a class, create a thinking tool such as a large concept map that shows your class's knowledge of equations.

LEARNING SEQUENCE

13.1 Overview
13.2 Solving equations using trial and error
13.3 Using inverse operations
13.4 Building up expressions
13.5 Solving equations using backtracking
13.6 Checking solutions
13.7 Keeping equations balanced
13.8 Review

13.2 Solving equations using trial and error

13.2.1 Writing equations

- By writing an expression equal to a constant or another expression, we create an **equation**; for example, $2x + 5 = 7$, $3y - 4 = y + 6$ are equations.
- We can use equations to describe a problem that we want to solve.
- To write a problem as an equation, use a pronumeral to stand for the unknown number.
- To solve an equation means to find the value of the pronumeral.

WORKED EXAMPLE 1

Write an equation to represent each of these puzzles.
I am thinking of a number.
a When I multiply the number by 8, the answer is 24.
b When I divide the number by 5, the answer is 7.
c When I divide 60 by the number, the answer is 10.
d When I subtract 7 from the number, the answer is 25.
e When I subtract the number from 72, the answer is 52.
f When I square the number, the result is 36.

THINK	WRITE
a 1 Use a pronumeral to describe the number.	**a** Let m be the number.
2 Multiply the number by 8.	$m \times 8 = 8m$
3 Write the equation.	$8m = 24$
b 1 Use a pronumeral to describe the number.	**b** Let t be the number.
2 Divide the number by 5.	$t \div 5 = \frac{t}{5}$
3 Write the equation.	$\frac{t}{5} = 7$
c 1 Use a pronumeral to describe the number.	**c** Let s be the number.
2 Divide 60 by the number.	$60 \div s = \frac{60}{s}$
3 Write the equation.	$\frac{60}{s} = 10$
d 1 Use a pronumeral to describe the number.	**d** Let l be the number.
2 Subtract 7 from the number (that is, take 7 away from the number).	$l - 7$
3 Write the equation.	$l - 7 = 25$
e 1 Use a pronumeral to describe the number.	**e** Let a be the number.
2 Subtract the number from 72 (that is, take the number away from 72).	$72 - a$
3 Write the equation.	$72 - a = 52$

f 1 Use a pronumeral to describe the number.	**f** Let z be the number.
2 Square the number (that is, multiply the number by itself).	$z \times z = z^2$
3 Write the equation.	$z^2 = 36$

- Sometimes the solution to an equation is obvious after close inspection.

WORKED EXAMPLE 2

Solve the following equations by inspection.
a $\frac{w}{3} = 4$ **b** $h - 9 = 10$

THINK	WRITE
a 1 Write the equation.	**a** $\frac{w}{3} = 4$
2 Think of a number which when divided by 3 gives 4. Try 12.	$12 \div 3 = 4$
3 So w must be 12.	$w = 12$
b 1 Write the equation.	**b** $h - 9 = 10$
2 Think of a number which equals 10 when 9 is subtracted from it. Try 19.	$19 - 9 = 10$
3 So h must be 19.	$h = 19$

13.2.2 Guess, check and improve

- We can try to guess the solution to an equation.
- By noting the result when we substitute each guess, we can improve the guess and come closer to the solution.

WORKED EXAMPLE 3

Use guess, check and improve to solve the equation $2x + 21 = 4x - 1$.

THINK

1 Set up a table with four columns displaying the value of x, the value of the left-hand and right-hand side equations (after substitution) and a comment on how these two values compare.

2 Substitute the first guess, say $x = 1$, into the LHS and RHS equations and comment on results.

3 Repeat step 2 for $x = 6$, $x = 10$ until the correct answer is obtained.

4 State the solution.

WRITE

	Check		
Guess x	$2x + 21$	$4x - 1$	Comment
1	23	3	$4x - 1$ is too small
6	33	23	This is closer.
10	41	39	Very close
11	43	43	That's it!

The solution is $x = 11$.

- This method also works when we need to find two variables.

WORKED EXAMPLE 4

Find two numbers whose sum is 31 and whose product is 150.

THINK

1 The numbers add up to 31 so guess two numbers that do this. Then check by finding their product.

2 Guess 1 and 30.

3 Guess 10 and 21.
Try a number between 1 and 10 for the first number.
Try a number between 5 and 10 for the first number.
Try a number between 5 and 8 for the first number.

4 State the solution.

WRITE

Guess	Check	
Sum (small number first)	Product (P)	Comment
1, 30	$1 \times 30 = 30$	P is too low.
10, 21	$10 \times 21 = 210$	P is too high.
5, 26	$5 \times 26 = 130$	P is too low.
8, 23	$8 \times 23 = 184$	P is too high.
6, 25	$6 \times 25 = 150$	That's it!

The numbers are 6 and 25.

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Complete this digital doc: SkillSHEET: Completing number sentences (doc-6571)
Complete this digital doc: SkillSHEET: Writing number sentences from written information (doc-6572)
Complete this digital doc: SkillSHEET: Applying the four operations (doc-6573)
Complete this digital doc: Spreadsheet: Solving equations (doc-1966)

Exercise 13.2 Solving equations using trial and error

assessON

Individual pathways

PRACTISE
Questions:
1, 2, 3, 4, 5, 6, 7, 12

CONSOLIDATE
Questions:
1, 2, 3, 4, 6e, h, 5, 6 column 1, 7, 10, 12

MASTER
Questions:
1, 2, 3, 4e, f, i, 5, 6 column 2, 6–13

Individual pathway interactivity: int-4372

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To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note*: Question numbers may vary slightly.

Fluency

1. WE1 Write an equation to represent each of these puzzles. I am thinking of a number.
a. When I add 7, the answer is 11.
b. When I add 3, the answer is 5.
c. When I add 12, the answer is 12.
d. When I add 5, the answer is 56.
e. When I subtract 7, the answer is 1.
f. When I subtract 11, the answer is 11.

g. When I subtract 4, the answer is 7.
h. When I subtract 8, the answer is 0.
i. When I multiply by 2, the answer is 12.
j. When I multiply by 6, the answer is 30.
k. When I multiply by 5, the answer is 30.
l. When I multiply by 6, the answer is 12.
m. When I divide by 7, the answer is 1.
n. When I divide by 3, the answer is 100.
o. When I divide by 5, the answer is 2.
p. When I divide by 7, the answer is 0.
q. When I subtract the number from 15, the answer is 2.
r. When I subtract the number from 52, the answer is 8.
s. When I divide 21 by the number, the answer is 7.
t. When I square the number, the answer is 100.

2. WE2 Solve the following equations by inspection.

a. $x + 7 = 18$	**b.** $y - 8 = 1$	**c.** $3m = 15$	**d.** $\frac{m}{10} = 3$
e. $w + 25 = 26$	**f.** $m - 1 = 273$	**g.** $4w = 28$	**h.** $\frac{k}{5} = 0$
i. $b + 15 = 22$	**j.** $\frac{w}{3} = 6$	**k.** $5k = 20$	**l.** $h - 14 = 11$
m. $b - 2.1 = 6.7$	**n.** $\frac{c}{3} = 1.4$	**o.** $5x = 14$	

3. WE3 Use guess, check and improve to solve the equation $3x + 11 = 5x - 1$. The process has been started for you.

Guess	Check		
x	$3x + 11$	$5x - 1$	Comment
1	14	4	$5x - 1$ is too small.
10	41	49	$5x - 1$ is too big.
5			

4. Use guess, check and improve to solve the following equations. *Note*: There are 2 solutions for parts **f** and **g**.

a. $5x + 15 = x + 27$	**b.** $2x + 12 = 3x - 2$	**c.** $x + 20 = 3x$
d. $12x - 18 = 10x$	**e.** $10(x + 1) = 5x + 25$	**f.** $x(x + 1) = 21x$
g. $x(x + 7) = 12x$	**h.** $6(x - 2) = 4x$	**i.** $3(x + 4) = 5x + 4$

5. WE4 Find two numbers whose sum is 21 and whose product is 98. The process has been started for you.

Guess	Check	
Sum (small number first)	Product (P)	Comment
1, 20	$1 \times 20 = 20$	P is too low.
10, 11	$10 \times 11 = 110$	P is too high.

6. Use guess, check and improve to find two numbers whose sum and product are given:

a. sum = 26, product = 165	**b.** sum = 27, product = 162
c. sum = 54, product = 329	**d.** sum = 45, product = 296
e. sum = 178, product = 5712	**f.** sum = 104, product = 2703

g. sum = 153, product = 4662
h. sum = 242, product = 14 065
i. sum = 6.1, product = 8.58
j. sum = 8, product = 14.79
k. sum = 978, product = 218 957
l. sum = 35, product = 274.89

Understanding

7. Copy and complete this table by substituting each x-value into $x^2 + 4$ and $4x + 1$. The first row has been completed for you. Use the table to find a solution to the equation $x^2 + 4 = 4x + 1$. (Remember, x^2 means $x \times x$.)

x	$x^2 + 4$	$4x + 1$
0	4	1
1		
2		9
3	13	
4		

Reasoning

8. A football team won 4 more games than it lost. The team played 16 games. How many did it win?
9. Lily is half the age of Pedro. Ross is 6 years older than Lily and 6 years younger than Pedro. How old is Pedro?
10. A plumber cut a 20-metre pipe into two pieces. One of the pieces is three times as long as the other. What are the lengths of the two pieces of pipe?
11. Julie has the same number of sisters as brothers. Her brother Todd has twice as many sisters as brothers. How many children are in the family?

Problem solving

12. Angus is the youngest in his family and today he and his Dad share a birthday. Both their ages are prime numbers. Angus's age has the same two digits as his Dad's but in reverse order. In 10 years' time, Dad will be three times as old as Angus. How old will each person be when this happens?
13. Sam accidentally divided two numbers on his calculator and got 0.6, when he should have added the two numbers and got 16. What are the two numbers?

Reflection

Why is it a good idea to write our guesses down when using guess, check and improve to solve equations?

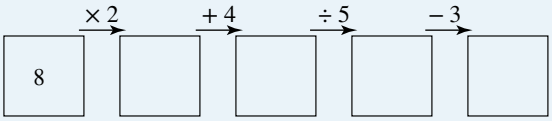
13.3 Using inverse operations

13.3.1 Using inverse operations

- We can use **flowcharts** to help us organise harder equations.
- The first number is called the **input number**.
- The last number is called the **output number**.
- Inverse operations mean 'doing the opposite operation'.
  For example, + is the inverse of ÷.
  − is the inverse of +.
  × is the inverse of ÷.
  ÷ is the inverse of ×.

WORKED EXAMPLE 5

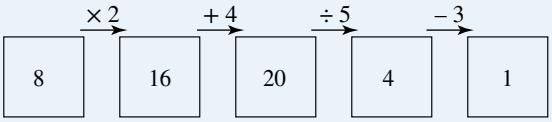
Complete the flowchart to find the output number.



THINK

1 Follow the steps and fill in the boxes.
$8 \times 2 = 16$
$16 + 4 = 20$
$20 \div 5 = 4$
$4 - 3 = 1$

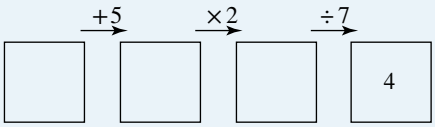
WRITE



- It is possible to travel in either direction through a flowchart. Working backwards, against the arrows, is called **backtracking**.
- To work backwards, carry out **inverse operations**, or undo each step.

WORKED EXAMPLE 6

Use backtracking and inverse operations to find the input number in this flowchart.



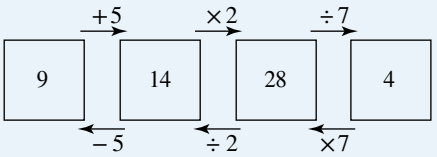
THINK

1 Fill in the numbers as you backtrack. The inverse of ÷7 is ×7 ($4 \times 7 = 28$).

2 The inverse of ×2 is ÷2 ($28 \div 2 = 14$).

3 The inverse of +5 is −5 ($14 - 5 = 9$).

WRITE



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Complete this digital doc: SkillSHEET: Inverse operations (doc-6574)

Complete this digital doc: Spreadsheet: Backtracking (doc-1967)

Exercise 13.3 Using inverse operations

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–4, 7	Questions: 1–5, 7, 8	Questions: 1–8

Individual pathway interactivity: int-4373

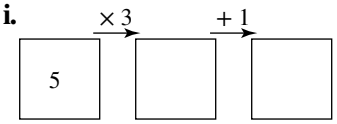
learnon ONLINE ONLY

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note*: Question numbers may vary slightly.

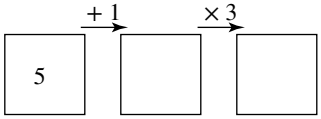
Fluency

1. WE5 Complete the following flowcharts to find the output number.

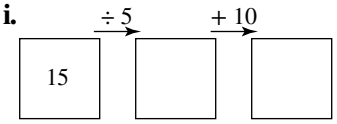
a. i.



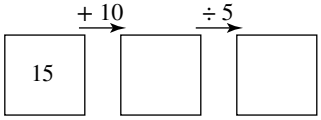
ii.



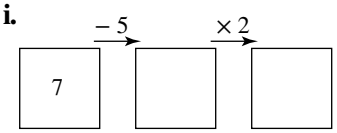
b. i.



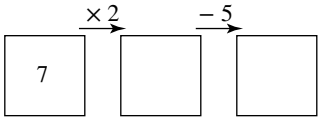
ii.



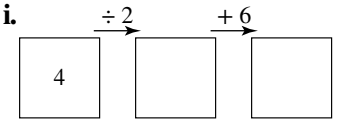
c. i.



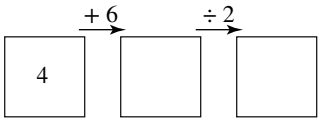
ii.



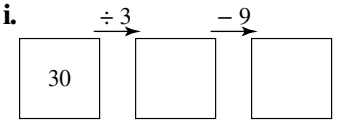
d. i.



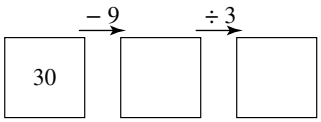
ii.



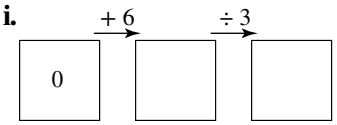
e. i.



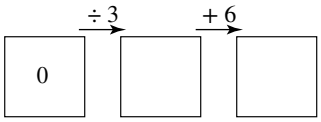
ii.



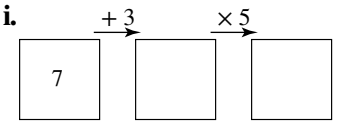
f. i.



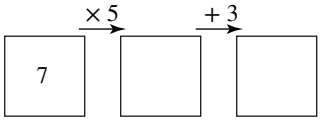
ii.



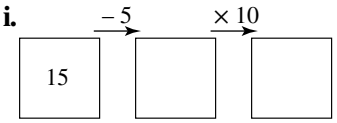
g. i.



ii.



h. i.



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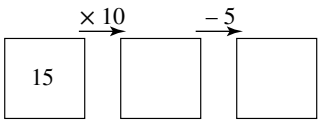
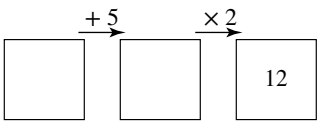
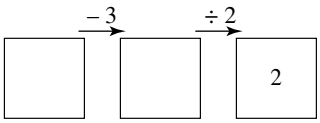


2. WE6 Use backtracking and inverse operations to find the input number in each of these flowcharts.

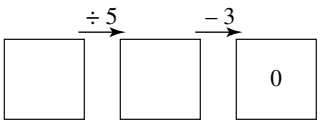
a.



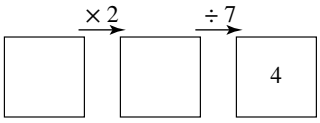
b.



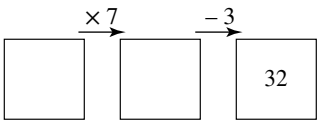
c.



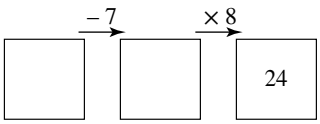
d.

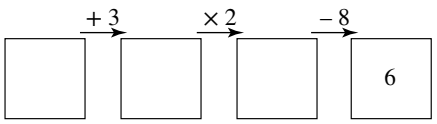


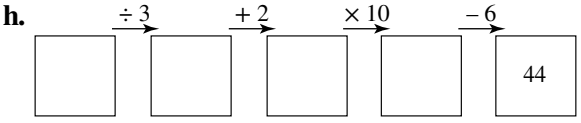
e.

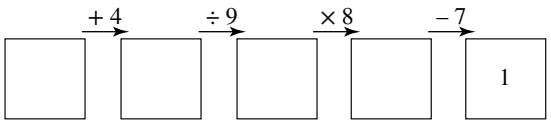


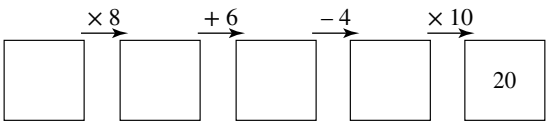
f.

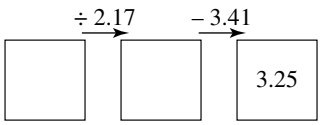


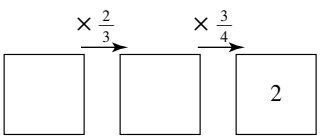
g. 

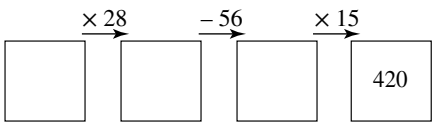
h. 

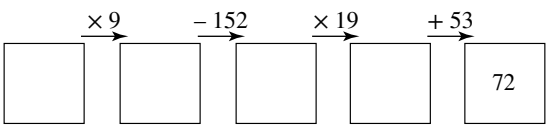
i. 

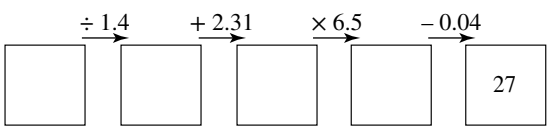
j. 

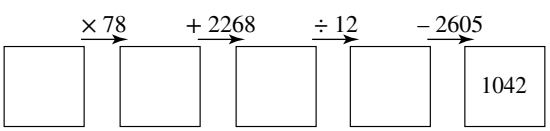
k. 

l. 

m. 

n. 

o. 

p. 

Understanding

3. a. In question **1**, what did you notice about each pair of flowcharts?

b. Does changing the order of operations affect the end result?

4. Complete the two statements below.

a. Adding and _____ are inverse operations.

b. _____ and dividing are inverse operations.

Reasoning

5. a. Explain with the use of examples why addition and subtraction are inverse operations. Start with $x + 3 = 7$ by trying to cancel down the $+3$.

b. Explain with the use of examples why multiplication and division are inverse operations. Start with $2x = -8$ by trying to cancel down the 2.

6. a. Which of the following statements is correct and which one would you use to calculate the value of x?

i. If $x + 6 = -11$, then $x = -11 - 6$.

ii. If $x + 6 = -11$, then $6 = -11 - x$.

b. Which of the following statements is correct and which one would you use to calculate the value of x?

i. If $-7x = 56$, then $-7 = \frac{56}{x}$.

ii. If $-7x = 56$, then $x = \frac{56}{-7}$.

Problem solving

7. Form an equation from the following statements.
I thought of a number and added 5. Three times the result was equal to 27.
Use your equation to find the number that I first thought of.

8. In an amusement park, 0.3 of the children were girls. If there were 80 more boys than girls, how many children were there in the park?

Reflection

When you're backtracking, does the order of the operations matter?

13.4 Building up expressions

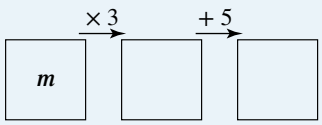
13.4.1 Flowcharts

- Flowcharts can be used to construct algebraic expressions.

WORKED EXAMPLE 7

Complete the flowcharts below to find the output number.

a



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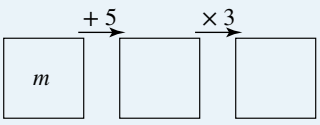


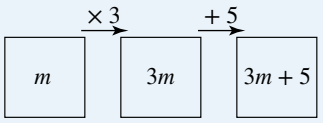
THINK

a $m \times 3 = 3m$
Adding 5 to $3m$ gives $3m + 5$.

b Adding 5 to m results in $m + 5$. To multiply all of $m + 5$ by 3, $m + 5$ will need to be inside brackets.

WRITE

a



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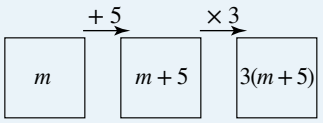


WORKED EXAMPLE 8

Draw a flowchart whose input number is m and whose output number is given by the expressions:

a $2m - 11$ **b** $\frac{m+9}{5}$ **c** $4\left(\frac{m}{3}+2\right)$.

THINK

a 1 The first step is to obtain $2m$; that is, multiply m by 2.

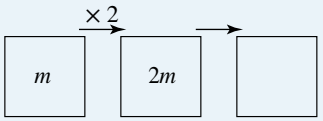
2 This is followed by subtracting 11.

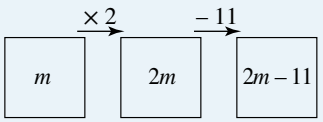
b 1 The expression $m + 9$ is grouped as in a pair of brackets, so we must obtain this part first. Therefore add 9 to m.

2 Then the whole expression is divided by 5.

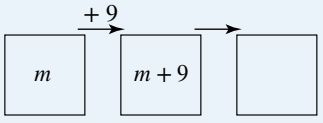
WRITE

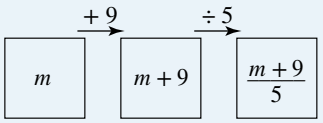
a





b

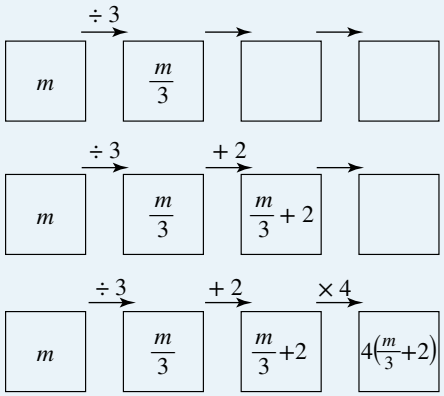




c **1** The pair of brackets indicates we must first work from within the brackets:
a divide m by 3
b then add 2 to this result.

2 Multiply the final result obtained in step 1 by 4.

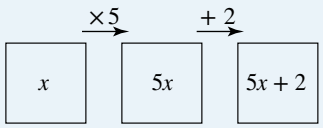
c



- Using flowcharts, we can backtrack to our input number using inverse operations.

WORKED EXAMPLE 9

Complete the flowchart at right by writing in the operations which must be carried out in order to backtrack to x.



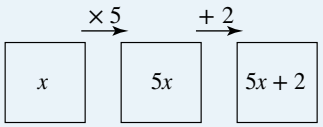
THINK

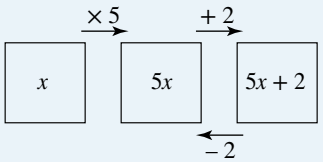
1 Copy the flowchart and look at the operations that have been performed.

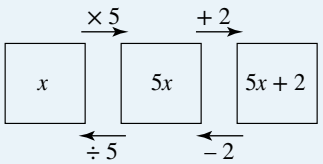
2 The inverse operation of +2 is −2. Show this on the flowchart.

3 The inverse operation of ×5 is ÷5. Show this on the flowchart.

WRITE







learnON RESOURCES — ONLINE ONLY

Watch this eLesson: Backtracking (eles-0010)

Exercise 13.4 Building up expressions

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1, 2, 3, 4, 5, 6, 7, 9	Questions: 1a, c, e, g, 2a, c, e, g, 3a, c, e, g, i, k, 4, 5a, c, 6, 7, 8, 9, 11	Questions: 1d, f, h, 2b, d, f, h, 3b, f, l, 4, 5b, d, 6, 7, 8, 10, 11

Individual pathway interactivity: int-4374

learnON ONLINE ONLY

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note*: Question numbers may vary slightly.

Fluency

1. WE7 Build up an expression by following the instructions on the flowcharts.

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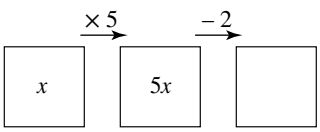

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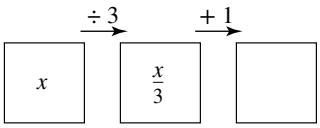

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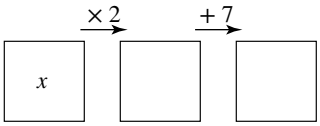

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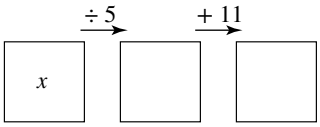

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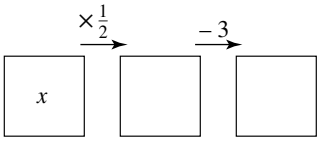

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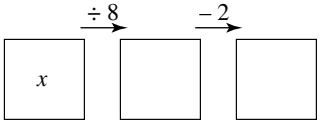

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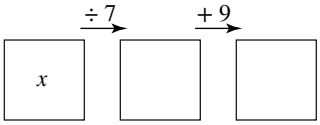

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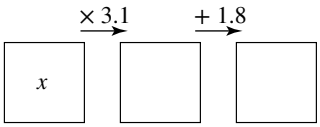
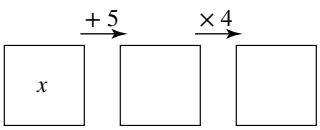


2. Build up an expression by following the instructions on the flowcharts. Use a grouping device, such as a pair of brackets or a vinculum; for example, $2(x + 3)$ or $\frac{x - 5}{4}$.

a.



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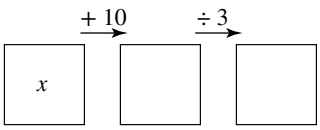

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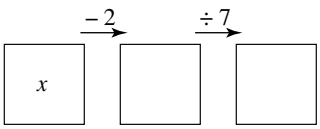

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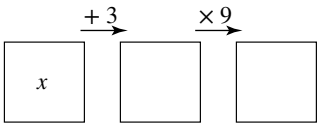

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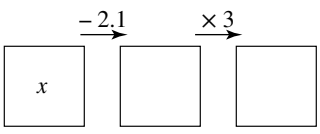

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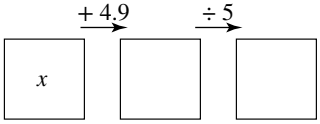

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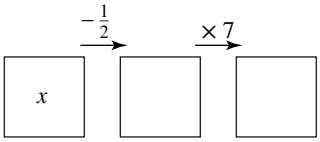

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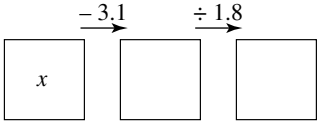

3. Copy and complete the following flowcharts by filling in the missing expressions.

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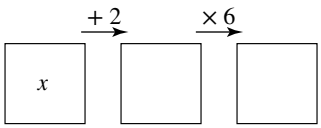

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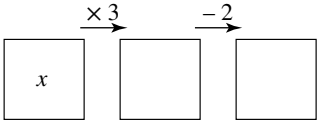

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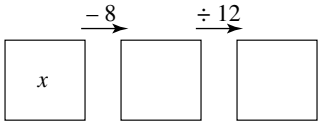

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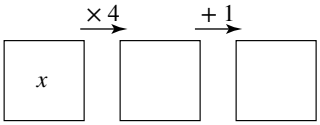

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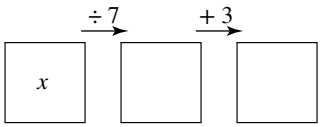

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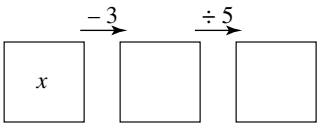

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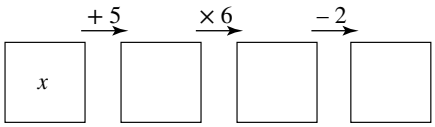

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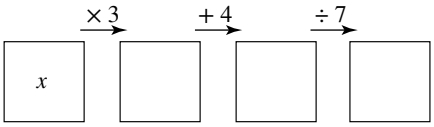

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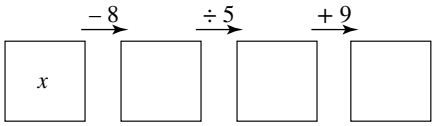
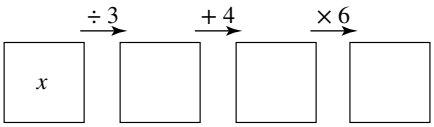
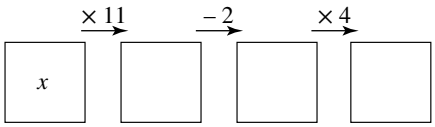


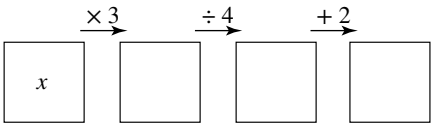
j.



k.



l.



4. **WE8** Draw a flowchart whose input number is x and whose output is given by the following expressions.

a. $5x + 9$

b. $2(x + 1)$

c. $\frac{x}{6} + 4$

d. $\frac{x - 8}{7}$

e. $12(x - 7)$

f. $\frac{x}{5} - 2$

g. $7x - 12$

h. $\frac{x + 6}{3}$

i. $3(x + 7) - 5$

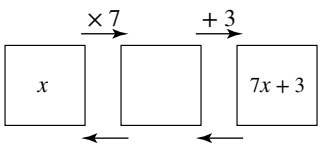
j. $\frac{3x + 7}{2}$

k. $4(3x + 1)$

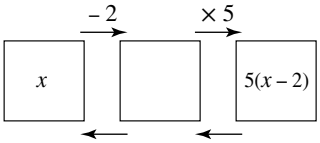
l. $3\left(\frac{x}{5} + 6\right)$

5. **WE9** Complete the following flowcharts by writing in the operations that must be carried out in order to backtrack to x.

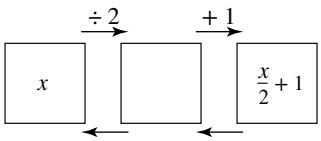
a.



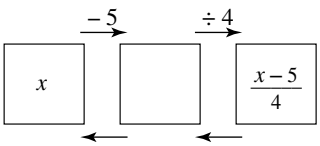
b.



c.



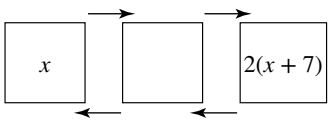
d.



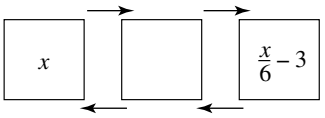
Understanding

6. Add the operations to complete these flowcharts.

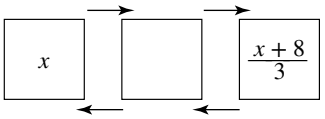
a.



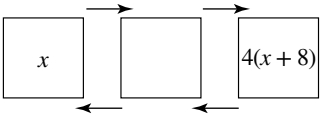
b.



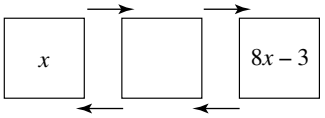
c.



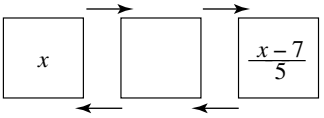
d.



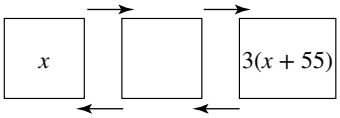
e.



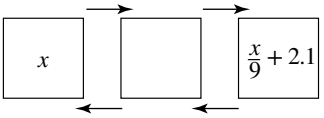
f.



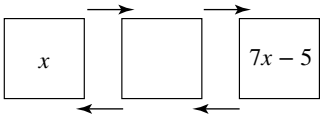
g.



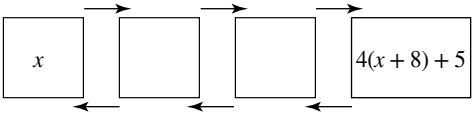
h.



i.



j.



Reasoning

7. I think of a number, and add 8 to it. I multiply the result by 5 and then divide the result by 4. The answer is 30. Build up an expression to write the above in the form of an equation. Show all your working.

8. I think of a number, multiply it by 5 and add 15. The result is 3 less than 4 times the original number. If the original number is n, write down an equation to show the relation. Show all your working.

Problem solving

9. Rachel and Jackson are 7 years apart in age. Jackson is older than Rachel. The sum of their ages is 51. Find Rachel's age.

10. Marcus and Melanie pooled their funds and purchased shares on the Stock Exchange. Melanie invested $350 more than Marcus. Together they invested $2780. Find how much Marcus invested.

11. The sum of two numbers is 32 and their product is 247. What are the two numbers?

Reflection

Do the order of operation laws apply when constructing flowcharts?

CHALLENGE 13.1

Clearly, the equation 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 = 100 is incorrect. By removing some + signs, including – signs and joining some of the digits to make larger numbers, see if you can make the equation true. Remember, only + and – signs are allowed and the equation must equal 100. One example has been included below. How many can you find?

123 + 45 – 67 + 8 – 9 = 100

13.5 Solving equations using backtracking

13.5.1 Using backtracking

- Backtracking an algebraic expression can be used to find the input number.

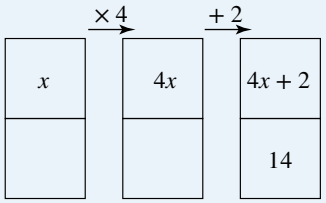
WORKED EXAMPLE 10

Draw a flowchart to represent the following puzzle and then solve it by backtracking. I am thinking of a number. When I multiply it by 4 and then add 2 the answer is 14.

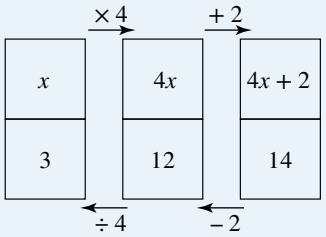
THINK

1 Build an expression using x to represent the number.
Start with x, multiply by 4 and add 2. The output number is 14.

WRITE



2 Backtrack to find the value of x. The inverse operation of $+2$ is -2 $(14 - 2 = 12)$.
The inverse operation of $\times 4$ is $\div 4$ $(12 \div 4 = 3)$.



3 State the answer.

So $x = 3$. The number is 3.

WORKED EXAMPLE 11

Solve the following equations by backtracking.

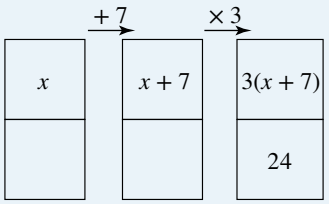
a $3(x + 7) = 24$ **b** $\frac{x}{3} + 5 = 6$

THINK

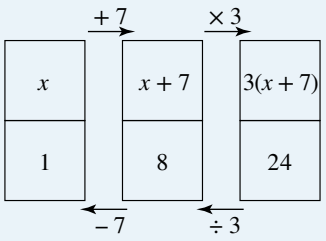
WRITE

a 1 Build the expression on the left-hand side of the equation.
Start with x, add 7 and then multiply by 3.
The output number is 24.

a



2 Backtrack to find x.
The inverse operation of $\times 3$ is $\div 3$ $(24 \div 3 = 8)$.
The inverse operation of $+7$ is -7 $(8 - 7 = 1)$.

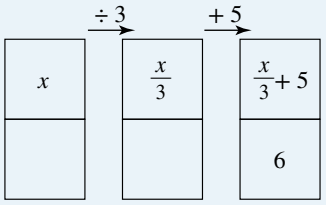


3 State the answer.

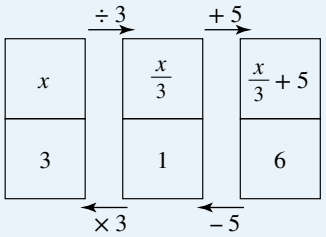
$x = 1$

b 1 Build the expression on the left-hand side of the equation.
Start with x, divide by 3 and then add 5. The output number is 6.

b



2 Backtrack to solve for x.
The inverse operation of $+5$ is -5 $(6 - 5 = 1)$.



3 State the answer.

$x = 3$

WORKED EXAMPLE 12

Simplify and then solve the following equation by backtracking.
$5x + 13 + 2x - 4 = 23$

THINK

1 Simplify by adding the like terms together on the left-hand side of the equation. $5x + 2x = 7x$, $13 - 4 = 9$

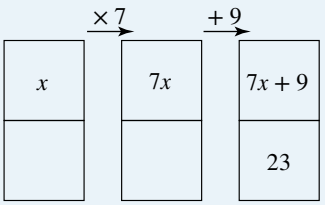
2 Draw a flowchart and build the expression $7x + 9$. Start with x, multiply by 7 and add 9. The output number is 23.

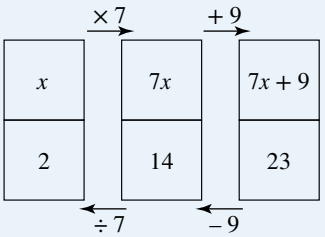
3 Backtrack to solve for x.
The inverse operation of $+9$ is -9 $(23 - 9 = 14)$.
The inverse operation of $\times 7$ is $\div 7$ $(14 \div 7 = 2)$.

4 State the answer.

WRITE

$5x + 13 + 2x - 4 = 23$
$7x + 9 = 23$





$x = 2$

learnon RESOURCES — ONLINE ONLY

Complete this digital doc: SkillSHEET: Combining like terms (doc-6575)

Complete this digital doc: WorkSHEET: Linear equations I (doc-1946)

Exercise 13.5 Solving equations using backtracking

assesson

Individual pathways

PRACTISE
Questions:
1–7, 9, 14, 21

CONSOLIDATE
Questions:
1–3, 4b, c, e, f, h, i, 5, 6 column 1, 7b, d, h, 8–22

MASTER
Questions:
1–3, 4c, f, i, 5, 6 column 2, 7a, g, h, 8–23

Individual pathway interactivity: int-4375

learnon ONLINE ONLY

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note*: Question numbers may vary slightly.

Fluency

1. WE10 Draw a flowchart to represent each of the following puzzles and then solve them by backtracking.
I am thinking of a number.

a. When I multiply it by 2 and then add 7 the answer is 11.
b. When I add 3 to it and then multiply by 5 the answer is 35.
c. When I divide it by 4 and then add 12 the answer is 14.
d. When I add 5 to it and then divide by 3 the answer is 6.
e. When I subtract 7 from it and then multiply by 6 the answer is 18.
f. When I multiply it by 8 and then subtract 11 the answer is 45.
g. When I subtract 4 from it and then divide by 9 the answer is 7.
h. When I divide it by 11 and then subtract 8 the answer is 0.
i. When I add 5 to it and then multiply by 2 the answer is 12.
j. When I multiply it by 6 and then add 4 the answer is 34.
k. When I multiply it by 5 and then subtract 10 the answer is 30.
l. When I subtract 3.1 from it and then multiply by 6 the answer is 13.2.
m. When I divide it by 8 and then subtract 0.26 the answer is 0.99.
n. When I divide it by 3.7 and then add 1.93 the answer is 7.62.
o. When I add $\frac{2}{5}$ to it and then divide by 6 the answer is $\frac{4}{5}$.
p. When I subtract $\frac{3}{4}$ from it and then divide by $\frac{2}{3}$ the answer is $\frac{1}{6}$.

2. Draw a flowchart and use backtracking to find the solution to the following equations.

a. $5x + 7 = 22$ **b.** $9y - 8 = 1$ **c.** $3m - 7 = 1$
d. $4x + 12 = 32$ **e.** $8w + 2 = 26$ **f.** $11m - 1 = 274$
g. $4w - 5.2 = 28$ **h.** $6b - \frac{5}{9} = \frac{2}{3}$ **i.** $2a + \frac{1}{3} = \frac{3}{5}$

3. WE11a Solve the following equations by backtracking.

a. $3(x + 7) = 24$ **b.** $2(x - 7) = 22$ **c.** $5(x - 15) = 40$
d. $11(x + 5) = 99$ **e.** $6(x + 9) = 72$ **f.** $3(x - 11) = 3$
g. $4(w + 5.2) = 26$ **h.** $6(b - \frac{5}{9}) = \frac{2}{3}$ **i.** $8(x - \frac{1}{4}) = \frac{4}{5}$

4. WE11b Solve the following equations by backtracking.

a. $\frac{x}{3} + 5 = 6$ **b.** $\frac{x}{9} - 2 = 3$ **c.** $\frac{x}{4} + 7 = 10$
d. $\frac{x}{2} - 11 = 6$ **e.** $\frac{x}{7} - 5 = 6$ **f.** $\frac{x}{3} + 1 = 1$
g. $\frac{x}{5} + 2.3 = 4.9$ **h.** $\frac{x}{4} - \frac{3}{11} = \frac{4}{11}$ **i.** $\frac{x}{2} + \frac{1}{9} = \frac{4}{9}$

5. Solve the following equations by backtracking.

a. $\frac{x + 4}{3} = 6$ **b.** $\frac{x - 8}{7} = 3$ **c.** $\frac{x - 8}{7} = 10$
d. $\frac{x + 11}{2} = 6$ **e.** $\frac{x - 5}{7} = 0$ **f.** $\frac{x + 100}{17} = 23$
g. $\frac{x + 2.21}{1.4} = 4.9$ **h.** $\frac{x - 1}{5} = \frac{4}{7}$ **i.** $\frac{x + 1}{4} = \frac{3}{8}$

6. Use backtracking to find the solution to the following equations.

a. $3x - 7 = 23$ **b.** $4(x + 7) = 40$ **c.** $\frac{x + 6}{9} = 6$

d. $\frac{x}{5} - 2 = 8$ **e.** $5(x - 3) = 15$ **f.** $\frac{x + 3}{8} = 3$

g. $6(x - 4) = 18$ **h.** $\frac{x}{3} + 10 = 12$ **i.** $\frac{x}{2.1} - 1.7 = 3.6$

j. $4x + \frac{3}{5} = 1$ **k.** $\frac{x + 5}{3} - 3 = 7$ **l.** $3(2x + 5) = 21$

m. $4(x - 2) + 5 = 21$ **n.** $3\left(\frac{x}{2} + 1\right) = 15$ **o.** $2(3x + 4) - 5 = 15$

7. WE12 Simplify and then solve the following equations by backtracking.

a. $2x + 7 + 3x + 5 = 27$ **b.** $x + x + 1 + x + 2 = 18$ **c.** $3x + 9 + x - 4 = 17$

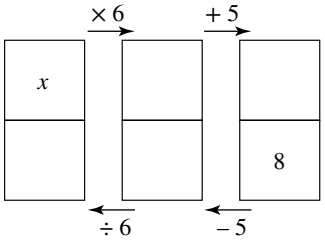
d. $3x + 5x + 2x = 40$ **e.** $6x + 6 - x - 4 = 37$ **f.** $3x - 11 + 4x = 17$

g. $5x - 2x + 5 - x = 19$ **h.** $2x + 3x + 4x + 5 = 7$ **i.** $7x - 4x + 8 - x = 10$

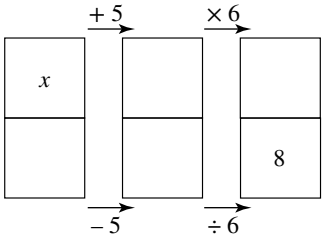
Understanding

8. MC The correct flowchart required to solve the equation $6x + 5 = 8$ is:

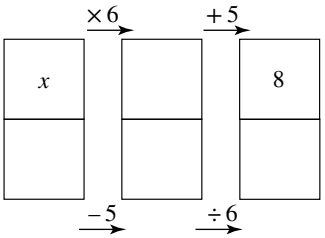
A.



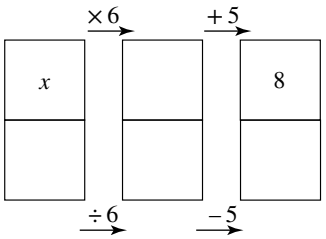
B.



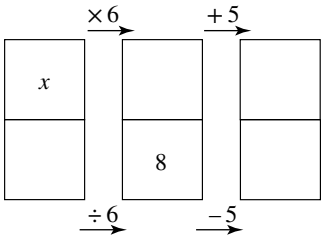
C.



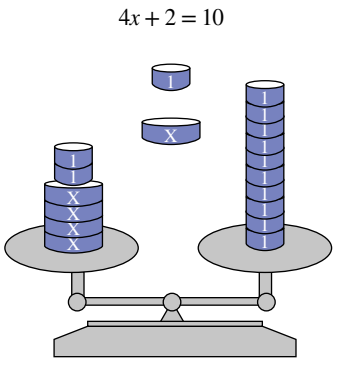
D.



E.



9. Given the following balance, what operations do you need to do to find the value of x? What is the value of x?



10. Chloë wrote the following explanation to solve the equation $3(x - 6) + 5 = 8$ by the backtracking method.

To solve this equation, work backwards and do things in the reverse order. First add 5, *then subtract* 6, *then finally multiply by* 3.

Explain why her instructions are not correct.

Reasoning

11. Imraan is 5 years older than his brother Gareth and the sum of their ages is 31 years. How old is Gareth? (Let x represent Gareth's age.) Show your working.

12. In three basketball games Karina has averaged 12 points each game. In the first game she scored 11 points, in the second she scored 17 points, and in the third game she scored x points.

a. From the given information, what is the average of 11, 17 and x?

b. Write an equation using the answer to part **a**.

c. Solve the equation.

d. How many points did Karina score in the third game?

13. Melanie and Callie went tenpin bowling together. Melanie scored 15 more pins than Callie, and their total score was 207. What did Callie score? Show your working.

14. The sum of three consecutive whole numbers is 51. Find the numbers. (*Hint:* Let the smallest number equal x.)

15. The sum of three consecutive odd numbers is 27. What are the numbers?

16. Three consecutive multiples of 5 add up to 90. What are the 3 numbers?

17. David is 5 years younger than his twin brothers. If the sum of their ages is 52, then how old is David?

18. In the high jump event Chris leapt 12 centimetres higher than Tim, but their two jumps made a total of 3 metres. How high did Chris jump?

19. Daniel and Travis are twins, but their sister Phillipa is 3 years older. If the sum of their three ages is 36, how old are the twins? Show your working.

20. Mel loves playing 'Think of a number' with friends. Here's an example of one of her puzzles.

- Think of a number.
- Double it.
- Add 10.
- Divide by 2.
- Take away the number you first thought of.
- Your answer is … 5!

Let's investigate to see why the answer is always 5, whatever number you first thought of. We can form expressions for each of the steps, using a variable as the starting value.

- Think of a number..... n
- Double it..... $n \times 2 = 2n$
- Add 10..... $2n + 10$
- Divide by 2.....
  $(2n + 10) \div 2 = n + 5$
- Take away the number you first thought of.....
  $n + 5 - n = 5$
- Your answer is..... 5

So, your answer will always be 5, for any starting number.

a. Write expressions for each step in the following, showing that you can determine the answer in each case.

Puzzle 1

- Take the year in which you were born.
- Subtract 500.
- Multiply by 2.
- Add 1000.
- Divide by 2.
- Your answer is … your birth year!

Puzzle 2

- Take your age (in years).
- Add 4.
- Multiply by 10.
- Subtract 10.
- Divide by 5.
- Subtract your age.
- Take away 6.
- Your answer will be … your age!

Puzzle 3

- Think of a number.
- Divide it by 2.
- Add 2.
- Multiply by 4.
- Take away your original number.
- Subtract your original number again.
- Your answer should be … 8.

b. Write some 'Think of a number' puzzles yourself. Try them out on friends. They will marvel at your mystical powers!

Problem solving

21. Using the six consecutive numbers from 4 to 9, complete the magic square at right so that each row, column and diagonal totals 15.

	1	
3		
		2

22. The sum of 3 consecutive odd numbers is 39. What are the 3 numbers?

23. Form an equation for the following statement.
One-third of a certain number is 12.
Solve the equation to find the number.

Reflection

Can you think of some situations where you might need to be able to solve equations?

13.6 Checking solutions

13.6.1 Checking solutions

- It is always a good idea to check if your solution to an equation is correct.
- The solution is the answer obtained when solving an equation.

WORKED EXAMPLE 13

For each of the following, determine whether $x = 7$ is the solution to the equation.

a $\frac{x+5}{3} = 4$ **b** $2x - 8 = 10$

THINK

a 1 Write the equation.

2 Write the left-hand side (LHS) of the equation and substitute $x = 7$.

3 Perform the calculation.

4 Compare this with the right-hand side (RHS) of the equation.

5 Comment on the result.

b 1 Write the equation.

2 Write the left-hand side of the equation and substitute $x = 7$.

3 Perform the calculation.

4 Compare this with the right-hand side of the equation.

5 Comment on the result.

WRITE

a $\frac{x+5}{3} = 4$

If $x = 7$, LHS $= \frac{x+5}{3}$

$= \frac{7+5}{3}$

$= \frac{12}{3}$

$= 4$

RHS $= 4$

The solution is $x = 7$, since LHS = RHS.

b $2x - 8 = 10$

If $x = 7$, LHS $= 2x - 8$

$= 2(7) - 8$

$= 14 - 8$

$= 6$

RHS $= 10$

Therefore, $x = 7$ is not the solution since LHS $\neq$ RHS.

WORKED EXAMPLE 14

For each equation below there is a solution given. Is the solution correct?

a $\frac{x+2}{3} = 2x - 12,\ x = 10$ **b** $3x - 7 = 2x + 3,\ x = 10$

THINK

a 1 Write the equation.

2 Write the left-hand side of the equation and substitute $x = 10$.

3 Perform the calculation.

WRITE

a $\frac{x+2}{3} = 2x - 12$

If $x = 10$,

LHS $= \frac{x+2}{3}$

$= \frac{10+2}{3}$

$= \frac{12}{3}$

$= 4$

THINK	WRITE
4 Write the right-hand side of the equation and substitute $x = 10$.	RHS $= 2x - 12$ $= 2(10) - 12$
5 Perform the calculation.	$= 20 - 12$ $= 8$
6 Comment on the results.	Therefore $x = 10$ is not the solution, since LHS $\neq$ RHS.
b 1 Write the equation.	**b** $3x - 7 = 2x + 3$
2 Write the left-hand side of the equation and substitute $x = 10$.	If $x = 10$, LHS $= 3x - 7$
3 Perform the calculation.	$= 3(10) - 7$ $= 30 - 7$ $= 23$
4 Write the right-hand side of the equation and substitute $x = 10$.	RHS $= 2x + 3$ $= 2(10) + 3$
5 Perform the calculation.	$= 20 + 3$ $= 23$
6 Comment on the results.	The solution is $x = 10$, since LHS = RHS.

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Complete this digital doc: SkillSHEET: Checking solutions by substitution (doc-6576)

Complete this digital doc: WorkSHEET: Linear equations II (doc-1947)

Exercise 13.6 Checking solutions

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–4, 9	Questions: 1–5, 7, 9	Questions: 1a, c, e, g, i, k, 2a, c, e, h, j, 3–8, 10

Individual pathway interactivity int-4376

learnON ONLINE ONLY

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note*: Question numbers may vary slightly.

Fluency

1. WE13 For each of the following determine whether:

a. $x = 3$ is the solution to the equation $x + 2 = 6$

b. $x = 3$ is the solution to the equation $2x - 1 = 5$

c. $x = 5$ is the solution to the equation $2x + 3 = 7$

d. $x = 4$ is the solution to the equation $6x - 6 = 24$

e. $x = 10$ is the solution to the equation $3x + 5 = 20$

f. $x = 5$ is the solution to the equation $4(x - 3) = 8$

g. $x = 7$ is the solution to the equation $3(x - 2) = 25$
h. $x = 8$ is the solution to the equation $5(x + 1) = 90$
i. $x = 12$ is the solution to the equation $6(x - 5) = 42$
j. $x = 81$ is the solution to the equation $3x - 53 = 80$
k. $x = 2.36$ is the solution to the equation $5x - 7 = 4.8$
l. $x = 4.4$ is the solution to the equation $7x - 2.15 = 18.64$.

2. WE14 For each equation below there is a solution given. Is the solution correct?

a. $2x + 1 = 3x - 5$	$x = 6$	**b.** $5x + 1 = 2x - 7$	$x = 8$
c. $3x - 5 = x + 8$	$x = 10$	**d.** $5x = 2x + 12$	$x = 4$
e. $4x = 3x + 8$	$x = 8$	**f.** $3x = x + 20$	$x = 15$
g. $3x - 1.2 = x + 2.9$	$x = 1.9$	**h.** $6x + 1.5 = 2x + 41.5$	$x = 10$
i. $2.4(x + 1) = 9.6$	$x = 3$	**j.** $1.2(x + 1.65) = 0.2(x + 9.85)$	$x = 0.45$

Understanding

3. Complete the table below to find the value of $2x + 3$ when $x = 0, 1, 2, 3, 4$.

x	0	1	2	3	4
$2x + 3$					

a. For what value of x does $2x + 3 = 11$?
b. What is the solution (that is, the value of x) for $2x + 3 = 11$?
c. What is the solution (that is, the value of x) for $2x + 3 = 5$?

4. Complete the table below to find the value of $5(x - 2)$ when $x = 2, 3, 4, 5, 6$.

x	2	3	4	5	6
$5(x - 2)$					

a. What is the solution (that is, the value of x) to $5(x - 2) = 10$?
b. What is the solution (that is, the value of x) to $5(x - 2) = 20$?
c. What do you guess is the solution (that is, the value of x) to $5(x - 2) = 30$? Check your guess.

5. a. Copy and complete the table below.

x	$2x + 1$	$3x - 5$
3	7	
4		
5		10
6		
7		

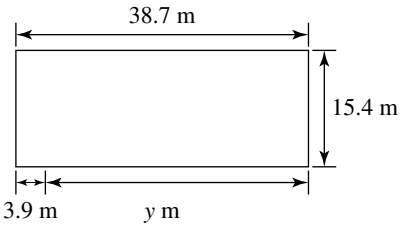
b. What is the solution to $2x + 1 = 3x - 5$?

6. a. Copy and complete the table below.

x	$\frac{x+3}{2}$	$2x-6$
3	3	
5		
7		8
9		
11		

b. What is the solution to $\frac{x+3}{2} = 2x - 6$?

Reasoning



7. Consider the diagram shown.

a. Do you need to know both dimensions to be able to calculate the value of y?

b. Calculate the value of y.

c. Explain how you would check the solution.

8. Carol is making a quilt for her granddaughter. The quilt pattern requires that $\frac{1}{7}$ of the quilt is made of a pink fabric. Carol has $0.5\,m^2$ of pink fabric. She intends to use all the pink material in the quilt. How much more fabric does she need for the entire quilt?

a. Write an equation for this problem.

b. Calculate the value of the unknown.

c. Check the solution.

Problem solving

9. $125 is shared between James and Alison, but Alison is to receive $19 more than James. Set up an equation to describe this situation. How much money do James and Alison each receive?

10. Six consecutive numbers add to 393. Find the value of the smallest of these numbers.

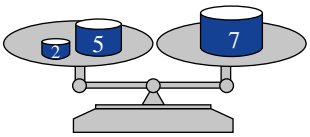
Reflection

Is it possible for an equation to have a decimal solution?

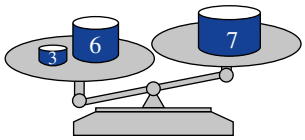
13.7 Keeping equations balanced

13.7.1 Keeping equations balanced

- Equations are mathematical statements that show two equal expressions; that is, the left-hand side and the right-hand side of an equation *are equal*. For example, $2 + 5 = 7$.
- A **pan balance scale** can be used to show whether things have equal mass. If the contents of the two pans have the same mass, the pans balance at the same level and the arms of the scale are parallel to the ground. If the contents of the two pans are different in mass, the arms tip so that the side with the greater mass is lower.

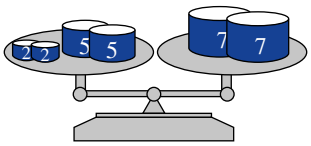


Equal



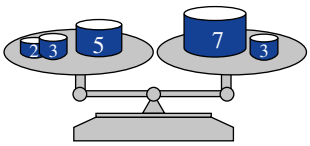
Not equal

- Any change that is made to one side must also be made to the other side if balance is to be maintained.



Equal

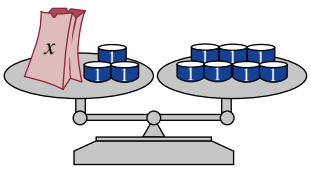
If we double what is on the left-hand side, we must double the amount on the right-hand side.



Equal

If we add 3 to the left-hand side, we must also add 3 to the right-hand side.

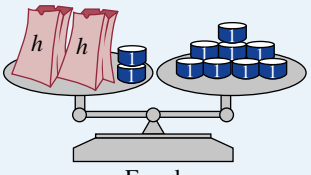
- Pronumerals in expressions can be represented on a pan balance scale using bags. Each bag contains a particular number of weights that is equal to the value of the variable shown on the front of the bag. For the example, the scale on the right shows the equation $x + 3 = 7$.
- Making changes to both sides of the scale allows you to work out how many weights are in the bag. This is the same as working out the value of the pronumeral. On the scale above, if we remove 3 weights from each pan, the bag with x weights in it weighs the same as 4 weights, so $x = 4$.



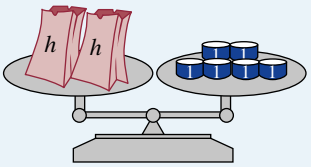
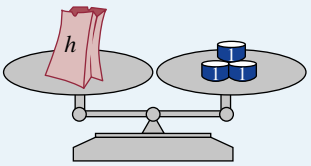
Equal

WORKED EXAMPLE 15

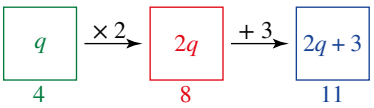
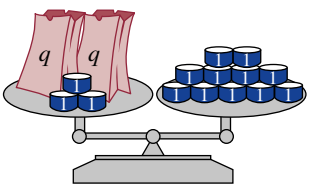
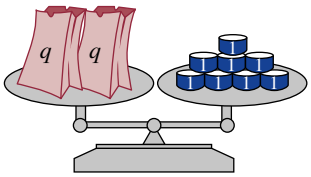
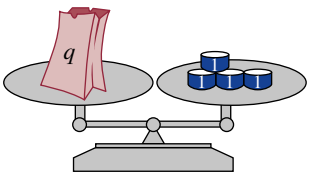
For the following pan balance scale:
a write the equation represented by the scale
b calculate the value of the variable.



Equal

THINK	WRITE
a On the LHS, there are 2 bags with h weights in each, and 2 weights outside the bags. On the RHS, there are 8 weights. Write this as an equation.	**a** $2h + 2 = 8$
b 1 Remove 2 weights from both sides so that the balance will be maintained (-2). This leaves the 2 bags on the LHS and 6 weights on the RHS.	**b**  Equal $2h = 6$
2 Remove half of the contents of each pan $(\div 2)$.	 Equal $h = 3$ Each bag contains 3 weights. Therefore, $h = 3$.

- Both backtracking and the balance method of solving equations use inverse operations to find the value of a pronumeral. For example, the equation $2q + 3 = 11$ can be solved as follows.

Backtracking	Balance method
 The equivalent equations made when backtracking are: $2q + 3 = 11$ $2q = 8$ $2q = 4$	 $2q + 3 = 11$ Equal Remove three counters from both sides (-3).  $2q = 8$ Equal
	Halve the contents of each pan $(\div 2)$.  $q = 4$ Equal

- In both methods:
  – the operations and the order in which they are performed to solve the equation are the same
  – the equivalent equations created on the way to the solution are the same
  – the aim is to get the pronumeral by itself on one side of the equation
  – the last operation performed on the pronumeral when building the equation is the first operation undone.

WORKED EXAMPLE 16

Use inverse operations to solve the following equations.

a $2y + 3 = 11$ **b** $\frac{h}{5} + 1 = 3$ **c** $2(k - 4) = 4$

THINK	WRITE
a 1 The last operation performed when building $2y + 3 = 11$ was +3. Subtract 3 from both sides and then simplify.	**a** $2y + 3 = 11$ $2y + 3 - 3 = 11 - 3$ $2y = 8$
2 To get y by itself, divide both sides by 2 and then simplify.	$\frac{2^1y}{2^1} = \frac{8^4}{2^1}$ $y = 4$
b 1 The last operation performed when building $\frac{h}{5} + 1 = 3$ was +1. Subtract 1 from both sides and then simplify.	**b** $\frac{h}{5} + 1 = 3$ $\frac{h}{5} + 1 - 1 = 3 - 1$ $\frac{h}{5} = 2$
2 To get h by itself, multiply both sides by 5 and then simplify.	$\frac{h}{5^1} \times 5^1 = 2 \times 5$ $\frac{h}{5} = 10$
c 1 The last operation performed when building $2(k - 4) = 4$ was ×2. Divide both sides by 2 and then simplify.	**c** $2(k - 4) = 4$ $\frac{2^1(k - 4)}{2^1} = \frac{4^2}{2^1}$ $k - 4 = 2$
2 To get k by itself, add 4 to both sides and then simplify.	$k - 4 + 4 = 2 + 4$ $k = 6$

13.7.2 Equations with rational number solutions

- When solving equations, sometimes the solution is a rational number.

WORKED EXAMPLE 17

Solve the following equations.

a $5d = 4$ **b** $3c + 1 = 6$

THINK	WRITE
a To get d by itself, divide both sides by 5 and then simplify.	**a** $5d = 4$ $\frac{5^1d}{5^1} = \frac{4}{5}$ $d = \frac{4}{5}$

b 1 The last operation performed when building $3c + 1 = 6$ was +1. Subtract 1 from both sides and simplify.

2 To get c by itself, divide both sides by 3 and then simplify.

b
$$3c + 1 = 6$$
$$3c + 1 - 1 = 6 - 1$$
$$3c = 5$$
$$\frac{\cancel{3}^{1}c}{\cancel{3}^{1}} = \frac{5}{3}$$
$$c = \frac{5}{3}$$
$$= 1\frac{2}{3}$$

13.7.3 Negative integers

- When the pronumeral is being subtracted, it is difficult to represent the equation using backtracking or the balance method.
- By adding the pronumeral to both sides, the equation can be solved as normal.

WORKED EXAMPLE 18

Solve the following equations.
a $5 - v = 2$ **b** $17 - 2a = 11$

THINK

a 1 Undo subtracting v by adding v and then simplify.

2 To get v by itself, subtract 2 from both sides and simplify.

b 1 Undo subtracting $2a$ by adding $2a$ and then simplify.

2 The last operation performed when building $17 = 11 + 2a$ was +11. Subtract 11 from both sides and simplify.

3 To get a by itself, divide both sides by 2 and then simplify.

WRITE

a
$$5 - v = 2$$
$$5 - v + v = 2 + v$$
$$5 = 2 + v$$
$$5 - 2 = 2 + v - 2$$
$$3 = v$$
$$v = 3$$

b
$$17 - 2a = 11$$
$$17 - 2a + 2a = 11 + 2a$$
$$17 = 11 + 2a$$
$$17 - 11 = 11 + 2a - 11$$
$$6 = 2a$$
$$\frac{\cancel{6}^{3}}{\cancel{2}^{1}} = \frac{\cancel{2}^{1}a}{\cancel{2}^{1}}$$
$$3 = a$$

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Try out this interactivity: Balancing equations (int-0077)

Exercise 13.7 Keeping equations balanced

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–9	Questions: 1–10	Questions: 1–8, 10, 11, 12

Individual pathway interactivity: int-4377

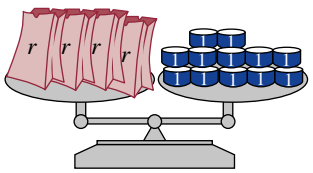
learnON ONLINE ONLY

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note*: Question numbers may vary slightly.

Fluency

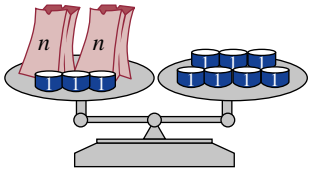
1. WE15 For each of the following pan balance scales:
- **i.** write the equation represented by the scale
- **ii.** calculate the value of the pronumeral.

a.



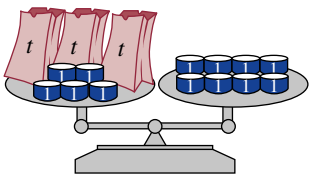
Equal

b.



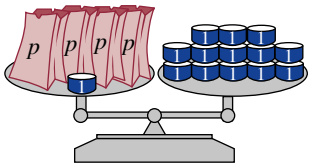
Equal

c.



Equal

d.



Equal

2. WE16 Use inverse operations to solve the following.

a. $3g + 7 = 10$ **b.** $4m - 6 = 6$ **c.** $\frac{q}{3} + 8 = 11$

d. $2(n - 5) = 8$ **e.** $\frac{g - 2}{4} = 3$ **f.** $4(y + 1) = 16$

3. WE17 Solve the following equations.

a. $3h = 7$ **b.** $2k = 5$ **c.** $2w + 1 = 8$

d. $3t - 4 = 9$ **e.** $6h - 3 = 10$ **f.** $3l + 4 = 8$

4. WE18 Solve the following equations.

a. $6 - m = 2$ **b.** $4 - d = 1$ **c.** $12 - 3v = 6$

d. $13 - 2s = 7$ **e.** $19 - 3g = 4$ **f.** $30 - 5k = 20$

Understanding

5. Solve the following equations.

a. $2(x + 3) = 11$ **b.** $3(5 - y) = 12$ **c.** $p + 3 = 6\frac{1}{4}$ **d.** $5(8 - 2h) = 15$

6. The formula $C = \frac{5(F - 32)}{9}$ is used to convert degrees Fahrenheit to degrees Celsius. Use the formula to find:

a. 45° Fahrenheit in degrees Celsius **b.** 45° Celsius in degrees Fahrenheit.

Reasoning

7. A taxi company charges a $2.70 flag fall. An additional $1.87 per km is charged for the journey.
   a. Calculate the cost of a 4.6-km journey.
   b. If the journey cost $111.16, how far was the journey? Show your working.

8. While shopping for music online, Olivia found an album she liked. She could purchase the entire album for $17.95 or songs from the album for $1.69 each. How many songs would Olivia want to buy for it to be cheaper to buy the entire album? Explain how you calculated the answer.

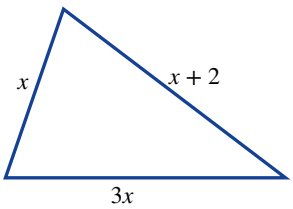
Problem solving

9. A class of 25 students has 7 more boys than girls. How many boys are there?

10. Form an equation to represent the following statement. When 12 is subtracted from 7 times a number, the result is 9. Solve your equation to find the number.

11. A number, x, becomes the fifth member of the set of numbers $\{3, 7, 9, 13\}$ to make the mean of the five numbers equal to their median. Find the possible values for x.

12. Given that the perimeter of the following triangle is 22 cm, find the value of x.



Reflection

How will you decide the order to undo the operations?

CHALLENGE 13.2

The scales show that two bricks are equal to 2 kilograms and half a brick. How much does each brick weigh?

13.8 Review

13.8.1 Review questions

Fluency

1. Solve these equations by inspection.

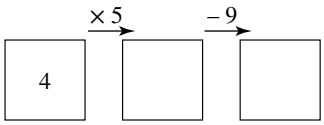
a. $m + 7 = 12$ **b.** $5h = 30$ **c.** $s - 12 = 7$ **d.** $\frac{d}{5} = 4$

2. Use guess, check and improve to find two numbers whose sum and product are given.

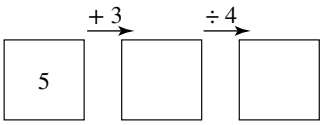
a. Sum = 83, product = 1632 **b.** Sum = 86, product = 1593

3. Complete these flowcharts to find the output number.

a.



b.



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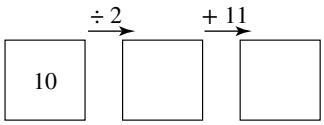

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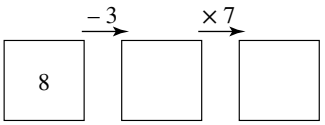
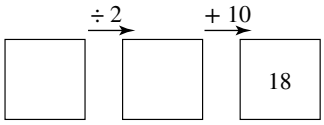


4. Use backtracking and inverse operations to find the input number in each of these flowcharts.

a.



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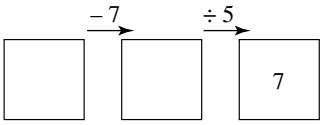

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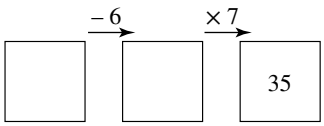

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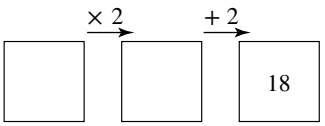
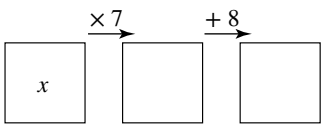


5. Build up an expression by following the instructions on the flowchart.

a.



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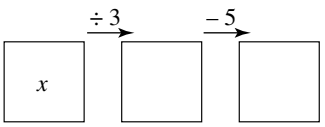

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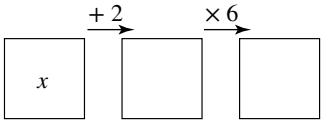

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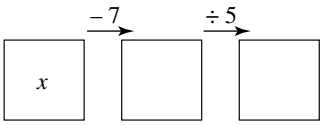

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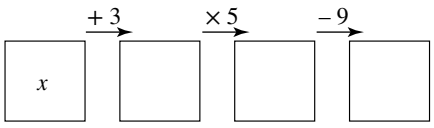

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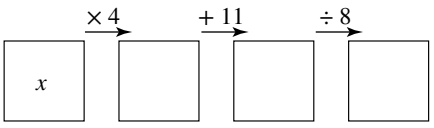
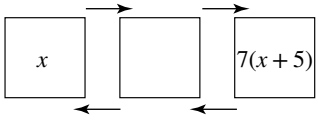


6. Draw the flowchart whose input is x and whose output is given by the expression.

a. $5(x + 7)$ **b.** $\frac{x}{4} - 3$ **c.** $6x - 14$ **d.** $\frac{x + 2}{5}$

7. Complete these flowcharts by writing in the operations which must be carried out in order to backtrack to x.

a.



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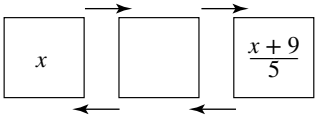

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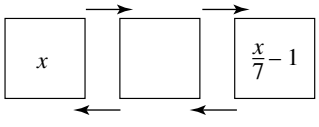
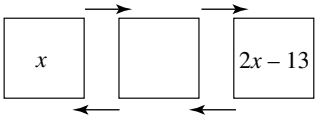


d.



8. Draw a flowchart and use backtracking to find the solution to the following equations.

a. $7x + 6 = 20$ **b.** $9(y - 8) = 18$ **c.** $\frac{m}{5} - 3 = 9$ **d.** $\frac{s + 7}{5} = 5$

9. Use backtracking to find the solution to these equations.

a. $3(d + 1) = 15$ **b.** $\frac{t}{4} - 11 = 14$ **c.** $6d - 3 = 15$ **d.** $\frac{a + 6}{4} = 3$

10. Simplify the expression and then solve the equation for each of the following.

a. $7v + 3 + 3v + 4 = 37$ **b.** $6c + 15 - 5c - 8 = 19$

11. For each equation below there is a solution given. Is the solution correct?

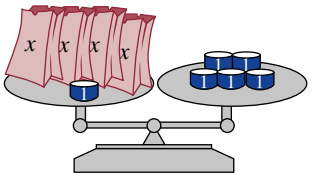
a. $5x - 7 = 2x + 2 \quad x = 3$ **b.** $\frac{x + 9}{2} = 2x - 7 \quad x = 5$

12. For each of the following pan balance scales:

i. write the equation represented by the scale

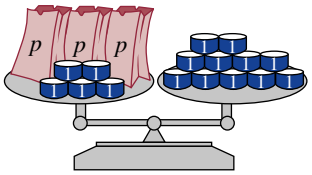
ii. calculate the value of the pronumeral.

a.



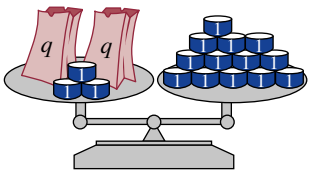
Equal

b.



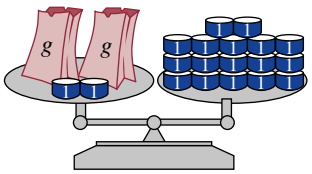
Equal

c.



Equal

d.



Equal

13. Use inverse operations to solve the following.

a. $2v + 1 = 7$ **b.** $\frac{x}{2} + 3 = 4$ **c.** $4w - 3 = 9$ **d.** $2(g + 1) = 10$

14. Solve the following equations. (Make sure you check your solutions.)

a. $4k + 3 = 10$ **b.** $2p - 3 = 8$ **c.** $3q + 6 = 7$ **d.** $5t + 1 = 3$

15. Solve the following equations. (Make sure you check your solutions.)

a. $3 - x = 1$ **b.** $7 - 2p = 3$ **c.** $10 - 4r = 2$ **d.** $13 - 3z = 7$

Problem solving

16. Sophie and Jackie each have a collection of football cards. Jackie has 5 more cards than Sophie, and together they have 67 cards. By writing and solving an equation, find out how many cards Sophie owns.

17. Andreas has completed 2 more pieces of homework than Richard who submitted x pieces of homework for the semester. If the total number of pieces of homework submitted by the two boys is 12, how many pieces of homework did Andreas submit?

18. Keith is 6 years younger than his twin brothers. If the sum of their ages is 48, how old is Keith?

19. Margaret bought six 2-hour tickets and four daily tickets for $28. If the cost of one daily ticket is $4, write an equation and solve it to find the cost of a 2-hour ticket.

20. Jetski hire is $30 deposit and $10 per hour.

a. How much will it cost to hire a jetski for:

i. 1 hour? **ii.** 2 hours? **iii.** 3 hours?

b. Write a rule that could be used to calculate the cost of hiring a jetski for h hours.

c. Use the rule to calculate the cost of hiring a jetski for 8 hours.

d. You have $85 to spend. Write an equation to help you work out for how many hours you could hire a jetski. Solve this equation.

e. Work out how much money (if any) you would have left over after you pay the hire charge, if you hire the jet ski for 5 hours.

f. After a great day on the water, you return the jetski with no damage. Would you have enough money for a hamburger on the way home?

21. Suppose it cost $30 for an adult and $15 for a child to enter the Sydney Royal Easter Show.

a. How much does it cost two adults and three children to enter?

b. How much does it cost one adult and five children to enter?

c. What is the maximum number of people who could get into the Show for $100?

d. What combination of adults and children could get in for $300?

22. Judy is watching her daughter play in the park. She notices that some children are riding tricycles and some are riding bicycles. Altogether, 19 children are on cycles. She counts that there are 43 wheels on the cycles. How many children are on tricycles and how many are on bicycles?

23. Two angles in a triangle have the same measure. The sum of the measures of these angles is the measure of the third angle. Find the measure of the angles. Is there something special about the triangle?

24. A stage screen is 4 times as long as it is wide. If it were 5 metres wider and 4 metres shorter it would be a square. What are the dimensions of the stage screen?

25. The largest angle in a triangle is 65 degrees more than the smallest angle. The third angle is 10 degrees more than the smallest angle. Find the size of the smallest angle.

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Try out this interactivity: Sudoku: (int-3173)

Complete this digital doc: Concept map: Topic 13 (doc-10741)

Language

It is important to learn and be able to use correct mathematical language in order to communicate effectively. Create a summary of the topic using the key terms below. You can present your summary in writing or using a concept map, a poster or technology.

backtracking
equation
flowchart
input number
inverse
inverse operation
operations
output number
pan balance scale
pronumeral
simplify
solution
solve

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Link to assessON for questions to test your readiness **FOR** learning, your progress **AS** you learn and your levels **OF** achievement.

assessON provides sets of questions for every topic in your course, as well as giving instant feedback and worked solutions to help improve your mathematical skills.

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Investigation | Rich task

Equations at the Olympic Games

The Olympic games is a sporting event held every four years for competing nations throughout the world. You may recall that Sydney was the host for the year 2000 Olympic games. A feature of the sporting events is that records continue to be broken — competitors run faster, lift heavier weights and so on. The spirit to strive to become better urges competitors to train harder, hoping to become a record holder.

This activity looks at running times for the men's and women's 100-metre running event. The running times can be approximated using the following equations:

Men's 100-metre event

$$t = -0.0094y + 28.73$$

Women's 100-metre event

$$t = -0.0173y + 45.31$$

where t represents the running time (in seconds) and y represents the year.

For example, the men's time in 1996 can be approximated by substituting 1996 into the equation as shown.

$$\begin{aligned} t &= -0.0094y + 28.73 \\ &= -0.0094 \times 1996 + 28.73 \\ t &= 9.9676 \text{ seconds} \end{aligned}$$

1. Use the equations provided to calculate the men's and women's running times for the 100-metre event in the year:

- 1928
- 1968
- 1988
- 2008.

The table below shows the winners and their running times for the 100-metre event final at four Olympic games.

Year	Men's winner	Time taken (seconds)	Women's winner	Time taken (seconds)
1928	Percy Williams	10.80	Elizabeth Robinson	12.20
1968	James Ray Hines	9.95	Wyomia Tyus	11.08
1988	Carl Lewis	9.92	Florence Griffith-Joyner	10.54
2008	Usain Bolt	9.69	Shelly-Ann Fraser	10.78

2. How well do these actual results compare with the times you calculated using the equations? What does this say about the equations?

3. Predict the times for both men and women at the 2016 Olympic games.

When making predictions about the future, we must remember that these predictions are based on the assumption that the patterns we observe now will continue into the future.

Your answers in question 1 show you that men run faster times than women in the 100-metre event. However, closer inspection of the times shows that women are making greater improvements in their times over the years.

4. If the running times continue to follow these patterns in future Olympic games, is it possible that women's times will become equal to men's times? Discuss your answer.

5. Why do you think the running times are coming down? Will they follow this pattern forever?

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Complete this digital doc: Code puzzle: Joke time (doc-14333)

Answers

Topic 13 Linear equations

Exercise 13.2 Solving equations using trial and error

1. a. $x + 7 = 11$ **b.** $x + 3 = 5$ **c.** $x + 12 = 12$ **d.** $x + 5 = 56$ **e.** $x - 7 = 1$

f. $x - 11 = 11$ **g.** $x - 4 = 7$ **h.** $x - 8 = 0$ **i.** $2x = 12$ **j.** $6x = 30$

k. $5x = 30$ **l.** $6x = 12$ **m.** $\frac{x}{7} = 1$ **n.** $\frac{x}{3} = 100$ **o.** $\frac{x}{5} = 2$

p. $\frac{x}{7} = 0$ **q.** $15 - x = 2$ **r.** $52 - x = 8$ **s.** $\frac{21}{x} = 7$ **t.** $x^2 = 100$

2. a. $x = 11$ **b.** $y = 9$ **c.** $m = 5$ **d.** $m = 30$ **e.** $w = 1$

f. $m = 274$ **g.** $w = 7$ **h.** $k = 0$ **i.** $b = 7$ **j.** $w = 18$

k. $k = 4$ **l.** $h = 25$ **m.** $b = 8.8$ **n.** $c = 4.2$ **o.** $x = 2.8$

3. $x = 6$

4. a. $x = 3$ **b.** $x = 14$ **c.** $x = 10$ **d.** $x = 9$ **e.** $x = 3$

f. $x = 20$ or $x = 0$ **g.** $x = 5$ or $x = 0$ **h.** $x = 6$ **i.** $x = 4$

5. 7 and 14

6. a. 11 and 15 **b.** 9 and 18 **c.** 7 and 47 **d.** 8 and 37

e. 42 and 136 **f.** 51 and 53 **g.** 42 and 111 **h.** 97 and 145

i. 2.2 and 3.9 **j.** 2.9 and 5.1 **k.** 347 and 631 **l.** 11.9 and 23.1

7.

x	$x^2 + 4$	$4x + 1$
0	4	1
1	5	5
2	8	9
3	13	13
4	20	17

$x = 1$ and $x = 3$ are both solutions.

8. 10 **9.** 24 years old **10.** 5 metres, 15 metres **11.** 7

12. Angus will be 27 and his father will be 81. **13.** 6 and 10

Exercise 13.3 Using inverse operations

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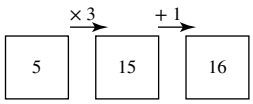

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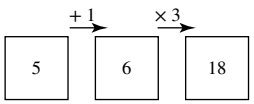

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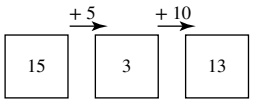

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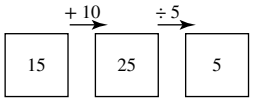

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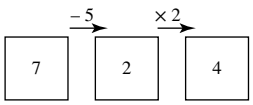

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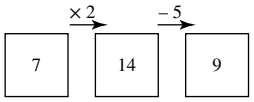

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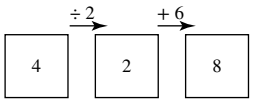

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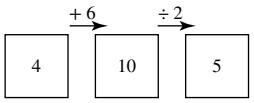

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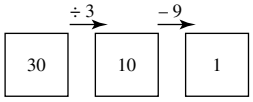

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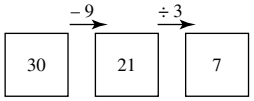

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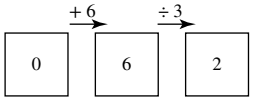

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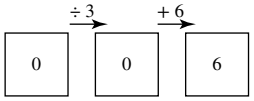

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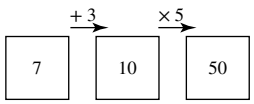

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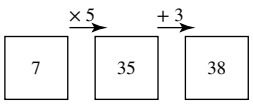

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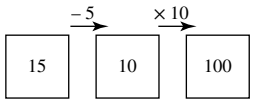
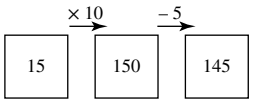


ii.



2. a. 1 **b.** 7 **c.** 15 **d.** 14 **e.** 5 **f.** 10 **g.** 4 **h.** 9
i. 5 **j.** 0 **k.** 14.4522 **l.** 4 **m.** 3 **n.** 17 **o.** 2.59 **p.** 532

3. a. Each pair of flowcharts has the same operations, but in the reverse order.

b. The results are quite different.

4. a. Adding and subtracting are inverse operations.

b. Multiplying and dividing are inverse operations.

5. a. $x + 3 = 7$
$x + 3 - 3 = 7 - 3$
$x + 0 = 4$
$x = 4$
Addition and subtraction are inverse operations because $+ a - a = 0$ and $-a + a = 0$; they undo each other.

b. $2x \div 2 = -8 \div 2$
$x = -4$
Multiplication and division are inverse operations because $a \div a = 1$ and $\frac{1}{a} \times a = 1$; they undo each other.

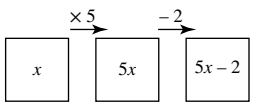
6. a. Both **i** and **ii** are correct. **i** should be used because the unknown x is isolated.

b. Both **i** and **ii** are correct. **ii** should be used because the unknown x is isolated.

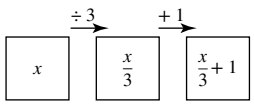
5. $3(x + 5) = 27$; 4 **6.** 200

Exercise 13.4 Building up expressions

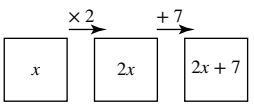
1. a.



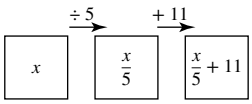
b.



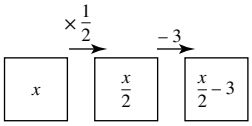
c.



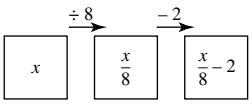
d.



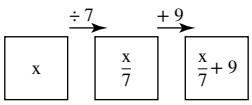
e.



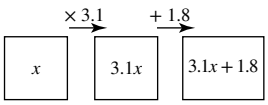
f.



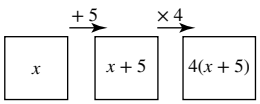
g.



h.



2. a.



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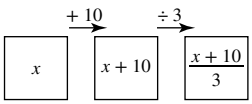
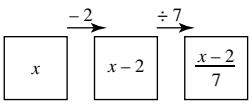
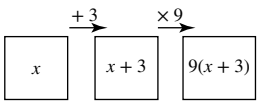


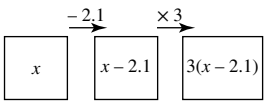
c.



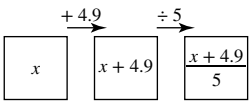
d.



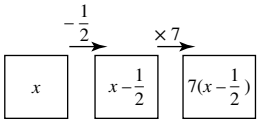
e.



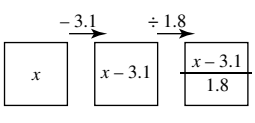
f.



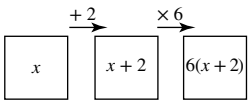
g.



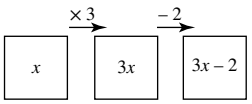
h.



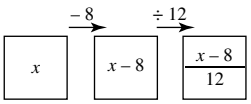
3. a.



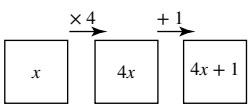
b.



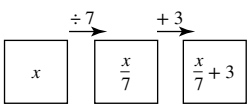
c.



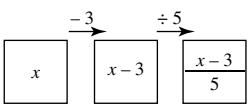
d.



e.



f.



g. $x \xrightarrow{+5} x+5 \xrightarrow{\times 6} 6(x+5) \xrightarrow{-2} 6(x+5)-2$

h. $x \xrightarrow{\times 3} 3x \xrightarrow{+4} 3x+4 \xrightarrow{\div 7} \frac{3x+4}{7}$

i. $x \xrightarrow{-8} x-8 \xrightarrow{\div 5} \frac{x-8}{5} \xrightarrow{+9} \frac{x-8}{5}+9$

j. $x \xrightarrow{\div 3} \frac{x}{3} \xrightarrow{+4} \frac{x}{3}+4 \xrightarrow{\times 6} 6(\frac{x}{3}+4)$

k. $x \xrightarrow{\times 11} 11x \xrightarrow{-2} 11x-2 \xrightarrow{\times 4} 4(11x-2)$

l. $x \xrightarrow{\times 3} 3x \xrightarrow{\div 4} \frac{3x}{4} \xrightarrow{+2} \frac{3x}{4}+2$

4. a. $x \xrightarrow{\times 5} 5x \xrightarrow{+9} 5x+9$

b. $x \xrightarrow{+1} x+1 \xrightarrow{\times 2} 2(x+1)$

c. $x \xrightarrow{\div 6} \frac{x}{6} \xrightarrow{+4} \frac{x}{6}+4$

d. $x \xrightarrow{-8} x-8 \xrightarrow{\div 7} \frac{x-8}{7}$

e. $x \xrightarrow{-7} x-7 \xrightarrow{\times 12} 12(x-7)$

f. $x \xrightarrow{\div 5} \frac{x}{5} \xrightarrow{-2} \frac{x}{5}-2$

g. $x \xrightarrow{\times 7} 7x \xrightarrow{-12} 7x-12$

h. $x \xrightarrow{+6} x+6 \xrightarrow{\times 3} \frac{x+6}{3}$

i. $x \xrightarrow{+7} x+7 \xrightarrow{\times 3} 3(x+7) \xrightarrow{-5} 3(x+7)-5$

j. $x \xrightarrow{\times 3} 3x \xrightarrow{+7} 3x+7 \xrightarrow{\div 2} \frac{3x+7}{2}$

k. $x \xrightarrow{\times 3} 3x \xrightarrow{+1} 3x+1 \xrightarrow{\times 4} 4(3x+1)$

l. $x \xrightarrow{\div 5} \frac{x}{5} \xrightarrow{+6} \frac{x}{5}+6 \xrightarrow{\times 3} 3(\frac{x}{5}+6)$

5. a. $x \xrightarrow{\times 7} 7x \xrightarrow{+3} 7x+3$ (reverse: $\div 7$, -3)

b. $x \xrightarrow{-2} x-2 \xrightarrow{\times 5} 5(x-2)$ (reverse: $+2$, $\div 5$)

c. $x \xrightarrow{\div 2} \frac{x}{2} \xrightarrow{+1} \frac{x}{2}+1$ (reverse: $\times 2$, -1)

d. $x \xrightarrow{-5} x-5 \xrightarrow{\div 4} \frac{x-5}{4}$ (reverse: $+5$, $\times 4$)

6. a. $x \xrightarrow{+7} x+7 \xrightarrow{\times 2} 2(x+7)$ (reverse: -7, $\div 2$)

b. $x \xrightarrow{\div 6} \frac{x}{6} \xrightarrow{-3} \frac{x}{6}-3$ (reverse: $\times 6$, $+3$)

c. $x \xrightarrow{+8} x+8 \xrightarrow{\div 3} \frac{x+8}{3}$ (reverse: -8, $\times 3$)

d. $x \xrightarrow{+8} x+8 \xrightarrow{\times 4} 4(x+8)$ (reverse: -8, $\div 4$)

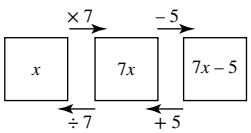
e. $x \xrightarrow{\times 8} 8x \xrightarrow{-3} 8x-3$ (reverse: $\div 8$, $+3$)

f. $x \xrightarrow{-7} x-7 \xrightarrow{\div 5} \frac{x-7}{5}$ (reverse: $+7$, $\times 5$)

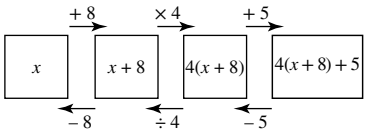
g. $x \xrightarrow{+55} x+55 \xrightarrow{\times 3} 3(x+55)$ (reverse: -55, $\div 3$)

h. $x \xrightarrow{\div 9} \frac{x}{9} \xrightarrow{+2.1} \frac{x}{9}+2.1$ (reverse: $\times 9$, -2.1)

i.



j.



7. $\frac{5(x+8)}{4} = 30$ **8.** $5n + 15 = 4n - 3$ **9.** 22 years old **10.** $1215

11. The two numbers are 19 and 13.

Challenge 13.1

Teacher to check. Some examples are:
$12 + 3 - 4 + 5 + 67 + 8 + 9 = 100$
$123 + 4 - 5 + 67 - 89 = 100$
$123 - 45 - 67 + 89 = 100$.

Exercise 13.5 Solving equations using backtracking

1. a. 2 **b.** 4 **c.** 8 **d.** 13 **e.** 10 **f.** 7 **g.** 67 **h.** 88
i. 1 **j.** 5 **k.** 8 **l.** 5.3 **m.** 10 **n.** 21.053 **o.** $4\frac{2}{5}$ **p.** $\frac{31}{36}$

2. a. $x = 3$ **b.** $y = 1$ **c.** $m = 6$ **d.** $x = 5$ **e.** $w = 3$
f. $m = 25$ **g.** $w = 5.7$ **h.** $b = \frac{11}{54}$ **i.** $a = \frac{2}{15}$

3. a. $x = 1$ **b.** $x = 18$ **c.** $x = 23$ **d.** $x = 4$ **e.** $x = 3$
f. $x = 12$ **g.** $w = 1.3$ **h.** $b = \frac{2}{3}$ **i.** $x = \frac{7}{20}$

4. a. $x = 3$ **b.** $x = 45$ **c.** $x = 12$ **d.** $x = 34$ **e.** $x = 77$
f. $x = 0$ **g.** $x = 13$ **h.** $x = \frac{28}{11}$ **i.** $x = \frac{2}{3}$

5. a. $x = 14$ **b.** $x = 29$ **c.** $x = 78$ **d.** $x = 1$ **e.** $x = 5$
f. $x = 291$ **g.** $x = 4.65$ **h.** $x = \frac{27}{7}$ **i.** $x = \frac{1}{2}$

6. a. $x = 10$ **b.** $x = 3$ **c.** $x = 48$ **d.** $x = 50$ **e.** $x = 6$
f. $x = 21$ **g.** $x = 7$ **h.** $x = 6$ **i.** $x = 11.13$ **j.** $x = \frac{1}{10}$
k. $x = 25$ **l.** $x = 1$ **m.** $x = 6$ **n.** $x = 8$ **o.** $x = 2$

7. a. $x = 3$ **b.** $x = 5$ **c.** $x = 3$ **d.** $x = 4$ **e.** $x = 7$
f. $x = 4$ **g.** $x = 7$ **h.** $x = \frac{2}{9}$ **i.** $x = 1$

8. A

9. First subtract 2 from both sides, then divide both side by 4; $x = 2$.

10. Inverse operations must be performed in the reverse order.

The forward order is:

First subtract 6, then multiply by 3 and finally add 5.

The reverse order is:

Subtract 5, divide by 3, then add 6.

11. Gareth is 13 years old.

12. a. 12 **b.** $\frac{x+28}{3} = 12$ **c.** $x = 8$

d. Karina scored 8 points in the third game.

13. Callie scored 96 pins.

14. 16, 17 and 18

15. 7, 9 and 11

16. 25, 30 and 35

17. David is 14 years old.

18. Chris jumped 156 centimetres.

19. The twins are 11 years old.

20. a. Puzzle 1 $[(n - 500) \times 2 + 1000] \div 2 = n$

Puzzle 2 $[(n + 4) \times 10 - 10] \div 5 - n - 6 = n$

Puzzle 3 $(n \div 2 + 2) \times 4 - n - n = 8$

b. Ask your teacher.

21.

8	1	6
3	5	7
4	9	2

22. 11, 13, 15

23. 36

Exercise 13.6 Checking solutions

1. a. No **b.** Yes **c.** No **d.** No **e.** No **f.** Yes
g. No **h.** No **i.** Yes **j.** No **k.** Yes **l.** No

2. a. Yes **b.** No **c.** No **d.** Yes **e.** Yes
f. No **g.** No **h.** Yes **i.** Yes **j.** No

3.

x	0	1	2	3	4
$2x + 3$	3	5	7	9	11

a. $x = 4$ **b.** $x = 4$ **c.** $x = 1$

4.

x	2	3	4	5	6
$5(x + 2)$	0	5	10	15	20

a. $x = 4$ **b.** $x = 6$ **c.** $x = 8$

5. a.

x	$2x + 1$	$3x - 5$
3	7	4
4	9	7
5	11	10
6	13	13
7	15	16

b. $x = 6$

6. a.

x	$\frac{x + 3}{2}$	$2x - 6$
3	3	0
5	4	4
7	5	8
9	6	12
11	7	16

b. $x = 5$

7. a. No. Length is enough.
b. 34.8 m
c. Adding 3.9 and the value of y. The answer should be 38.7 m.

8. a. $\frac{1}{7}x = 0.5$ **b.** $x = 3.5$

9. James \$53 and Alison \$72

10. 63

Exercise 13.7 Keeping equations balanced

1. a. **i.** $4r = 12$ **ii.** $r = 3$
b. **i.** $2n + 3 = 7$ **ii.** $n = 2$
c. **i.** $3t + 5 = 8$ **ii.** $t = 1$
d. **i.** $4p + 1 = 13$ **ii.** $p = 3$

2. a. $g = 1$ **b.** $m = 3$ **c.** $q = 9$ **d.** $n = 9$ **e.** $g = 14$ **f.** $y = 3$

3. a. $h = 2\frac{1}{3}$ **b.** $k = 2\frac{1}{2}$ **c.** $w = 3\frac{1}{2}$ **d.** $t = 4\frac{1}{3}$ **e.** $h = 2\frac{1}{6}$ **f.** $l = 1\frac{1}{3}$

4. a. $m = 4$ **b.** $d = 3$ **c.** $v = 2$ **d.** $s = 3$ **e.** $g = 5$ **f.** $k = 2$

5. a. $x = 2\frac{1}{2}$ **b.** $y = 1$ **c.** $p = 3\frac{1}{4}$ **d.** $h = 2\frac{1}{2}$

6. a. $7\frac{2}{9}$ °C **b.** 113 °F

7. a. \$11.30 **b.** 58 km

8. 11 songs **9.** 16 boys **10.** 3 **11.** 3, 8, 13 **12.** 4 cm

Challenge 13.2

$1\frac{1}{3}$ kilograms

13.8 Review

1. a. $m = 5$ **b.** $h = 6$ **c.** $s = 19$ **d.** $d = 20$

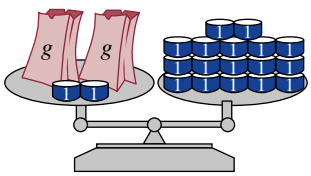
2. a. 32, 51 **b.** 27, 59

3. a. 11 **b.** 2 **c.** 16 **d.** 35

4. a. 16 **b.** 42 **c.** 11 **d.** 8

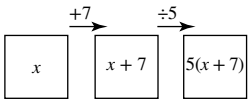
5. a. $7x + 8$ **b.** $\frac{x}{3} - 5$ **c.** $6(x + 2)$ **d.** $\frac{x - 7}{5}$ **e.** $5(x + 3) - 9$ **f.** $\frac{4x + 11}{8}$

6. a.

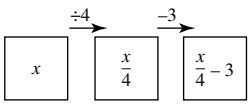


Equal

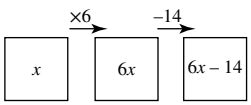
b.



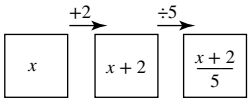
c.



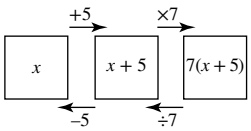
d.



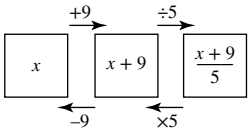
7. a.



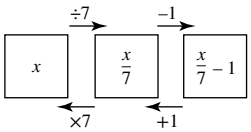
b.



c.



d.



8. a. $x = 2$ **b.** $y = 10$ **c.** $m = 60$ **d.** $s = 18$

9. a. $d = 4$ **b.** $t = 100$ **c.** $d = 3$ **d.** $a = 6$

10. a. $v = 3$ **b.** $c = 12$

11. a. Yes **b.** No

12. a. **i.** $4x + 1 = 5$
ii. $x = 1$

b. **i.** $3p + 5 = 11$
ii. $p = 2$

c. **i.** $2q + 3 = 13$
ii. $q = 5$

d. **i.** $2g + 2 = 17$
ii. $g = 7\frac{1}{2}$

13. a. $v = 3$ **b.** $x = 2$ **c.** $w = 3$ **d.** $g = 4$

14. a. $k = 1\frac{3}{4}$ **b.** $p = 5\frac{1}{2}$ **c.** $q = \frac{1}{3}$ **d.** $t = \frac{2}{5}$

15. a. $x = 2$ **b.** $p = 2$ **c.** $r = 2$ **d.** $z = 2$

16. Sophie owns 31 cards

17. 7 pieces of homework

18. 12 years old

19. $2

20. a. i. $40

ii. $50

iii. $60

b. Cost = 30 + 10h

c. $110

d. 85 = 30 + 10 h; h = 5.5 hours

e. $5

f. Your deposit is returned ($30). You could buy a hamburger on the way home.

21. a. $105 **b.** $105 **c.** 6 children

d. 10 adults; 9 adults and 2 children; 8 adults and 4 children; 7 adults and 6 children; 6 adults and 8 children; 5 adults and 10 children; 4 adults and 12 children; 3 adults and 14 children; 2 adults and 16 children; 1 adult and 18 children; 20 children

22. 14 bicycles and 5 tricycles

23. 45°, 45°, 90°. This is a right-angled isosceles triangle.

24. Length 12 m, width 3 m

25. 35°

Investigation — Rich task

1. Men's: 1928, 10.607 s; 1968, 10.231 s; 1988, 10.043 s; 2008, 9.855 s

Women's: 1928, 11.956 s; 1968, 11.264 s; 1988, 10.918 s; 2008, 10.527 s

2. All calculated times are well within 0.4 seconds of the actual running times.

3. Using the formula: Men's time = 9.817 s;

Women's time = 10.502 s.

4. Women's times are coming down faster than men's. It is possible only if these patterns continue.

5. Different training programs, fitter athletes, better shoes, etc. There must be some levelling out, as the times cannot keep coming down forever; it will be impossible to run 100 metres in 0 seconds.

TOPIC 14 Representing and interpreting data

14.1 Overview

Numerous **videos** and **interactivities** are embedded just where you need them, at the point of learning, in your learnON title at www.jacplus.com.au. They will help you to learn the content and concepts covered in this topic.

14.1.1 Why learn this?

Understanding data helps us to make sense of graphs, charts and advertising material. The media often present statistics such as temperature charts, share market information and advertising claims. An understanding of statistics helps us to understand this information.

14.1.2 What do you know?

assessON

1. **THINK** List what you know about data. Use a thinking tool such as a concept map to show your list.
2. **PAIR** Share what you know with a partner and then with a small group.
3. **SHARE** As a class, create a thinking tool such as a large concept map that shows your class's knowledge of data.

LEARNING SEQUENCE

14.1 Overview
14.2 Classifying data
14.3 Displaying data in tables
14.4 Measures of centre and spread
14.5 Representing data graphically
14.6 Comparing data
14.7 Review

14.2 Classifying data

14.2.1 Classifying data

- Each day, people in all types of professions are presented with various forms of information, called data, which will assist them in answering questions and planning for the future.
- Statistics is the branch of mathematics that deals with the *collection*, *organisation*, *display*, *analysis* and *interpretation* of data. These data are usually presented in numerical form.
- Data may be classified in the following ways.

<table>
<tr><td colspan="4">Data</td></tr>
<tr><td colspan="2">Qualitative</td><td colspan="2">Quantitative</td></tr>
<tr><td colspan="2">Data which are placed in categories; that is, non-numerical form; such as hair colour, type of vehicle, and so on.</td><td colspan="2">Data which are in numerical form; such as height, number of children in the family, and so on.</td></tr>
<tr><td>Nominal</td><td>Ordinal</td><td>Discrete</td><td>Continuous</td></tr>
<tr><td>Need sub-groups to complete the description, such as hair colour: blonde, brown and so on.</td><td>Need a ranking to order the description, such as achievement levels: very high, high, satisfactory and so on.</td><td>Counted in exact values, such as goals scored in a football match, shoe size and so on. Values are often, but not always, whole numbers.</td><td>Measured in a continuous decimal scale, such as mass, temperature, length and so on.</td></tr>
</table>

WORKED EXAMPLE 1

Classify each of the following data using two selections from the following descriptive words: qualitative, quantitative, nominal, ordinal, discrete and continuous.

a The number of students absent from school
b The types of vehicle using a certain road
c The various pizza sizes available at a local takeaway
d The room temperature at various times during a particular day

THINK	WRITE
a 1 Determine whether the data are qualitative or quantitative.	**a** The data are quantitative as absences are represented by a number.
2 Determine whether the data are discrete or continuous.	The data are discrete as the number of absences can be counted and are exact values.
b 1 Determine whether the data are qualitative or quantitative.	**b** The data are qualitative as the types of vehicle need to be placed in non-numerical categories.
2 Determine whether the data are nominal or ordinal.	The data are nominal as there is no ranking or order involved.
c 1 Determine whether the data are qualitative or quantitative.	**c** The data are qualitative as the pizza sizes need to be ranked in order ranging from small to family.

2 Determine whether the data are nominal or ordinal.

The data are ordinal as pizzas are ranked in order of size.

d 1 Determine whether the data are qualitative or quantitative.

d The data are quantitative as room temperature is represented by a number.

2 Determine whether the data are discrete or continuous.

The data are continuous as temperature can assume any value and measurement is involved.

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Complete this digital doc: SkillSHEET: Distinguishing qualitative from quantitative data (doc-6578)

Complete this digital doc: SkillSHEET: Distinguishing discrete from continuous data (doc-6579)

Exercise 14.2 Classifying data

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–8, 10	Questions: 1–8, 10, 11	Questions: 1–11

Individual pathway interactivity: int-4378

learnON ONLINE ONLY

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note*: Question numbers may vary slightly.

Fluency

1. Match each word with its correct meaning:

a. discrete	**i.** placed in categories or classes
b. qualitative	**ii.** counted in exact values
c. ordinal	**iii.** data in the form of numbers
d. continuous	**iv.** needs further names to complete the description
e. quantitative	**v.** needs a ranking order
f. nominal	**vi.** measured in decimal numbers.

Understanding

2. WE1 Classify each of the following data using two words selected from the following descriptive words: qualitative, quantitative, nominal, ordinal, discrete and continuous.

a. The population of your town or city

b. The types of motorbike in a parking lot

c. The heights of people in an identification line-up

d. The masses of babies in a group

e. The languages spoken at home by students in your class

f. The time spent watching TV
g. The number of children in the families in your suburb
h. The air pressure in your car's tyres
i. The number of puppies in a litter
j. The types of radio program listened to by teenagers
k. The times for swimming 50 metres
l. The quantity of fish caught in a net
m. The number of CDs you own
n. The types of shops in a shopping centre
o. The football competition ladder at the end of each round
p. The lifetime of torch batteries
q. The number of people attending the Big Day Out
r. Final Year 12 exam grades
s. The types of magazine sold at a newsagency
t. Hotel accommodation rating

3. Write a sentence explaining the difference between discrete and continuous data. Give an example of each.

4. List two examples of each of the following types of data:
a. quantitative, discrete
b. qualitative, ordinal
c. quantitative, continuous
d. qualitative, nominal.

5. MC Data representing shoe (or rollerblade) sizes can be classified as:
A. quantitative, continuous
B. qualitative, nominal
C. qualitative, ordinal
D. quantitative, discrete
E. none of the above

6. MC The data, points scored in a basketball game, can best be described as:
A. discrete
B. continuous
C. qualitative
D. ordinal
E. nominal

7. MC An example of qualitative, ordinal data would be the:
A. heights of buildings in Melbourne
B. number of pets in households
C. type of pets in households
D. birthday month of students in Year 7
E. number of hours spent playing sport

Reasoning

8. A fisheries and wildlife officer released 200 tagged trout into a lake. A week later, the officer took a sample of 50 trout and found that 8 of them were tagged. The officer can use this information to estimate the population of trout in the lake. How many trout are in the lake? Explain how you reached this answer.

9. Explain why data such as postal codes, phone numbers and driver's licence numbers are not numerical data.

Problem solving

10. The following questions would collect categorical data. Rewrite the questions so that you could collect numerical data.
a. Do you read every day?
b. Do you play sport every day?
c. Do you play computer games every day?

11. The following questions would collect numerical data. Rewrite the questions so that you could collect categorical data.

a. On average, how many minutes per week do you spend on Maths homework?

b. How many books have you read this year?

c. How long does it take you to travel to school?

Reflection

Why is it necessary to classify data into different categories?

14.3 Displaying data in tables

- Data can be displayed in a variety of forms.
- Generally, data is first organised into a table; then, a suitable graph is chosen as a visual representation.

14.3.1 Frequency distribution tables

- One way of presenting data is by using a frequency distribution table.
- **Frequency** is the number of times a result or piece of data occurs.
- The **frequency distribution table** consists of three columns: score, tally and frequency.

WORKED EXAMPLE 2

A particular class was surveyed to find out the number of pets per household and the data were recorded. The raw data were:
0, 3, 1, 2, 0, 1, 0, 1, 2, 4, 0, 6, 1, 1, 0, 2, 2, 0, 1, 3, 0, 1, 2, 1, 1, 2.

a Organise the data into a frequency distribution table.
b How many households were included in the survey?
c How many households have fewer than 2 pets?
d Which is the most common number of pets?
e How many households have 3 or more pets?
f What fraction of those surveyed had no pets?

THINK

a 1 Draw a frequency distribution table comprising three columns, headed score (that is, the number of pets), tally and frequency.

2 In the first column list the possible number of pets per household (that is, 0 to 6).

3 Place a stroke in the tally column each time a particular score is noted.
Note: A score of 5 is denoted as a 'gate post' (that is, four vertical strokes and one diagonal stroke 𝍸).

WRITE

a

Score	Tally	Frequency
0	𝍸 ||	7
1	𝍸 ||||	9
2	𝍸 |	6
3	||	2
4	|	1
5		0
6	|	1
	Total	26

	4 Write the total tally strokes for each pet in the frequency column.	
	5 Calculate the total of the frequency column.	
b	The total of the frequency column gives the number of households surveyed.	**b** Twenty-six households were surveyed.
c 1	Calculate the number of households which have fewer than 2 pets. *Note:* Fewer than 2 means 0 pets or 1 pet.	**c** Fewer than two pets $= 7 + 9$ $= 16$
	2 Answer the question.	Sixteen households have fewer than 2 pets.
d 1	Make a note of the highest value in the frequency column and check which score it corresponds to.	**d** The score with the highest frequency (that is, 9) corresponds to one pet.
	2 Answer the question.	The most common number of pets is one.
e 1	Calculate the number of households which have 3 or more pets. *Note:* 3 or more means 3, 4, 5 or 6.	**e** 3 or more pets $= 2 + 1 + 0 + 1$ $= 4$
	2 Answer the question.	Four households have 3 or more pets.
f 1	Write the number of households with no pets.	**f** Households with no pets $= 7$
	2 Write the total number of households surveyed.	Total number of households surveyed $= 26$
	3 Define the fraction and substitute the known values into the rule.	$\frac{\text{Households with no pets}}{\text{Total number of households surveyed}} = \frac{7}{26}$
	4 Answer the question.	Of the households surveyed $\frac{7}{26}$ have no pets.

Sometimes, the data may contain too many numerical values to list them all individually in the 'score' column. In this case, we use a range of values, called a *class interval*, as our category. For example, the range 100–104 may be used to cater for all the values that lie within the range, including 100 and 104.

WORKED EXAMPLE 3

The data below show the ages of a number of mobile phone owners:
12, 11, 21, 12, 30, 26, 13, 15, 29, 16, 17, 17, 17, 21, 19, 12, 14, 16, 43, 18, 51, 25, 30, 28, 33, 62, 39, 40, 30, 18, 19, 41, 22, 21, 48, 31, 33, 33, 34, 41, 18, 17, 31, 43, 42, 17, 46, 23, 24, 33, 27, 31, 53, 52, 25

a Draw a frequency table to classify the given data. Use a class interval of 10; that is, ages 11–20, 21–30 and so on, as each category.
b How many people were surveyed?
c Which age group had the largest number of people with mobile phones?
d Which age group had the least number of people with mobile phones?
e How many people in the 21–30 age group own a mobile phone?

THINK	WRITE
a 1 Draw a frequency distribution table.	**a**

Age group	Tally	Frequency
11–20	𝍸 𝍸 𝍸 IIII	19
21–30	𝍸 𝍸 𝍸	15
31–40	𝍸 𝍸	10
41–50	𝍸 II	7
51–60	III	3
over 60	I	1
	Total	55

2 In the first column, list the possible age groups; that is, 11–20, 21–30 etc.	
3 Systematically go through the results and place a stroke in the tally column each time a particular age group is noted.	
4 Write the total tally of strokes for each age group in the frequency column.	
5 Calculate the total of the frequency column.	
b The total of the frequency column gives us the number of people surveyed.	**b** A total of 55 people were surveyed
c 1 Make note of the highest value in the frequency column and check which age group it corresponds to.	**c** The 11–20 age group has the highest frequency; that is, a value of 19.
2 Answer the question.	The 11–20 age group has the most number of people with mobile phones.
d 1 Make note of the lowest value in the frequency column and check which age group it corresponds to. *Note:* There may be more than one answer.	**d** The over 60 age group has the lowest frequency; that is, a value of 1.
2 Answer the question.	The over 60 age group has the least number of people with mobile phones.
e 1 Check the 21–30 age group in the table to see which frequency value corresponds to this age group.	**e** The 21–30 age group has a corresponding frequency of 15.
2 Answer the question.	Fifteen people in the 21–30 age group own a mobile phone.

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Complete this digital doc: Investigation: How many red M&Ms? (doc-3438)

Complete this digital doc: Spreadsheet: Frequency tally tables (doc-3437)

Complete this digital doc: WorkSHEET: Representing and interpreting data I (doc-1978)

Exercise 14.3 Displaying data in tables

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1, 2, 3, 5, 6, 7, 16	Questions: 1, 2, 3, 4, 5, 7, 9–11, 14, 16	Questions: 1, 2, 3–16

Individual pathway interactivity: int-4379

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To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note*: Question numbers may vary slightly.

Fluency

1. WE2 The number of children per household in a particular street is surveyed and the data recorded. The raw data are:

0, 8, 6, 4, 0, 0, 0, 2, 1, 3, 3, 3, 1, 2, 3,
2, 3, 2, 1, 2, 1, 3, 0, 2, 2, 4, 2, 3, 5, 2.

a. Organise the data into a frequency distribution table.
b. How many households are included in the survey?
c. How many households have no children?
d. How many households have at least 3 children?
e. Which is the most common number of children?
f. What fraction of those surveyed have 4 children?

2. WE3 Draw a frequency table to classify the following data on house prices. Use a class interval of 10 000; that is prices $100 000 to $109 000 and so on for each category. The values are:
$100 000, $105 000, $110 000, $150 000, $155 000, $106 000, $165 000, $148 000, $165 000, $200 000, $195 000, $138 000, $142 000, $153 000, $173 000, $149 000, $182 000, $186 000.

Understanding

3. Rosemary decided to survey the participants of her local gym about their preferred sport. She asked each participant to name one preferred sport and recorded her results: hockey, cricket, cricket, tennis, scuba diving, netball, tennis, netball, swimming, netball, tennis, hockey, cricket, lacrosse, lawn bowls, hockey, swimming, netball, tennis, netball, cricket, tennis, hockey, lacrosse, swimming, lawn bowls, swimming, swimming, netball, netball, tennis, golf, hockey, hockey, lacrosse, swimming, golf, hockey, netball, swimming, scuba diving, scuba diving, golf, tennis, cricket, cricket, hockey, lacrosse, netball, golf.
a. Can you see any problems with the way Rosemary has displayed the data?
b. Organise Rosemary's results into a frequency table to show the participants' preferred sports.
c. From the frequency table, find:
i. the most preferred sport
ii. the least preferred sport.
d. Did any sport(s) have the same frequency?

4. Complete a frequency distribution table for each of the following.

a. Andrew's scores in Mathematics tests this semester are: 6, 9, 7, 9, 10, 7, 6, 5, 8, 9.

b. The number of children in each household of a particular street are:
2, 0, 6, 1, 1, 2, 1, 3, 0, 4, 3, 2, 4, 1, 0, 2, 1, 0, 2, 0.

c. The masses (in kilograms) of students in a certain Year 7 class are:
46, 60, 48, 52, 49, 51, 60, 45, 54, 54, 52, 58, 53, 51, 54,
50, 50, 56, 53, 57, 55, 48, 56, 53, 58, 53, 59, 57.

d. The heights of students in a particular Year 7 class are:
145, 147, 150, 150, 148, 145, 144, 144, 147, 149, 144, 150, 150, 152, 145, 149, 144, 145, 147, 143, 144, 145, 148, 144, 149, 146, 148, 143.

5. Use the frequency distribution table to answer the questions.

Score	Tally	Frequency
0	||	2
1	𝍸	5
2	|||	3
3	𝍸 𝍸 |	11
4	𝍸 |||	8
5	||||	4
	Total	

a. How many participated in the survey?

b. What was the most frequent score?

c. How many scored less than 3?

d. How many scored 3 or more?

e. What fraction of those surveyed scored 3?

6. A random sample of 30 families was surveyed to find the number of high-school-aged children in each family. Below are the raw data collected:
2, 1, 1, 0, 2, 0, 1, 0, 2, 0, 3, 1, 1, 0, 0, 0, 1, 4, 1, 0, 0, 1, 2, 1, 2, 0, 3, 2, 0, 1.

a. Organise the data into a frequency distribution table.

b. How many families have no children of high school age?

c. How many have 2 or more children of high school age?

d. Which score has the highest frequency?

e. What is the greatest number of high-school-aged children in the 30 families surveyed?

f. What fraction of families had 2 children of high school age?

7. Draw a frequency table to classify the following data on students' heights. Use a range of values (such as 140–144) as each category. The values are: 168 cm, 143 cm, 145 cm, 151 cm, 153 cm, 148 cm, 166 cm, 147 cm, 160 cm, 162 cm, 175 cm, 168 cm, 143 cm, 150 cm, 160 cm, 180 cm, 146 cm, 158 cm, 149 cm, 169 cm, 167 cm, 167 cm, 163 cm, 172 cm, 148 cm, 151 cm, 170 cm, 160 cm.

8. Complete a frequency table for all vowels in the following paragraphs.

Australian Rules Football is a ball game played by two teams of eighteen players with an ellipsoid ball on a large oval field with four upright posts at each end. Each team attempts to score points by kicking the ball through the appropriate posts (goals) and prevent their opponents from scoring. The team scoring the most points in a given time is the winner. Usually this period is divided into four quarters of play.

Play begins at the beginning of a quarter or after a goal, with a tap contest between two opposing players (rucks) in the centre of the ground after the umpire either throws the ball up or bounces it down.

Questions **9**, **10** and **11** refer to the following information. A real estate agent has listed all the properties sold in the area in the last month as shown below. She wants to know what has been the most popular type of property from the following: 2 bedroom house, 4 bedroom house, 3 bedroom house, 2 bedroom unit, 4 bedroom house, 1 bedroom unit, 3 bedroom house, 2 bedroom unit, 3 bedroom house, 1 bedroom unit, 2 bedroom unit, 3 bedroom house, 3 bedroom house, 3 bedroom house, 2 bedroom unit, 1 bedroom unit.

9. Complete a frequency table for the list given and work out which type of property was most popular.

10. MC The least popular type of property is the:

A. 1 bedroom unit **B.** 2 bedroom unit
C. 2 bedroom house **D.** 3 bedroom house
E. 4 bedroom house

11. MC The property which is half as popular as a 2 bedroom unit is the:

A. 4 bedroom house **B.** 3 bedroom house
C. 2 bedroom house **D.** 1 bedroom unit
E. none of these

12. MC The frequency column of a frequency table will:

a. add up to the total number of categories
b. add up to the total number of results given
c. add up to the total of the category values
d. display the tally
e. none of these

Reasoning

13. This frequency table shows the percentage occurrence of the vowels in a particular piece of text. Two pieces of data are missing — those for O and U.

Vowel	Percentage frequency
A	22.7
E	27.6
I	19.9
O	
U	

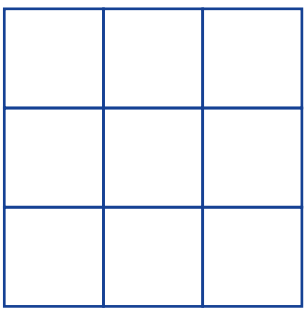
The occurrence of O is 2.6 times that of U. What are the two missing values? Show your working.

14. Explain why tallies are drawn in batches of four vertical lines crossed by a fifth one.

Problem solving

15. Four girls — Eugenie, Florence, Kim and Anthea — each prefer a different means of communication. Match the girls to one of the following: home phone, mobile with SMS messaging, email, mail. Use the following information: both Eugenie and Florence have no computer, though Florence is the only one with a mobile phone with SMS; also, Australia Post can collect mail from Kim and Florence.

16. Place 6 crosses in the grid at right so that there are exactly 2 crosses in each row and column. (There is more than one way to solve this problem.)



Reflection

What do you need to consider when selecting a class interval for a frequency distribution table?

CHALLENGE 14.1

A survey was conducted on 45 houses to find the number of children living in each household. Use the following information to determine the results and complete the table.

- The number of houses with no children is equal to the number of houses with 3 children.
- The number of houses with 1 child and 2 children were four times and five times the number of houses with no children respectively.
- There was one house with 4 children.

Score	Frequency
0	
1	
2	
3	
4	
Total	

14.4 Measures of centre and spread

14.4.1 Measures of centre

Three measures of centre are used to show how a set of data is grouped around a central point.

- The **mean** of a set of data is another name for the average. It is calculated by adding all the data values, and dividing by the number of values in the set of data.
- The symbol for the mean is $\bar{x}$.
- The **median** is the middle value of the data when the values are arranged in numerical order.
- The **mode** of a set of data is the most frequently occurring value.

14.4.2 Mean

The mean, or average, of a set of values is the sum of all the values divided by the number of values.

WORKED EXAMPLE 4

For each of the following sets of data, calculate the mean ($\bar{x}$).
a 5, 5, 6, 4, 8, 3, 4
b 0, 0, 0, 1, 1, 1, 1, 1, 4, 4, 4, 4, 5, 5, 5, 7, 7

THINK

a 1 Calculate the total of the given values.

2 Count the number of values.

3 Define the rule for the mean.

4 Substitute the known values into the rule and evaluate.

b 1 Calculate the total of the given values. Take note of the number of times each value occurs. That is,
0 occurs 3 times (3×0),
1 occurs 5 times (5×1),
4 occurs 4 times (4×4),
5 occurs 3 times (3×5),
7 occurs 2 times (2×7).

WRITE

a Total of values $= 5 + 5 + 6 + 4 + 8 + 3 + 4$
$= 35$

Number of values $= 7$

$$\text{Mean} = \frac{\text{total of values}}{\text{number of values}}$$
$$= \frac{35}{7}$$
$$= 5$$

b Total of values $= 3 \times 0 + 5 \times 1 + 4 \times 4 + 3 \times 5 + 2 \times 7$
$= 0 + 5 + 16 + 15 + 14$
$= 50$

2 Count the number of values. *Note:* Although zero has no numerical value, it is still counted as a piece of data and must be included in the number of values tally.	Number of values = 17
3 Define the rule for the mean.	$\text{Mean} = \frac{\text{total of values}}{\text{number of values}}$
4 Substitute the known values into the rule and evaluate.	$= \frac{50}{17}$ $= 2.941176471$
5 Round the answer to 1 decimal place. *Note:* The mean doesn't necessarily have to be a whole number or included in the original data.	$= 2.9$

14.4.3 Median

The median is the middle value for an odd number of data and the average of the two middle values for an even number of data.

When determining the median:

1. the values must be arranged in numerical order
2. there are as many values above the median as there are below it
3. for an even number of values, the median may not be one of the listed scores.

WORKED EXAMPLE 5

Find the median for the following sets of data:
a 5, 4, 2, 6, 3, 4, 5, 7, 4, 8, 5, 5, 6, 7, 5
b 8, 2, 5, 4, 9, 9, 7, 3, 2, 9, 3, 7, 6, 8.

THINK	WRITE
a 1 Arrange the values in ascending order.	**a**
2 Select the middle value. *Note:* There are an odd number of values; that is, 15. Hence, the eighth value is the middle number or median.	2, 3, 4, 4, 4, 5, 5, ⑤, 5, 5, 6, 6, 7, 7, 8
3 Answer the question.	The median of the scores is 5.
b 1 Arrange the values in ascending order.	**b**
2 Select the middle values. *Note:* There are an even number of values; that is, 14. Hence, the sixth and seventh values are the middle numbers.	2, 2, 3, 3, 4, 5, ⑥, ⑦, 7, 8, 8, 9, 9, 9

3 Obtain the average of the two middle values. *Note:* Add the two middle values and divide the result by 2.	Median $= \frac{6 + 7}{2}$ $= \frac{13}{2}$ (or $6\frac{1}{2}$)
4 Answer the question.	The median of the scores is $6\frac{1}{2}$ or 6.5.

14.4.4 Mode

The mode is the most common value in a set of data. Some sets of data have more than one mode, or no mode at all; that is, there is no value which corresponds to the highest frequency, as all values occur once only.

WORKED EXAMPLE 6

Find the mode of the following scores:
a 2, 3, 4, 5, 5, 6, 6, 7, 8, 8, 8, 9
b 12, 18, 5, 17, 3, 5, 2, 10, 12
c 42, 29, 11, 28, 21.

THINK	WRITE
a 1 Look at the set of data and highlight any values that have been repeated.	**a** 2, 3, 4, ⑤, ⑤, ⑥, ⑥, 7, ⑧, ⑧, ⑧, 9
2 Choose the value which has been repeated the most.	The numbers 5 and 6 occur twice. However, the number 8 occurs three times.
3 Answer the question.	The mode for the given set of values is 8.
b 1 Look at the set of data and highlight any values that have been repeated.	**b** ⑫, 18, ⑤, 17, 3, ⑤, 2, 10, ⑫
2 Choose the value(s) which have been repeated the most.	The number 5 occurs twice. The number 12 occurs twice.
3 Answer the question. *Note:* Some sets have more than one mode. The data set is called bimodal as two values were most common.	The mode for the given set of values is 5 and 12.
c 1 Look at the set of data and highlight any values that have been repeated.	**c** 42, 29, 11, 28, 21
2 Answer the question. *Note:* No mode is not the same as having a mode which equals 0.	No values have been repeated. The set of data has no mode since none of the scores correspond to a highest frequency. Each of the numbers occurs once only.

14.4.5 Measures of spread

These measures indicate how far data values are spread from the centre, or from each other. There are several measures, but the appropriate one to discuss at this stage is the range.

14.4.6 Range

The **range** of a set of values is the difference between the highest and lowest values.

WORKED EXAMPLE 7

Find the range of the following data.
12, 76, 35, 29, 16, 45, 56

THINK	WRITE
1 Obtain the highest and lowest values.	Highest value = 76 Lowest value = 12
2 Define the range.	Range = highest value − lowest value
3 Substitute the known values into the rule.	= 76 − 12
4 Evaluate.	= 64
5 Answer the question.	The set of values has a range of 64.

learnON RESOURCES — ONLINE ONLY

- **Complete this digital doc:** SkillSHEET: Finding the mean of ungrouped data (doc-6580)
- **Complete this digital doc:** SkillSHEET: Finding the median (doc-6581)
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- **Complete this digital doc:** Spreadsheets: Mean (doc-3434)
- **Complete this digital doc:** Spreadsheets: Median (doc-3435)
- **Complete this digital doc:** Spreadsheets: Mode (doc-3436)

Exercise 14.4 Measures of centre and spread

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–7, 10, 11, 15, 18, 28	Questions: 1–5 column 2, 7–10, 16, 20, 22, 24, 26, 27, 28	Questions: 1b, h, j, 2d, f, 3b, f, i, 4b, f, j, 5b, d, f, g, j, 7–28

Individual pathway interactivity: int-4380

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Fluency

1. WE4a For each of the following sets of data, calculate the mean.

a. 3, 4, 5, 5, 6, 7 **b.** 5, 6, 7, 5, 5, 8 **c.** 4, 6, 5, 4, 2, 3
d. 3, 5, 6, 8, 7, 7 **e.** 5, 4, 4, 6, 2, 3 **f.** 2, 2, 2, 4, 3, 5
g. 12, 10, 13, 12, 11, 14 **h.** 11, 12, 15, 17, 18, 11 **i.** 12, 15, 16, 17, 15, 15
j. 10, 14, 12, 12, 16, 14

2. WE4b For each of the following sets of data, calculate the mean.
Hint: Use the grouping of values to help you.

a. 9, 9, 7, 7 **b.** 2, 2, 2, 4, 4, 4 **c.** 4, 4, 3, 3, 5, 5
d. 1, 1, 2, 3, 3 **e.** 1, 2, 2, 4, 4, 5 **f.** 1, 2, 2, 5, 5, 6, 7
g. 9, 9, 8, 8, 7, 1, 1, 1, 1 **h.** 3, 3, 3, 1, 1, 1, 2, 2, 2 **i.** 4, 4, 5, 5, 8, 8, 1, 1, 9
j. 2, 2, 2, 3, 3, 3, 3, 6

3. WE5a Find the middle value (median) for the following sets of data, by carefully ordering the values first.

a. 3, 3, 4, 5, 5, 6, 7 **b.** 1, 2, 2, 3, 4, 8, 9 **c.** 411, 2, 5, 6, 8, 8, 9
d. 2, 2, 2, 3, 3, 4, 5 **e.** 5, 5, 6, 6, 7, 7, 8, 9, 9 **f.** 7, 7, 7, 10, 11, 12, 15, 15, 16
g. 4, 3, 5, 3, 4, 4, 3, 5, 4 **h.** 1, 2, 5, 4, 1, 1, 1, 2, 5 **i.** 1, 2.5, 5, 3.4, 1, 2.4, 5
j. 1.2, 1.5, 1.4, 1.8, 1.9

4. WE5b Find the middle value (median) for the following sets of data, by carefully ordering the values first. Note there is an even number of values.

a. 1, 1, 2, 2, 4, 4 **b.** 1, 2, 2, 2, 4, 5 **c.** 4, 5, 5, 5, 6, 7
d. 4, 5, 7, 7, 8, 9 **e.** 1, 2, 2, 3, 3, 4 **f.** 2, 4, 4, 6, 8, 9
g. 1, 5, 7, 8 **h.** 2, 4, 5, 7, 8, 8, 9, 9 **i.** 1, 4, 7, 8
j. 1, 5, 7, 8, 10, 15

5. WE6 Find the mode for each of the following sets of data.

a. 3, 3, 4, 4, 4, 5, 6 **b.** 2, 9, 8, 8, 4, 5 **c.** 1, 1, 2, 2, 2, 3
d. 4, 6, 4, 2, 7, 3 **e.** 2, 4, 3, 6, 2, 4, 2 **f.** 4, 8, 8, 3, 3, 4, 3, 3
g. 6, 2, 12, 10, 14, 14, 10, 12, 10, 12, 10, 12, 10 **h.** 7, 9, 4, 6, 26, 71, 3, 3, 3, 2, 4, 6, 4, 25, 4
i. 2, 2, 3, 4, 4, 9, 9, 9, 6 **j.** 3, 7, 4, 3, 4, 3, 6, 3

6. WE7 **a.** Find the range of the following: 15, 26, 6, 38, 10, 17.
b. Find the range of the following: 12.8, 21.5, 1.9, 12.0, 25.4, 2.8, 1.3.

Understanding

7. MC The mean for the data 5, 5, 6, 7, 2 would be found by:
A. adding all the results and multiplying by the number of results
B. adding all the results and dividing by the number of results
C. adding all the results
D. choosing the middle result
E. ordering the results, then choosing the middle result

8. MC When finding the mean of a set of data:
A. zeroes do not matter at all
B. zeroes must be counted in the number of results
C. zeroes must be added to the total as they will change it
D. zeroes will make the mean zero
E. none of these is true

9. MC For the following set of data, 2.6, 2.8, 3.1, 3.7, 4.0, 4.2:
A. the mean value for the data will be above 4.2
B. the mean value for the data will be below 2.6
C. the mean value for the data will be between 2.6 and 3.0
D. the mean value for the data will be between 3.0 and 4.0
E. the mean value for the data will be between 4.0 and 4.2

10. MC Which of the following is a correct statement?
A. The mean, median and mode for any set of data will always be the same value.
B. The mean, median and mode for any set of data will never be the same value.
C. The mean, median and mode for any set of data must always be close in value.
D. The mean, median and mode for any set of data are usually close in value.
E. None of these statements is true.

11. MC The range of the following set of numbers: 16, 33, 24, 48, 11, 30, 15, is:
A. 48 **B.** 59 **C.** 37 **D.** 20 **E.** 11

12. Eleanor wanted to know what her mathematics test average was. Her teacher said that she used the mean of her test results to calculate the end-of-year mark. Eleanor's test results (percentages) were: 89, 87, 78, 75, 89, 94, 82, 93, 78. What was her mathematics test mean?

13. The number of shoes inspected by a factory worker in an hour was counted over a number of days' work. The results are as follows:
105, 102, 105, 106, 103, 105, 105, 102, 108, 110, 102, 103, 106, 107, 108, 102, 105, 106, 105,104, 102, 99, 98, 105, 102, 101, 97, 100

What is the mean number of shoes checked by this worker in one hour? Round your answer to the nearest whole number.

14. The number of students in the cafeteria each lunchtime was surveyed for 2 weeks. The results were as follows:
52, 45, 41, 42, 53, 45, 47, 32, 52, 56.
What was the mean number of students in the cafeteria at lunchtime in that fortnight? Round your answer to the nearest whole number.

15. A cricketer had scores of 14, 52, 35, 42 and 47 in her last 5 innings. What is her mean score?

16. Tom thinks that the petrol station where he buys his petrol is cheaper than the one where his friend Sarah buys her petrol. They begin to keep a daily watch on the prices for 4 weeks and record the following prices (in cents per litre).

Tom: 75.2, 72.5, 75.2, 75.3, 75.4, 75.6, 72.8, 73.1, 73.1, 73.2, 73.4, 75.8, 75.6, 73.4, 73.4, 75.6, 75.4, 75.2, 75.3, 75.4, 76.2, 76.2, 76.2, 76.3, 76.4, 76.4, 76.2, 76.0

Sarah: 72.6, 77.5, 75.6, 78.2, 67.4, 62.5, 75.0, 75.3, 72.3, 82.3, 75.6, 72.3, 79.1, 70.0, 67.8, 67.5, 70.1, 67.8, 75.9, 80.1, 81.0, 58.5, 68.5, 75.2, 68.3, 75.2, 75.1, 72.0

a. Calculate the mean petrol prices for Tom and Sarah.
b. Which station sells cheaper petrol on average?
c. Why might Tom have been misled?

17. Peter has calculated his mean score for history to be 89%, based on five tests. If he scores 92% in the sixth test, what will his new mean score be?

18. Kim has an average (mean) score of 72 in Scrabble. He has played six games. What must he score in the next game to keep the same average?

19. A clothing company wanted to know the size of jeans that should be manufactured in the largest quantities. A number of shoppers were surveyed and asked their jeans size. The results were:
13, 12, 14, 12, 15, 16, 14, 12, 15, 14, 12, 14, 13, 14, 11, 10, 12,
13, 14, 14, 10, 12, 14, 12, 12, 10, 8, 16, 17, 12, 11, 13, 12, 15,
14, 12, 17, 8, 16, 11, 12, 13, 12, 12.

a. What is the mode of these data?
b. Why would the company be more interested in the mode than the mean or median values?

20. Jennifer wants to ensure that the mean height of her jump in the high jump for 10 jumps is over 1.80 metres.
a. If her jumps so far have been (in metres) 1.53, 1.78, 1.89, 1.82, 1.53, 1.81, 1.75, 1.86, 1.82, what is her current mean?
b. What height must she jump on the tenth jump to achieve a mean of 1.80?
c. Is this likely, given her past results?

21. The local football team has been doing very well. They want to advertise their average score (to attract new club members). You suggest that they use the mean of their past season's game scores. They ask you to find that out for them. Here are the results. Game scores for season (totals):
110, 112, 141, 114, 112, 114, 95, 75, 58, 115, 116, 115, 75, 114, 78, 96, 78, 115, 112, 115, 102, 75, 79, 154, 117, 62.
a. What was their mean score?
b. Would the mode or median have been a 'better' average to use for the advertisement?

Reasoning

22. A group of three children have a mean height of 142 cm. The middle height is the same as the mean. The tallest child leaves the group, and is replaced by a child with the same height as the shortest child. The mean height of this group of three children is now 136 cm.
What are the heights of the four children? Explain how you reached the answer.

23. Find five whole numbers that have a mean of 10 and a median of 12.

24. The mean of 5 different test scores is 15. What are the largest and smallest possible test scores, given that the median is 12? All test scores are whole numbers.

25. The mean of 5 different test scores is 10. What are the largest and smallest possible values for the median? All test scores are whole numbers.

26. The mean of 9 different test scores that are whole numbers and range from 0 to 100 is 85. The median is 80. What is the greatest possible range between the highest and lowest possible test scores?

Problem solving

27. The club coach at a local cycling track was overheard saying that he felt at least half the cyclists were cycling at a speed of 30 km/h or more. The speeds (in km/h) of the club cyclists were recorded as follows.
31, 22, 40, 12, 26, 39, 49, 23, 24, 38, 27, 16, 25, 37, 19, 25, 45, 23, 17, 20, 34, 19, 24, 15, 40, 39, 11, 29
33, 44, 29, 50, 18, 22, 51, 24, 19, 20, 30, 40, 49, 29, 17, 25, 37, 25, 18, 34, 21, 20, 18
Is the coach correct in making this statement? First round each of these speeds to the nearest 5 km/h.

28. Gavin records the amount of rainfall in millimetres each day over a two-week period. Gavin's results are:
11, 24, 0, 6, 15, 0, 0, 0, 12, 0, 0, 127, 15, 0.
a. What is the mean rainfall for the two-week period?
b. What is the median rainfall?
c. What is the mode of the rainfall?
d. Which of the mean, median and mode is the best measure of the typical rainfall? Explain your choice.

Reflection

Why do we need to summarise data by calculating measures of centre and spread?

14.5 Representing data graphically

14.5.1 Representing data graphically

- Graphs are a useful way of displaying data, or numerical information. Newspapers, magazines and TV frequently display data as graphs.
- All graphs should have the following features:
  1. a *title* — to tell us what the graph is about
  2. clear *labels* for the axes — to explain what is being shown
  3. *evenly scaled axes* — if the graph has numerical axes, they must have a scale, which must stay constant for the length of the axes and the units that are being used should be indicated
  4. **legends** — these are not always necessary, but are necessary when any symbols or colours are used to show some element of the graph.

14.5.2 Column and bar graphs

- Columns and bar graphs use categories to divide the results into groups.
- The frequency for each category determines the length of the bar, or height of the column.
- It is easiest to graph the data from a frequency table.

Column graphs

Column graphs should be presented on graph paper and have:
1. a title
2. labelled axes which are clearly and evenly scaled
3. columns of the same width
4. an even gap between each column
5. the first column beginning half a unit (that is, half the column width) from the vertical axis.

WORKED EXAMPLE 8

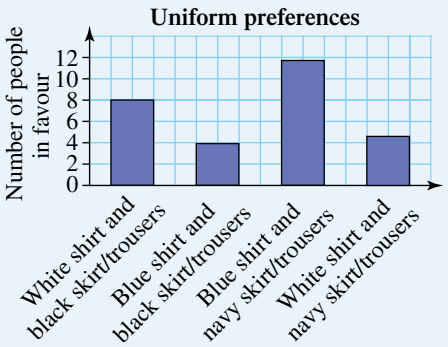
Beth surveyed the students in her class to find out their preferences for the school uniform. Her results are shown in the table below. Construct a column graph to display the results.

Type of uniform	Tally	Frequency
White shirt and black skirt/trousers	𝍸 III	8
Blue shirt and black skirt/trousers	III	4
Blue shirt and navy skirt/trousers	𝍸 𝍸 II	12
White shirt and navy skirt/trousers	𝍸	5
	Total	**29**

THINK

1. Rule a set of axes on graph paper. Provide a title for the graph. Label the horizontal and vertical axes.
2. Scale the horizontal and vertical axes. *Note:* Leave a half interval at the beginning and end of the graph; that is, begin the first column half a unit from the vertical axis.
3. Draw the first column so that it reaches a vertical height corresponding to 8 people. Label the section of the axis below the column as 'White shirt and black skirt/ trousers'.
4. Leave a gap (measuring one column width) between the first column and the second column.
5. Repeat steps **3** and **4** for each of the remaining uniform types.

DRAW



Bar graphs

Bar graphs are drawn in a similar manner to column graphs. However, there is one major difference. To draw a bar graph, numbers are placed on the horizontal axis and categories on the vertical axis. Therefore, instead of having vertical columns we have horizontal bars.

When drawing bar graphs, they should be presented on graph paper and have:

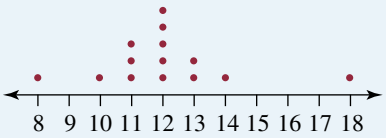
1. a title
2. labelled axes which are clearly and evenly scaled
3. horizontal bars of the same width
4. an even gap between each horizontal bar
5. the first horizontal bar beginning half a unit (that is, half the bar width) above the horizontal axis.

14.5.3 Dot plots

- **Dot plots** can be likened to picture graphs where each piece of data or score is represented by a single dot.
- Dot plots consist of a horizontal axis that is labelled and evenly scaled, and each data value is represented by a dot.
- Dot plots give a quick overview of a particular distribution. They show clustering, extreme values, and help to determine whether data should be grouped.
- If a score is repeated in a dot plot, a second dot is placed directly above the previous one. Once all values have been recorded, the data points, if neatly drawn and evenly spaced, resemble columns placed over a number line.
- Sometimes extreme values occur in a data set. They appear to be not typical of the rest of the data, and are called **outliers**. Sometimes they occur because measurements of the data have been incorrectly recorded. They serve as a reminder to always check the data collected.

WORKED EXAMPLE 9

Over a 2-week period, the number of packets of potato chips sold from a vending machine each day was recorded: 10, 8, 12, 11, 12, 18, 13, 11, 12, 11, 12, 12, 13, 14.
a Draw a dot plot of the data.
b Comment on the distribution.

THINK	WRITE
a 1 Use a scaled number line to include the full range of data recorded.	**a** 
2 Place a dot above the appropriate scale number for each value recorded.	
b Comment on interesting features of the dot plot, such as the range, clustering, extreme values and any practical conclusions that fit the situation.	**b** For the given dot plot: The scores extend from 8 to 18, that is, a range of ten. Mostly between 11 to 13 packets were sold. Sales of 8 and 18 packets of chips were extremely low. A provision of 20 packets of chips each day should cover the most extreme demand.

14.5.4 Stem-and-leaf plots

- When data are being displayed, a stem-and-leaf plot may be used as an alternative to the frequency distribution table.
- Sometimes 'stem-and-leaf plot' is shortened to 'stem plot'.
- Each piece of data in a stem plot is made up of two components: a stem and a leaf. For example, the value 28 is made up of a tens component (the stem) and the units component (the leaf) and would be written as:

Stem	*Leaf*
2	8

- It is important to provide a key when drawing up stem plots, as the plots may be used to display a variety of data, that is, values ranging from whole numbers to decimals.
- Ordered stem plots are drawn in ascending order.

WORKED EXAMPLE 10

Prepare an ordered stem plot for each of the following sets of data.
a 129, 148, 137, 125, 148, 163, 152, 158, 172, 139, 162, 121, 134
b 1.6, 0.8, 0.7, 1.2, 1.9, 2.3, 2.8, 2.1, 1.6, 3.1, 2.9, 0.1, 4.3, 3.7, 2.6

THINK

a 1 Rule two columns with the headings 'Stem' and 'Leaf'.

2 Include a key to the plot that informs the reader of the meaning of each entry.

3 Make a note of the smallest and largest values of the data (that is, 121 and 172 respectively). List the stems in ascending order in the first column (that is, 12, 13, 14, 15, 16, 17).
Note: The hundreds and tens components of the number represent the stem.

4 Systematically work through the given data and enter the leaf (unit component) of each value in a row beside the appropriate stem.
Note: The first row represents the interval 120–129, the second row represents the interval 130–139 and so on.

5 Redraw the stem plot so that the numbers in each row of the leaf column are in ascending order.

b 1 Rule the stem and leaf columns and include a key.

2 Make a note of the smallest and largest values of the data (that is, 0.1 and 4.3 respectively). List the stems in ascending order in the first column (that is, 0, 1, 2, 3, 4).
Note: The units components of the decimal represent the stem.

3 Systematically work through the given data and enter the leaf (tenth component) of each decimal in a row beside the appropriate stem.
Note: The first row represents the interval 0.1 − 0.9, the second row represents the interval 1.0 − 1.9 and so on.

WRITE

a Key: 12 | 1 = 121

Stem	*Leaf*
12	9 5 1
13	7 9 4
14	8 8
15	2 8
16	3 8
17	2

Key: 12 | 1 = 121

Stem	*Leaf*
12	1 5 9
13	4 7 9
14	8 8
15	2 8
16	3 8
17	2

b Key: 0 | 1 = 0.1

Stem	*Leaf*
0	8 7 1
1	6 2 9 6
2	3 8 1 9 6
3	1 7
4	3

4 Redraw the stem plot so that the numbers in each row of the leaf column are in ascending order to produce an ordered stem plot.

Key: 0 | 1 = 0.1

Stem	*Leaf*
0	1 7 8
1	2 6 6 9
2	1 3 6 8 9
3	1 7
4	3

The advantage of using a stem plot compared with a grouped frequency distribution table is that all the original data are retained. It is therefore possible to identify smallest and largest values, as well as repeated values. Measures of centre (such as mean, median and mode) and spread (range) are able to be calculated. This cannot be done when values are grouped in class intervals. When two sets of data are related, we can present them as back-to-back stem plots.

WORKED EXAMPLE 11

The ages of male and female groups using a ten-pin bowling centre are listed.

Males: 65, 15, 50, 15, 54, 16, 57, 16, 16, 21, 17, 28, 17, 27, 17, 22, 35, 18, 19, 22, 30, 34, 22, 31, 43, 23, 48, 23, 46, 25, 30, 21.

Females: 16, 60, 16, 52, 17, 38, 38, 43, 20, 17, 45, 18, 45, 36, 21, 34, 19, 32, 29, 21, 23, 32, 23, 22, 23, 31, 25, 28.

Display the data as a back-to-back stem plot and comment on the distribution.

THINK

1 Rule three columns, headed Leaf (female), Stem and Leaf (male).

2 Make a note of the smallest and largest values of both sets of data (15 and 65). List the stems in ascending order in the middle column.

3 Beginning with the males, work through the given data and enter the leaf (unit component) of each value in a row beside the appropriate stem.

4 Repeat step **3** for the females' set of data.

WRITE

Key: 1 | 5 = 15

Leaf (female)	*Stem*	*Leaf (male)*
9 8 7 7 6 6	1	5 5 6 6 6 7 7 7 8 9
8 5 3 2 3 3 1 9 1 0	2	1 8 7 2 2 2 3 3 5 1
1 2 2 4 6 8 8	3	5 0 4 1 0
5 5 3	4	3 8 6
2	5	0 4 7
0	6	5

5 Include a key to the plot that informs the reader of the meaning of each entry.

6 Redraw the stem plot so that the numbers in each row of the leaf columns are in ascending order. *Note:* The smallest values are closest to the stem column and increase as they move away from the stem.

Key: 1 | 5 = 15

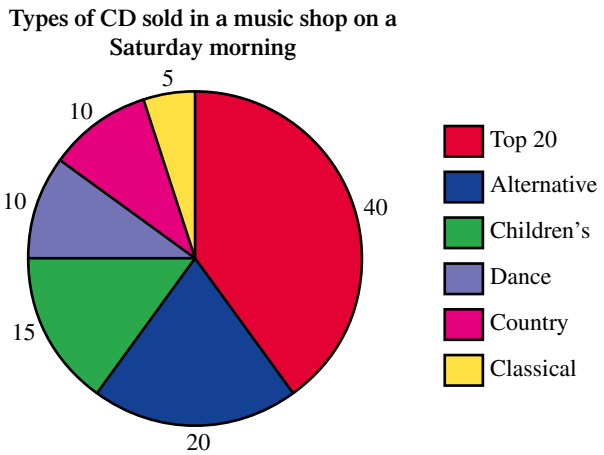
Leaf (female)	*Stem*	*Leaf (male)*
9 8 7 7 6 6	1	5 5 6 6 6 7 7 7 8 9
9 8 5 3 3 3 2 1 1 0	2	1 1 2 2 2 3 3 5 7 8
8 8 6 4 2 2 1	3	0 0 1 4 5
5 5 3	4	3 6 8
2	5	0 4 7
0	6	5

7 Comment on any interesting features.

The youngest male attending the ten-pin bowling centre is 15 and the oldest 65; the youngest and oldest females attending the ten-pin bowling centre are 16 and 60 respectively. Ten-pin bowling is most popular for men in their teens and 20s, and for females in their 20s and 30s.

14.5.5 Pie graphs

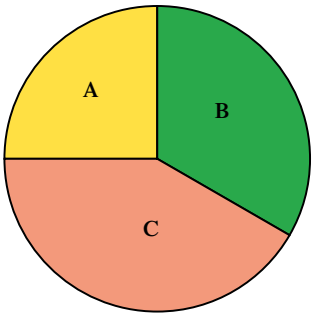
- Pie graphs are also called pie charts or sector graphs because they are made up of sectors of a circle.



- The pie graph shown above has six different sectors.
- Sectors should be ordered from largest to smallest in a clockwise direction (from 12 o'clock).
- Each sector must be labelled appropriately either on the graph or using a legend.

Calculating the angle for each sector:
The angle for each sector is determined by first calculating the fraction for each category.

For example, in the following pie graph:



- category A contains $\frac{1}{4}$ of the data, so it occupies $\frac{1}{4}$ of the circle
- category B contains $\frac{4}{12}$ or $\frac{1}{3}$ of the data, so it occupies $\frac{4}{12}$ or $\frac{1}{3}$ of the circle.

Category	Frequency	Fraction of total	Angle at centre of circle
A	3	$\frac{3}{12} = \frac{1}{4}$	$\frac{1}{4} \times 360° = 90°$
B	4	$\frac{4}{12} = \frac{1}{3}$	$\frac{1}{3} \times 360° = 120°$
C	5	$\frac{5}{12}$	$\frac{5}{12} \times 360° = 150°$
Total	12		

- A protractor is needed to draw the angles for each sector.
- Remember that there are 360° in a circle.

WORKED EXAMPLE 12

Of 120 people surveyed about where they would prefer to spend their holidays this year, 54 preferred to holiday in Australia, 41 preferred to travel overseas and 25 preferred to stay at home. Represent the data as a pie graph.

THINK

1 Draw a table to collate the data and calculate the angle for each sector.
- Find the total number of people surveyed. The total of the category frequencies is 120, so 120 people were surveyed. Write 120 as the total of the 'Frequency' column, as shown in blue.
- Find the fraction that each category is of the total number of people surveyed. The category 'Australia' contains 54 out of the 120 people surveyed, so write $\frac{54}{120}$ in the 'Fraction of data set' column, as shown in red.
- Find the fractional amount of 360° for each category. The 'Australia' category contains 54 of the 120 of the people surveyed, so it will take up $\frac{54}{120}$ of the circle or $\frac{54}{120}$ of 360°, as shown in green.

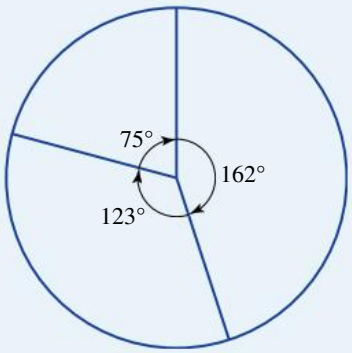
WRITE

Category	Frequency	Fraction of data set	Angle at centre of circle
Australia	54	$\frac{54}{120}$	$\frac{54}{{}^{1}\cancel{120}} \times \frac{{}^{3}\cancel{360}°}{1} = 162°$
Overseas	41	$\frac{41}{120}$	$\frac{41}{{}^{1}\cancel{120}} \times \frac{{}^{3}\cancel{360}}{1} = 123°$
At home	25	$\frac{25}{120}$	$\frac{25}{{}^{1}\cancel{180}} \times \frac{{}^{3}\cancel{360}}{1} = 75°$
Total	120		

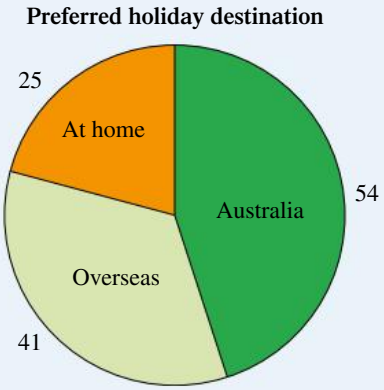
2 Check that the total of the angles is 360° (or within 2° of 360).

$162° + 123° + 75° = 306°$

3
- Draw a circle with a pair of compasses and mark the centre.
- Use a ruler to draw a vertical line running from the centre directly upwards to the edge of the circle; that is, in the 12 o'clock position.
- Use a protractor to measure an angle of 162° (the largest angle) in a clockwise direction from this line, and then rule in the next line to form a sector.
- From this new line, continue in a clockwise direction to measure the next largest angle (123°), and then rule in the next line.
- Continue until all sectors have been constructed.



4 Label each sector and include a title for the completed pie graph. Write the frequency for each sector next to the section.



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Complete this digital doc: Spreadsheet: Bar graphs (doc-3442)

Complete this digital doc: Spreadsheet: Dot plots (doc-3443)

Exercise 14.5 Representing data graphically

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–13, 23	Questions: 1–4, 5a, d, 6, 7, 8a, c, e, 9–13, 17, 23, 24	Questions: 1–4, 5c, d, 6, 8e, 9c, 10–24

Individual pathway interactivity: int-4381

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To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note*: Question numbers may vary slightly.

Fluency

1. WE8 Beth surveyed the students in her class to find out their method of travelling to school. Her results are shown in Table 1. Construct a column graph to display the data.

TABLE 1

Transport	Tally	Frequency
Car	𝍸 𝍸 𝍸	15
Tram	𝍸 ||||	9
Train	𝍸 𝍸 𝍸 |||	18
Bus	𝍸 |||	8
Bicycle	|||	3
	Total	53

2. Construct a column graph to display the data shown in Table 2, showing the mean daily maximum temperatures for each month in Cairns, Queensland.

3. The data in Table 3 show the number of students absent from school each day in a fortnight. Construct a bar graph to display the data.

4. WE9 Over a 2-week period, the number of packets of potato chips sold from a vending machine each day was recorded as follows:
15, 17, 18, 18, 14, 16, 17, 6, 16, 18, 16, 16, 20, 18.
a. Draw a dot plot of the data.
b. Comment on the distribution.

5. Draw a dot plot for each of the following sets of data:
a. 2, 0, 5, 1, 3, 3, 2, 1, 2, 3
b. 18, 22, 20, 19, 20, 21, 19, 20, 21
c. 5.2, 5.5, 5.0, 5.8, 5.3, 5.2, 5.6, 5.3, 6.0, 5.5, 5.6
d. 49, 52, 60, 55, 57, 60, 52, 66, 49, 53, 61, 57, 66, 62, 64, 48, 51, 60.

6. WE10a The following data give the number of fruit that have formed on each of 40 trees in an orchard:
29, 37, 25, 62, 73, 41, 58, 62, 73, 67, 47, 21, 33, 71, 92, 41, 62, 54, 31, 82, 93, 28, 31, 67, 29, 53, 62, 21, 78, 81, 51, 25, 93, 68, 72, 46, 53, 39, 28, 40
Prepare an ordered stem plot that displays the data.

TABLE 2

Month	Mean daily maximum temperature (°C)
January	31.8
February	31.5
March	30.7
April	29.3
May	27.6
June	25.9
July	25.6
August	26.4
September	27.9
October	29.7
November	30.8
December	31.8

TABLE 3

Day	Number of students absent
Monday	15
Tuesday	17
Wednesday	20
Thursday	10
Friday	14
Monday	16
Tuesday	14
Wednesday	12
Thursday	5
Friday	14

7. The number of errors made each week by 30 machine operators is recorded below:
12, 2, 0, 10, 8, 16, 27, 12, 6, 1, 40, 16, 25, 3, 12, 31, 19, 22,
15, 7, 17, 21, 18, 32, 33, 12, 28, 31, 32, 14

Prepare an ordered stem plot that displays the data.

8. Prepare an ordered stem plot for each of the following sets of data:

a. 132, 117, 108, 129, 165, 172, 145, 189, 137, 116, 152, 164, 118
b. 131, 173, 152, 146, 150, 171, 130, 124, 114
c. 196, 193, 168, 170, 199, 186, 180, 196, 186, 188, 170, 181, 209
d. 207, 205, 255, 190, 248, 248, 248, 237, 225, 239, 208, 244
e. 748, 662, 685, 675, 645, 647, 647, 708, 736, 691, 641, 735

9. WE10b Prepare an ordered stem plot for each of the following sets of data:

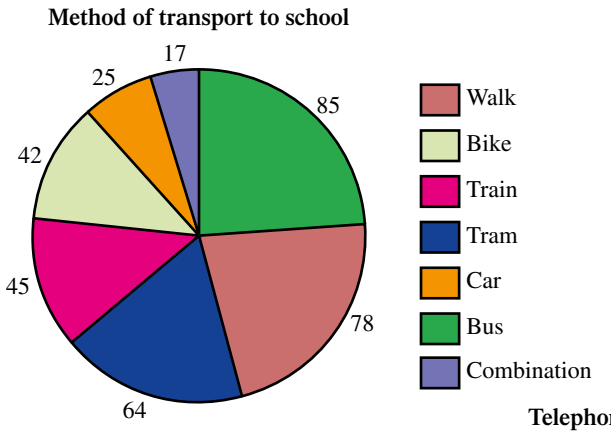
a. 1.2, 3.9, 5.8, 4.6, 4.1, 2.2, 2.8, 1.7, 5.4, 2.3, 1.9
b. 2.8, 2.7, 5.2, 6.2, 6.6, 2.9, 1.8, 5.7, 3.5, 2.5, 4.1
c. 7.7, 6.0, 9.3, 8.3, 6.5, 9.2, 7.4, 6.9, 8.8, 8.4, 7.5, 9.8
d. 14.8, 15.2, 13.8, 13.0, 14.5, 16.2, 15.7, 14.7, 14.3, 15.6, 14.6, 13.9, 14.7, 15.1, 15.9, 13.9, 14.5
e. 0.18, 0.51, 0.15, 0.02, 0.37, 0.44, 0.67, 0.07

10. WE11 The number of goals scored in football matches by Mitch and Yani were recorded as follows:

Mitch	0	3	1	0	1	2	1	0	0	1
Yani	1	2	0	1	0	1	2	2	1	1

Display the data as a back-to-back stem plot and comment on the distribution.

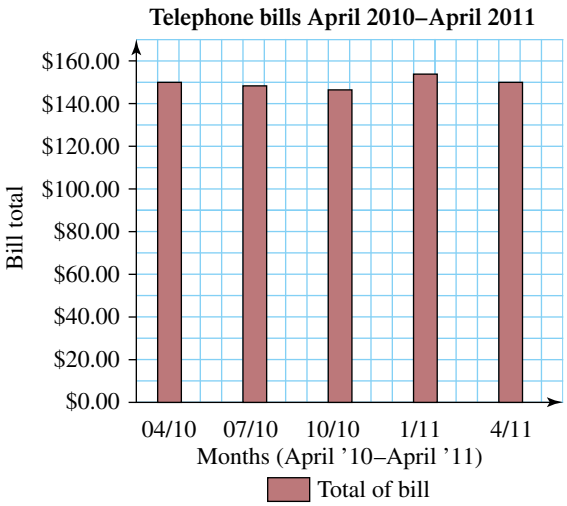
11. WE12 A survey was conducted of a group of students to determine their method of transport to school each day. The following pie graph displays the survey's results.



a. How many students were surveyed?
b. What is the most common method of transport to school?

Understanding

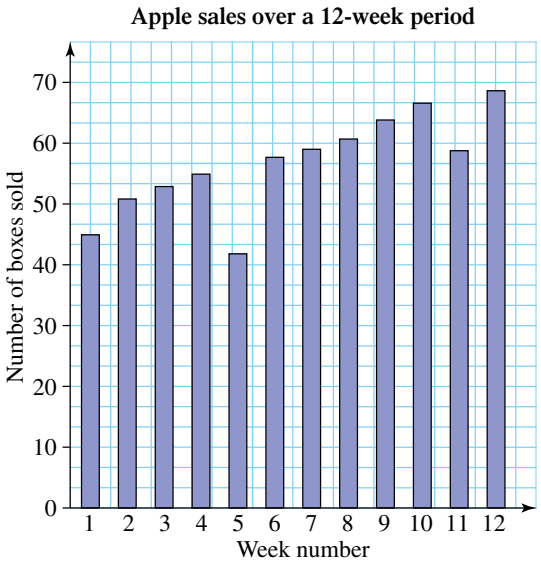
12. Telephone bills often include a graph showing your previous bill totals. Use the Telephone bills column graph to answer the following questions.



a. What is the title of this graph?
b. What is the horizontal axis label?
c. What is the vertical axis label?
d. How often does this person receive a phone bill?

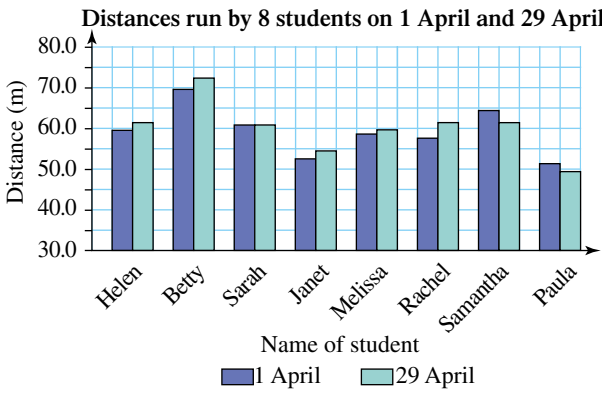
e. In which month was the bill the highest?
f. Was each bill for roughly the same amount?
g. If you answered yes to part **f**, approximately how much was the amount?
h. Why would it be useful to receive a graph like this with your phone bill?
i. If the next bill was for $240.09, would this be normal? Why?
j. How much (approximately) do phone calls from this phone cost per month?

13. An apple producer records his sales for a 12-week period.



a. How many boxes were sold in the first week?
b. How many boxes were sold in the fifth week?
c. How many boxes were sold in the eighth week?
d. The values for some weeks may be unusual. Which ones might be unusual? Explain your answer.
e. What might cause unusual values in a graph like this?
f. Does the graph indicate that apple sales are improving? Explain your answer.

14. In their physical education class the girls in a Year 7 class were asked to sprint for 10 seconds. The teacher recorded their results on 2 different days. The following graph displays the results collected.



a. Why are there 2 columns for each girl?
b. Which girl ran the fastest on either day?
c. How far did she run on each day?
d. Which girl improved the most?
e. Were there any students who did not improve? Who were they?
f. Could this graph be misleading in any way? Explain your answer.
g. Why might the graph's vertical axis start at 30 m?

15. The following table shows the different sports played by a group of Year 7 students.

a. Copy and complete the table. **b.** Draw a pie graph to display the data.

Sport	Number of students	Fraction of students	Angle at centre of circle
Basketball	55	$\frac{55}{180}$	$\frac{55}{\cancel{180}^{1}} \times \frac{\cancel{360^{\circ}}^{3}}{1} = 110^{\circ}$
Netball	35	$\frac{35}{180}$	
Soccer	30		
Football	60		
Total	180		

16. Compare and comment on the range, clustering and extreme values (if any) for the dot plots in question **5**.

17. The following stem plot gives the age of members of a theatrical group.

Key: 2 | 4 = 24

Stem	*Leaf*
1	7 8 8 9 9
2	2 4 7 9
3	1 3 3 8
4	0 2 2 2 6 6
5	5 7
6	4

a. How many people are in the theatrical group?
b. What is the age of the youngest member of the group?
c. What is the age of the oldest member of the group?
d. How many people are over 30 years of age?
e. What age is the most common in the group?
f. How many people are over 65 years of age?

18. Swim times, in seconds, over 100 metres were recorded for a random sample of 20 swimmers: 10.8, 11.0, 12.0, 13.2, 12.4, 13.9, 11.8, 12.8, 14.0, 15.0, 11.2, 12.6, 12.5, 12.8, 13.6, 11.5, 13.6, 10.9, 14.1, 13.9.
a. Show the data as a stem plot.
b. Comment on the range of performance and other interesting points.
c. What conclusions could be drawn about the swimmers' performance?

19. Answer the following questions for the back-to-back stem plot in question **10**.
a. How many times did each player score more than 1 goal?
b. Who scored the greatest number of goals in a match?
c. Who scored the greatest number of goals overall?
d. Who is the more consistent performer?

20. Percentages in a mathematics exam for two classes were as follows:

9A	32	65	60	54	85	73	67	65	49	96	57	68
9B	46	74	62	78	55	73	60	75	73	77	68	81

a. Construct a back-to-back stem plot of the data.
b. What percentage of each group scored above 50?
c. Which group had more scores over 80?
d. Compare the clustering for each group.
e. Comment on extreme values.
f. Calculate the *average* percentage for each group.
g. Show a back-to-back dot plot of the data (use colour).
h. Compare class performances by reference to both graphs.

Reasoning

21. Ten randomly chosen students from Class A and Class B each sit for a test in which the highest possible mark is 10. The results of the ten students from the two classes are:

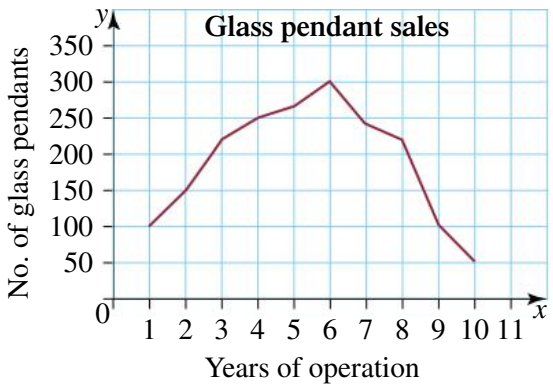
Class A:	1	2	3	4	5	6	7	8	9	10
Class B :	1	2	2	3	3	4	4	5	9	10

a. Graphically display the data on a dot plot.
b. Calculate measures of centre and spread.
c. Explain any similarities or differences between the results of the two classes.

22. Explain and give an example of the effect that outliers in a set of data have on the:
a. mean **b.** median **c.** mode **d.** range.

Problem solving

23. Lisa created a line graph to illustrate the number of glass pendants she has sold for each of the last ten years. She intends to use the graph for a presentation at her bank, in order to obtain a loan.
a. In what year of operation did she sell the most pendants? How many did she sell?
b. In what year of operation did she sell the least number of pendants? How many did she sell?
c. Is there a trend in the sales of the pendants?
d. Will this trend help Lisa obtain the loan?



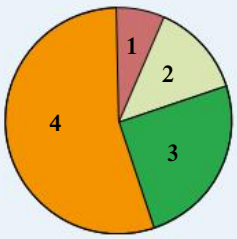
24. State which type of data is represented by the following graphs: column and bar graphs, dot plots, stem-and-leaf plots and pie graphs.

Reflection

Why is it important to use a key with all stem plots? Give an example to illustrate.

CHALLENGE 14.2

The pie graph shown has four sectors representing different amounts. Starting at the smallest sector, each new sector is double the size of the previous one. Without using a protractor, what is the third sector as a percentage? Show your working.



14.6 Comparing data

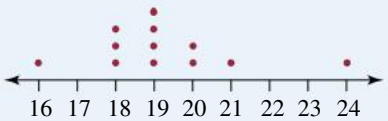
14.6.1 Determining measures of centre and spread from graphs

We have considered the calculation of measures of centre and spread from listed data. We also need to know how to calculate these measures from graphs of individual data. We can then make comparisons between data presented in listed form and graphical form.

When data are displayed graphically, their spread may be obvious, but we often need to calculate their measures of centre so that we can understand them better. We also need to be able to determine which measure of centre best represents the data.

WORKED EXAMPLE 13

Consider this dot plot.



a Use the dot plot to determine the:
i mean **ii median**
iii mode **iv range.**
b Comment on the most suitable measure of centre for the data.

THINK	WRITE
a i 1 Find the total of the values.	**a** Total of values $= 16 + 3 \times 18 + 4 \times 19 + 2 \times 20 + 21 + 24$ $= 231$
2 Count the number of values.	There are 12 values.
3 Find the mean by dividing the total by the number of values.	$\text{Mean} = \frac{\text{total of values}}{\text{number of values}}$ $= \frac{231}{12}$ $= 19.25$
ii 1 The values are already in order. The median is the middle value. There are 12 values, so the middle one is the average of the 6th and 7th values. Locate these.	The middle position of the 12 values is between the 6th and 7th values. These are both 19.
2 Calculate the average of these.	The median value is 19.
iii The mode is the most common value. Look for the one which occurs most frequently.	The mode is 19.
iv The range is the difference between the highest value and the lowest value.	Range $= 24 - 16$ $= 8$
b Look at the measures of mean, median and mode to see which best represents the values in terms of their closeness to the centre.	**b** The values of mean (19.25), median (19) and mode (19) are all quite close together, so any of these measures could be used to represent the data. There appear to be two outliers (16 and 24). These two tend to cancel out the effect of each other.

WORKED EXAMPLE 14

Consider this stem plot.

Key: 1 | 8 = 18

Stem	*Leaf*
1	**8 9**
2	**2 2 5 7 7 8**
3	**0 1 4 6 7**
4	**0 5**

a Use the stem plot to determine the:
- **i mean**
- **ii median**
- **iii mode**
- **iv range.**

b Comment on the most suitable measure of centre for the data.

THINK	WRITE
a i 1 Find the total of the values.	**a** Total of values $= 18 + 19 + 22 + 22 + 25 + 27 + 27 + 28 + 30 + 31 + 34 + 36 + 37 + 40 + 45$ $= 441$

2 Count the number of values.

There are 15 values.

3 Find the mean by dividing the total by the number of values.

$$\text{Mean} = \frac{\text{total of values}}{\text{number of values}} = \frac{441}{15} = 29.4$$

ii The values are already in order. The median is the middle value. There are 15 values, so the middle one is the 8th value. Locate this.

The middle position of the 15 values is the 8th value. This is 28.
The median value is 28.

iii The mode is the most common value. Look for the one which occurs most frequently.

There are two modes (it is bimodal) — 22 and 27.

iv The range is the difference between the highest value and the lowest value.

$\text{Range} = 45 - 18 = 27$

b Look at the measures of mean, median and mode to see which best represents the values in terms of their closeness to the centre.

b The values of mean (29.4), median (28) and modes (22 and 27) are quite different in this case. There do not appear to be any outliers. The mean or median could be used to represent the centre of this set of data.

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Exercise 14.6 Comparing data

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–7, 11, 12, 17	Questions: 1–9, 10, 12, 14, 16, 17	Questions: 1–17

Individual pathway interactivity: int-4382

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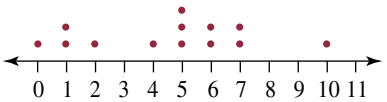
To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note*: Question numbers may vary slightly.

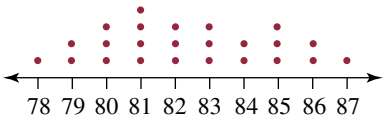
Fluency

1. WE13 Consider this dot plot.



a. Use the dot plot to determine the:
i. mean **ii.** median **iii.** mode **iv.** range.
b. Disregard the score of 10, and recalculate each of these values.
c. Discuss the differences/similarities in your two sets of results.

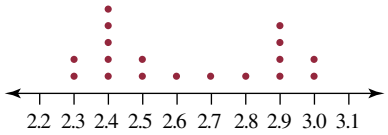
2. Consider this dot plot.



a. Use the dot plot to determine the:
i. mean **ii.** median **iii.** mode **iv.** range.

b. Comment on the most suitable measure of centre for the data.

3. Consider this dot plot.



a. Use the dot plot to determine the:
i. mean **ii.** median
iii. mode **iv.** range.

b. Comment on the most suitable measure of centre for the data.

4. WE14 Consider this stem plot.

Stem	*Leaf*
1	0 2
2	1 3 3 5
3	
4	4

Key: 1 | 0 = 10

a. Use the stem plot to determine the:
i. mean **ii.** median **iii.** mode **iv.** range.

b. Disregard the score of 44, and recalculate each of these values.

c. Discuss the differences/similarities in your two sets of results.

5. Consider this stem plot.

Stem	*Leaf*
6.1	8 8 9
6.2	0 5 6 8
6.3	0 1 2 4 4 4

Key: 6.1 | 8 = 6.18

a. Use the graph to determine the:
i. mean **ii.** median **iii.** mode **iv.** range.

b. Comment on the most suitable measure of centre for the data.

Understanding

6. A survey of the number of people in each house in a street produced these data: 2, 5, 1, 6, 2, 3, 2, 1, 4, 3, 4, 3, 1, 2, 2, 0, 2, 4.

a. Prepare a frequency distribution table with an $f \times x$ column and use it to find the average (mean) number of people per household.

b. Draw a dot plot of the data and use it to find the median number per household.

c. Find the modal number per household.

d. Which of the measures would be most useful to:
i. real estate agents renting out houses
ii. a government population survey
iii. an ice-cream mobile vendor?

7. A small business pays these wages (in thousands of dollars) to its employees: 18, 18, 18, 18, 26, 26, 26, 35, 80 (boss).

a. What is the wage earned by most workers?

b. What is the average wage?

c. Find the median of the distribution.

d. Which measure might be used in wage negotiations by:
i. the union, representing the employees (other than the boss)
ii. the boss?
Explain each answer.

8. The mean of 12 scores is 6.3. What is the total of the scores?

9. Five scores have an average of 8.2. Four of those scores are 10, 9, 8 and 7. What is the fifth score?

10. MC **a.** The score that shows up most often is the:

A. median **B.** mean **C.** mode **D.** average **E.** frequency

b. The term *average* in everyday use suggests:

A. the mean **B.** the mode **C.** the median

D. the total **E.** none of these

c. The measure affected by outliers (extreme values) is:

A. the middle **B.** the mode **C.** the median

D. the mean **E.** none of these

11. MC The back-to-back stem plot displays the heights of a group of Year 7 students.

Key: 13 | 7 = 137 cm

Leaf (boys)	*Stem*	*Leaf (girls)*
9 8	13	7 8
9 8 8 7 6	14	3 5 6
9 8 8	15	1 2 3 7
7 6 6 5	16	3 5 6
8 7 6	17	1

a. The total number of Year 7 students is:

A. 13 **B.** 17 **C.** 30 **D.** 36 **E.** 27

b. The tallest male and shortest female heights respectively are:

A. 186 cm and 137 cm **B.** 171 cm and 148 cm **C.** 137 cm and 188 cm

D. 178 cm and 137 cm **E.** none of these

Reasoning

12. A class of 26 students had a median mark of 54 in Mathematics; however, no-one actually obtained this result.

a. Explain how this is possible.

b. Explain how many must have scored below 54.

13. A soccer team had averaged 2.6 goals per match after 5 matches. After their sixth match, the average had dropped to 2.5. How many goals did they score in that latest match? Show your working.

14. A tyre manufacturer selects 48 tyres at random from the production line for testing. The total distance travelled during the safe life of each tyre is shown in the following table.

Distance in km ('000)	82	78	56	52	50	46
Number of tyres	2	4	10	16	12	4

a. Calculate the mean, median and mode.

b. Which measure best describes average tyre life? Explain.

c. Recalculate the mean with the 6 longest-lasting tyres removed. By how much is it lowered?

d. If you selected a tyre at random, what tyre life would it most likely have?

e. In a production run of 10 000 tyres, how many could be expected to last for a maximum of 50 000 km?

f. As the manufacturer, for what distance would you be prepared to guarantee your tyres? Why?

Problem solving

15. A clothing store records the dress sizes sold during a day in order to cater for the popular sizes. The results for a particular day are:

12, 14, 10, 12, 8, 12, 16, 10, 8, 12,
10, 12, 18, 10, 12, 14, 16, 10, 12, 12,
12, 14, 18, 10, 14, 12, 12, 14, 14, 10

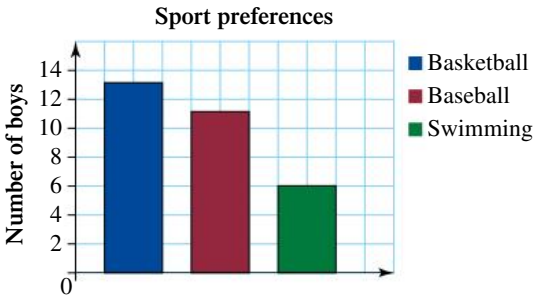
Rebecca is in charge of marketing and sales. She uses these figures in ordering future stock. From these figures she decided on the following ordering strategy. Order:

- the same number of size 8, 16 and 18 dresses
- three times this number of size 10 and size 14 dresses
- five times as many size 12 dresses as size 8, 16 and 18.

Comment on Rebecca's strategy.

16. In a survey, a group of boys was asked to name their favourite sport. Part of the data collected is shown in the bar chart below.

a. On the same chart, draw a bar to show that 10 boys named soccer as their favourite sport.



b. A boy is chosen at random. What is the probability that his favorite sport is baseball?

Reflection

Why do we need to be able to compare sets of data?

14.7 Review

14.7.1 Review questions

Fluency

1. MC The classification of data describing the number of iPods sold during the year is:

A. quantitative, discrete **B.** qualitative, nominal **C.** qualitative, ordinal
D. quantitative, continuous **E.** none of these

2. MC The classification of data which describe a person's mass is:

A. quantitative, discrete **B.** qualitative, nominal **C.** qualitative, ordinal
D. quantitative, continuous **E.** none of these

3. MC The data which describe sandwich types at a takeaway outlet are:

A. qualitative, nominal **B.** quantitative, discrete **C.** qualitative, ordinal

D. quantitative, continuous **E.** none of these

4. Eighty students in a school of 800 participated in a survey to find their favourite drinks. The results were: Coke: 20, Pepsi: 14, other soft drink: 10, milk/milk-based: 16, coffee: 9, tea: 4 and fruit juice: 7.

a. What was the most popular beverage?

b. What was the least popular beverage?

5. Use the following frequency distribution table to answer the following questions.

a. How many were in the class?

b. What was the most frequent score?

c. What was the least frequent score?

d. How many scored, at most, 15?

e. How many scored at least 16?

Score	Frequency
13	2
14	9
15	3
16	5
17	6
18	1

6. A random sample of 24 families was surveyed to determine the number of vehicles in each household. Here are the raw data collected:

2, 0, 3, 2, 1, 0, 2, 3, 4, 2, 2, 1, 0, 1, 3, 2, 1, 0, 0, 0, 2, 2, 3, 3.

a. Organise the data into a frequency distribution table.

b. How many families have no vehicles in their household?

c. How many families have 2 or more vehicles in their household?

d. Which score has the highest frequency?

e. Which is the highest score?

f. What fraction of families had 2 vehicles in their household?

7. For the following sets of data, find:

i. the mean **ii.** the mode **iii.** the median.

a. 4, 4, 4, 3, 2, 2, 3 **b.** 2, 4, 2, 4, 3, 7, 4 **c.** 4, 2, 6, 6, 5, 4, 6

d. 4, 5, 6, 6, 5, 5, 3 **e.** 1, 2, 1, 4, 1, 5, 1 **f.** 5, 8, 1, 7, 7, 5, 2, 7, 5, 5

g. 1, 5, 4, 5, 7 **h.** 2, 4, 2, 5, 7, 6, 3 **i.** 1, 7, 5, 8, 9, 5, 3

j. 1, 5, 8, 7, 4, 8, 5, 6, 8

8. Look at the graph at right, showing the test marks of a group of students.

a. What is the title for this graph?

b. What is the label on the vertical axis?

c. What is the label on the horizontal axis?

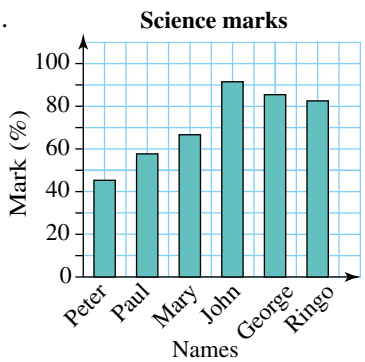
d. What scale is used on the vertical axis?

e. Which student has the highest mark?

f. Which student(s) would have failed if the pass mark was 50%?

g. What was Paul's mark?

h. What was George's mark?



9. Look at the graph at right, showing air quality.

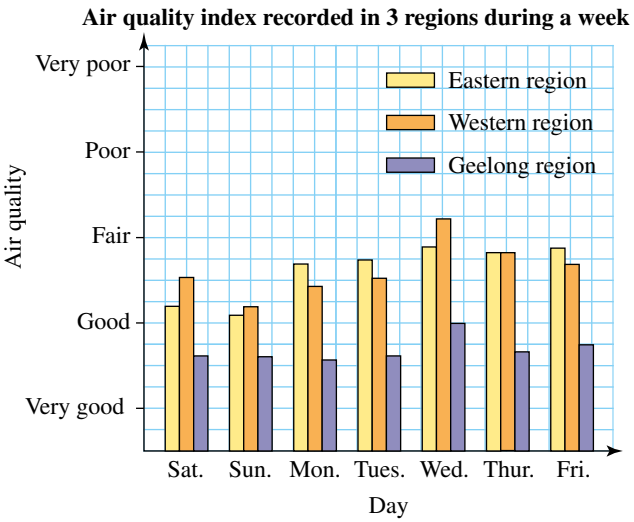
a. What does the horizontal axis represent?

b. What does the vertical axis represent?

c. Which region had a rating that ranged from very good to good every day?

d. Which day did the Western region have a fair air quality rating?

e. Which region had a good rating every day?



10. The following data give the speed of 30 cars recorded by a roadside speed camera along a stretch of road where the speed limit is 80 km/h. 75, 90, 83, 92, 103, 96, 110, 92, 102, 93, 78, 94, 104, 85, 88, 82, 81, 115, 94, 84, 87, 86, 96, 71, 91, 91, 92, 104, 88, 97

Present the data as an ordered stem plot.

11. Comment on the data recorded by the roadside speed camera in question **10**.

12. Complete the specified tasks for the given sets of data.

Set A	64	30	59	1	57	13	45	28
Set B	3	41	38	68	29	32	30	31

a. Compile a back-to-back stem plot of the two sets of data.

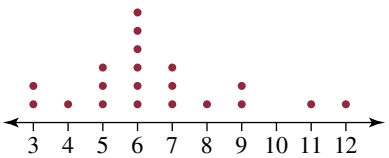
b. For each data set, find the:

i. lowest score **ii.** highest score **iii.** mean

iv. median **v.** range.

c. Compare the two data sets.

13. Consider this dot plot, which represents the number of soft drinks sold at lunch time over a 20 − day period from a vending machine in a cafeteria.



a. Use the dot plot to determine the:

i. mean **ii.** median **iii.** mode **iv.** range.

b. Comment on the distribution of the data.

14. Consider this stem plot, which represents the ages of participants in an aerobics class at a gym.

Key: 1 | 7 = 17

Stem	*Leaf*
1	4 7 8 8 9
2	0 1 3 5 6 7 9
3	2 4 5 6 7 8
4	2 5
5	3

a. Use the graph to determine the:

i. mean age **ii.** median age **iii.** modal age **iv.** range of ages.

b. Comment on the distribution of the data.

15. A group of 150 people were surveyed about the types of breakfast that they ate that morning. Of the people surveyed, 62 ate cereal, 70 ate toast, 12 ate a cooked breakfast and 6 ate no breakfast. Represent the data as a pie graph.

Problem solving

16. This table shows the maximum and minimum daily temperatures in a city over a one-week period.

Day	1	2	3	4	5	6	7
Maximum (°C)	12	13	10	11	9	10	8
Minimum (°C)	3	3	2	1	0	4	2

Use the table to answer the following questions.

a. What was the maximum temperature on day 3?
b. Which day had the lowest minimum temperature?
c. Which day was the coldest?
d. Which day had the warmest overnight temperature?
e. What was the temperature range (variation) on day 2?
f. Which day had the smallest range of temperatures?

18. Look at the following part of a teacher's mark book. Use it to answer the questions at the top of the following page.

Name	**Test 1**	**Test 2**
John	85	94
Peter	85	63
Mark	95	58
James	82	67
David	76	95
Rachel	62	85
Mary	87	75
Eve	94	63
Esther	68	68

a. What was the class mean for test 1?
b. What was the class mean for test 2?
c. In which test did the class do better?
d. How many students scored above the mean in test 1?
e. How many students scored above the mean in test 2?
f. What was the mode score for test 1?
g. What was the mode score for test 2?
h. What was the median score for test 1?
i. What was the median score for test 2?
j. Calculate the mean score for the two tests for each student and list them.
k. What is the range of values for these means?

19. Look at the following results of height measurements (in cm) for the students in class 9A:
145, 152, 148, 152, 163, 148, 165, 158, 159, 162, 145, 153, 156, 158, 157, 159, 169, 156, 156, 156, 152, 154, 128, 141, 154, 153, 156, 156, 165, 168.

a. What is the range of heights?
b. What is the mean height of the class?
c. What is the mode for the heights?
d. What is the median height of the class?
e. Are these three values (mean, mode, median) similar?
f. Why might this be so?

20. Use the table of values below to answer the questions on the following page.

Month	Number of flies (thousands)
January	24
February	28
March	26
April	20
May	18
June	15
July	12
August	10
September	11
October	12
November	15
December	19

a. In drawing a column graph using these data, what would be on the horizontal axis?
b. In drawing a column graph, what would be on the vertical axis?
c. What is the minimum value required on the vertical axis?
d. What is the maximum value required on the vertical axis?
e. What scale could be used for the vertical axis?
f. Draw a column graph to display these data.

21. Use the following table of values of students enrolled in elective classes to answer the questions below.

Subject	Number enrolled
Modern history	15
China in focus	12
Beginner's photography	16
First aid and fitness	18
Business enterprises	15
Consumer studies	12
Outdoor education	16

a. What is the minimum value that would be required on the horizontal axis if a bar graph was to be drawn using these data?
b. What is the maximum value that would be required on the horizontal axis if a bar graph was to be drawn?
c. Which class has the largest number of students?
d. If the maximum number of students for each of these electives is 15 students, which ones can take more students?
e. Which electives have too many students?
f. Draw a bar graph of the data.

22. The number of pets cared for by each of 20 families was surveyed, giving these data:
1, 2, 3, 2, 0, 1, 0, 2, 5, 3, 2, 1, 2, 0, 2, 0, 1, 3, 2, 1.
a. Organise the data into a frequency distribution table.
b. Draw a dot plot of the data and comment on the distribution.
c. Find the mode, median and mean.

23. a. For the given set of scores, 9, 3, 8, 5, 6, 9, 4, 5, 5, 25, find the:

i. mode **ii.** median **iii.** mean **iv.** outliers, if any

v. mean if outliers are omitted

vi. mean and median if 10 is added to each of the given scores.

b. Comment on any discoveries from part **a**.

24. This back-to-back stem plot shows the heights of a group of boys and girls.

Key: 13 | 7 = 137 cm

Leaf (boys)	*Stem*	*Leaf (girls)*
9 8	13	7 8
9 8 8 7 6	14	3 5 6
9 8 8	15	1 2 3 7
7 6 6 5	16	3 5 6
8 7 6	17	1

Give a full description of the heights of the boys and the girls.

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- **Try out this interactivity:** Word search: Topic 14 (int-2607)
- **Try out this interactivity:** Crossword: Topic 14 (int-2608)
- **Try out this interactivity:** Sudoku: Topic 14 (int-3174)
- **Complete this digital doc:** Concept map: Topic 14 (doc-10743)

Language

It is important to learn and be able to use correct mathematical language in order to communicate effectively. Create a summary of the topic using the key terms below. You can present your summary in writing or using a concept map, a poster or technology.

analysis
bar graph
categories
class interval
column graph
continuous
data collection
discrete
display
dot plot
frequency
frequency distribution table
graph
interpretation
legend
mean
median
mode
nominal
numerical
ordinal
organisation
outlier
quantitative
range
stem-and-leaf plot
systematically
values

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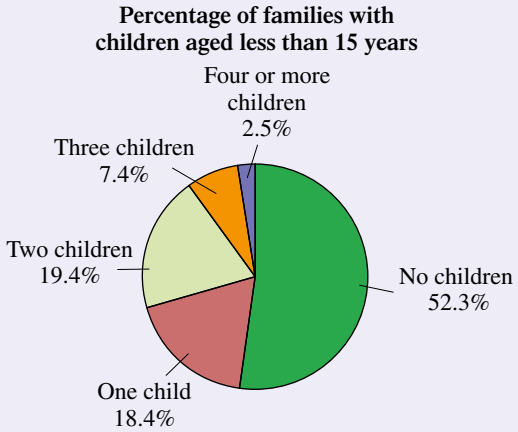
Investigation | Rich task

Families with children

Surveys help us to find out about lots of different things. They can be conducted by collecting information from an entire population or smaller groups. The information collected in a survey, known as data, can be examined and presented in many different ways. Graphs are commonly used to present the results of a survey.

The graph below shows the percentage of families in Australia with children aged less than 15 years, taken from the 2011 census.



Source: Australian Bureau of Statistics.

1. What type of graph has been used to display the information?
2. What is the most common category of Australian families with children aged less than 15 years?
3. What other type of graph can be used to present this information? Give an example.

Conduct your own survey on the number of children aged less than 15 in the families of your classmates. Compare your results with the results obtained from the 2011 census.

4. Record your survey results in the following frequency distribution table.

Number of children	Tally	Frequency	Percentage
1			
2			
3			
4 or more			
	Total		

5. Is the survey you conducted an example of a census or a sample? Explain.
6. Are the data you collected classified as categorical data or numerical data?
7. Present the information from your survey as a column graph. Use the percentage values on the vertical axis and number of children on the horizontal axis.
8. How do the results of your class compare with the results obtained in the 2011 census? What is the major difference in your results?
9. Design and conduct a new survey on a topic of interest. Carry out the survey on members of your class or expand it to include a larger target audience. Present your data as a poster with an appropriate graph to display the findings of your survey.

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Complete this digital doc: Code puzzle: What did Jacques Cousteau invent in 1942? (doc-14335)

Answers

Topic 14 Representing and interpreting data

Exercise 14.2 Classifying data

1. **a.** ii **b.** i **c.** v **d.** vi **e.** iii **f.** iv

2. **a.** Quantitative, discrete **b.** Qualitative, nominal **c.** Quantitative, continuous
d. Quantitative, continuous **e.** Qualitative, nominal **f.** Quantitative, continuous
g. Quantitative, discrete **h.** Quantitative, continuous **i.** Quantitative, discrete
j. Qualitative, nominal **k.** Quantitative, continuous **l.** Quantitative, discrete
m. Quantitative, discrete **n.** Qualitative, nominal **o.** Qualitative, ordinal
p. Quantitative, continuous **q.** Quantitative, discrete **r.** Qualitative, ordinal
s. Qualitative, nominal **t.** Qualitative, ordinal

3. Discrete data deal with values which are exact and must be counted, such as the number of people attending a football match. Continuous data deal with values which are measured and may assume decimal form such as the length of each football quarter.

4. Discuss with your teacher.

5. D **6.** A **7.** D **8.** 1250

9. Although represented by digits, this type of data is categorical because they do not represent numerical ordered values.

10. Possible answers:
a. How many minutes, on average, do you read per day? **b.** How many times a week do you play sport?
c. How many times a week do you play computer games?

11. Possible answers:
a. Do you do Maths homework every night? **b.** What books have you read this year?
c. What suburb do you live in?

Exercise 14.3 Displaying data in tables

1. **a.**

Score	Tally	Frequency
0	𝍸	5
1	𝍸	4
2	𝍸 𝍸	9
3	𝍸 𝍸	7
4	𝍸	2
5	𝍸	1
6	𝍸	1
7		0
8	𝍸	1
	Total	30

b. 30 **c.** 5 **d.** 12 **e.** 2 **f.** $\frac{2}{30} = \frac{1}{15}$

2.

Value (thousand dollars)	Tally	Frequency
100–109	𝍸	3
110–119	𝍸	1
120–129		0
130–139	𝍸	1
140–149	𝍸	3
150–159	𝍸	3
160–169	𝍸	2
170–179	𝍸	1
180–189	𝍸	2
190–199	𝍸	1
200–209	𝍸	1
	Total	18

3. a. The list is messy; it is difficult to see how many different sports there are and to gauge how many people prefer a particular sport.

b.

Sport	Tally	Frequency
Hockey	𝍸 III	8
Cricket	𝍸 I	6
Tennis	𝍸 II	7
Netball	𝍸 IIII	9
Swimming	𝍸 II	7
Golf	IIII	4
Scuba diving	III	3
Lacrosse	IIII	4
Lawn bowls	II	2
	Total	50

c. i. Netball **ii.** Lawn bowls

d. Yes, tennis and swimming had a frequency of 7 and golf and lacrosse had a frequency of 4.

4. a.

Score	Tally	Frequency
5	I	1
6	II	2
7	II	2
8	I	1
9	III	3
10	I	1
	Total	10

b.

Score	Tally	Frequency
0	𝍸	5
1	𝍸	5
2	𝍸	5
3	II	2
4	II	2
5		0
6	I	1
	Total	20

c.

Score	Tally	Frequency
45	I	1
46	I	1
47		0
48	II	2
49	I	1
50	II	2
51	II	2
52	II	2
53	IIII	4
54	III	3
55	I	1
56	II	2
57	II	2
58	II	2
59	I	1
60	II	2
	Total	28

d.

Score	Tally	Frequency
143	II	2
144	𝍸 I	6
145	𝍸	5
146	I	1
147	III	3
148	III	3
149	III	3
150	IIII	4
151		0
152	I	1
	Total	28

5. a. 33 **b.** 3 **c.** 10 **d.** 23 **e.** $\frac{1}{3}$

6. a.

Score	Tally	Frequency
0	𝍸 𝍸 |	11
1	𝍸 𝍸	10
2	𝍸 |	6
3	||	2
4	|	1
	Total	30

b. 11 **c.** 9 **d.** 0 **e.** 4 **f.** $\frac{1}{5}$

7.

Height	Tally	Frequency
140–144	||	2
145–149	𝍸 |	6
150–154	𝍸	4
155–159	|	1
160–164	𝍸	5
165–169	𝍸 |	6
170–174	||	2
175–179	|	1
180–184	|	1
	Total	28

8.

Vowel	Tally	Frequency
A	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 |	42
E	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸	55
I	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 |	38
O	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸	40
U	𝍸 𝍸 𝍸	15
	Total	190

9.

Type	Tally	Frequency
1 bedroom unit	𝍸	3
2 bedroom unit	𝍸	4
2 bedroom house	|	1
3 bedroom house	𝍸 |	6
4 bedroom house	||	2
	Total	181

A 3 bedroom house was the most popular.

10. C **11.** A **12.** B **13.** O 21.5%, U 8.3%

14. Makes it easier to collect and read the data.

15. Phone, Eugenie; SMS, Florence; email, Anthea; mail, Kim

16. Solution 1 (cross in centre)

X		X
X	X	
	X	X

Solution 2 (centre empty)

X	X	
X		X
	X	X

Challenge 14.1

Score	Frequency
0	4
1	16
2	20
3	4
4	1
Total	45

Exercise 14.4 Measures of centre and spread

1. a. 5 **b.** 6 **c.** 4 **d.** 6 **e.** 4
f. 3 **g.** 12 **h.** 14 **i.** 15 **j.** 13

2. a. 8 **b.** 3 **c.** 4 **d.** 2 **e.** 3
f. 4 **g.** 5 **h.** 2 **i.** 5 **j.** 3

3. a. 5 **b.** 3 **c.** 6 **d.** 3 **e.** 7
f. 11 **g.** 4 **h.** 2 **i.** 2.5 **j.** 1.5

4. a. 2 **b.** 2 **c.** 5 **d.** 7 **e.** 2.5
f. 5 **g.** 6 **h.** 7.5 **i.** 5.5 **j.** 7.5

5. a. 4 **b.** 8 **c.** 2 **d.** 4 **e.** 2
f. 3 **g.** 10 **h.** 4 **i.** 9 **j.** 3

6. a. 32 **b.** 24.1

7. B **8.** B **9.** D **10.** D **11.** C

12. 85% **13.** 104 shoes **14.** 47 students **15.** 38

16. a. Tom's 75.0, Sarah's 72.8 **b.** Sarah's sells cheaper petrol.

c. Tom may have seen the price on a very expensive day. There's a greater range of values at Sarah's petrol station.

17. 89.5% **18.** 72

19. a. Mode = 12

b. The mode shows which size to order more of. The mean and median would not show the more common sizes and would give only an indication of the middle of the range.

20. a. 1.75 m **b.** 2.21 m

c. Based on her past performances and her past range of values, she cannot jump this high.

21. a. 101.9

b. Both the mode (115) and median (112) give a better impression of how the team has performed, even though they could give a 'misleading' impression of the team's performance.

22. 133 cm, 133 cm, 142 cm, 151 cm

23. One possible answer is 5, 6, 12, 13, 14. If the five numbers are in ascending order, the third number must be 12 and the other 4 numbers must total 38.

24. Highest score is 49 and lowest score is 0. The scores would be 0, 1, 12, 13, 49.

25. Largest value for median is 15. Scores would be 0, 2, 15, 16, 17. Smallest value for median is 2. Scores would be 0, 1, 2, a, b where $a + b = 47$.

26. 43; highest score: 100; lowest score: 57. The scores would be 57, 77, 78, 79, 80, 97, 98, 99, 100.

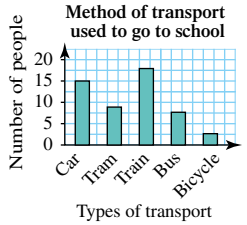
27. The coach's statement was not correct.

28. a. 15 **b.** 3 **c.** 0

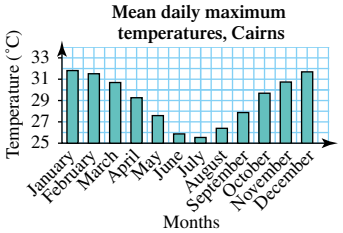
d. The median is the best measure as the mode is the lowest of all scores and the mean is inflated by one much larger figure.

Exercise 14.5 Representing data graphically

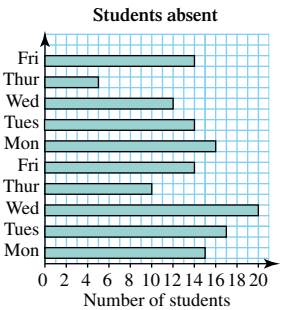
1.



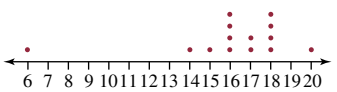
2.



3.



4. a.

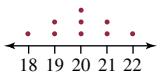


b. The scores lie between and include 6 to 20; that is, a range of fourteen.

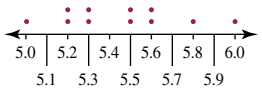
Mostly 16 to 18 packets were sold. Sales of 6 and 20 packets of chips were extremely low. A provision of 20 packets of chips each day should cover the most extreme demands.

5. a.

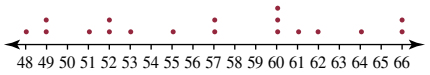
b.



c.



d.



6. Key: 2 | 7 = 27

Stem	*Leaf*
2	1 1 5 5 8 8 9 9
3	1 1 3 7 9
4	0 1 1 6 7
5	1 3 3 4 8
6	2 2 2 2 7 7 8
7	1 2 3 3 8
8	1 2
9	2 3 3

7. Key: 3 | 6 = 36

Stem	*Leaf*
0	0 1 2 3 6 7 8
1	0 2 2 2 2 4 5 6 6 7 8 9
2	1 2 5 7 8
3	1 1 2 2 3
4	0

8. a. Key: 12 | 7 = 127

Stem	*Leaf*
10	8
11	6 7 8
12	9
13	2 7
14	5
15	2
16	4 5
17	2
18	9

b. Key: 13 | 2 = 132

Stem	*Leaf*
11	4
12	4
13	0 1
14	6
15	0 2
16	
17	1 3

c. Key: 18 | 5 = 186

Stem	*Leaf*
16	8
17	0 0
18	0 1 6 6 8
19	3 6 6 9
20	9

d. Key: 23 | 7 = 237

Stem	*Leaf*
19	0
20	5 7 8
21	
22	5
23	7 9
24	4 8 8 8
25	5

e. Key: 65 | 2 = 652

Stem	*Leaf*
64	1 5 7 7
65	
66	2
67	5
68	5
69	1
70	8
71	
72	
73	5 6
74	8

9. a. Key: 1 | 7 = 1.7

Stem	*Leaf*
1	2 7 9
2	2 3 8
3	9
4	1 6
5	4 8

b. Key: 2 | 6 = 2.6

Stem	*Leaf*
1	8
2	5 7 8 9
3	5
4	1
5	2 7
6	2 6

c. Key: 6 | 7 = 6.7

Stem	*Leaf*
6	0 5 9
7	4 5 7
8	3 4 8
9	2 3 8

d. Key: 13 | 7 = 13.7

Stem	*Leaf*
13	0 8 9 9
14	3 5 5 6 7 7 8
15	1 2 6 7 9
16	2

e. Key: 1 | 5 = 0.15

Stem	*Leaf*
0	2 7
1	5 8
2	
3	7
4	4
5	1
6	7

10. Key: 0 | 0 = 0

Leaf (Mitch)	*Stem*	*Leaf (Yani)*
3 2 1 1 1 1 0 0 0 0	0	0 0 1 1 1 1 1 2 2 2

Mitch scored between 0 and 3 goals inclusive.

Yani scored between 0 and 2 goals inclusive.

11. a. 356 **b.** Bus

12. a. Telephone bills April 2010 to April 2011

b. Months **c.** Bill total **d.** Every 3 months (quarterly)

e. January 2011 **f.** Yes **g.** $150

h. A graph like this is useful to monitor the spending pattern over time and to decide whether there have been any unusual increases.

i. It would not be normal, but there might be reasons to explain why it was so much higher, such as overseas phone calls.

j. This phone costs about $50 per month.

13. a. 45 **b.** 42 **c.** 61

d. 5 and 11

For these weeks the sales decreased in comparison to other weeks.

e. Sometimes an unusual result may be caused by seasonal effects, or unusual situations in the particular survey or area.

f. It does indicate an improvement, despite these two low weeks, because there is an overall upward trend throughout the time period.

14. a. There are two columns for each girl because each girl ran on two occasions.

b. Betty

c. 1 April — 70 m, 29 April — 73 m

d. Rachel

e. Yes. Sarah — remained the same. Samantha and Paula sprinted at a slower pace.

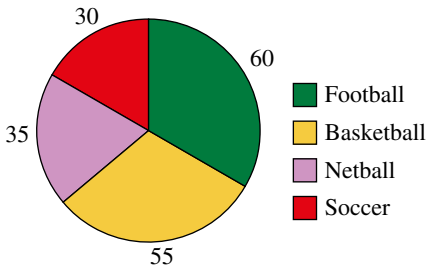
f. Yes, the graph could be misleading as the vertical scale begins at 30 m and not at 0 m. If the vertical scale was not read properly, it might appear that the girls covered a greater distance in a shorter period of time.

g. The graph's vertical axis starts at 30 m because no-one ran less than that distance and the teacher wanted to use all the space to show small differences in the distances.

15. a.

Sport	Number of students	Fraction of students	Angle at centre of circle
Basketball	55	$\frac{55}{180}$	$\frac{55}{\cancel{180}^{1}} \times \frac{\cancel{360}^{3}}{1} = 110°$
Netball	35	$\frac{35}{180}$	$\frac{35}{\cancel{180}} \times \frac{\cancel{360}^{2}}{1} = 70°$
Soccer	30	$\frac{30}{180}$	$\frac{30}{\cancel{180}} \times \frac{\cancel{360}^{2}}{1} = 60°$
Football	60	$\frac{60}{180}$	$\frac{60}{\cancel{180}} \times \frac{\cancel{360}^{2}}{1} = 120°$
Total	180		

b. Sports played by Year 7 students



16. Check with your teacher.

17. a. 22 **b.** 17 **c.** 64 **d.** 13 **e.** 42 **f.** 0

18. a. Key: 10 | 8 = 10.8

Stem	*Leaf*
10	8 9
11	0 2 5 8
12	0 4 5 6 8 8
13	2 6 6 9 9
14	0 1
15	0

b. Range = 4.2 Discuss in class.

c. Discuss in class.

19. a. Mitch: 2, Yani: 3 **b.** Mitch

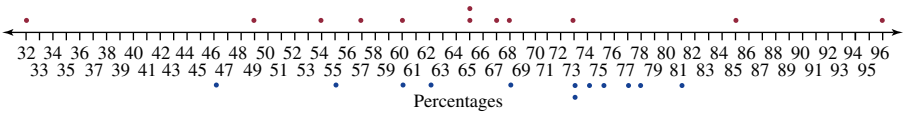
c. Yani: 11 **d.** Yani, as he scored goals in more games.

20. a. Key: 3 | 2 = 32

Leaf (9A)	*Stem*	*Leaf (9B)*
2	3	
9	4	6
7 4	5	5
8 7 5 5 0	6	0 2 8
3	7	3 3 4 5 7 8
5	8	1
6	9	

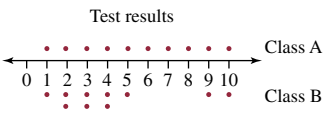
b. 9A: 83%; 9B: 92% **c.** 9A: 2 **d.** Discuss in class. **e.** Discuss in class. **f.** 9A: 64.3;9B: 68.5

g.



h. Discuss in class.

21. a.



b. Class A: Mean 5.5; median 5.5; no mode; range 9

Class B: Mean 4.3; median 3.5; modes 2, 3 and 4; range 9

c. Class A has a higher mean and median than Class B, while their range is the same. The results for Class A are scattered throughout the whole range, while those for Class B are concentrated more towards the lower end.

22. a–d. Class discussion. Generally, outliers affect the mean and range, having little effect on the median, and no effect on the mode.

23. a. In year 6 she sold 300. **b.** In year 10 she sold 50.

c. There are two trends. The sales of the glass pendants gradually increase from year 1 to year 6. However, after year 6 the sales decrease quickly.

d. No, the second trend is down, and will not help Lisa obtain the loan because the lowest sales are her most recent ones.

24. Categorical data: column and bar graphs and pie graphs. Numerical data: dot plots, stem-and-leaf plots.

Challenge 14.2

$26\frac{2}{3}\%$

Exercise 14.6 Comparing data

1. a. i. 4.5 **ii.** 5 **iii.** 5 **iv.** 10

b. i. 4.1 **ii.** 5 **iii.** 5 **iv.** 7

c. The mean and range are affected by the outlier. The median and mode have not been affected.

2. a. i. 82.4 **ii.** 82 **iii.** 81 **iv.** 9

b. The mean, median and mode are all quite close, so any could be used as a measure of centre.

3. a. i. 2.63 **ii.** 2.55 **iii.** 2.4 **iv.** 0.7

b. This distribution is quite spread out with a significant number at the lower end and towards the top. For this reason, the mode is probably the best measure of centre.

4. a. i. 22.6 **ii.** 23 **iii.** 23 **iv.** 34

b. i. 19 **ii.** 22 **iii.** 23 **iv.** 15

c. The mean and range are most affected by the outlier. The median is affected slightly, while the mode is not affected.

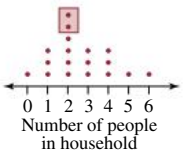
5. a. i. 6.27 **ii.** 6.28 **iii.** 6.34 **iv.** 0.16

b. The mean or median would probably be the best measures of centre in this case.

6. a.

Score x	Frequency f	Freq. × score $f \times x$
0	1	0
1	3	3
2	6	12
3	3	9
4	3	12
5	1	5
6	1	6
	$n = 18$	$\sum fx = 47$

b.



c. 2

d. i. Median **ii.** Mean **iii.** Mode

7. a. $18 000 **b.** $29 444 **c.** $26 000

d. i. Mode **ii.** Mean

8. 75.6 **9.** 7

10. a. C **b.** A **c.** D

11. a. C **b.** D

12. a. The median was calculated by taking the average of the 2 middle scores.

b. 13

13. 2

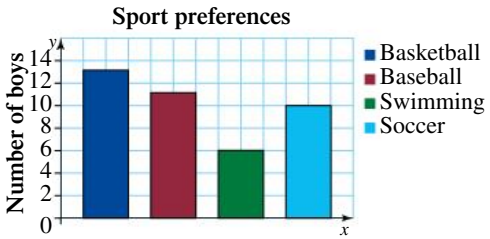
14. a. 55 250 km 52 000 km, 52 000 km **b.** Discuss in class.

c. 51 810 km; It is reduced by 3440 km. **d.** 52 000 km

e. 3333 **f.** 50 000 km; 92% last that distance or more.

15. Rebecca's strategy seems reasonable.

16. a.



b. $\frac{11}{40}$

Exercise 14.7 Review

1. A **2.** D **3.** A

4. a. Coke **b.** Tea

5. a. 26 **b.** 14 **c.** 18 **d.** 14 **e.** 12

6. a.

Cars	Frequency
0	6
1	4
2	8
3	5
4	1
Total	24

b. 6 **c.** 14 **d.** 2 **e.** 4 **f.** $\frac{1}{3}$

7. a. i. 3.14 **ii.** 4 **iii.** 3 **b. i.** 3.71 **ii.** 4 **iii.** 4
c. i. 4.71 **ii.** 6 **iii.** 5 **d. i.** 4.86 **ii.** 5 **iii.** 5
e. i. 2.14 **ii.** 1 **iii.** 1 **f. i.** 5.2 **ii.** 5 **iii.** 5
g. i. 4.4 **ii.** 5 **iii.** 5 **h. i.** 4.14 **ii.** 2 **iii.** 4
i. i. 5.43 **ii.** 5 **iii.** 5 **j. i.** 5.78 **ii.** 8 **iii.** 6

8. a. Science marks **b.** Percentage **c.** Names **d.** 1 unit 10%
e. John **f.** Peter **g.** 59% **h.** 89%

9. a. Days of week **b.** Air quality rating **c.** Geelong region
d. Wednesday **e.** Eastern region

10. Key: 3 | 6 = 36
11 | 5 = 11.5

Stem	*Leaf*
7	1 5 8
8	1 2 3 4 5 6 7 8
9	0 11 2 2 2 3 4 4 6 6 7
10	2 3 4 4
11	0 5

11. Only 3 cars were travelling under the 80 km/h speed limit. The slowest speed recorded was 71 km/h, while the fastest speed recorded was 115 km/h. Most of the recorded speeds were in the 90–99 km/h class interval. The most common (modal) speed recorded was 92 km/h.

12. a. Key: 3 | 1 = 31

Leaf Set A	*Stem*	*Leaf Set B*
1	0	3
3	1	
8	2	9
0	3	0 1 2 8
5	4	1
9 7	5	
4	6	8

b. Set A? Set B?
i. 1 **i.** 3
ii. 64 **ii.** 68
iii. 37.1 **iii.** 34
iv. 37.5 **iv.** 31.5
v. 63 **v.** 65

c. The two sets have similar minimum and maximum values, giving them a similar range. The mean of Set A is slightly higher than that of Set B. The scores of Set A are spread out more evenly than those in Set B, which tend to be clustered towards the middle.

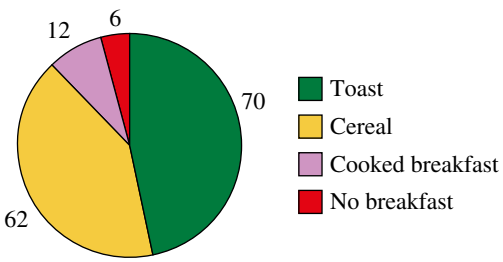
13. a. i. 6.55 **ii.** 6 **iii.** 6 **iv.** 9

b. There appear to be two outliers (11 and 12). The mean, median and mode values are quite close, indicating that it is common for 6 soft drinks to be sold quite frequently. The minimum number sold over the period is 3, while the maximum is 12.

14. a. i. 29 years **ii.** 27 years **iii.** 18 years **iv.** 39 years

b. The ages of the group range from 14 years to 53 years, with most of the group being under 40. The mean or median values would be a good representation of the age of the group.

15. People who ate different types of breakfast



16. a. 10°C **b.** Day 5 **c.** Day 7 **d.** Day 6 **e.** 10°C **f.** Days 6 and 7

17. a. 81.56 **b.** 74.22 **c.** Test 1 **d.** 6 **e.** 4

f. 85 **g.** 63 **h.** 85 **i.** 68

j. John 89.5, Peter 74, Mark 76.5, James 74.5, David 85.5, Rachel 73.5, Mary 81, Eve 78.5, Esther 68.

k. 21.5

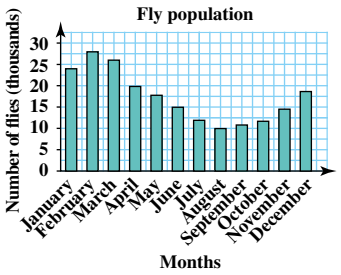
18. a. 41 cm **b.** 154.8 cm **c.** 156 cm **d.** 156 cm **e.** Yes

f. There are quite a lot of values which are very similar, so the mean, median and mode are fairly close.

19. a. Months **b.** Number of flies **c.** 10 000 **d.** 28 000

e. 1 cm 5000 flies (if beginning from 0 flies) or 1 cm 2000 flies (if beginning from 10 000 flies)

f.

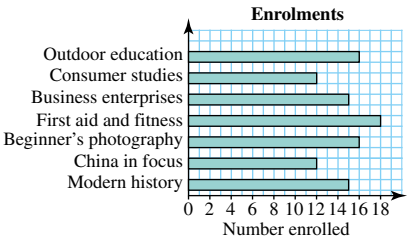


20. a. 12 **b.** 18

c. First aid and fitness **d.** China in focus, Consumer studies

e. Beginners' photography, First aid and fitness, Outdoor education

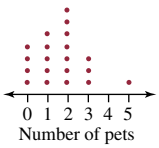
f.



21. a.

x	f
0	4
1	5
2	7
3	3
4	0
5	1
Total	20

b.



c. 2, 2, 1.65

22. a. i. 5 **ii.** 5.5 **iii.** 7.9 **iv.** 25 **v.** 6 **vi.** 17.9, 15.5

b. Discuss in class.

23. The boys have a mean height of 158 cm, with a median of 158 cm and a mode of 148 cm. They range in height from 138 cm to 178 cm.

The girls have a mean height of 153 cm, with a median of 152 cm and a mode of 137 cm. They range in height from 137 cm to 171 cm.

The shortest person is a girl, while the tallest person is a boy. On the whole, the boys are taller than the girls.

Investigation — Rich task

1. Pie graph

2. Families with no children in this age range is the most common category.

3. Column of bar graph

4. Teacher to check.

5. Sample. The entire population was not surveyed.

6. Numerical data

7. Teacher to check.

8. Teacher to check. The major difference is that students will receive no results for 'no children' as they are all in the less than 15 years age range.

9. Teacher to check.

TOPIC 15
Coordinates and the Cartesian plane

15.1 Overview

Numerous **videos** and **interactivities** are embedded just where you need them, at the point of learning, in your learnON title at www.jacplus.com.au. They will help you to learn the content and concepts covered in this topic.

15.1.1 Why learn this?

Using coordinate systems enables us to solve problems about locations in different settings. Scientists and geographers often use coordinates when they analyse data. Reading and interpreting maps involves us knowing about coordinates. Computer programmers use coordinates when creating art work. A computer screen is made up of tiny points of light called pixels. Programmers use coordinates to identify which pixels to light when creating art.

15.1.2 What do you know?

assessON

1. **THINK** List what you know about plotting coordinates. Use a thinking tool such as a concept map to show your list.
2. **PAIR** Share what you know with a partner and then with a small group.
3. **SHARE** As a class, create a thinking tool such as a large concept map that shows your class's knowledge of plotting coordinates.

LEARNING SEQUENCE

15.1 Overview
15.2 Alphanumeric grid references on a map
15.3 The Cartesian plane
15.4 Plotting simple linear relationships
15.5 Interpreting graphs
15.6 Review

15.2 Alphanumeric grid references on a map

15.2.1 Alphanumeric grid references on a map

- Areas of land can be displayed by using a grid reference map, where the map is divided into squares.
- Because grid references refer to a square region rather than a point on a map, they are often called **area references**.
- The reference for each square on a map is given by two coordinates — a letter and a number (hence the name *alphanumeric*).
- Maps with alphanumeric grid references are usually found in a street directory or an atlas.
- An example of a map with an alphanumeric grid is shown below.

© Gregory's Road Atlas of Australia 2003, Universal Publishers Pty Ltd

WORKED EXAMPLE 1

Joe is holidaying in Brisbane and has planned to visit various sites. Using the map of the Gold Coast on the previous page, give the grid reference for each of the following sites Joe plans to visit.

a Southport **b Movie World** **c Bond University**

THINK	WRITE
a 1 Locate Southport.	**a**
2 Read the horizontal and vertical coordinates.	
3 Write the answer.	C5
b 1 Locate Movie World.	**b**
2 Read the horizontal and vertical coordinates.	
3 Write the answer.	B4
c 1 Locate Bond University.	**c**
2 Read the horizontal and vertical coordinates.	
3 Write the answer.	C6

WORKED EXAMPLE 2

Name the major feature located at each of the following grid references in the preceding Gold Coast map.

a D3 **b E8** **c B3**

THINK	WRITE
a 1 Look for the major feature in column D and row 3.	**a**
2 Write the answer.	South Stradbroke Island
b 1 Look for the major feature in column E and row 8.	**b**
2 Write the answer.	Tweed Heads
c 1 Look for the major feature in column B and row 3.	**c**
2 Write the answer.	Dreamworld

learnON RESOURCES — ONLINE ONLY

Complete this digital doc: SkillSHEET: Finding locations (doc-6586)

Complete this digital doc: SkillSHEET: Reading scales (doc-6587)

Exercise 15.2 Alphanumeric grid references on a map

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1, 2, 3, 4, 5, 6, 10	Questions: 1, 2, 3, 4, 5, 6, 7, 8, 10	Questions: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10

Individual pathway interactivity: int-4383

learnON ONLINE ONLY

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Fluency

Use the grid map of Victoria and its surrounding states to answer questions **1** and **2**.

1. WE1 Give the grid reference for each of the following.

a. Mt Kosciuszko
b. Port Campbell
c. Lake Tyrrell
d. Wilsons Promontory
e. The Grampians
f. Phillip Island
g. Mt Hotham
h. Portland Bay
i. Ninety Mile Beach

2. WE2 Name the major place(s) located at each of the following grid references.

a. C1 **b.** B2 **c.** C2 **d.** B1 **e.** A1 **f.** B3 **g.** D2 **h.** A2 **i.** D3

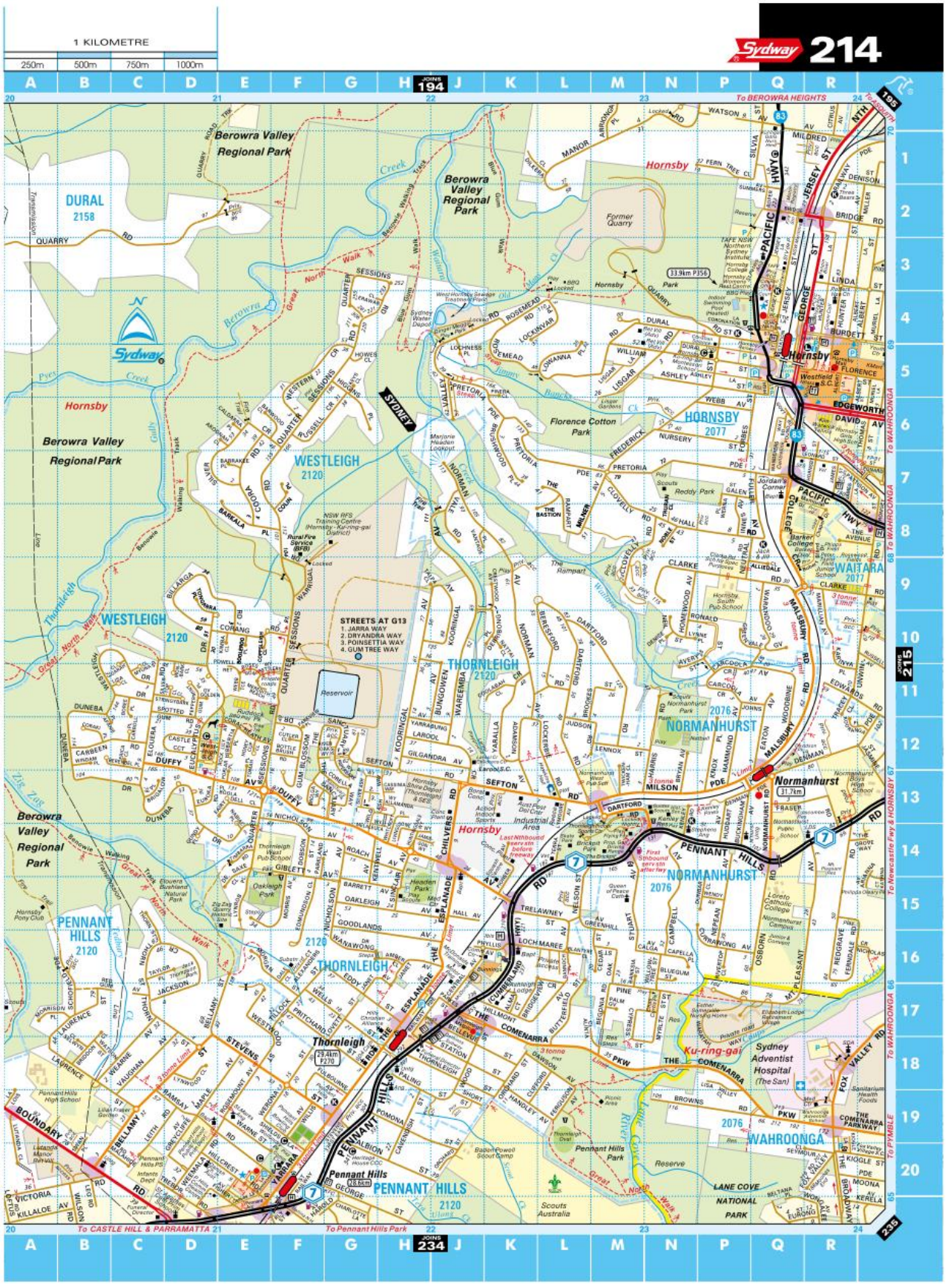
Use the Sydney map on the following page to answer questions **3** and **4**.

3. In which square would you find:

a. Normanhurst Station
b. Sydney Adventist Hospital
c. Fox Valley Road?

4. What is the name of the longest street passing through:

a. N14 **b.** L18 **c.** K13?


Sydway 214
1 KILOMETRE
250m
500m
750m
1000m
Berowra Valley Regional Park
DURAL 2158
Hornsby
HORNSBY 2077
WESTLEIGH 2120
THORNLEIGH 2120
NORMANHURST 2076
WAITARA 2077
PENNANT HILLS 2120
WAHROONGA 2076
Florence Cotton Park
Hornsby Park
Pennant Hills Park
Ku-ring-gai
LANE COVE NATIONAL PARK
Sydney Adventist Hospital (The San)
STREETS AT G13
1. JARRA WAY
2. DRYANDRA WAY
3. POINSETTIA WAY
4. GUM TREE WAY
Reservoir
To BEROWRA HEIGHTS
To CASTLE HILL & PARRAMATTA
To Pennant Hills Park
To Newcastle Fwy & HORNSBY
To WAHROONGA
To PYMBLE
SYDNEY
Normanhurst
Thornleigh
JOINS 194
JOINS 215
JOINS 234
JOINS 235

Understanding

5. A group of tourists will start from the Tourist Information Office in the Rocks and visit Cadman's Cottage, St Mary's Cathedral, Mrs Macquarie's Chair, Sydney Tower, Sydney Opera House and The Mint Museum.

a. Use the following map to list the map references for each of the above attractions.

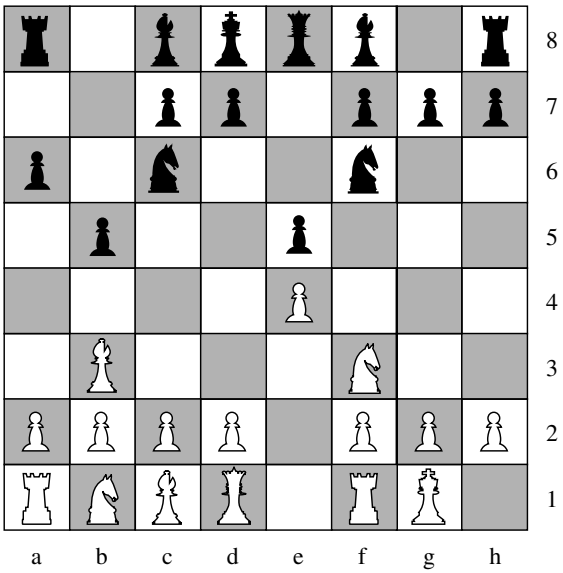
b. Since they have limited time, the tour group wants to visit each one but not pass any of the attractions more than once. They will start and finish at the Tourist Information Office in the Rocks. Suggest possible travel routes they could take.

6. Consider the following chess board and answer the questions that follow.

a. One of the black rooks is on a8. Give the location of the white rooks.

b. Which chess piece is on c6?

c. One of the white bishops is on c1. Where is the second white bishop at the moment?



d. A knight makes an L-shaped move (that is, one square to the side and two vertically, or two squares to the side and one vertically).

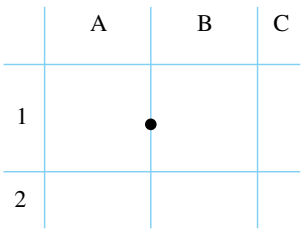
i. If it is White's turn, which pawn can the knight capture in the next move? (Specify the move.)

ii. If it is Black's turn, which pawn can the knight capture in the next move? (Specify the move.)

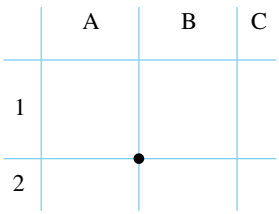
e. A bishop can move diagonally any number of (unoccupied) squares at a time. Complete the following sentence: 'With its next move, the bishop at b3 could capture the …….. (name the chess piece) at …….. (name the position).'

Reasoning

7. What would the map reference be of a place on the map located on a line between two squares?



8. What would the map reference be of a place on the map located on a point of intersection?



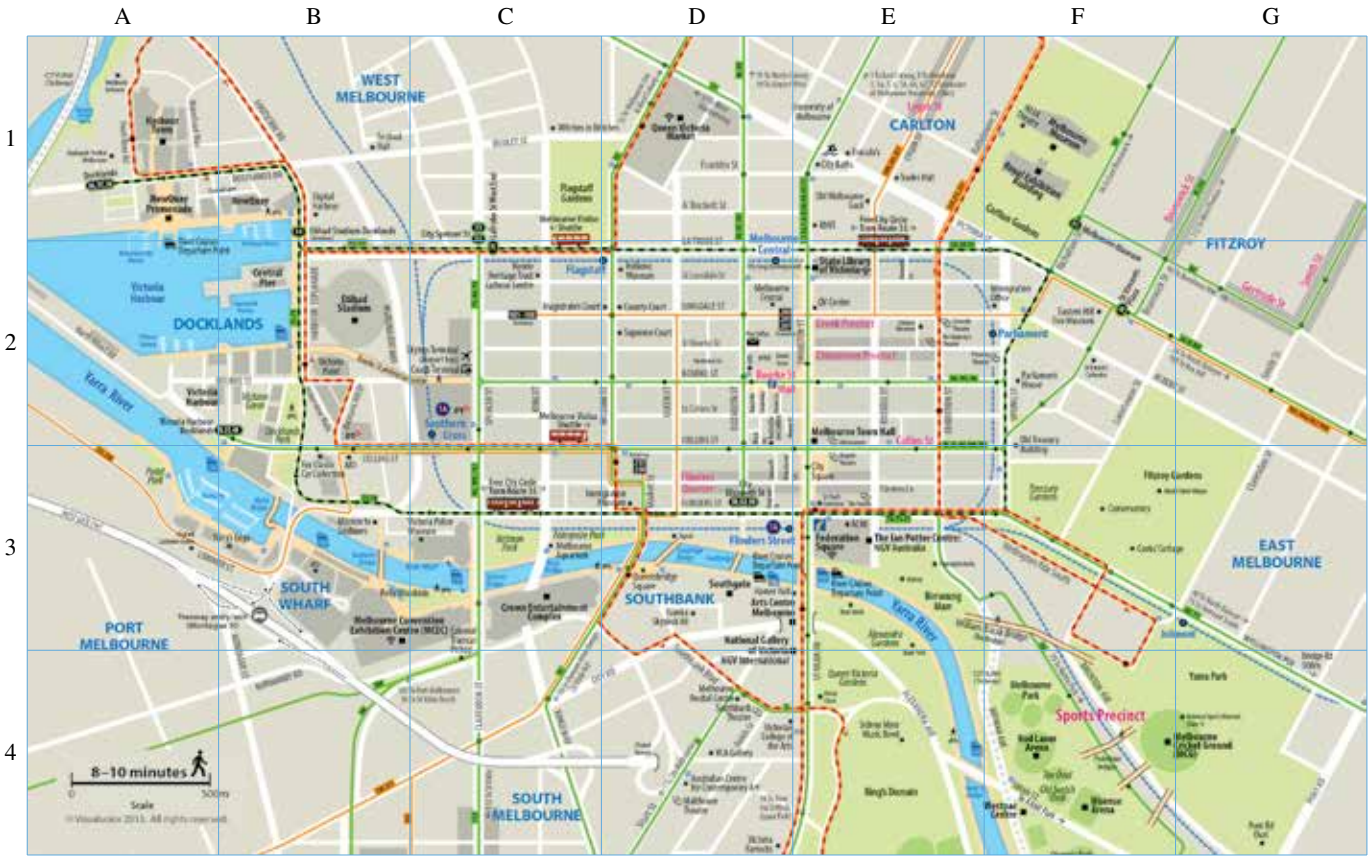
Problem solving

9. A group of students from country Victoria are visiting Melbourne for three days. They have three itineraries to follow.

a. *Itinerary 1*: They start from the Melbourne City Tourist Shuttle and visit the Immigration museum, Queensbridge Square, Eureka Skydeck, Melbourne Recital Centre and Australian Centre for Contemporary Art. They will enjoy some quiet time in the Queen Victoria Gardens before visiting Federation Square.
Use the map shown to list the map references for each of these places.

b. *Itinerary 2*: They start from Southern Cross and visit Etihad Stadium, Flagstaff Gardens, Queen Victoria Market and the University of Melbourne. They will enjoy some quiet time in Carlton Gardens before visiting the IMAX Theatre and the Melbourne Museum.
Use the map shown to list the map references for each of these places.

c. *Itinerary 3*: Suggest an itinerary of your choice.



10. Use the map on the following page to list the map references for each of these places:

a. WhiteWater World: The Wedgie, Super Tubes HydroCoaster, The Green Room, BRO — Blue Ring Octopus, The Rip.

b. Dreamworld: The Claw, Cyclone, Wipeout, BuzzSaw, Tower of Terror II, Motocoaster, The Giant Drop, Wiggles World.

c. Make your own list of favourite attractions and state the path you would follow to explore all these attractions.

WORLD OF THRILLS

DREAMWORLD

- ◆A J7 **The Claw** Q&U Swinging 78 km/hr while spinning 360°
- ◆A M3 **Cyclone** Q&U Gravity rollercoaster with a 360° loop
- ◆A K2 **Wipeout** Spins, twists and tumbles
- ◆K E9 **BuzzSaw** Q&U 15 storey roller coaster relaxes you plummet down at speeds of up to 100km/hr
- ◆A I5 **Tower of Terror II** Q&U Go backwards at speeds of 161 km/hr in 7 seconds before plummeting back down, face first
- ■S F5 **Mick Doohan's Motocoaster** Q&U Motorbike rollercoaster
- ◆A C7 **The Giant Drop** 120m free fall ride
- ◆A J5 **Pandamonium** Q&U Spin, swing and go upside down, reaching 2.8G forces
- ◆E M6 **Zombie Evilution** The Battle for Earth Will Join the laser tag war - zombies vs. soldiers!
- ◆HoMM **FlowRider** The fastest growing board sport in the world, flowriding is born from the fusion of surfing, skateboarding and body boarding

WHITEWATER WORLD

- ◆K U8 **The Wedge** Experience the thrill of Australia's first drop and loop waterslide
- ■I U9 **Super Tubes HydroCoaster** Blasted from the 18m tower down the 220m track of terror
- ◆D T8 **The Green Room** Accelerates riders from a 70m tunnel to a 10m tunnel
- ◆D R4 **BRO - Blue Ring Octopus** Riders hurtle down one of eight, 120m tentacles
- ◆D G4 **The Rip** 12m of pitch black terror before blasting into a massive bowl

WORLD OF FAMILY

DREAMWORLD

- ■D K7 **ShockWave** Q&U A racing, friendly open air adventure that accelerates to leave you spinning
- ■B K7 **Reef Diver** A calm but satisfying ride of fun
- ●C F6 **Dreamworld Railway** A scenic ride on the Dreamworld Express
- ●B J7 **Ogre-Go-Round** Spin in an ogre carriage or on a bottle shield
- ●E A3 **Avis Vintage Cars** Hand turn driving your own mini vintage car
- ■E D9 **Thunder River Rapids Ride** Q&U Go on this water carving and watch the excitement unfold
- ■B B7 **Rocky Hollow Log Ride** Q&U Plunge through the water storms
- C C9 **Big Brother House - Limited time only** Get your photo in the Big Brother fishy room shot and go behind the glass to explore the house
- ○F R8 **Bloodsuck Bumper Cars** Drive human slugs on a venom dodge, electrified enclosure
- ★C J6 **Kung Fu Academy** Practice your Kung Fu skills in the training academy
- ■A J5 **Pandamonium** Q&U Spin, swing and go upside down

WHITEWATER WORLD

- ■D O5 **The Temple of Huey** Board rafts along the lazy river. Take it for the ride or throw a friend in one
- ◆D R5 **The Little Rippers** Floating tyre ride down water slide
- ●C T6 **Cave of Waves** Sit it out in a tube or take on the surging wave
- R6 **The Shell** Timed shallow splash spot
- ● S8 **WhiteWater World Cabanas** Located throughout & Administration. Rental point at Guest Services

WORLD OF KIDS

DREAMWORLD

- ●B H3 **Big Red Boat Ride** Wiggly boating adventure
- ●C I3 **Big Red Car Ride** Cruise through the Wiggles house
- ★C H4 **Fun Spot Activity Centre** Sing, dance and play
- ★C H4 **Captain Feathersword's Ship SS Featherwood**
- ●B H3 **Dorothy's Roxy Tea Cup Ride** Jump in a giant coloured tea cup then twist and twirl
- ■G I5 **Escape from Madagascar** Q&U The wildest rollercoaster around
- ●B J5 **Puss in Boots Sword Swing** Fly through the air on a quest for adventure
- ★B J7 **MAD Jungle Jam** Join in the fun with hundreds of squidgy balls, air cannons, foam fountains, rope bridges & twisted towers
- ●H J5 **Gingy's Glider** Lie on your tummy as you fly amongst lollipops & gummy drops
- ●E J7 **Donkey Flyers** Take to the skies on the back of Donkey

WHITEWATER WORLD

- ■I O6 **Pipeline Plunge** Run wild on four flume waterslides, play with water sprays, geysers and cannons, or just chill out under the massive Pipeline bucket, dumping over 1000L of water onto guests
- ●L O7 **Wiggle Bay** Mini slides, interactive sprays, colourful water cannons and giant instruments

TIGER ISLAND & DREAMWORLD CORROBOREE

DREAMWORLD

- ★C K4 **Tiger Island** Dreamworld Tigers
- ★C L4 **The Lair** Get up close to our Tigers
- ★C E1 **Rainforest People & Wildlife** and reptiles
- ★C B1 **Wetlands People & Wildlife** Kangaroos, Koalas & other Australian wildlife
- ★C D2 **Torres Strait Wetland People & Wildlife** Massive saltwater Crocodiles
- ★C C4 **Arid Wildlife** Meet some of the Red Centre dwellers including Wombats, Emus & Dingoes
- ★C E2 **Dreamworld Corroboree Theatre**
- ★C B2 **Bilby House** Representation of life in the desert
- ★C F1 **Entry into Corroboree** Jingeri
- ★C C1 **Koala (Ben Hi) Photos** Cuddle a Koala
- ★C D4 **Yunning Circle** Face painting & make traditional music
- ★C F1 **Weapons Demonstration** Spears, Boomerangs & Shields

Q4U AVOID THE QUEUES

Dreamworld and WhiteWater World are smoke free environments. Designated smoking areas are provided throughout the park. Shrek, Madagascar and Kung Fu Panda © 2018 DreamWorks Animation L.L.C.

OPERATING HOURS

DREAMWORLD

Open 10am to 5pm*

Closed Christmas Day (25 December) and ANZAC Day (25 April). Hours may be extended during school holidays.

*Times subject to change. Refer to website for updates.

dreamworld.com.au

WHITEWATER WORLD

Open 10am to 4pm* (minimum)

Closed Christmas Day (25 December) and ANZAC Day (25 April). Hours may be extended during Summer.

*Times subject to change. Refer to website for updates.

whitewaterworld.com.au

Please don't ask extra risk your safety. Dreamworld and WhiteWater World safety precautions rides for your protection based on speed, motion and other special features. Restrictions for guests are posted at rides. Guests with disabilities should assess each ride for suitability before proceeding.

Participant responsibility. You should be in good health to ride safely. If you suspect your health could be at risk, or you could aggravate a pre-existing condition of any kind, DO NOT RIDE! Staff are legally responsible for enforcing safety regulations and can and make exceptions.

- Lifeguards and staff are unable to supervise children. Adults must actively supervise their children at all times.
- Some rides have height and weight restrictions, check height before entering ride queue lines.
- Suitable waterproof nappies or toddlers must be worn prior to entering pools.
- Please use shower facilities prior to entering the attractions.
- Guests with cuts, bruises, ear wax or any hard or sharp objects are not permitted on any rides.

KEEP WATCH @ PUBLIC POOLS

Information · Upgrade Booth · Telephone · Mobile Charge Station · First Aid · Toilets · Disabled Toilets · Strollers & Wheelchairs · Parenting Room · Lockers · Food & Beverage Outlet · Drinking Fountain · Prayer Room · Photographers · Cash Withdrawals (ATM) · Designated Smoking Areas

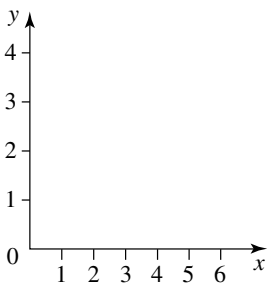
Reflection

When we specify an alphanumeric grid reference, does order matter? (That is, does it matter whether the letter is stated first and the number second, or the other way around?)

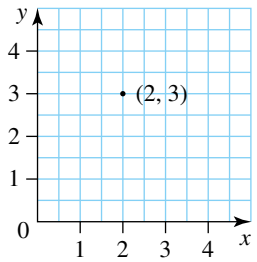
15.3 The Cartesian plane

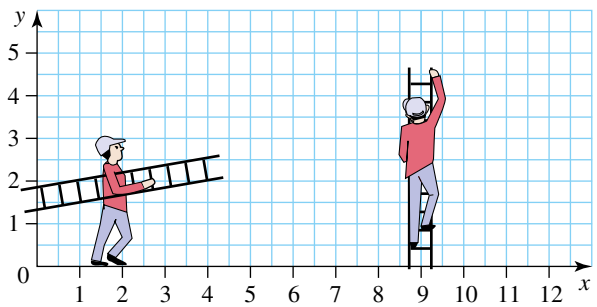
15.3.1 The Cartesian plane

- The **Cartesian plane** is named after its inventor, mathematician René Descartes. It is a visual means of describing locations on a plane by using two numbers as coordinates (rather than a letter and a number).
- The Cartesian plane is formed by two perpendicular lines. The horizontal line is called the x-axis, while the vertical line is referred to as the y-axis. The point where the two axes intersect is called the origin.
- Both axes must be marked (with marks being evenly spaced) and numbered. The distance between each mark is one unit.



- To locate any point on the Cartesian plane we use a pair of numbers called *Cartesian coordinates*. The numbers are written in a set of brackets and are separated by a comma. The first number in brackets is called the x-coordinate of the point; this shows how far across from the origin the point is located. The second number is called the y-coordinate; this shows how far up (or down) from the origin the point is.
- If the Cartesian coordinates of the point are known, it can be easily located by moving across and up (or down) from the origin the specified number of units. For example, to find the point with coordinates (2, 3), start from the origin and move 2 units to the right and 3 units up.
  Hint: To help remember the order in which Cartesian coordinates are measured, think about using a ladder. Remember we must always walk across with our ladder and then climb up it.





WORKED EXAMPLE 3

Draw a Cartesian plane with axes extending from 0 to 6 units. Mark the following points with a dot, and label them.

a $(2, 4)$ **b** $(5, 0)$ **c** $(0, 2)$ **d** $(3\frac{1}{2}, 2)$

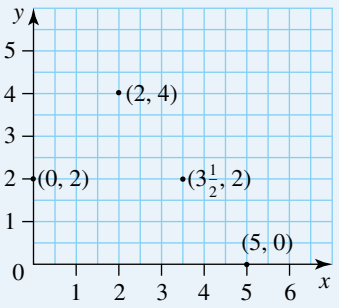
THINK

1 First rule up and label the axes.

2 Mark each point.

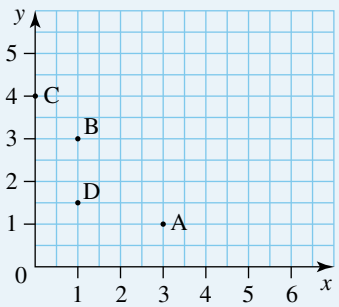
- **a** $(2, 4)$ means starting at the origin, go across 2 units, and then up 4 units.
- **b** $(5, 0)$ means go across 5 units and up 0 units. It lies on the x-axis.
- **c** $(0, 2)$ means go across 0 units and up 2 units. It lies on the y-axis.
- **d** $(3\frac{1}{2}, 1)$ means go across $3\frac{1}{2}$ units and up 1 unit. Label each point.

DRAW



WORKED EXAMPLE 4

Find the Cartesian coordinates for each of the points A, B, C and D.



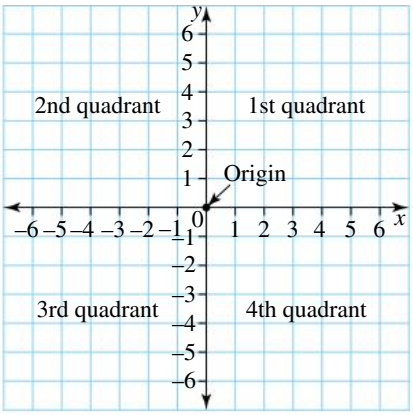
THINK	WRITE
Point A is 3 units across and 1 unit up.	A is at $(3, 1)$
Point B is 1 unit across and 3 units up.	B is at $(1, 3)$
Point C is 0 units across and 4 units up.	C is at $(0, 4)$
Point D is 1 unit across and $1\frac{1}{2}$ units up.	D is at $(1, 1\frac{1}{2})$

15.3.2 Extending the axes

- The Cartesian **axes** can extend infinitely in both directions.
- On the x-axis, the values to the left of the origin are negative and decreasing. Likewise, on the y-axis the values below the origin are negative and decreasing.
- The axes divide the Cartesian plane into four sections called **quadrants**. The quadrants are numbered in an anti-clockwise direction, starting with the top right corner.
- If both x- and y-coordinates of the point are positive, it will be located in the first quadrant. If the x-coordinate is negative but the y-coordinate is positive, the point will be in the second quadrant. If

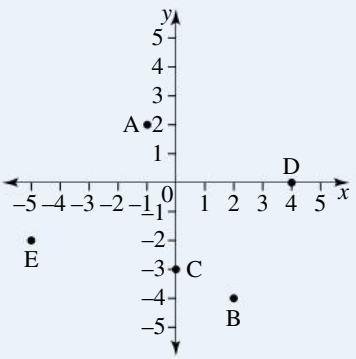
the point is in the third quadrant, both the x- and y-coordinates of the point will be negative. Finally, if the point is in the fourth quadrant, its x-coordinate is positive, while its y-coordinate is negative.

- If the point is located on the x-axis, its y-coordinate is always 0. Likewise, if the point is on the y-axis, its x-coordinate is always 0.



WORKED EXAMPLE 5

Plot the following points on the Cartesian plane.
A(−1, 2), B(2, −4), C(0, −3), D(4, 0), E(−5, −2)
State the location of each point on the plane (that is, the quadrant, or the axis on which it sits).

THINK	WRITE
1 Draw a set of axes, ensuring that they are long enough to fit all the values.	
2 Plot the points. The first point is one unit to the left and two units up from the origin. The second point is two units to the right and four units down from the origin (and so on).	
3 Look at the plane and state the location of each point. Remember that the quadrants are numbered in an anti-clockwise direction, starting at the top right. If the point is on the axis, specify which axis it is.	Point A is in the second quadrant. Point B is in the fourth quadrant. Point C is on the y-axis. Point D is on the x-axis. Point E is in the third quadrant.

learnon RESOURCES — ONLINE ONLY

Watch this eLesson: Coordinates and the Cartesian plane (eles-0008)
Complete this digital doc: SkillSHEET: Marking points on the line (doc-6588)
Complete this digital doc: Spreadsheet: Plotting points (doc-0002)
Complete this digital doc: WorkSHEET: Coordinates and the Cartesian plane I (doc-2004)

Exercise 15.3 The Cartesian plane

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1, 2, 3, 4, 5, 6, 7, 14	Questions: 1, 2, 3, 4, 5, 6, 7, 9, 11, 13, 14	Questions: 1, 2, 3, 4, 5, 6, 8–13, 15

Individual pathway interactivity: int-4384

learnON ONLINE ONLY

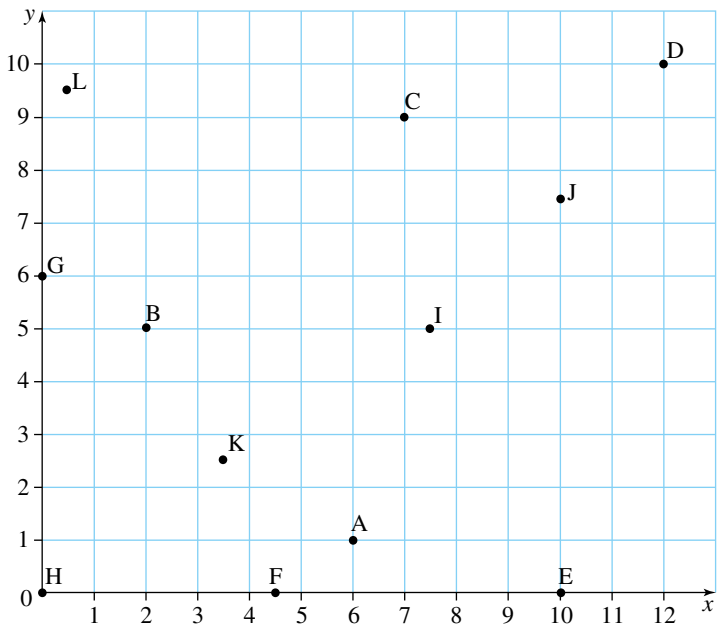
To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note*: Question numbers may vary slightly.

Fluency

1. WE3 Draw a Cartesian plane with axes extending from 0 to 5 units. Mark the following points with a dot, and label them.

a. (4, 3) **b.** (1, 4) **c.** (3, 3) **d.** (2, 0) **e.** (0, 4) **f.** (0, 0)

2. WE4 Find the Cartesian coordinates for each of the points A–L.



3. WE5 Plot the following points on the Cartesian plane.
A(−1, −3), B(−2, 5), C(3, −3), D(0, −4), E(−2, −2), F(−5, 0), G(3, 1), H(3, 0), I(−4, −2), J(4, −5)
State the location of each point on the plane (that is, the quadrant, or the axis it sits on).

Understanding

4. Each of the following sets of Cartesian axes (except one) has something wrong with it. From the list below, match the mistake in each diagram with one of the sentences.

A. The units are not marked evenly.
B. The y-xis is not vertical.
C. The axes are labelled incorrectly.
D. The units are not marked on the axes.
E. There is nothing wrong.

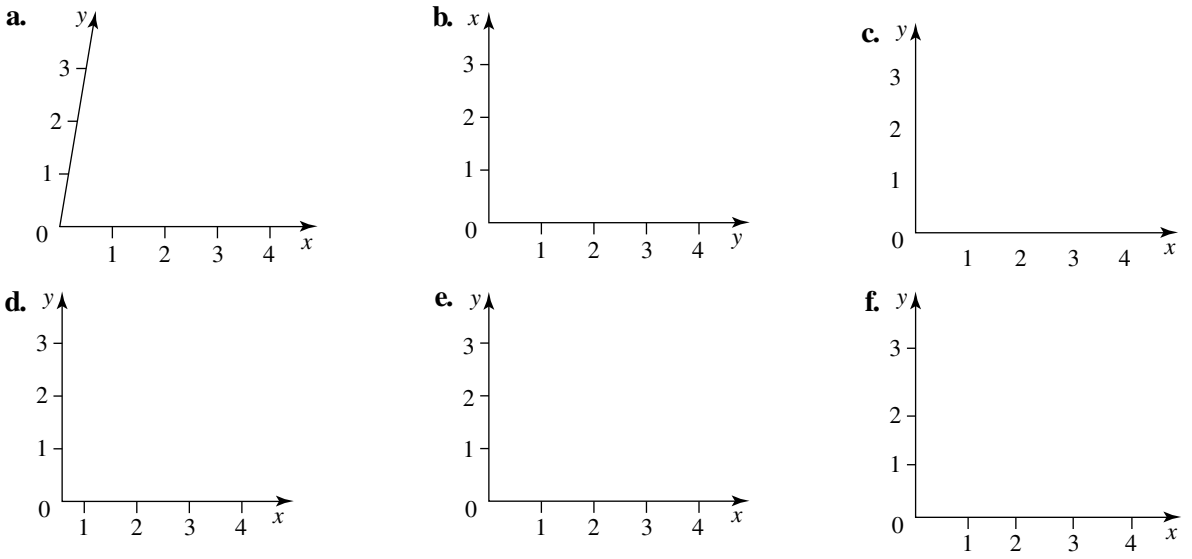


Figure 1

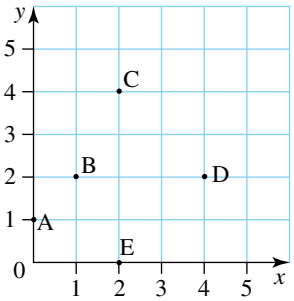
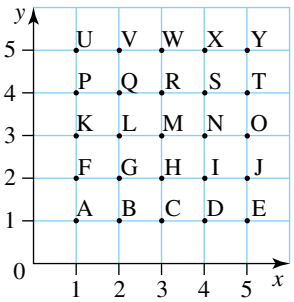


Figure 2



5. From Figure 1, write down the coordinates of 2 points which:

a. have the same x-coordinate

b. have the same y-coordinate.

6. Messages can be sent in code using a grid like the one shown in Figure 2, where the letter B is represented by the coordinates $(2, 1)$. Use the diagram to decode the answer to the following riddle.

Q Where did they put the man who was run over by a steamroller?

A $(4, 2)(4, 3)(3, 2)(5, 3)(4, 4)(1, 4)(4, 2)(5, 4)(1, 1)(2, 3)$
$(4, 2)(4, 3)(3, 5)(1, 1)(3, 4)(4, 1)(4, 4)(4, 4)(4, 2)(4, 5)$
$(4, 4)(5, 1)(2, 5)(5, 1)(4, 3)(5, 1)(4, 2)(2, 2)(3, 2)(5, 4)$
$(1, 1)(4, 3)(4, 1)(4, 3)(4, 2)(4, 3)(5, 1)$

7. Rule up a Cartesian plane with both axes extending from 0 to 10 units. Plot the following points and join them in the order given to make a geometric figure. Name each shape.

a. $(2, 2)-(5, 2)-(2, 6)-(2, 2)$

b. $(4, 4)-(8, 4)-(6, 8)-(4, 4)$

c. $(1, 1)-(10, 1)-(8, 9)-(2, 9)-(1, 1)$

d. $(0, 0)-(8, 0)-(10, 10)-(2, 10)-(0, 0)$

8. Here is an exercise which may require extra care and concentration. On graph paper or in your exercise book rule up a pair of Cartesian axes. The x-axis must go from 0 to 26 and the y-axis from 0 to 24. Plot the following points and join them in the order given.

$(0, 15)-(4, 17)-(9, 22)-(10, 21)-(12, 24)-(16, 22)-(15, 21)-(18, 19)-(20, 24)-$
$(22, 18)-(26, 12)-(26, 10)-(23, 4)-(20, 3)-(18, 4)-(14, 7)-(11, 7)-(4, 6)-(2, 7)-$
$(2\frac{1}{2}, 8)-(0, 15)$

Complete the picture by joining $(19, 2)-(21, 2)-(20, 0)-(19, 2)$.

9. What is the area of a rectangle formed by connecting the points $(2, 1)$, $(7, 1)$, $(7, 4)$ and $(2, 4)$ on a Cartesian plane?

10. Consider the following set of points:

A(2, 5) B(−4, −12) C(3, −7) D(0, −2) E(−10, 0) F(0, 0) G(−8, 15)
H(−9, −24) I(18, −18) J(24, 0).

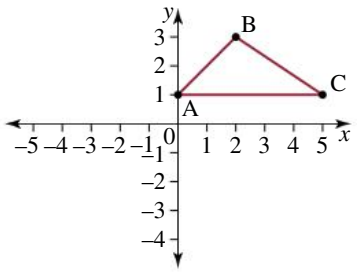
Which of the following statements is true?

a. Points A and J are in the first quadrant.

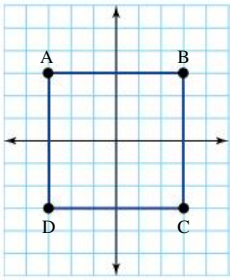
b. Points B and H are in the third quadrant.

c. Only point I is in the fourth quadrant.
d. Only one point is in the second quadrant.
e. Point F is at the origin.
f. Point J is not on the same axis as point E.
g. Point D is two units to the left of point F.

11. Consider the following triangle ABC.
a. State the coordinates of the vertices of the triangle ABC.
b. Find the area of the triangle.
c. Reflect the triangle in the *x*-axis. (You need to copy it into your workbook first.) What are the new coordinates of the vertices?
d. Now reflect the triangle you have obtained in part **c** into the *y*-axis, and state the new coordinates of the vertices.



Reasoning



12. Consider the square ABCD shown.
a. State the coordinates of the vertices of the square ABCD.
b. Calculate the area of the square ABCD.
c. Extend sides AB and CD 3 squares to the right and 3 squares to the left.
d. Calculate the area of the new shape.
e. Extend sides AD and BC 3 squares up and 3 squares down.
f. Calculate the area of the new shape.
g. Compare the three areas calculated in parts **b**, **d** and **f**. Explain the changes of area in relation to the change in side length.

13. a. Explain what would happen to a shape in quadrant 3 if it was reflected in:
i. the *x*-axis **ii.** the *y*-axis?
b. Explain what would happen to a shape in quadrant 4 if it was reflected in:
i. the *x*-axis **ii.** the *y*-axis?
c. Write a rule about reflections:
i. in the *x*-axis **ii.** in the *y*-axis.

Problem solving

14. A line AB passes through points A(–5, 6) and B(3, 6).
a. Draw line AB on a number plane.
b. What is the length of AB?
c. Find the coordinates of the middle point.
d. Draw another horizontal line and find the coordinates of the middle point.
e. Is there a formula to calculate the *x*-coordinate of the middle point of a horizontal line?

15. A line CD passes through points C(1, 7) and D(1, –5).
a. Draw line CD on a number plane.
b. What is the length of CD?
c. Find the coordinates of the middle point.
d. Draw another vertical line and find the coordinates of the middle point.
e. Is there a formula to calculate the *y*-coordinate of the middle point of a vertical line?

Reflection

Why must the *x*-coordinate always be written first and the *y*-coordinate second?

CHALLENGE 15.1

What are the area and perimeter of a rectangle formed by connecting the points (3, 3) (9, 2) (9, 5) and (3, 5) on a Cartesian plane?

15.4 Plotting simple linear relationships

15.4.1 Plotting simple linear relationships

- When a set of points is plotted on the Cartesian plane, a pattern may be formed. If a pattern forms a straight line, we call it a *linear pattern*.
- The coordinates of the points that form a pattern can be presented as a set, or in a table. If shown in a table (similar to the one below), the coordinates of each point should be read 'in columns'; that is, the top number in each column gives the x-coordinate and the bottom number gives the corresponding y-coordinate of the point. Consider, for example, the following table of values and set of points. Both show the same information.

x	0	1	2	3
y	8	6	4	2

$$(0, 8)(1, 6)(2, 4)(3, 2)$$

- The Cartesian coordinates of the points are ordered pairs. That is, the first number always represents the x-coordinate, and the second always represents the y-coordinate of a point. A set of ordered pairs forms a relation between x and y.
- If the points form a linear pattern when plotted, we say that the relation between x and y is linear.

WORKED EXAMPLE 6

Plot the following set of points on the Cartesian plane, and comment on any pattern formed.
(1, 3) (2, 4) (3, 5) (4, 6) (5, 7)

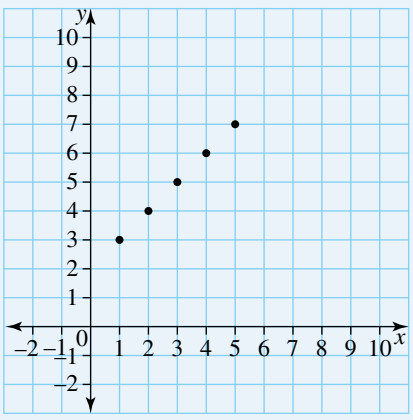
THINK

1 Look at the coordinates of the points in the set: the x-values range between 1 and 5, while the y-values range between 3 and 7. Draw a set of axes, ensuring they are long enough to fit all the values.

2 Plot the points.

3 Comment on the pattern that the points form.

WRITE



The points form a linear pattern.

WORKED EXAMPLE 7

Plot the following points on a Cartesian plane and decide whether or not the relationship is linear.

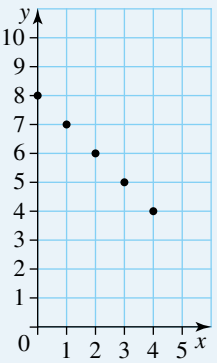
x	0	1	2	3	4
y	8	7	6	5	4

THINK

WRITE

1 The x-values range from 0 to 4; the y-values range from 4 to 8. Draw a set of axes that will include all these values. (The axes need not be the same length. In this case you can extend the y-axis a bit more than the x-axis, as the y-values are higher.)

2 The entries in the table can be rewritten as a set of points. The top number in each column gives the x-coordinate and the bottom number gives the corresponding y-coordinate of the point. Thus the first column gives the point (0, 8), the second (1, 7), and so on. Plot all points.



3 Consider the pattern formed by the points. It is a straight line. Draw a suitable conclusion about the relationship between x and y.

The relationship between x and y is linear.

learnON RESOURCES — ONLINE ONLY

Complete this digital doc: SkillSHEET: Drawing scales (doc-6589)

Complete this digital doc: SkillSHEET: Substituting values into the rule (doc-6590)

Complete this digital doc: SkillSHEET: Finding patterns (doc-6591)

Complete this digital doc: SkillSHEET: Establishing patterns connecting two numbers (doc-6592)

Exercise 15.4 Plotting simple linear relationships

assessON

Individual pathways

PRACTISE
Questions:
1, 2, 3, 4, 8, 12

CONSOLIDATE
Questions:
1b, d, F, 2b, d, 3a, e, 4, 5, 7, 8, 11, 12

MASTER
Questions:
1a, c, e, 2a, c, 3e, f, 4, 5, 6, 7, 8, 9, 10, 11, 12

Individual pathway interactivity: int-4385

learnON ONLINE ONLY

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note*: Question numbers may vary slightly.

Fluency

1. WE6 Plot the following sets of points on the Cartesian plane, and comment on any pattern formed.

a. (0, 1)(1, 2)(2, 3)(3, 4)(4, 5)

b. (0, 3)(1, 4)(2, 5)(3, 6)(4, 7)

c. (2, 2)(3, 3)(4, 4)(5, 5)(6, 6)

d. (0, 2)(1, 3)(2, 1)(3, 0)(4, 5)

e. (1, 10)(2, 8)(3, 6)(4, 4)(5, 2)

f. (2, 3)(3, 3)(4, 3)(5, 3)(6, 3)

2. Plot the following sets of points on the Cartesian plane. (Note that some coordinates are negative, so you will need to use all four quadrants.) Comment on any pattern formed.

a. (−2, 0)(−1, 2)(0, 4)(1, 6)(2, 8)

b. (−3, −3)(−2, 0)(−1, 3)(0, 6)(1, 9)

c. (−2, 7)(−1, 6)(0, 4)(1, 3)(2, 1)

d. (−6, 8)(−5, 6)(−4, 4)(−3, 2)(−2, 0)

3. WE7 Plot the points in each table on a Cartesian plane, and decide whether or not the relationship is linear.

a.

x	0	1	2	3	4
y	3	4	5	6	7

b.

x	0	1	2	3	4
y	0	1	4	9	16

c.

x	1	2	3	4	5
y	7	6	5	4	3

d.

x	1	2	3	4	6
y	12	6	4	3	2

e.

x	0	1	2	3	4
y	0	3	6	9	12

f.

x	2	3	4	5	6
y	0	−2	−4	−6	−8

Understanding

4. a. Plot the following points on the set of axes: (0, 10)(1, 8)(2, 6)(3, 4).

b. Describe the pattern formed by the points.

c. Extend the pattern by plotting the next two points.

d. What are the coordinates of the two points you have plotted?

5. The points in the table below form a linear pattern. The y-coordinate of the middle point is missing.

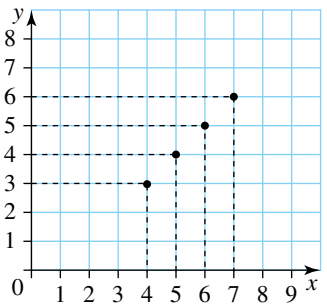
x	1	2	3	4	5
y	0	3		9	12

a. Plot the four points (whose x- and y-coordinates are both known) on the Cartesian plane.

b. Use your graph to predict the missing y-coordinate of the middle point.

c. Add the middle point to your graph and check whether it fits the pattern.

6. Consider the following linear pattern.



a. If the pattern is continued to the right, what will be the coordinates of the next point?

b. If the pattern is extended to the left, what will be the coordinates of the previous point?

7. Which of the following sets of points, when plotted, will form a linear pattern?

a. (0, 3)(1, 5)(2, 7)(3, 9)(4, 11)

b. (−1, 8)(−2, 7)(−3, 6)(−4, 5)(−5, 4)

c. (1, 6)(2, 5)(3, 7)(4, 8)(5, 9)

d. (2, 3)(3, 5)(4, 7)(5, 9)(6, 11)

e. (0, 2)(1, 4)(2, 4)(3, 5)(4, 6)

f. (10, 7)(9, 4)(8, 3)(7, 1)(6, 0)

8. Rachel planted a cherry tomato in her vegetable patch. She measured the height of her plant every week for four consecutive weeks, obtaining the following data: 20 mm, 24 mm, 28 mm, 32 mm. Rachel forgot to measure her plant in week 5, but the following week she found the height of the cherry tomato to be 40 mm.

a. Fill in the table of values below.

Week						
Height (mm)						

b. Plot the heights of the cherry tomato plant on a set of axes.

c. Is the relationship between the number of the week and the height of the plant linear? Explain your answer.

d. Assuming the pattern was the same for the first 6 weeks, what was the height of the tomato plant in week 5?

e. If the pattern continues, predict the height of the cherry tomato plant in two weeks.

Reasoning

9. The relation between the two variables x and y is described by the rule $y = 2x - 3$.

a. Complete the following statement: 'The rule describes a set of points such that the y-coordinate of each point is three less than …'

b. Construct a table of values for whole numbers $-2 \leq x \leq 2$ using the given rule.

c. Plot the points from the table on a set of axes.

d. Is the relation linear? Justify your answer.

e. The x-values of the relation form the *domain* of that relation, while the y-values form its *range*. The domain of the given relation is $-2 \leq x \leq 2$. What is its range?

10. a. On the same set of axes plot the following linear relationships: $y = x$, $y = 3x + 1$ and $y = 5x - 2$.

b. On the same set of axes plot the following linear relationships: $y = -x$, $y = -3x + 1$ and $y = -5x - 2$.

c. Explain the differences and similarities between the two sets of graphs.

Problem solving

11. a. Use the following information to complete the table of coordinates.

Point A is two units to the left of the origin and five units below the origin.
Point B is one unit to the left of the origin and two units above point A.
Point C is one unit below the origin on the vertical axis.
Point D is one unit to the right of C and two units above point C.
Point E is two units to the right of y-axis and three points above the x-axis.

x					
y					
	A	B	C	D	E

b. Plot these points on a Cartesian plane.

c. What is the value of the horizontal step between two adjacent points?

d. Is the relationship linear? Explain your answer.

12. The table of values shown displays the relationship between time, in minutes, and parking fee, in dollars.

Time, minutes	30	60	90	120	
Parking fee, $	1.20	2.40		4.80	6.00

a. Write a rule for this relationship.

b. Use the rule found in part **a** to calculate the missing values.

Reflection

Is there a way to predict whether the points will form a linear pattern without plotting them?

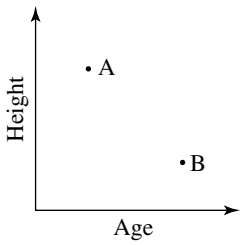
CHALLENGE 15.2

There is the same number of desks in each row in my classroom, and the rows are straight. My desk is fourth from the front and fourth from the back. It has one desk to its left and two desks to its right. How many desks are there in the classroom?

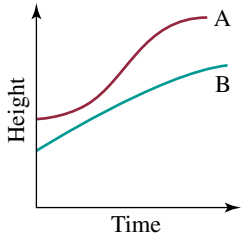
15.5 Interpreting graphs

15.5.1 Interpreting graphs

- Graphs are found in many areas of everyday life; therefore, being able to interpret graphs is a useful skill.
- When interpreting graphs, consider the following points:
  - Look at the overall picture. Where are the points situated? Does the line go up from left to right or go down from left to right?
  - Look at what horizontal and vertical axes represent, the values included (if any) and their units of measurement.
  - The values are increasing as you move up the vertical axis and to the right along the horizontal axis.
- A point graph shows pieces of information only. A line graph can show how the information changes. Time is often used on the horizontal axis.



Point graph



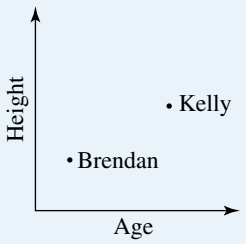
Line graph

WORKED EXAMPLE 8

Use the graph shown to answer the following questions about Brendan and Kelly.

a Who is taller, Brendan or Kelly?

b Who is younger?



THINK

a Height is on the vertical axis. Kelly is above Brendan, so Kelly is taller.

b The horizontal axis shows age. Brendan is to the left of Kelly, so Brendan is younger.

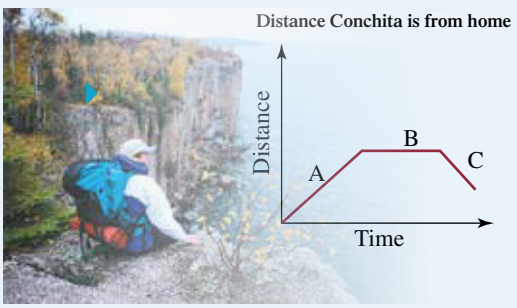
WRITE

a Kelly is taller.

b Brendan is younger.

WORKED EXAMPLE 9

This graph shows the distance Conchita is from home when she is out hiking. Describe what each straight line section of the graph is showing.



THINK	WRITE
1 As you move along section A, the distance is increasing and time is increasing, so Conchita is moving further away from home.	Section A: Conchita is walking away from home.
2 As you move along section B, the distance is staying the same but time is still increasing.	Section B: Conchita could be resting.
3 As you move along section C, the distance from home is decreasing and time is increasing.	Section C: Conchita is walking towards home.

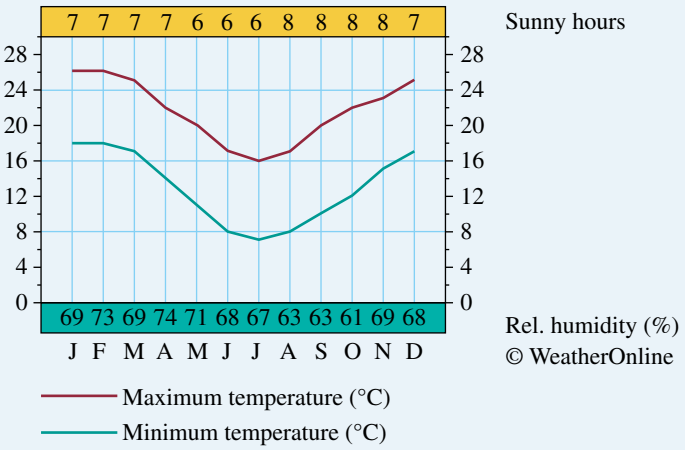
WORKED EXAMPLE 10

This graph shows the average maximum and minimum monthly temperatures for Sydney. Use the graph to answer the following questions.

a What month was the hottest and what was the average maximum temperature?

b What month was the coldest and what was the average minimum temperature?

c Which months had an average minimum temperature of 12 °C



THINK	WRITE
a Temperature is on the vertical axis and months are on the horizontal axis. We can read the hottest month and highest temperature from the graph.	**a** Hottest month is January and the average maximum temperature is 26 °C.
b We can read the coldest month and lowest temperature from the graph.	**b** Coldest month is July and the average minimum temperature is 7 °C.
c We can read the average minimum temperature of 12 °C from the vertical axis.	**c** Average minimum temperature of 12 °C was in May and October.

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Complete this digital doc: WorkSHEET: Coordinates and the Cartesian plane II (doc-6696)

Exercise 15.5 Interpreting graphs

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 15, 19	Questions: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13, 15, 18, 19	Questions: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 17, 18, 20

Individual pathway interactivity: int-4386

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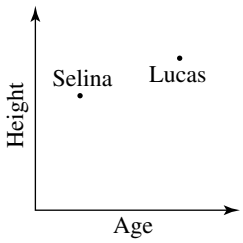
To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note*: Question numbers may vary slightly.

Fluency

1. WE8 Use the following graph to answer the following questions about Lucas and Selina.

a. Who is taller?

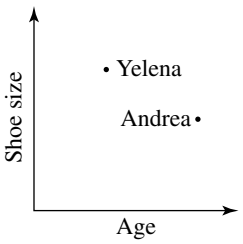
b. Who is younger?



2. Use the following graph to answer the following questions.

a. Does Yelena have a larger shoe size?

b. Is Yelena older than Andrea?

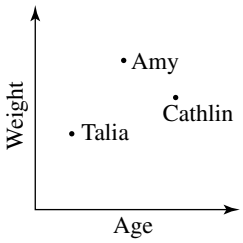


3. Use the following graph to answer the following questions.

a. Who is the youngest?

b. Who weighs the most?

c. Is the oldest person the heaviest?

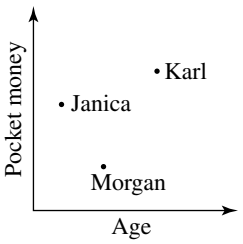


4. Use the following graph to answer the following questions.

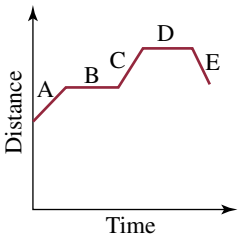
a. Who is given the most pocket money?

b. Is the youngest person given the least amount of pocket money?

c. Is the oldest person given the most pocket money?



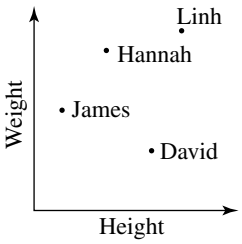
5. WE9 The following graph shows the distance Claire is from home when she is out rockclimbing. Describe what each straight line section of the graph is showing.



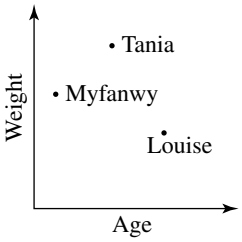
Understanding

6. a. Use the graph at right to answer the following questions.
- **i.** Who weighs the most?
- **ii.** Who is the tallest?
- **iii.** Does the shortest person weigh the least?

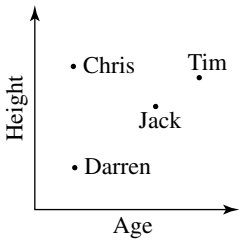
b. Copy this graph and plot the height and weight of Georgia, who is taller than Linh and weighs less than Hannah but more than James.



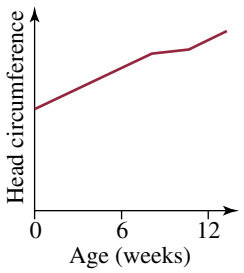
7. MC Which of the following is shown by this graph?
- **A.** Myfanwy is the lightest and youngest.
- **B.** Tania is the oldest.
- **C.** Myfanwy is older than Louise and younger than Tania.
- **D.** Tania and Louise are the same age.
- **E.** Louise weighs the least.



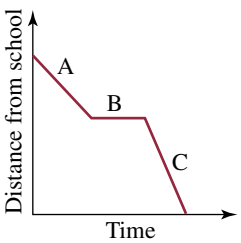
8. MC Which of the following is shown by this graph?
- **A.** Jack is the tallest.
- **B.** Chris is the oldest.
- **C.** Darren and Chris are the same age.
- **D.** Tim is the tallest.
- **E.** Darren is the same height as Chris.



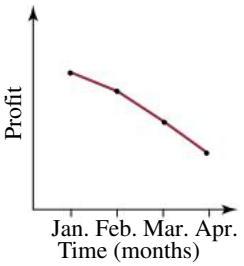
9. The graph below shows Thomas's head circumference for the first 12 weeks after birth.
- **a.** Why doesn't the graph start at zero?
- **b.** What has happened to his head circumference during the first 12 weeks?
- **c.** What would you expect the graph to look like beyond the first 12 weeks?



10. The graph at right shows Monique's journey as she walks to school.
- **a.** In which section of the graph is Monique not moving?
- **b.** The line in section C is steeper than the line in section A. What is this telling us about section C?
- **c.** Where is Monique at the end of section C?



11. Yupa owns an ice-cream shop. The following graph shows his profit for the first 4 months of the year.

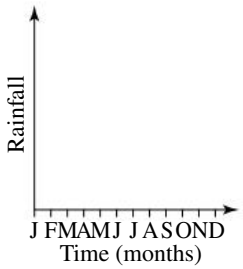


a. In which month did Yupa make the most profit?

b. In which month did Yupa make the least profit?

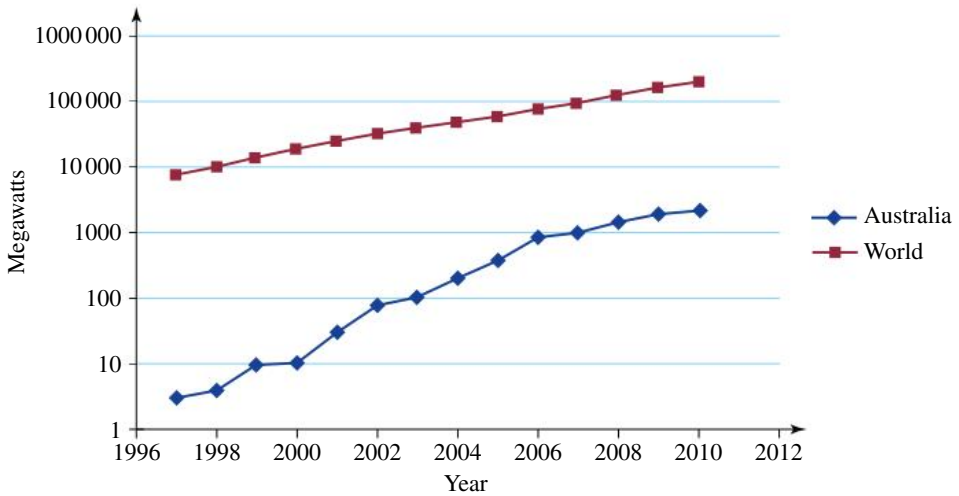
c. Describe what has happened to Yupa's profit over the 4 months.

12. Copy the following axis and draw a line graph that you think would show rainfall over one year.



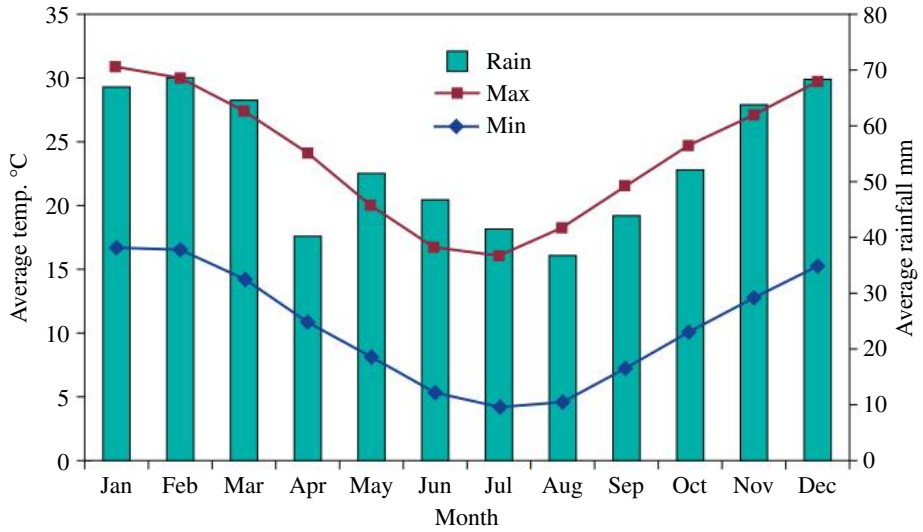
13. Cathy has been training hard for the 100 m. She has kept a record of the fastest time she has run each week. Draw a graph that shows the following sections:
Section A: Cathy increased her speed for the 100 m each week for the first 4 weeks.
Section B: After 4 weeks Cathy's speed stayed the same for the next 2 weeks.
Section C: Cathy had a cold and her speed decreased for 2 weeks.
Section D: Having recovered from her cold, Cathy trained hard and increased her speed again.

14. Looking at the following graph, what can you say about the future for wind power in Australia and the world?



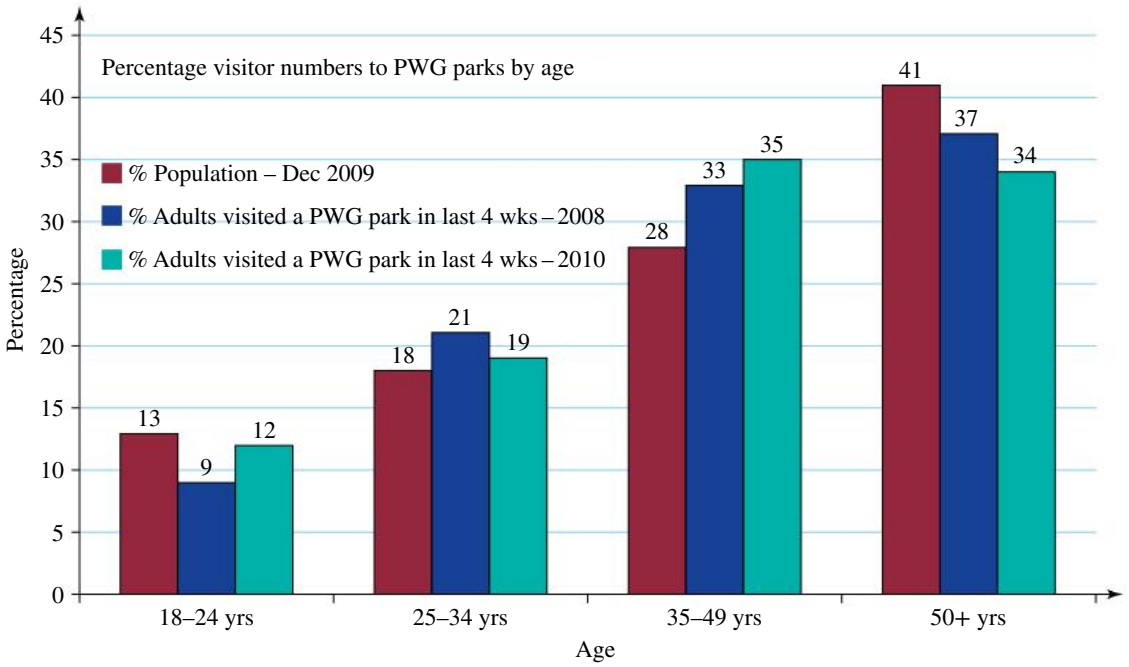
Installed world and Australia wind power

15. WE10 The following graph shows the average rainfall and average maximum and minimum temperatures for Muswellbrook and the Upper Hunter Valley.



a. Which two months had the lowest average rainfall?
b. What was the lowest average rainfall?
c. Which month had the lowest average minimum temperature?

16. The following graph shows the percentage of visitors to Parks and Wildlife Group–managed parks in New South Wales for 3 years.



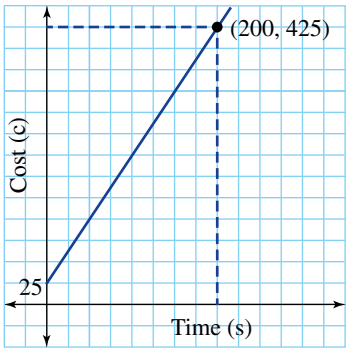
a. For the 25–34 age group, which year had the lowest percentage of visits?
b. Which age group visited the parks more frequently than any other group?
c. Suggest a reason why this age group visited the parks more often.

Reasoning

17. Peter and Sonja are walking home from school. A dog started chasing them and Peter started running away from the dog. Sonja noticed that this was her neighbour's dog so she stopped to pat the dog and then

started to walk towards Peter who was now waiting for her. Draw a graph to represent the relationship between distance travelled and time for both Peter and Sonja.

18. The cost C in cents of a phone call is displayed in the graph shown. Using the graph, explain the relationship between the cost and the time of a call.



Problem solving

19.

x	−4	−3	−2	−1	0	1	2	3	4
y	16	9	4	1	0	1	4	9	16

a. On a number plane plot the relationship shown in the table above and connect all points with a smooth line.

b. Complete the statement: 'As the value of x increases, the value of y …'

20.

x	−3	−2	−1	0	1	2	3
y	−27	−8	−1	0	1	8	27

a. On a number plane plot the relationship shown in the table above and connect all points with a smooth line.

b. Complete the statement: 'As the value of x increases, the value of y …'

Reflection

Think of some everyday situations where being able to interpret graphs would come in handy.

15.6 Review

15.6.1 Review questions

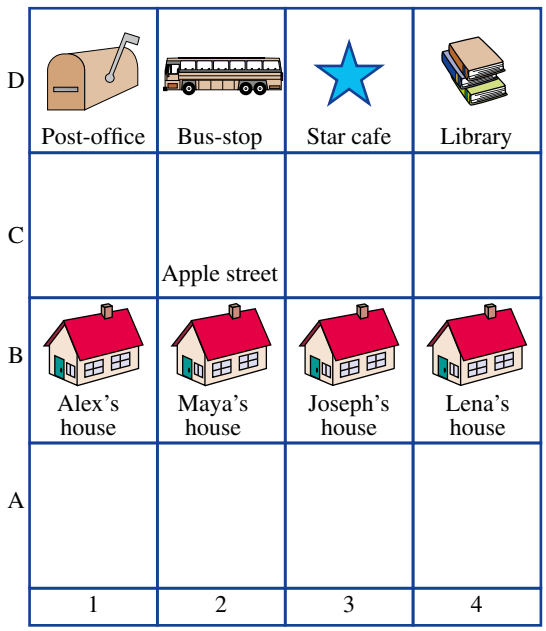
Fluency

1. The grid at right shows a simplified map that uses alphanumeric grid references.

a. Fill in the blanks in the table below.

b. A playground is to be built at A3. Mark its location on the grid with a X.

Location	Grid reference
	B3
Alex's house	
	D1
Lena's house	



2. The map on the following page is a street directory map of part of Sydney.

a. What is the name of the major road passing through F15?

b. What is the map reference of Glenwood Reserve?

© Reproduced with permission from Sydway Street Directory Edition 15

3. Give the grid reference for each of these places using the map of Australia below.

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a. Rockhampton
b. Canberra
c. Broome
d. Hobart

4. Use the tourist map of central London on the following page to answer the questions below.

a. Give the location of the following places of interest:
  i. Cleopatra's needle
  ii. Westminster Abbey
  iii. British Museum
b. Which rail stations are located in F9?
c. What can be found in A4?
d. Where is Buckingham Palace?

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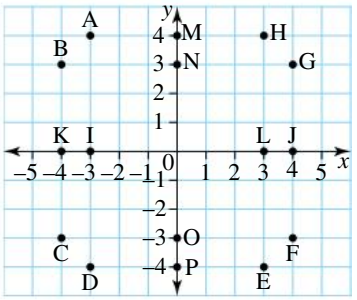
5. Study the diagram at right to answer the following questions.



a. Which letter in the diagram represents the point (−3, 4)?

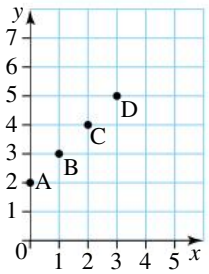
b. Which points are on the x-axis?

c. Name all points whose x-coordinate is zero.

d. Which points are in the second quadrant? Give their coordinates.

e. What are the coordinates of C and F?

6. Use the graph at right to answer the following questions.



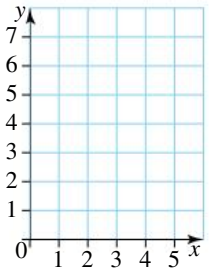
a. Fill in the blanks.

Coordinates	Point
(0, 2)	
	C
	D

b. Mark E (4, 6) on the graph. Join all the points with a straight line.

7. The table below gives the x and y coordinates of some points.

x	0	1	2	3
y	0	2	4	6



a. Using the axes at right, plot the points given in the table and join them.

b. Does the relation between x and y appear linear? Explain your answer.

Problem solving

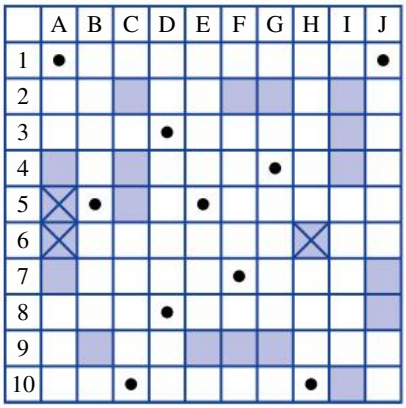
8. A game consists of a grid, and a dart that is thrown to land on a square of the grid.
The dart is thrown and lands on a letter that:

- has horizontal symmetry
- also has vertical symmetry
- and is not a vowel.

A	B	C	D	E
F	G	H	I	J
K	L	M	N	O
P	Q	R	S	T
U	V	W	X	Y

What are the possibilities for the landing position of the dart? Give instructions for locating any possibilities on the grid.

9. Rachel and Nathan are playing a battleship game. Rachel's field (in progress, up to this moment) is shown below.



a. For each of the following attempts, state whether Nathan will miss (M), sink (S) or hit (H) one of Rachel's ships.

i. A7 **ii.** F9 **iii.** I10 **iv.** C3

b. Specify the location of all of Rachel's one-berth ships.

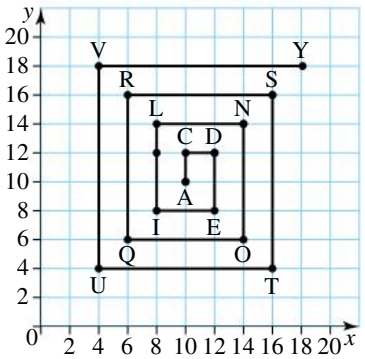
c. State the set of grid references that Nathan needs if he is to sink Rachel's largest ship.

10. Answer the following questions, then translate each letter of your answer into an ordered pair by using the grid at right.

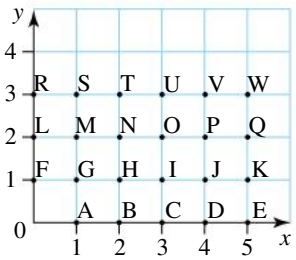
a. In which Australian state is the Great Barrier Reef?

b. Which Australian state is known as the Garden State?

c. The Opera House is in which city?



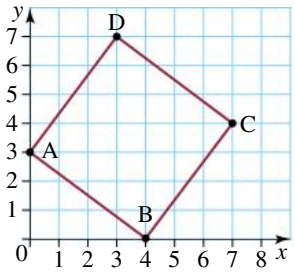
11. The diagram below gives the key to a code by matching letters to coordinates. Use the key to write in code this famous statement by Descartes: 'I think therefore I am.'



12. On a carefully ruled pair of Cartesian axes join the following points in the order given. It reveals a symbol used by an ancient mathematical secret society.

$$(1, 0) - (6, 4) - (0, 4) - (5, 0) - (3, 6) - (1, 0)$$

13. ABCD is a square drawn in the first quadrant as shown.
If A(0, 3) and B(4, 0) and C(7, 4) find the coordinates of D.



14. a. Plot the points A(3, 0), B(6, 7), C(10, 7), D(7, 0).
What type of quadrilateral is ABCD?
Find its area.

b. Draw a set of axes and on them plot points so that the encompassing triangle has an area of 20 square units. List the points and show why the area is 20 square units.

15. Four points — A, B, C and D — are drawn on a Cartesian plane, and are joined in order to form a square. A is the point $(-4, 0)$, B is $(1, 5)$ and C is $(6, 0)$. What are the coordinates of point D?

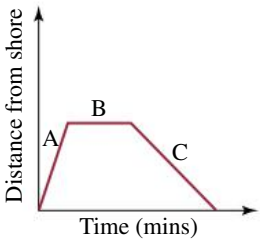
16. Does this table represent a relationship between x and y which is linear?

x	1	2	3	4
y	1	4	9	16

Explain why or why not.

17. Consider the following statement: 'A linear relation, when plotted on a Cartesian plane, produces a straight line that cuts the x-axis at one point, and the y-axis at one point.'
Discuss whether or not this statement is true.

18. The following graph shows Anton's distance from the shore during a surf life-saving drill.



a. Describe what each section of the graph is showing.

b. The line in section A is steeper than the line in section C. What does this tell you about Anton's swimming speed in section A compared to his speed in section C?

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- **Try out this interactivity:** Crossword: Topic 15 (int-2610)
- **Try out this interactivity:** Sudoku: Topic 15 (int-3175)
- **Complete this digital doc:** Concept map (doc-10745)

Language

It is important to learn and be able to use correct mathematical language in order to communicate effectively. Create a summary of the topic using the key terms below. You can present your summary in writing or using a concept map, a poster or technology.

alphanumeric
area references
axes
Cartesian plane
coordinates
domain
grid
grid reference
horizontal
interpret
linear
location
maximum
minimum
ordered pair
plot
quadrant
range
reference
route
vertical

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Investigation | Rich task

Battleships game

The goal of this two-person game is to sink all of your opponent's ships before they do the same to you.

Equipment: Each player has two 10 × 10 grids. The grids are labelled with letters across the top (A to J) and numbers down the side (1 to 10). One of the grids is labelled 'My grid' and the other is labelled 'Opponent's grid'.

Set-up: Each player decides where to place four ships on their 'My grid': a five-square battleship, a four-square cruiser, a three-square submarine and a two-square destroyer. All ships must be placed in straight lines either horizontally or vertically, but not diagonally. It is legal (but not required) for two or more ships to be adjacent to each other.

How to play: Players take turns taking shots at each other. A shot is taken by calling out the coordinates of a square on the 10 × 10 grid, such as A1 or H4. Each player takes one shot at a time. If the player calls the coordinates of a square where one of the opponent's ships is located, the opponent calls out 'Hit' and says which ship was hit; if the shot misses, the opponent says 'Miss'.

Players should mark the shots they take on their 'Opponent's grid' and whether each shot was a hit or a miss, so that they don't call any square more than once. Players should also mark their 'My grid' with shots taken by their opponent.

A ship is sunk when all of its squares have been hit. When this happens, the player whose ship was sunk says, for example, 'You sank my battleship.'

The first player to sink all of their opponent's ships wins the game.

Opponent's grid										
	A	B	C	D	E	F	G	H	I	J
1										
2										
3										
4										
5										
6										
7										
8										
9										
10										

My grid										
	A	B	C	D	E	F	G	H	I	J
1										
2										
3										
4										
5										
6										
7										
8										
9										
10										

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Complete this digital doc: Code puzzle: One of Galileo's achievements (doc-14337)

Answers

Topic 15 Coordinates and the Cartesian plane

Exercise 15.2 Alphanumeric grid references on a map

1. a. E2 **b.** B1 **c.** B3 **d.** D1 **e.** B2 **f.** C1 **g.** D2 **h.** A1 **i.** D1

2. a. Geelong **b.** Ballarat **c.** Melbourne
d. Warrnambool/Colac **e.** Portland **f.** Mildura
g. Albury/Wodonga/Wangaratta/Bairnsdale **h.** Mount Gambier **i.** Griffith/Wagga Wagga

3. a. Q12 **b.** Q18 **c.** R18

4. a. Pennant Hills Road **b.** The Comenarra Parkway **c.** Sefton Road

5. a. D3, P8, H3, E7, F3, F7 **b.** Answers will vary.

6. a. A1, F1 **b.** Black knight **c.** B3
d. i. F3 to E5 **ii.** F6 to E4
e. Pawn at F7

7. It could be either A1 or B1.

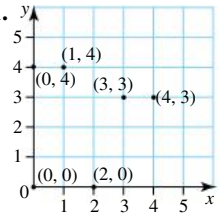
8. It could be one of A1, A2, B1 or B2

9. a. C2, D3, D3, D3, D4, D4, E4, E3 **b.** C2, B2, C1, D1, E1, F1, F1, F1 **c.** Various possible answers.

10. a. U8, U9, T8, R4, Q4 **b.** J7, M10, K8, E9, I5, E5, D6, G4 **c.** Various possible answers.

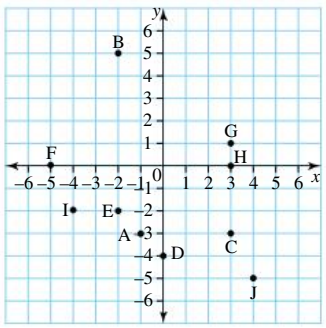
Exercise 15.3 The Cartesian plane

1.



2. A(6, 1) B(2, 5) C(7, 9) D(12, 10) E(10, 0) F($4\frac{1}{2}$, 0) G(0, 6) H(0, 0) I($7\frac{1}{2}$, 5) J(10, $7\frac{1}{2}$) K($3\frac{1}{2}$, $2\frac{1}{2}$) L($\frac{1}{2}$, $9\frac{1}{2}$)

3.



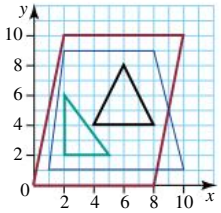
Quadrant I: point G; Quadrant II: point B; Quadrant III: points A, E, I; Quadrant IV: points C, J; x-axis: points F, H; y-axis: point D.

4. a. B **b.** C **c.** D **d.** A **e.** E **f.** A

5. a. E, C **b.** B, D

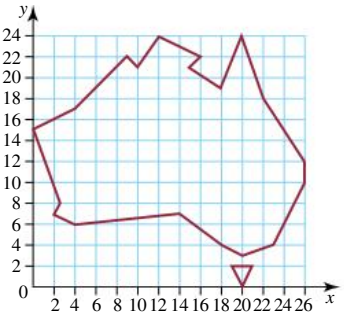
6. In hospital; in wards six, seven, eight and nine

7.



a. Right-angled triangle **b.** Isosceles triangle **c.** Trapezium **d.** Parallelogram

8.



9. 15 square units

10. **b, d, e**

11. a. A(0, 1) B(2, 3) C(5, 1)

b. 5 square units

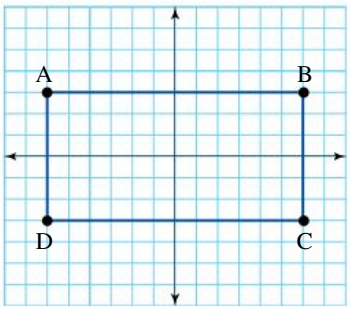
c. A(0, −1) B(2, −3) C(5, −1)

d. A(0, −1) B(−2, −3) C(−5, −1)

12. a. A(–3, 3), B(3, 3), C(3, –3), D(–3, –3)

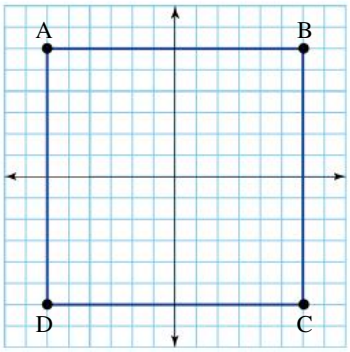
b. 36 square units.

c.



d. 72 squared units.

e.



f. 144 square units.

g. The length doubled, so the area doubled.
When both the length and the width doubled, the area became four times bigger.

13. a. i. It would move in quadrant 2.

ii. It would move in quadrant 4.

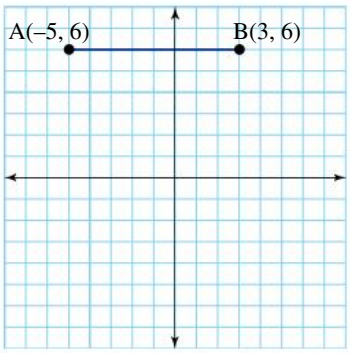
b. i. It would move in quadrant 1.

ii. It would move in quadrant 3.

c. i. Vertical reflection.

ii. Horizontal reflection.

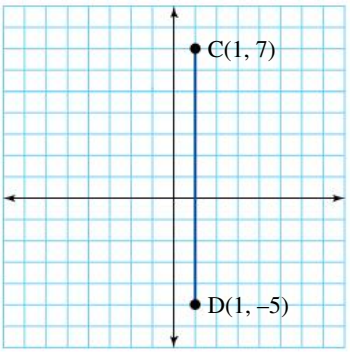
14. a.



b. 8 units **c.** $(-1, 6)$

d. Various possible answers. **e.** $\frac{x_1 + x_2}{2}$

15. a.



b. 12 units **c.** (1, 1)

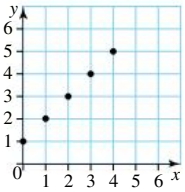
d. Various possible answers. **e.** $\frac{y_1 + y_2}{2}$

Challenge 15.1

Area is 18 square units, perimeter is 18 units.

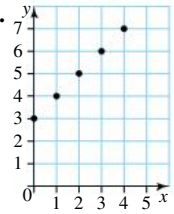
Execrcise 15.4 Plotting simple linear relationships

1. a.



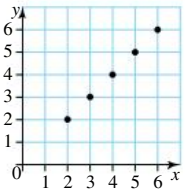
Linear pattern

b.



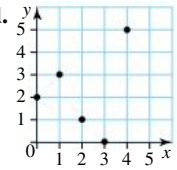
Linear pattern

c.



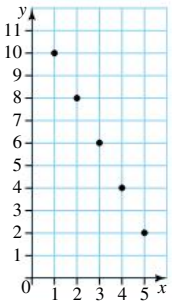
Linear pattern

d.



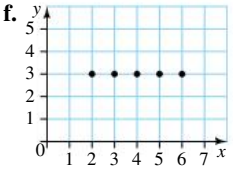
Non-linear pattern

e.



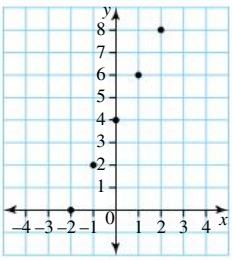
Linear pattern

f.



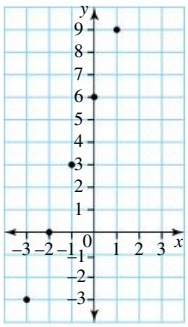
Linear pattern

2. a.

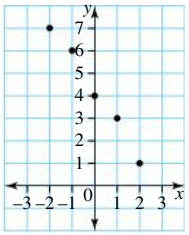


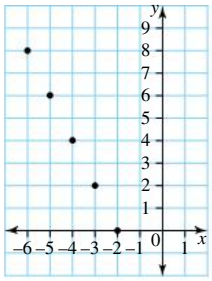
Linear pattern

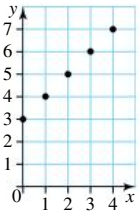
b.

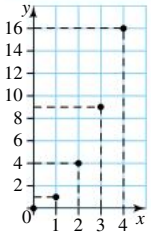


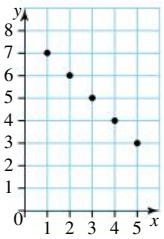
Linear pattern

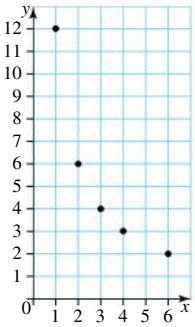
c.  Non-linear pattern

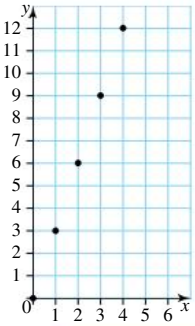
d.  Linear pattern

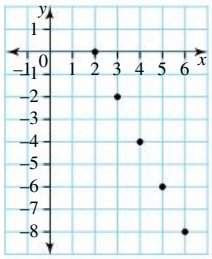
3. a.  Linear relationship

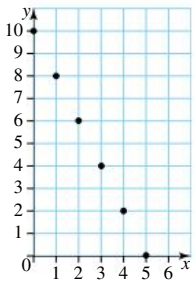
b.  Non-linear relationship

c.  Linear relationship

d.  Non-linear relationship

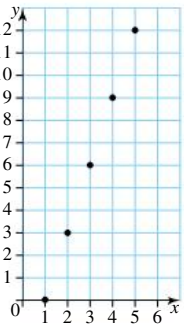
e.  Linear relationship

f.  Linear relationship

4. a, c 

b Linear pattern

d (4, 2), (5, 0)

5. a, c 

b $y = 6$

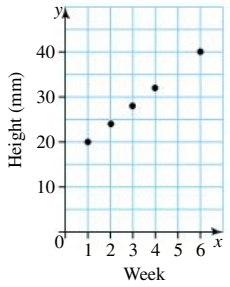
6. a. (8, 7) **b.** (3, 2)

7. a, b, d

8. a.

Week	1	2	3	4	5	6
Height (mm)	20	24	28	32		40

b.



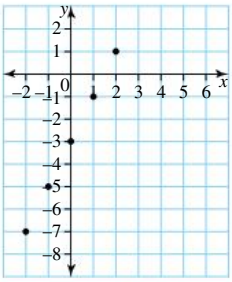
c. Yes, the points form a straight line. **d.** 36 mm **e.** 48 mm

9. a. The rule describes a set of points such that the y-coordinate of each point is three less than twice the x-coordinate.

b.

x	−2	−1	0	1	2
y	−7	−5	−3	−1	1

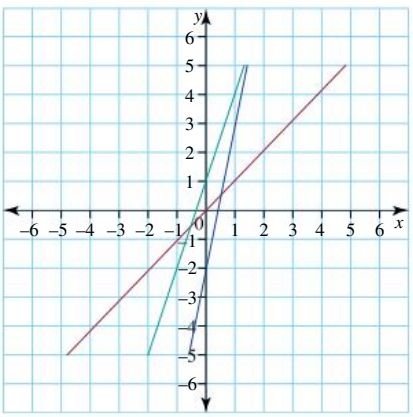
c.



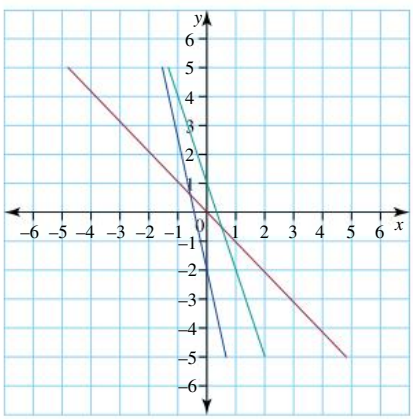
d. Yes; the points form a straight line.

e. $-7 \leq y \leq 1$

10. a.



b.



c. *Differences*: The coefficient of x is positive in the first set and negative in the second set.

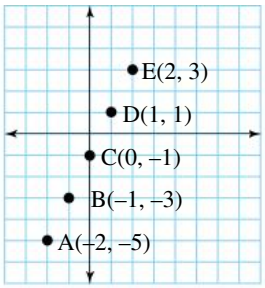
In the first set, the graphs are sloping up while in the second set, the graphs are sloping down.

Similarities: They are all straight lines.

11. a.

x	−2	−1	0	1	2
y	−5	−3	−1	1	3

b.



c. 1 unit

d. Yes, the points form a straight line.

11. a. Parking fee = 0.04 × time

b.

Time, minutes	30	60	90	120	150
Parking fee, $	1.20	2.40	3.60	4.80	6.00

Challenge 15.2

28 desks

Exercise 15.5 Interpreting graphs

1. a. Lucas **b.** Selina

2. a. Yes **b.** No

3. a. Talia **b.** Amy **c.** No

4. a. Karl **b.** No **c.** Yes

5. A: Claire is moving away from home.

B: Claire could be resting as she has stopped.

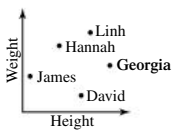
C: Claire is moving further away from home.

D: Claire could be resting again as she has stopped.

E: Claire is travelling home.

6. a. i. Linh **ii.** Linh **iii.** No

b.



7. E

8. C

9. a. A baby is not born with a zero head circumference.

b. The head circumference has increased steadily over the 12 weeks.

c. You would expect the graph to continue to show some increase in head circumference, but this would eventually slow down.

10. a. B

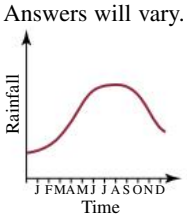
b. Monique is walking faster in section C; therefore, the distance from school is decreasing more quickly.

c. At school

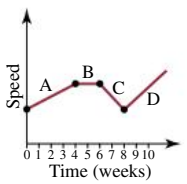
11. a. January **b.** April

b. Yupa's profit has gone down each month.

12. Answers will vary.



13.



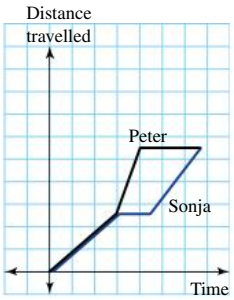
14. Windpower is increasing in Australia and worldwide.

15. a. April and August **b.** 40 mm **c.** July

16. a. 2009 **b.** 50 + yrs

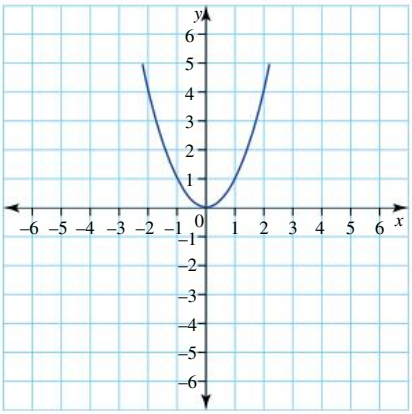
c. Retired people have more time available and want to increase fitness levels as they are getting older.

17.



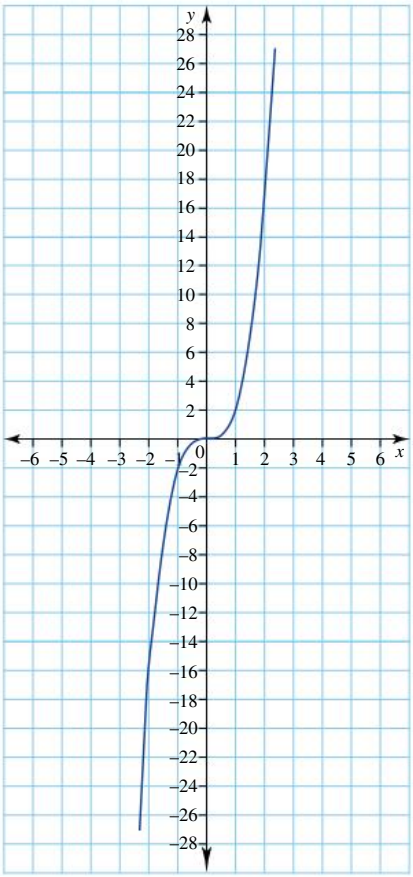
18. Cost 25 cents to make a phone call and then 2 cents for every second of the call.

19. a.



b. As the value of x increases, the value of y decreases to zero and then increases.

20. a.



b. As the value of x increases, the value of y increases.

15.6 Review

1. a. Joseph's House; B1; Post office; B4

b.

D	Post-office	Bus-stop	Star cafe	Library
C		Apple street		
B	Alex's house	Maya's house	Joseph's house	Lena's house
A				
	1	2	3	4

2. a. Balmoral Road **b.** C18

3. a. F3 **b.** E2 **c.** C4 **d.** E1

4. a. i. G5 **ii.** F2 **iii.** F7

b. St Pancras and King's Cross

c. Hyde Park

d. C3 – C4

5. a. A

b. K, I, L, J

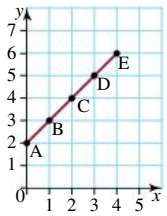
c. M, N, O, P

d. A(−3, 4) B(−4, 3)

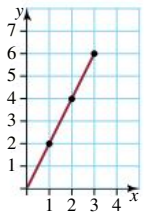
e. C(−4, −3) F(4, −3)

6. a. A(2, 4) (3, 5)

b.



7. a.



b. Yes, the points form a straight line.

1. H or X

H lies in column 3 and row 2.

X lies in column 4 and row 5.

2. a. i. H **ii.** H **iii.** S **iv.** M

b. B9, C2, H6, I10

c. A5 and A6 are already hit, so he needs A4 and A7 to sink it.

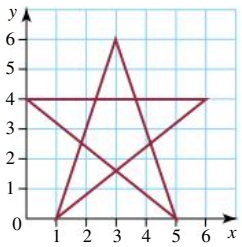
3. a. QUEENSLAND (6, 6), (4, 4), (12, 8), (12, 8), (14, 14), (16, 16), (8, 14), (10, 10), (14, 14), (12, 12)

b. VICTORIA (4, 18), (8, 8), (10, 12), (16, 4), (14, 6), (6, 16), (8, 8), (10, 10)

c. SYDNEY (16, 16), (18, 18), (12, 12), (14, 14), (12, 8), (18, 18)

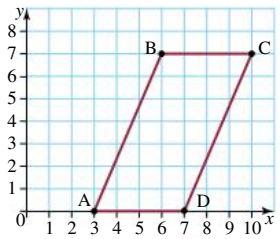
4. (3, 1) (2, 3) (2, 1) (3, 1) (2, 2) (5, 1) (2, 3) (2, 1) (5, 0) (0, 3) (5, 0) (0, 1) (3, 2) (0, 3) (5, 0) (3, 1) (1, 0) (1, 2)

5.

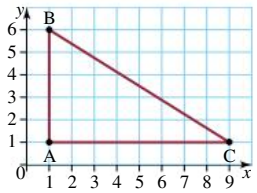


6. D is (3, 7)

7. a. Parallelogram area = 28square units



b.



One example could be Triangle ABC where A is (1, 1), B is (1, 6) and C is (9, 1).

The area of this triangle is $\frac{1}{2} \times$ base $\times$ height $= \frac{1}{2} \times 8 \times 5 = 20$ square units as required.

8. D(1, −5)

9. No; the relation is $y = x^2$,which is not in the form $y = mx + c$.

10. Not true, as a straight line could:

- be parallel to the y-axis and only cut the x-axis once
- be parallel to the x-axis and only cut the y-axis once
- pass through the origin, cutting the axes at only that point.

11. a. A: Anton is swimming out from the shore.

B: Anton is staying the same distance from the shore.

C: Anton is swimming back to the shore.

b. Anton is swimming faster in section A than in section C.

Investigation — Rich task

Teacher to check.

TOPIC 16 Transformations

16.1 Overview

Numerous **videos** and **interactivities** are embedded just where you need them, at the point of learning, in your learnON title at www.jacplus.com.au. They will help you to learn the content and concepts covered in this topic.

16.1.1 Why learn this?

Geometry is not only the study of figures and shapes; it is the study of movement of figures. Transformation is a general term to describe a change to an object or its position. Patterns present in modern design, ancient design and architecture show how transformations can be applied.

16.1.2 What do you know?

assessON

1. **THINK** List what you know about transforming shapes by translation, reflection and rotation. Use a thinking tool such as a concept map to show your list.
2. **PAIR** Share what you know with a partner and then with a small group.
3. **SHARE** As a class, create a thinking tool such as a large concept map that shows your class's knowledge of transformations.

LEARNING SEQUENCE

16.1 Overview
16.2 Axes of symmetry
16.3 Translations
16.4 Reflections
16.5 Rotations and combined transformations
16.6 Review

16.2 Axes of symmetry

16.2.1 Axes of symmetry

- If it is possible to draw a line which would divide a shape into two parts, so that when folded along that line the parts would coincide exactly, the shape is said to be symmetrical along that line.
- The line itself is called the **line** (or the **axis**) **of symmetry**.
- If a mirror is placed along the axis of symmetry, the half of the shape that faces the mirror, together with its reflection, will form the whole shape (that is, the mirror reflection will look exactly like the other half of the shape).
- We can think of the axis of symmetry as the line that acts like a mirror.
- Many things in nature are symmetrical. Some objects have more than one axis of symmetry, while others have none.

WORKED EXAMPLE 1

Decide whether the front of the house shown at right is symmetrical and, if so, show where the line of symmetry is.

THINK

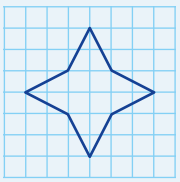
A line of symmetry divides a shape into two parts so that if the shape is folded along it, the two parts will coincide. Imagine folding the picture along a line drawn through the top point of the roof and the midpoint of the base of the house. The two halves will coincide exactly. Show this line of symmetry using a dotted line.

WRITE

The shape of the house is symmetrical. The dotted line shows the axis of symmetry.

WORKED EXAMPLE 2

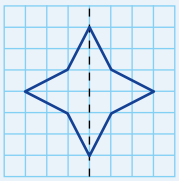
How many axes of symmetry does this shape have?



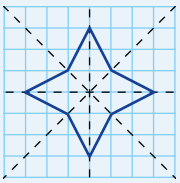
THINK

1 An axis of symmetry is a line that acts as a mirror. A vertical line through the top and bottom vertices of the given shape would divide it into halves that are mirror images of each other. So it is the axis of symmetry.

WRITE/DRAW



2 Look for other axes of symmetry. Indicate all the axes of symmetry with dotted lines. State the number.

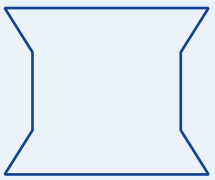


There are 4 axes of symmetry.

- A figure has rotational symmetry if it can be turned less than 360° to match the original figure.
- The number of times a match occurs within the 360° is known as the *order* (or *degree*) of rotational symmetry.

WORKED EXAMPLE 3

Determine if the figure at right has rotational symmetry and, if it does, state the order.



THINK

1 The shape is rotated less than 360° to match the original and so it has rotational symmetry.

2 Work out how many turns are needed to return to the initial position. Two turns are needed.

WRITE

The figure has rotational symmetry of order 2.

learnON RESOURCES — ONLINE ONLY

Watch this eLesson: Transformations (eles-0012)

Exercise 16.2 Axes of symmetry

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 14	Questions: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13, 14	Questions: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14

Individual pathway interactivity: int-4387

learnON ONLINE ONLY

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note*: Question numbers may vary slightly.

Fluency

1. WE1 Decide whether each of the following is symmetrical and, if so, show the line of symmetry.

a.

b.

c.

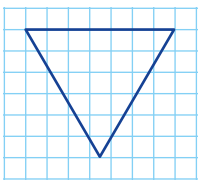
d.

e.

f.

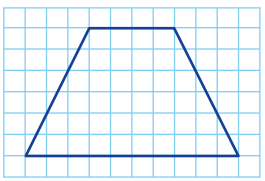
2. Copy each of the following shapes onto grid or squared paper. Carefully cut out each shape. Fold it to find the axes of symmetry. State the number of axes of symmetry for each shape. (Some shapes will have more than one axis of symmetry.)

a.

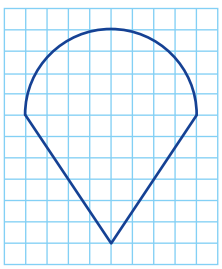


b.

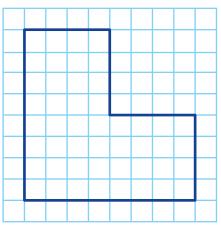
c.



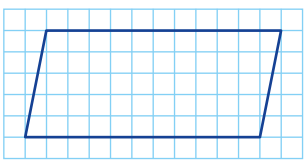
d.



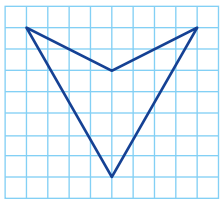
e.



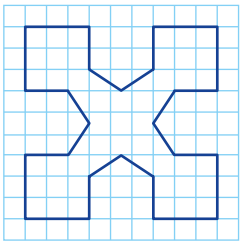
f.



g.



h.



3. WE2 How many axes of symmetry do each of the following shapes have?

a.

b.

c.

d.

e.

f.

4. WE3 Determine which of the figures in question **2** have rotational symmetry and, for those that do, state the order.

5. For each figure in question **3**, state the order of rotational symmetry.

Understanding

6.

A B C D E F G H I J K L M
N O P Q R S T U V W X Y Z

a b c d e f g h i j k l m
n o p q r s t u v w x y z

Copy the letters and draw in the axes of symmetry for each if appropriate.

a. Which upper case letters of the alphabet have a line of symmetry?

b. Which lower case letters of the alphabet have a line of symmetry?

c. Do any of the letters have more than one line of symmetry? If so, which ones?

7. MC The number of axes of symmetry in the following shape is:

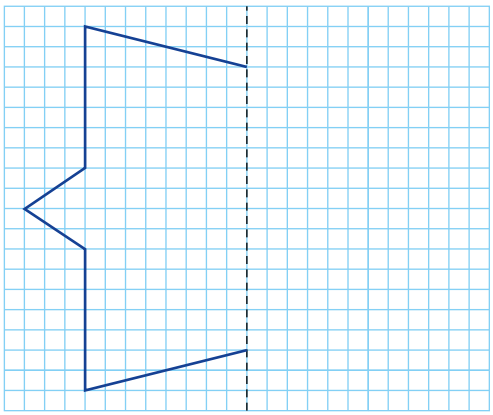
A. 0 **B.** 1 **C.** 2 **D.** 4 **E.** 3

8. MC The number of axes of symmetry in the following shape is:

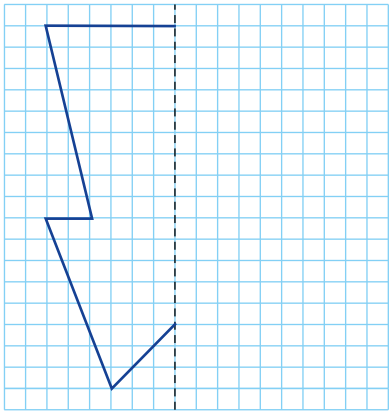
A. 0 **B.** 1 **C.** 2 **D.** 3 **E.** 4

9. For each of the following diagrams, use the dotted line as an axis of symmetry to complete the picture.

a.



b.



10. For each of the following, use the dotted line as an axis of symmetry to complete the picture.

a.

b.

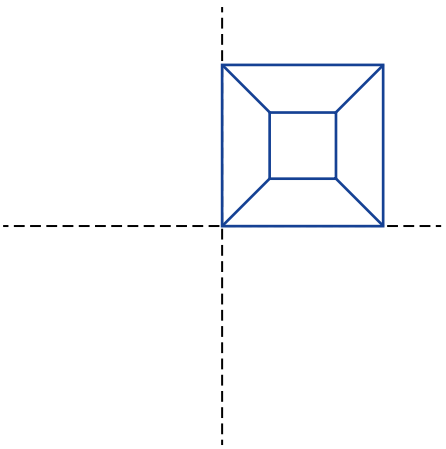
Reasoning

11. The following shapes are regular polygons.

a. How many axes of symmetry does each star have?

b. Is there a relationship between the number of vertices and the number of axes of symmetry? Explain.

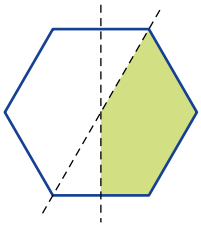
12. a. Complete the following pattern knowing that the horizontal and the vertical dotted lines are both axes of symmetry for the pattern.



b. How did you know where to draw the pattern? Explain using the concept of symmetry and its meaning.

Problem solving

13. Each of the dotted lines drawn on this rectangular hexagon is an axis of symmetry. Determine the fraction of the hexagon that is shaded.



14. a. How many axes of symmetry can you draw on the umbrella shown?

b. How many axes of symmetry does the Melbourne Star have?

Reflection

What strategy would you use to determine the number of axes of symmetry in a shape?

16.3 Translations

16.3.1 Transformations

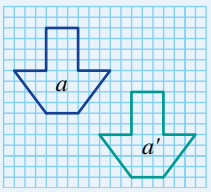
- Shapes can be shifted by an ordered set of moves from one place to another. Such ordered sets of moves are called **transformations**.
- Prior to transformation, the shape is usually called the *original object*; after the transformation, the shape is called the *image*.
- When dealing with various transformations, it is customary to denote the image with the same letters as the original shape, but with an added apostrophe. For example, if the original object is a rectangle, named ABCD, then its image after the transformation is usually named A′B′C′D′.
- If, after the transformation, the image has exactly the same size and shape as the original object, such a transformation is called **isometric**.
- Isometric transformations discussed in this chapter include **translations**, **reflections** and **rotations**. They result in **congruent figures**.

16.3.2 Translations

- A translation is the movement of an object up, down, left or right (U, D, L or R) without flipping, turning or changing size.
- After a translation the object has been carried across a plane in any direction, parallel to its original position. Translations are also known as 'slides'.

WORKED EXAMPLE 4

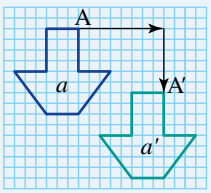
State the translation of shape *a* to *a′*.



THINK

1 Select any point on the boundaries of the original object, *a* (any vertex is good to choose) and name it A. Locate the corresponding point on the image and name it A′. From point A draw a horizontal line, until it is directly above A′; then draw a vertical line, so that it meets A′.

WRITE/DRAW



2 Count the number of units that the object has moved across (to the right) and down and, hence, record the translation that took place.

Translation: 8R 6D

- If an object has been translated several times in both horizontal and vertical directions, its final position (relative to the original one) can be described by the total horizontal and the total vertical translations.
- When finding the total horizontal translation, we treat translations (that is, the number of units) *to the right* as *positive numbers* and *translations to the left* as *negative*. The number of units the object has been moved to the right and to the left (with appropriate signs) can be added together to give the total horizontal translation.
- To find the total vertical translation, we add together the number of units the object has been moved upward and downward, treating *upward moves* as *positive* numbers and *downward moves* as *negative*.

WORKED EXAMPLE 5

State the final position of an object (relative to its original position) after the translation: 4U 2L, 2D 5L, 6U 2R and 3D 2L.

THINK	WRITE
1 Find the total vertical translation. *Note:* Think of upward movements as positive and downward movements as negative.	4U + 2D + 6U + 3D = 4 − 2 + 6 − 3 = 5 So, the vertical translation is 5U.
2 Find the total horizontal translation. *Note:* Treat movements to the right as positive and movements to the left as negative.	2L + 5L + 2R + 2L = − 2 − 5 + 2 − 2 = − 7 So, the horizontal translation is 7L.
3 State the final position of the object, relative to its position prior to translation.	The position after translation, relative to the original position, is 5U7L.

When you are drawing the image of an object after a series of translations, the following steps might be of assistance.

Step 1: Find the total vertical and the total horizontal translations.

Step 2: Select a point on the boundaries of the original object. (The vertices are usually the most convenient points to choose.)

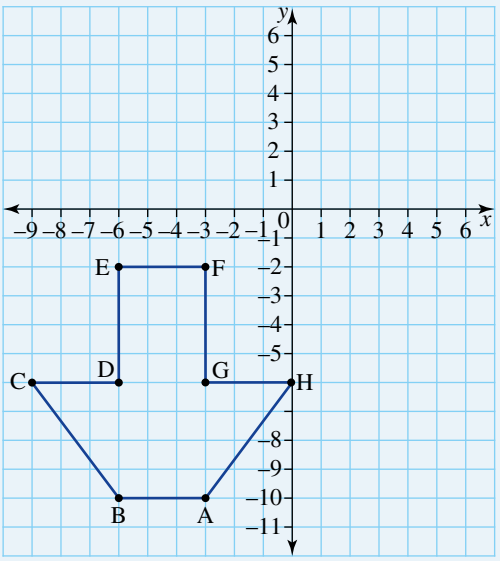
Step 3: Translate the selected point as required.

Step 4: If the shape is complicated, translate a few more points.

Step 5: Complete the image of the object, using the points translated previously as a guide.

WORKED EXAMPLE 6

a Give the coordinates of the points A to H on the object shown at right.
b Translate the object 5 units up, 2 units left, 2 units up and 8 units right.
c Give the coordinates of the new points, A′ to H′.



THINK	WRITE/DRAW
a Write the coordinates of the points A to H.	**a** The coordinates of the vertices are: A(−3, −10), B(−6, −10), C(−9, −6), D(−6, −6), E(−6, −2), F(−3, −2), G(−3, −6), H(0, −6).

b **1** Find the total vertical move.

b Vertical move: 5 up and 2 up $= 5 + 2$
$= 7$ (up)

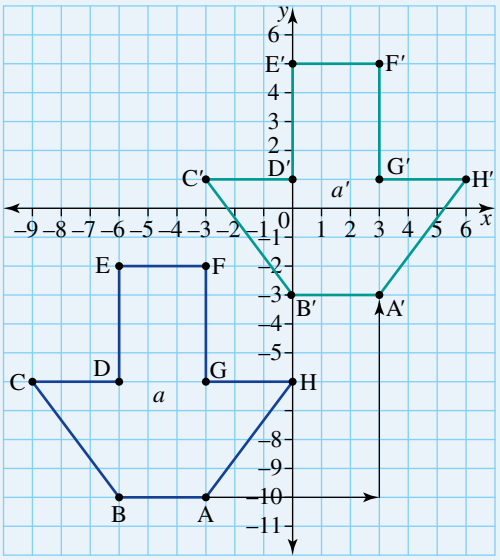
2 Find the total horizontal move.

Horizontal move:
2 to the left and 8 to the right $= -2 + 8$
$= 6$ (to the right)

3 State the total translation (that is, the simplified version of the required translation).

Translation:
7U 6R

4 Select any point on the object, say point A. Translate point A 7 units up and 6 units to the right. Call the translated (image) point A′. Complete the image of the object, using point A′ as a guide. Label the original object *a* and its image *a*′. (You may wish to highlight the image, using a different colour.)



c Write the coordinates of the points A′ to H′.

c The coordinates of the vertices are:
A′(3, −3), B′(0, −3), C′(−3, 1), D′(0, 1), E′(0, 5), F′(3, 5), G′(3, 1), H′(6, 1).

learnon RESOURCES — ONLINE ONLY

Complete this digital doc: SkillSHEET: Translation of a point (doc-6596)

Exercise 16.3 Translations

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1, 2, 3, 4, 6, 7, 11	Questions: 1, 2a, c, e, g, 3, 4, 5, 6, 8, 9, 11, 12	Questions: 1, 2a, e, h, 3, 4a, d, 5, 6, 8, 9, 10, 11, 12

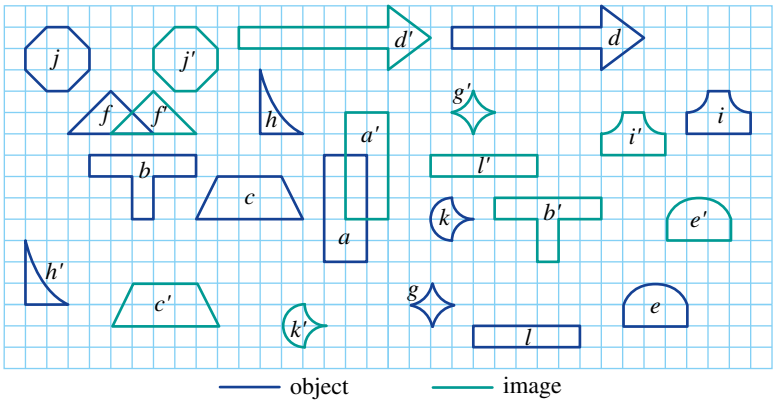
Individual pathway interactivity: int-4388

learnon ONLINE ONLY

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note*: Question numbers may vary slightly.

Fluency

1. WE4 State the translation that has occurred to each of the shapes labelled *a* to *l* in the figure below.

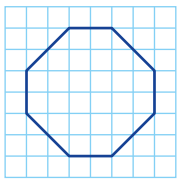


2. WE5 State the final position of an object (relative to its original position) after each of the following translations:

a. 4L 2U, 3R 4U
b. 6R 3D, 4L 5D
c. 8L 2U, 4R 3D, 6R 4U
d. 10R 3D, 4L 5D, 6R 8U
e. 12L 4D, 2L 3D, 8R 4D, 6L 2U
f. 3U, 4D 3R, 4D 6L, 2R, 7L 2U, 3L
g. 8L 2D, 4R 9U, 6L 2D, 4R 3U, 2L 9D
h. 6R 2U, 3L 5U, 3R 4D, 6L 3D, 2R 4U, 6L 3U

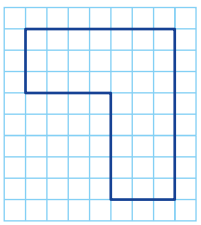
3. Draw the following translations to the shapes shown.

a.



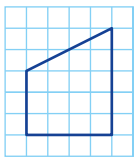
6L 2D

b.



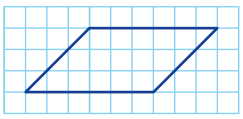
2R 3U

c.



8R 1D

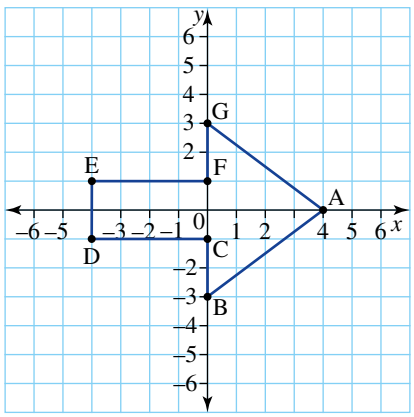
d.



2L 7U

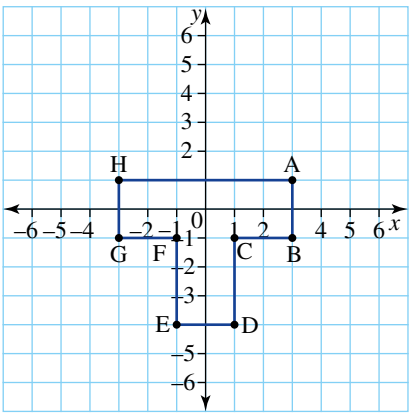
4. i. WE6 Translate the following objects as specified under the figures.

a.



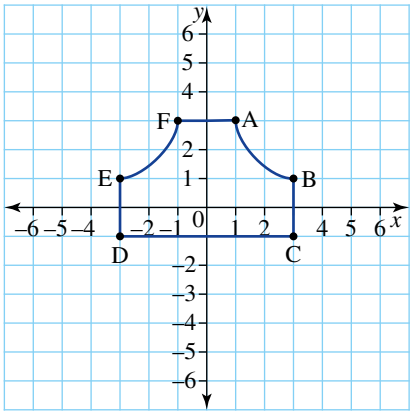
4L 2D, 3R 4U

b.



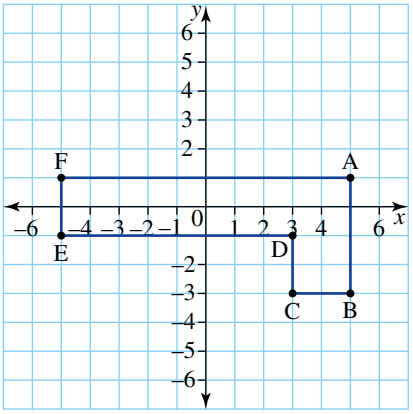
6L 2D, 8R 3D

c.



2R 4D, 6R 3D, 2R 2D

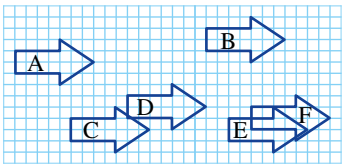
d.



7R 5U, 2R 3U, 8L 2D, 7L 1D

ii. Give the coordinates of the vertices of each object in its final position.

Understanding



5. Specify each of the following translations of the labelled arrows in the diagram.

a. A to C **b.** A to F
c. F to E **d.** E to F
e. D to B **f.** E to A

6. Using the diagram in question **5**, state which arrow has moved where (that is, which letter represents the object and which one represents the image), if the following translations have occurred.

a. 2U 5R **b.** 2D 5L **c.** 2U 9L
d. 2L 8U **e.** 4U 10L **f.** 14L

7. MC If an object is translated 3U 2L, 5R 4D, 6L, its final position (related to the original one) is:

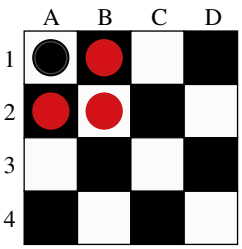
A. 7U 3R **B.** 1U 3L **C.** 7U 3L **D.** 1D 3L **E.** 1D 3R

8. MC If, relative to the original position, the final position of an object is 3L 4U, then the series of translations that could have occurred is:

A. 4R 2U, 3L 2U **B.** 4R 1U, 7L 3U **C.** 4R 1U, 7R 3U
D. 4R 2U, 3L 3U **E.** 4R 1U, 4L 3U

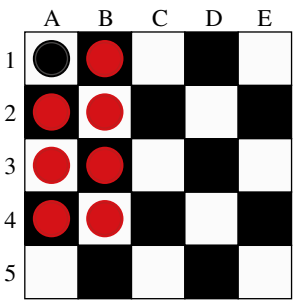
Reasoning

9. In a game of draughts, a counter is removed from the board when another counter jumps over it. Four counters are arranged on the corner of a draughts board as shown. Use three horizontal or vertical jumps to remove all three red counters, leaving the single black counter. Copy and complete the table to show your working.



Jump	From	To
1		
2		
3		

10. In another game, any counter can jump any other counter. Use seven horizontal or vertical jumps to remove all seven red counters, leaving the single black counter. Copy and complete the table to show your working.



Jump	From	To
1		
2		
3		
4		
5		
6		
7		

Problem solving

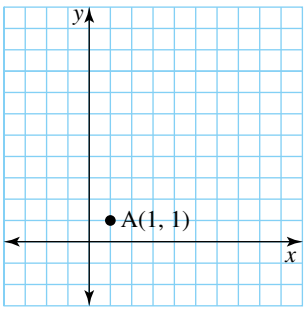
11. Alja likes to skip around her backyard.

a. If her starting position on a number plane is at point A(1, 1) and she skips two steps forward and one step to the right, draw her next four locations after every set of three steps.

b. Do the five points form a pattern?

c. From her last location, Alja decides to skip one step backwards and two steps to the left. She skips in this pattern four times.

d. Do these five points form a pattern?



12. A crane is moving a girder needed in the construction of a building 5 m up from an initial position of 3 m above the ground. How far up from ground level will the girder be at the end of the translation?

Reflection

What strategy would you use to translate an object from one position to another?

CHALLENGE 16.1

This array of nine 9s contains 8 straight lines, each totalling 27 (3 across, 3 down, 2 diagonal). Show how you would move two 9s to new positions to make 10 straight lines totalling 27.

9	9	9
9	9	9
9	9	9

16.4 Reflections

16.4.1 Reflections

- A reflection is the image of an object, as seen in the mirror.
- Reflections are often called 'mirror images' and the lines in which the objects are reflected — 'mirror' lines.
- Mirror images always have reverse orientation; that is, left appears to be right and vice versa.
- The points of the image and the corresponding points of the object lie on the same line, which is perpendicular to the mirror line.
- Each point of the image is the same distance from the mirror line as is the object on the other side of the mirror.
- Reflections are also known as 'flips'.

When reflecting shapes in the given line, the following steps can be of assistance:

Step 1: Select some key points on the original object (the vertices are usually convenient to choose).

Step 2: From each point draw a line, perpendicular (that is, at right angles) to the mirror line. Extend each line beyond the mirror line.

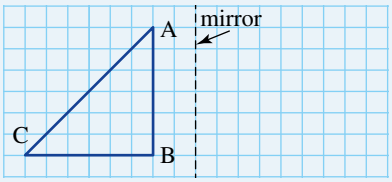
Step 3: For each selected point, measure its distance along the line from the mirror. Then measure out exactly the same distance along the line on the other side of the mirror to find the position of the corresponding point of the image.

Step 4: Complete the image, using the previously reflected points as a guide.

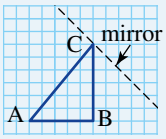
WORKED EXAMPLE 7

For each of the following shapes, find the reflected image in the line given.

a



b



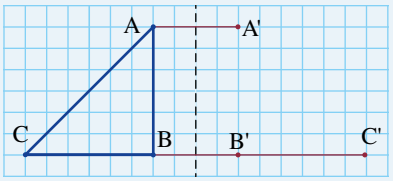
THINK

a 1 From each vertex of the given triangle, draw the lines perpendicular to and extending beyond the mirror line.

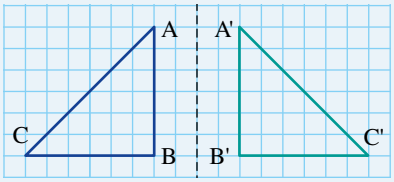
DRAW

a

2 Points A and B are both 2 units from the mirror to the left of it. Since the image is reversed, the vertices A′ and B′ are 2 units from the mirror on the right side of it. Furthermore, point C is 8 units to the left of the mirror. The corresponding point of the image C′ is 8 units to the right of the mirror.



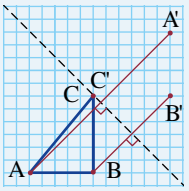
3 Join the vertices A′, B′ and C′ to complete the image.



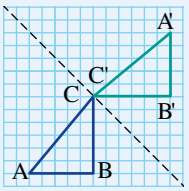
b 1 Draw the lines from points A and B so that they extend beyond the mirror line and are perpendicular to it. (Point C of the given triangle is already on the mirror line, so nothing needs to be done to it.)

b

2 Any point of the object that is on the mirror line will reflect onto itself. So C′ will coincide with point C, since point C is on the mirror line. Points A′ and B′ are the same distance from the mirror as points A and B respectively, but on the other side of it. (The distances must be measured along the perpendicular lines.)



3 Join the vertices A′, B′ and C′ to complete the image.

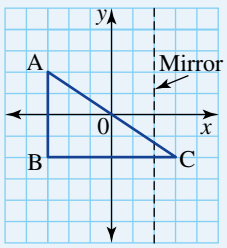


Sometimes the line in which the object is to be reflected is placed over the object so that the object is divided in two parts. Since the image is reversed, the part of the object that is to the left of the mirror line when reflected will appear to the right of it. The part of the object to the right of the mirror line will appear to the left of it after the reflection.

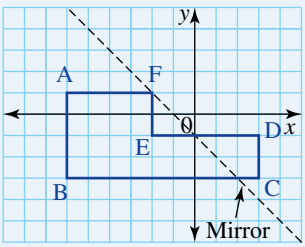
WORKED EXAMPLE 8

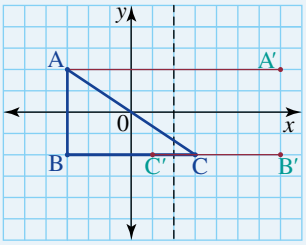
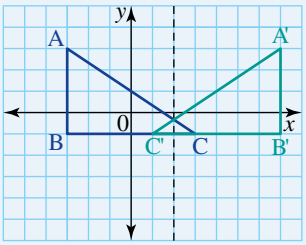
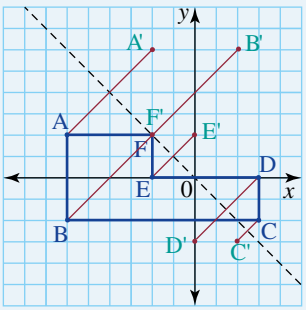
For each of the following objects:
i give the coordinates of the vertices of the object
ii draw the image reflected in the mirror
iii give the coordinates of the vertices of the image.

a

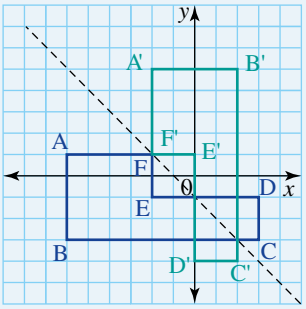


b



THINK	WRITE/DRAW
a i Write the coordinates of each of the vertices of the object.	**a** The coordinates of the vertices of the object are: A(−3, 2), B(−3, −2), C(3, −2)
ii 1 Extend line BC (it is perpendicular to the mirror line). From point A, draw the line extending beyond the mirror and perpendicular to it.	
2 The mirror line is placed over the object, so one part of the image will appear to the left of the mirror and the other part to the right of it. Points A and B are both 5 units to the left of the mirror, so A′ and B′ will be 5 units to the right of it. Point C is 1 unit to the right of the mirror, so its image, C′, will be 1 unit to the left of the mirror line.	
3 Complete the image by joining the reflected points appropriately.	
iii Write the coordinates of each of the vertices of the image.	The coordinates of the vertices of the image are: A′(7, 2), B′(7, −2), C′(1, −2)
b i Write the coordinates of each of the vertices of the object.	**b** The coordinates of the vertices of the object are: A(−6, 1), B(−6, −3), C(3, −3), D(3, −1), E(−2, −1), F(−2, 1)
ii 1 From every vertex of the object, draw the lines extending beyond and perpendicular to the mirror line.	
2 The mirror is placed over the object, so parts of the image will appear on either sides of the mirror line. Points A, B and E are to the left of the mirror, so their corresponding images (that is, A′, B′ and E′) will be the same distances from the mirror line, but to the right of it. Points D and C are to the right of the mirror, so their images (D′ and C′) will appear to the left of the mirror line. Finally, F′ will coincide with point F, as F is exactly on the mirror line.	

3 Complete the image, using the reflected vertices as a guide.



iii Write the coordinates of each of the vertices of the image.

The coordinates of the vertices of the image are:

$A'(-2, 5), B'(2, 5), C'(2, -4), D'(0, -4), E'(0, 1), F'(-2, 1)$

learnON RESOURCES — ONLINE ONLY

Complete this digital doc: SkillSHEET: Reflection of a point (doc-6597)

Complete this digital doc: SkillSHEET: Reflection of an object (doc-6598)

Complete this digital doc: Transformations I (doc-2024)

Exercise 16.4 Reflections

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1, 2, 3, 4, 5a, 9, 10	Questions: 1, 2a, c, e, 3, 4a, d, 5, 8, 9, 10	Questions: 1, 2a, d, 3, 4a, d, 5, 6, 7, 8, 9, 10

Individual pathway interactivity: int-4389

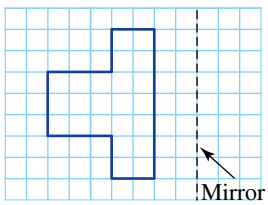
learnON ONLINE ONLY

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note*: Question numbers may vary slightly.

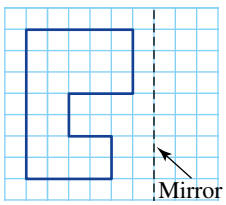
Fluency

1. WE7 For each of the following shapes, find the reflected image in the line given.

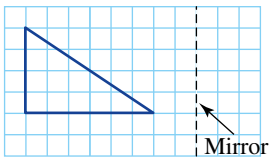
a.



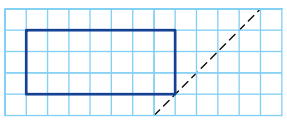
b.



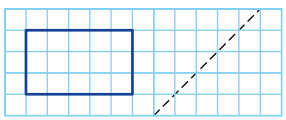
c.



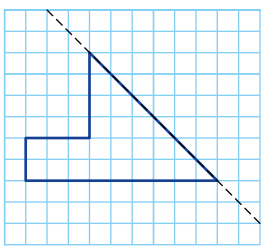
d.



e.



f.

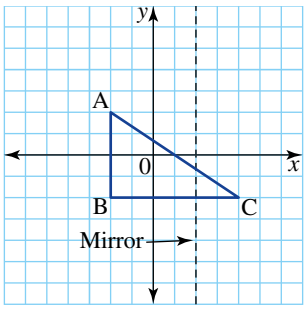


2. WE8 For each of the following shapes:

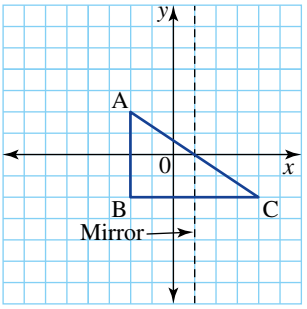
i. draw the image reflected in the mirror

ii. give the coordinates of the vertices of the image.

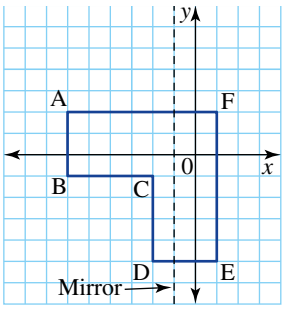
a.



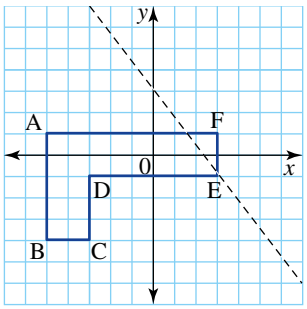
b.



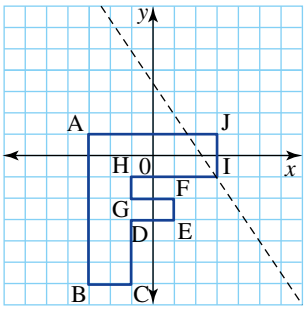
c.



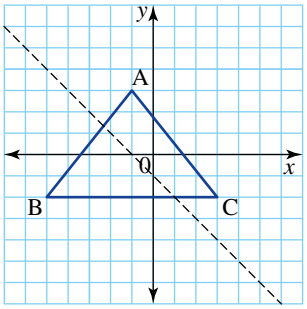
d.



e.

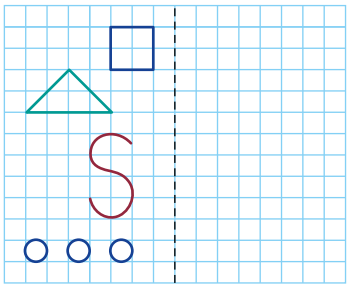


f.

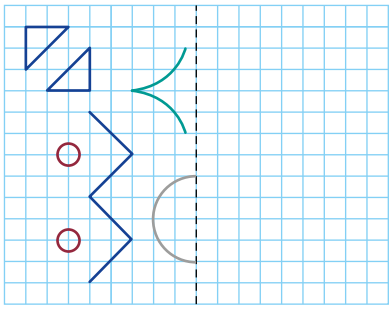


3. Reflections in mirrors transform objects. Complete each of the patterns by finding images of the objects reflected in a single mirror, as shown in the following.

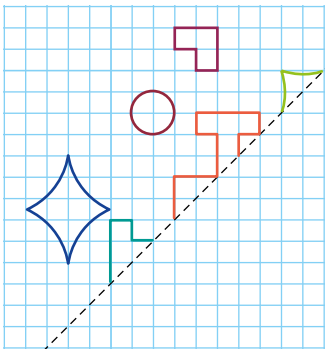
a.



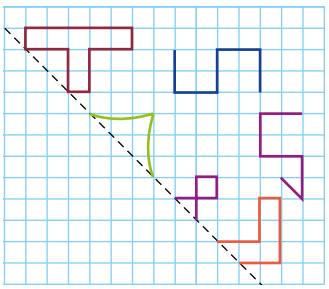
b.



c.



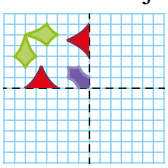
d.



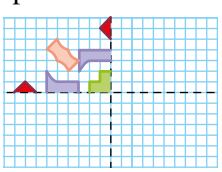
Understanding

4. A kaleidoscope uses more than one mirror to create wonderful patterns. To help design a kaleidoscope, reflect each object in each of the mirrors to complete the pattern in each of these figures.

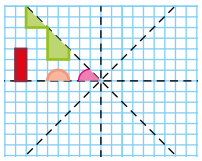
a.



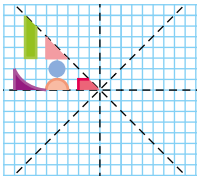
b.



c.

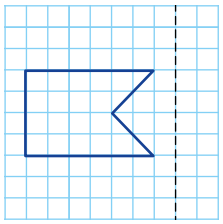


d.



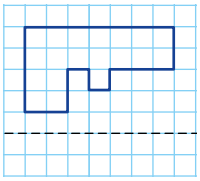
5. Find the image for each of the following objects, using the transformations specified under the figures.

a.



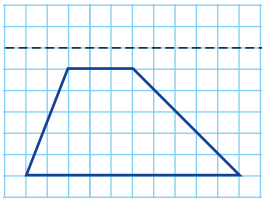
Reflected in the mirror and then translated 2 units down and 3 units left

b.



Translated 3U 4R and then reflected in the mirror

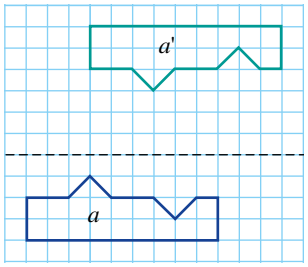
c.



Translated 1U 4L, reflected in the mirror and then translated 6U 2R

6. MC The diagram shows an object and its image after a certain set of transformations. The object was:

A. reflected in the mirror, then translated 3R 2D
B. reflected in the mirror, then translated 4R 2D
C. translated 3R 2D, then reflected in the mirror
D. translated 4R 2D, then reflected in the mirror
E. translated 3R 2U, then reflected in the mirror.



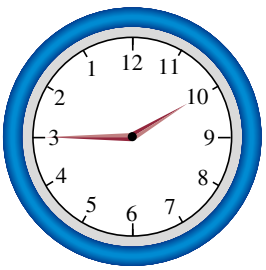
Reasoning

7. In a reflection, the axis of reflection becomes the axis of symmetry if the object and its image are next to each other. Explain the previous statement.

8. Explain the difference between reflection in horizontal axis and reflection in a vertical axis.

Problem solving

9. This is a backward clock. It has the numbers in reverse order, and the hands sweep anticlockwise. What time does the clock show?



10. Which one of the four strawberries is a reflection in the dotted line of the strawberry marked as original? Explain.

Original

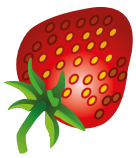


Image 1

Image 2

Image 3

Image 4

Reflection

What strategy would you use to draw the reflection of a shape in a mirror?

16.5 Rotations and combined transformations

16.5.1 Rotations

- A figure can be rotated about a point called the **centre of rotation**.
- To specify the rotation, we need to give the angle through which the object is to be turned and the direction of the rotation (clockwise or anticlockwise).
- Rotations are also known as 'turns'.

To perform a rotation, the following steps could be of assistance:

Step 1: Select any point on the object (vertices are most convenient to choose).

Step 2: Join the selected point and the centre of rotation with a straight line.

Step 3: Using a protractor and a ruler (or a pair of compasses), find the image of the selected point after the required rotation. The image point is as far from the centre of rotation as is its corresponding point on the object.

Step 4: Rotate some other points if necessary.

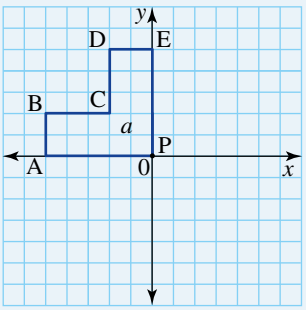
Step 5: Complete the image, using previously rotated points as a guide. Label the image appropriately.

WORKED EXAMPLE 9

Consider the shape shown at right.

a Show the image of the shape after a rotation of 90° (quarter turn) anticlockwise about point P.

b Compare the coordinates of the vertices of the object with those of its image.



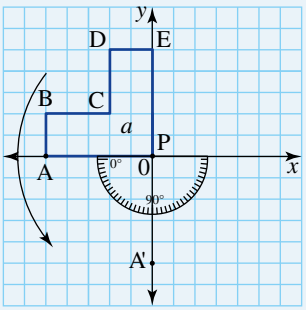
THINK

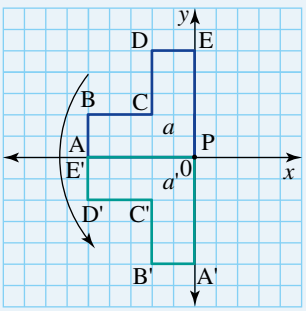
a 1 Select any point on the object, say A. Join point A with the centre of rotation, point P. (In this case the points are already joined by the side of the shape.) Place the protractor so that its centre is on P, its zero is on the line AP and its scale increases in an anticlockwise direction. Measure the 90° angle. The image point A′ is as far from the centre of rotation, P, as is point A (5 units in this case).

2 Repeat step **1** for some other points. Use these points as a guide to complete the image. Label the image *a′*. (You may wish to highlight it, using a different colour.)

WRITE/DRAW

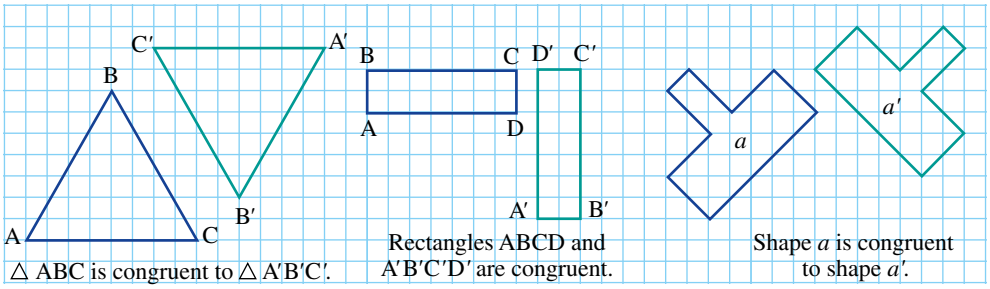
a





b 1 Write the coordinates of the object and the image.	**b** The coordinates of each point of the object and its corresponding position on the image are: $A(-5, 0) \rightarrow A'(0, -5)$ $B(-5, 2) \rightarrow B'(-2, -5)$ $C(-2, 2) \rightarrow C'(-2, -2)$ $D(-2, 5) \rightarrow D'(-5, -2)$ $E(0, 5) \rightarrow E'(-5, 0)$ The coordinates of P do not change as it is the point of rotation.
2 Compare the values of the coordinates.	The x-coordinate of the object is the same as the y-coordinate of its image, while the x-coordinate of the image is the negative value of the y-coordinate of the object.

Note: This relationship does not apply to all rotations. The relationship depends on the type of rotation, and the position of the object on the Cartesian plane.



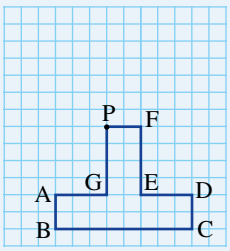
During a rotation the figures do not change their shape or size; therefore, the images are said to be congruent to their object. Congruent shapes are identical.

Instead of the lengthy *congruent*, the sign $\cong$ is used. For example, in the above diagram, $\Delta ABC \cong \Delta A'B'C'$.

WORKED EXAMPLE 10

a Show the image of the shape after a rotation about point P by 180° (half turn) in a clockwise direction. Label the image appropriately.

b State whether the image is congruent to the object.

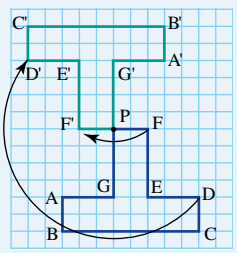


THINK

a Rotate the vertices of the object 180° in a clockwise direction. Label the image of point A, using A′, the image of point B with B′ and so on. Join the vertices appropriately to complete the image.

WRITE/DRAW

a

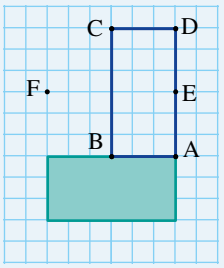


b Determine whether the two figures are exactly the same size and shape, and answer the question.

b Yes, the object and its image are congruent; therefore, ABCDEFG ≅ A′B′C′D′E′F′G′.

WORKED EXAMPLE 11

The rectangle shown has been rotated 90° about one of the points A, B, C, D, E or F in an anticlockwise direction. State the centre of rotation that has been used to form the shaded image.

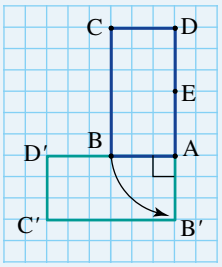


THINK

a As the image has not moved from point A, test rotation about this point. That is, copy the rectangle ABCD and perform the rotation of 90° in an anticlockwise direction about point A.

b Answer the question.
Note: The image obtained by rotating the object about point A is exactly the same as the given one. Therefore, A is the centre of rotation that has been used.

WRITE/DRAW



The rectangle has been rotated about point A.

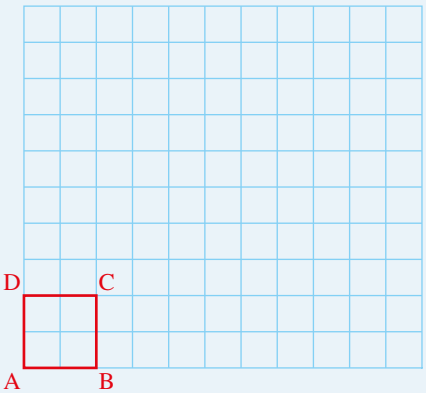
16.5.2 Combined transformations

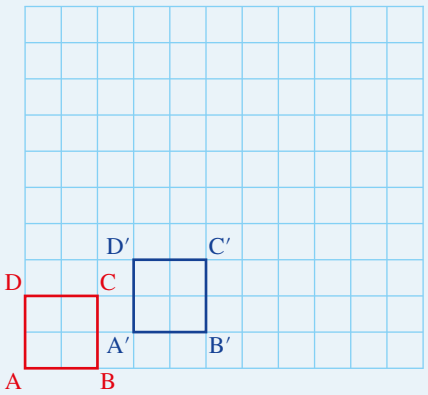
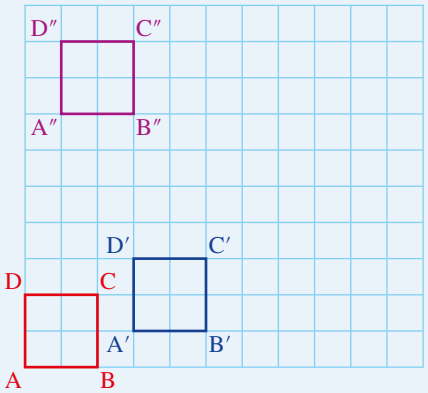
- Transformations can be combined by performing two or more transformations one after the other.

WORKED EXAMPLE 12

a Translate the square ABCD 3 units right and 1 unit up, then translate the image 2 units left and 6 units up.

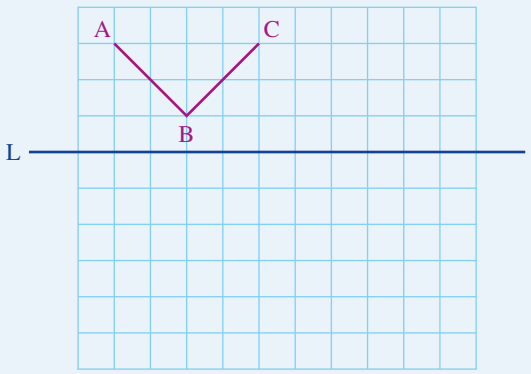
b Determine a single transformation that would have produced the same result.

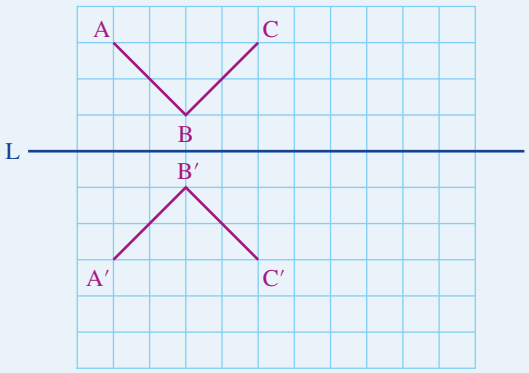
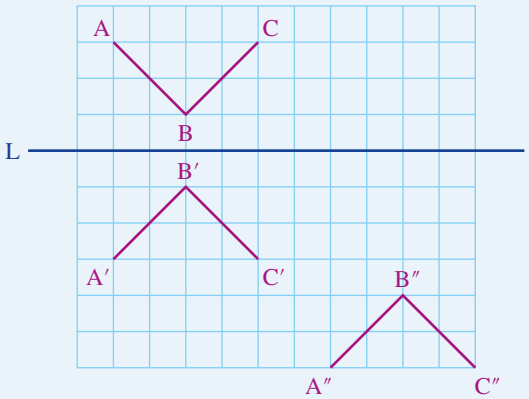
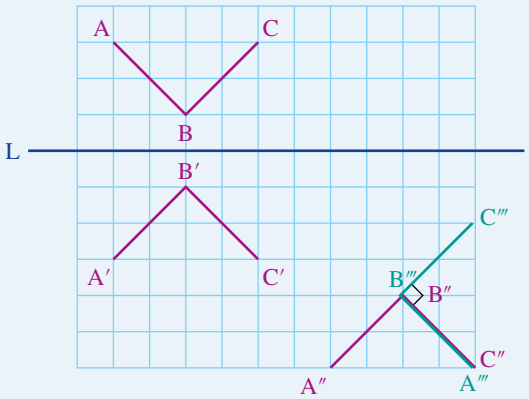


THINK	DRAW
a 1 Take each corner point of the square and translate it 3 units to the right and 1 unit up. Mark each new corner with the appropriate label to represent the image. So, A is translated to A′ and so on.	**a** 
2 Take each image point from the previous step and translate it 2 units left and 6 units up. Mark each new corner with the appropriate label to represent the image. So, A′ is translated to A″ and so on.	
b 1 Starting at the original square ABCD, count the number of units up and to the right from A to A″. Check that this translation works for each corner.	**b** 1 unit right and 7 units up.
2 State the combined transformation.	The combined transformation is the same as a translation of 1 unit right and 7 units up.

WORKED EXAMPLE 13

Reflect the V in the line L, then translate the image 6 units right and 3 units down, and then rotate the image 90° anti-clockwise.



THINK	DRAW
1 Reflect the V in the line L and label the vertices A', B', C'.	
2 Translate the image 6 units to the right and 3 units down. Label this new image A'', B'', C''.	
3 Rotate this image 90° anti-clockwise. Label this new image A''', B''', C'''.	

learnON RESOURCES — ONLINE ONLY

- **Try out this interactivity:** Multiple transformations (int-2745)
- **Complete this digital doc:** SkillSHEET: Rotation of a point (doc-6599)
- **Complete this digital doc:** SkillSHEET: Rotation of an object through 180° (doc-6600)
- **Complete this digital doc:** WorkSHEET: Transformations II (doc-2025)
- **Complete this digital doc:** Investigation: Designing a patchwork quilt (doc-3444)

Exercise 16.5 Rotations and combined transformations

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1, 2, 3, 4, 5, 7, 8, 10, 16	Questions: 1, 2, 3, 4, 5, 6, 7a, c, e, 8, 10, 12, 15, 16	Questions: 1, 2, 3, 4, 5, 6, 7a, c, f, 8–17

Individual pathway interactivity: int-4390

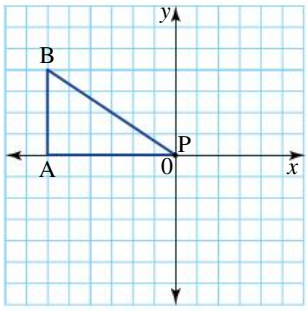
learnON ONLINE ONLY

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note*: Question numbers may vary slightly.

Fluency

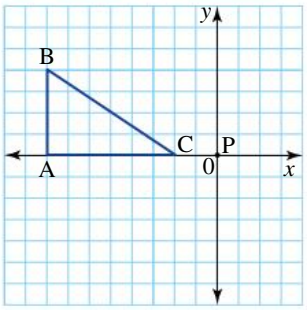
1. i. WE9 Show the image of the following shapes, rotated about point P.

a.



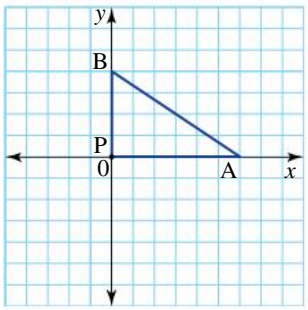
Rotated 90° clockwise

b.



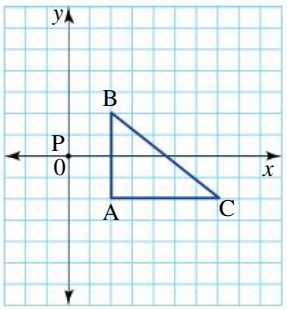
Rotated 90° anticlockwise

c.



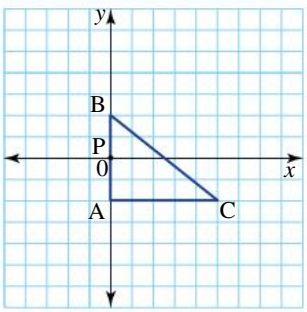
Rotated 180° anticlockwise

d.



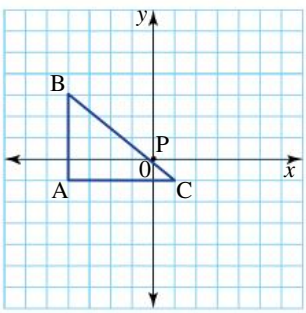
Rotated 270° ($\frac{3}{4}$ turn) clockwise

e.



Rotated 270° anticlockwise

f.

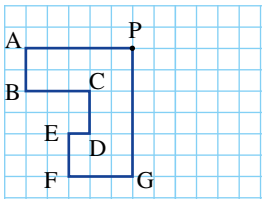


Rotated 450° ($1\frac{1}{4}$ turns) clockwise

ii. Compare the coordinates of the vertices of the object with those of its image.

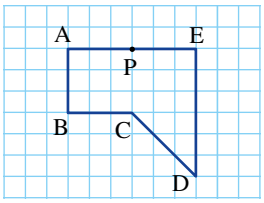
2. WE10 Show the image of each of the following shapes after rotation about point P. Label the images appropriately. In each case state whether the image is congruent to the object.

a.



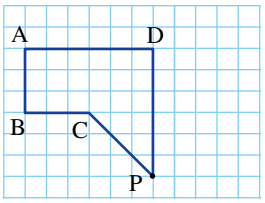
Rotated 90° anticlockwise

b.



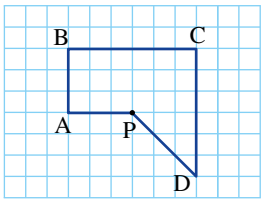
Rotated 270° clockwise

c.



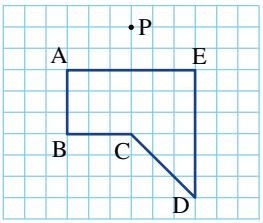
Rotated 180° anticlockwise

d.



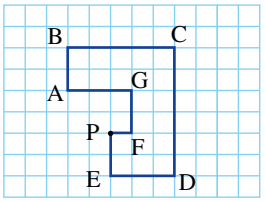
Rotated 90° clockwise

e.



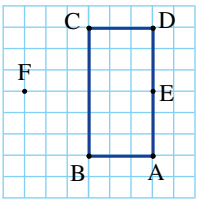
Rotated 270° anticlockwise

f.

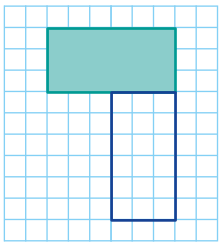


Rotated 450° clockwise

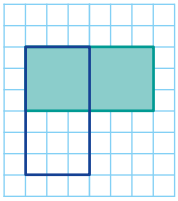
3. WE11 The rectangle at right has been rotated 90° about A, B, C, D, E or F in a clockwise direction. State the centre of rotation that has been used to form each of the following images (shaded).



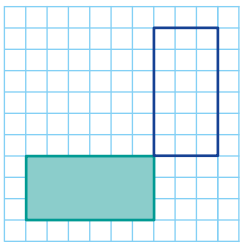
a.



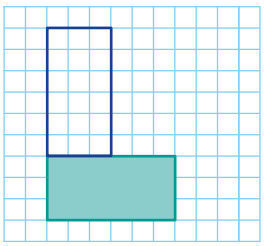
b.



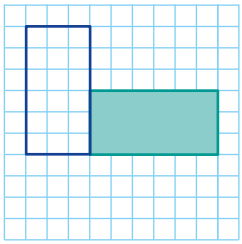
c.



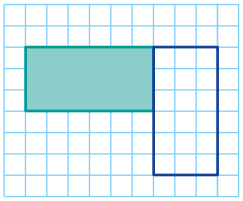
d.



e.



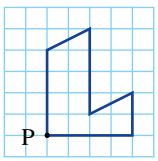
f.



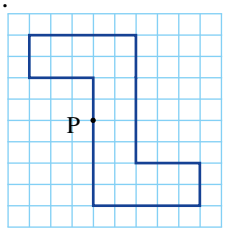
Understanding

4. Patterns can be made by rotating images. For each of these shapes, show the pattern after a 180° turn in a clockwise direction about point P.

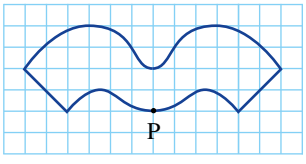
a.



b.

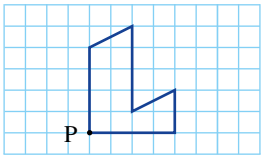


c.

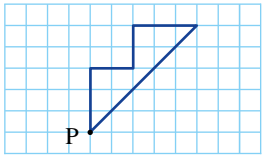


5. Some patterns can be made by rotating images several times. For each of the following, rotate the object *three times* about point P, 90° in a clockwise direction. That is, rotate the object first, then rotate the image of the object and, finally, the image of the image, so that you have 4 shapes altogether.

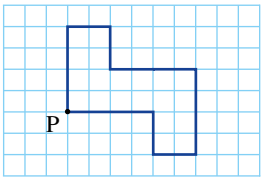
a.



b.



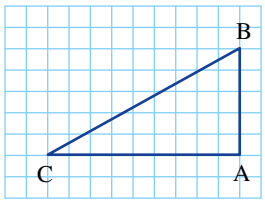
c.



6. Create your own designs by drawing an interesting shape and rotating it several times.

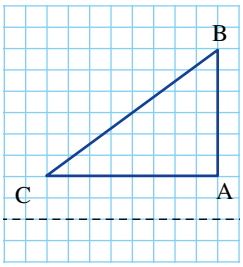
7. Show the final image after the following sets of transformations.

a.



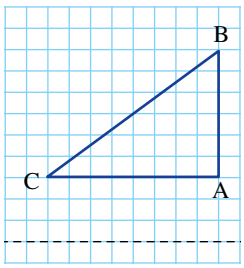
Translated 2U 2R, rotated 90° about the image of C in a clockwise direction

b.



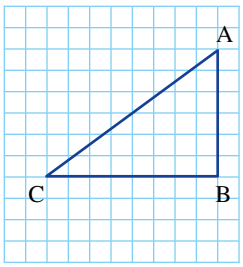
Reflected in the mirror line and then rotated 180° clockwise about the image of C

c.



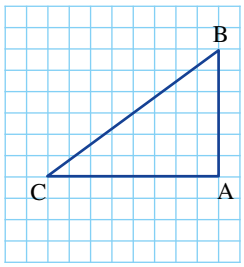
Rotated 270° anticlockwise about C and then reflected in the mirror line

d.



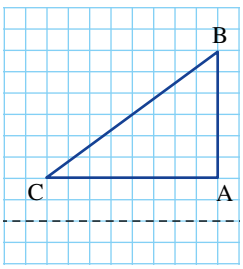
Rotated 90° anticlockwise about C, then translated 2D 5L

e.



Translated 3D 4L; then rotated 180° clockwise about the image of C

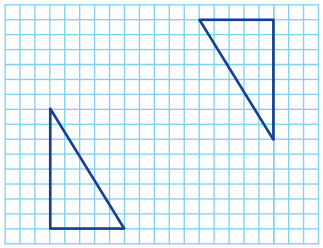
f.



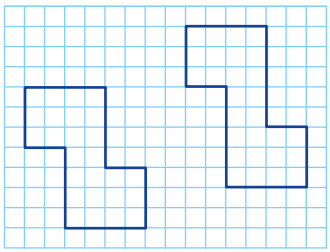
Rotated 180° about C, translated 5R 3U and then reflected in the mirror line

8. Which of the following pairs of shapes are congruent?

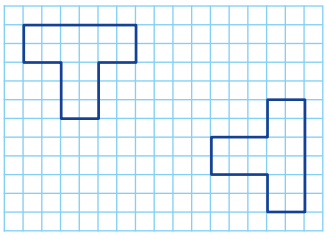
a.



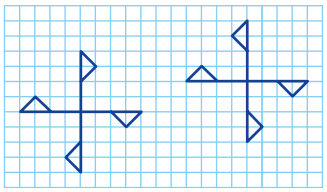
b.



c.



d.



9. State whether the following statements are True or False.

a. A shape that has been translated is congruent to the original one.

b. A shape that has been rotated is congruent to the original one.

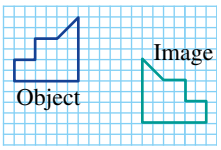
c. A shape that has been reflected is congruent to the original one.

d. A shape that has been translated and then rotated is not congruent to the original one.

e. A shape that has been enlarged is congruent to the original one.

f. A shape that has been reduced is not congruent to the original one.

10. MC The diagram at right shows the object and its image after a certain transformation(s). The object was:

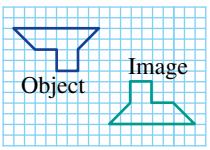


A. reflected **B.** translated

C. rotated **D.** reflected and then translated

E. reflected and then rotated

11. MC To obtain the image shown in the diagram at right, the object was:



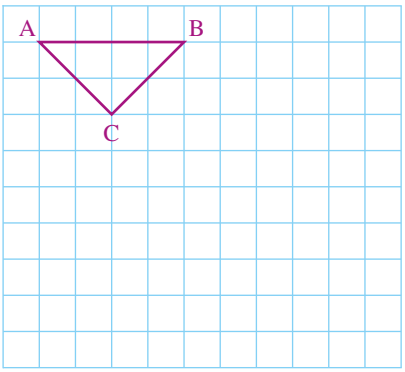
A. reflected **B.** translated

C. rotated **D.** rotated and then reflected

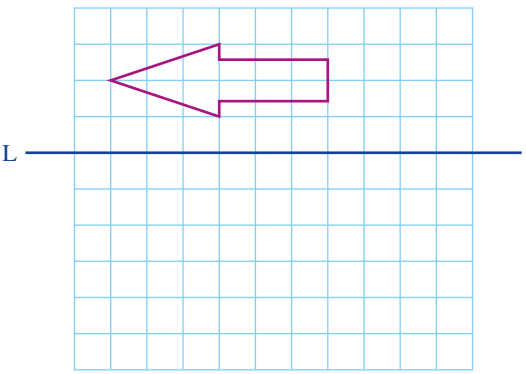
E. rotated and then translated

12. WE12 **a.** Translate the triangle ABC 6 units right and 7 units down and then translate the image 4 units left and 2 units up.

b. Determine a single transformation that would have produced the same result.



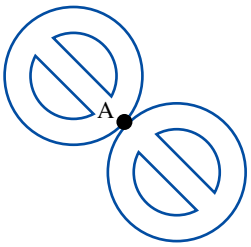
13. WE13 Reflect the arrow in the line L, then translate the image 4 units right and 2 units down and then rotate this image 90° anticlockwise.



Reasoning

14. In the photograph shown, a spoke of a wheel undergoes rotation about the centre of the wheel. What angle has spoke A rotated through to move to the position of spoke B?

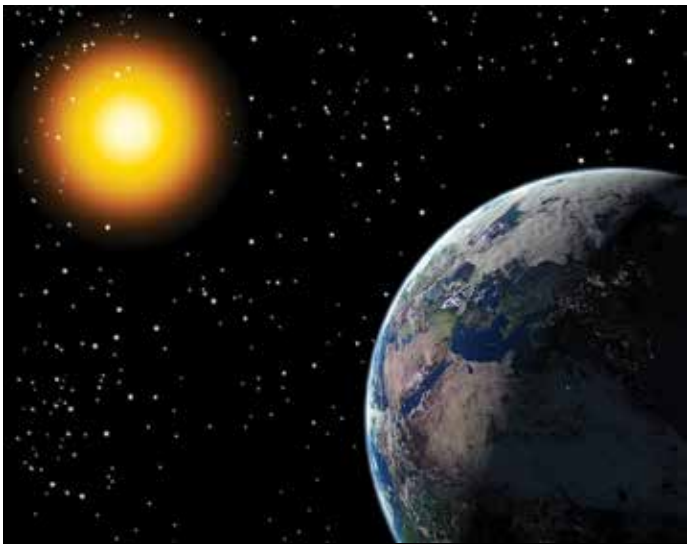
15. The following object has been rotated 180° about point A. Is there another transformation that would produce the same image?



Problem solving

16. Earth completes a full rotation around the Sun in approximately one year. What is the approximate angle of rotation of a point on Earth in one day?

17. An object was translated seven units to the right and four units down, rotated 180° clockwise about the initial position, then reflected in a vertical line passing through the initial position, and then translated seven units to the left and four units down. What is the new position of the object at the end of the transformations relative to the initial position? Draw a diagram to represent the transformations.



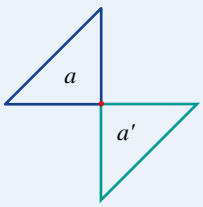
Reflection

What strategy would you use when drawing the rotated image of an object?

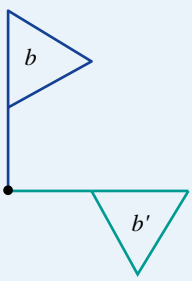
CHALLENGE 16.2

Specify the rotation that has taken place in each of the following. Give two possible answers in each case.

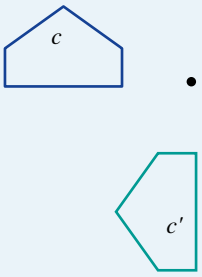
a.



b.



c.

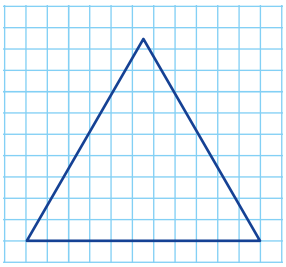


16.6 Review

16.6.1 Review questions

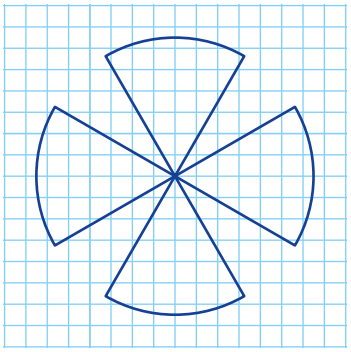
Fluency

1. How many axes of symmetry does this shape have?

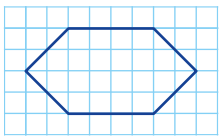


a. 0 **b.** 1 **c.** 2 **d.** 3 **e.** 4

2. Copy the shape below and draw in its axes of symmetry.



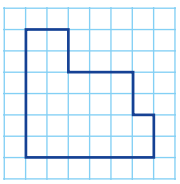
3. a. Copy the shape below and draw in its axes of symmetry.



b. Rotate 90° anticlockwise.

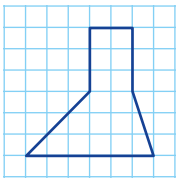
4. Show the following translations by copying the shapes and producing an image on a grid.

a.



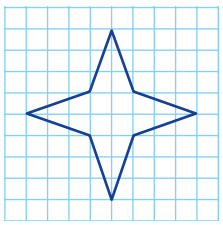
7R 2U

b.



2L 3D

c.



5R 2D

5. State the final position after each set of translations given below.

a. 3U 2R, 4D 6R, 2U 3L

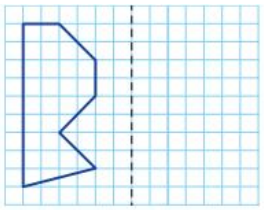
b. 2D 3L, 4U 5R, 2D 6L, 3D 2R

c. 3L 2U, 9R 5D, 2L 3D, 4R 7U

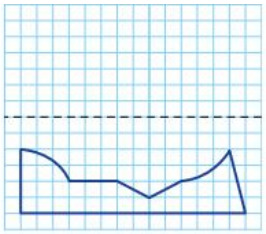
d. 12L 3U, 4R 2D, 6L 5U, 2R 2D

6. For each of the following shapes, find the reflected image in the mirror line shown.

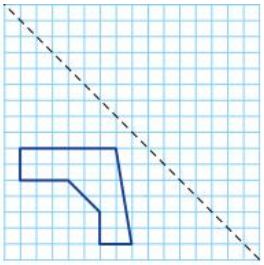
a.



b.

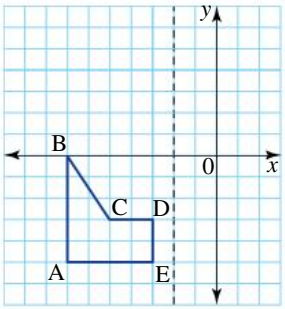


c.



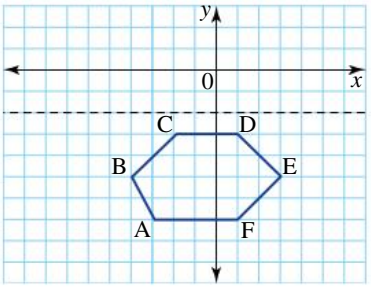
7. i. Find the image after the sets of transformations shown in these objects.

a.



Translated 3L 4U; then reflected in the mirror

b.



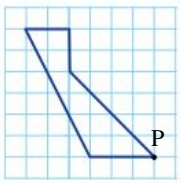
Reflected in the mirror; then translated 3U 5L

ii. Give the coordinates of the vertices of the object and its image.

iii. Discuss the relationship between these coordinates in light of the transformation that took place.

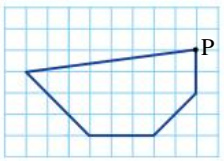
8. For each of the following shapes, show the image after the rotation about point P as specified below the figures.

a.



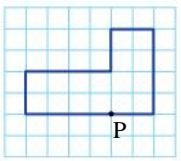
90° clockwise

b.



180° anticlockwise

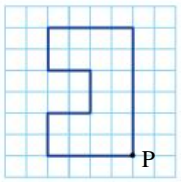
c.



270° clockwise

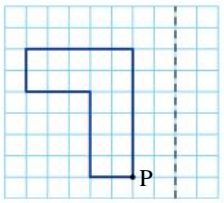
9. Show the final image after each of the following shapes have gone through these sets of transformations.

a.



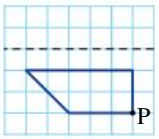
Translated 3U 6L; then rotated 90° anticlockwise around the image of P.

b.



Reflected in the mirror; then rotated 90* clockwise around the image of P.

c.

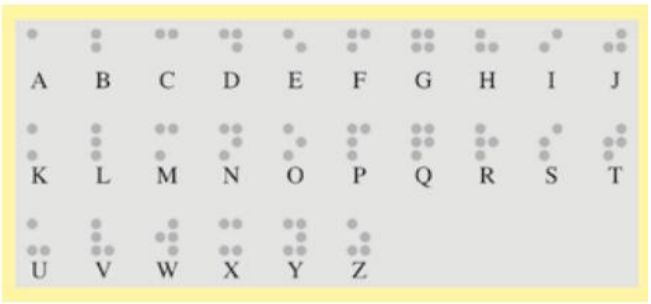


Translated 2D 4R, rotated 180° clockwise around the image of P and then reflected.

Problem solving

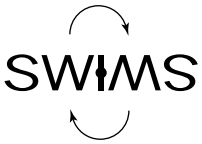
10. Braille is a code of raised dots that can be read by touch. It was developed by a 15-year-old blind French student named Louis Braille. The Braille alphabet is based on a cell three dots high and two dots wide.

a. Compare the first 10 letters of the Braille alphabet with the second 10 letters. What patterns do you see?



b. Which Braille letters are reflections of each other?
c. Which Braille letters are rotations of each other?
d. There are no Braille letters that are translations of each other. Can you explain why?
e. A word like MUM has reflection symmetry. Find two words that have reflection symmetry when written in Braille.

f. A word like SWIMS has rotational symmetry. Find two words that have rotational symmetry when written in Braille.



11. Design an image on grid paper. Perform three successive transformations on this image.

learnON RESOURCES — ONLINE ONLY

- **Try out this interactivity:** Word search: Topic 16 (int-2611)
- **Try out this interactivity:** Crossword: Topic 16 (int-2612)
- **Try out this interactivity:** Sudoku: Topic 16 (int-3176)
- **Complete this digital doc:** Concept map: Topic 16 (doc-10747)

Language

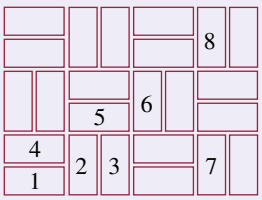
It is important to learn and be able to use correct mathematical language in order to communicate effectively. Create a summary of the topic using the key terms below. You can present your summary in writing or using a concept map, a poster or technology.

anticlockwise
axes
axis
centre of rotation
clockwise
congruent
coordinates
diagonal
identical
image
isometric
reflections
rotations
rotational symmetry
symmetry
symmetrical
transformations
translations
version
vertices

assessON
Link to assessON for questions to test your readiness FOR learning, your progress AS you learn and your levels OF achievement.
assessON provides sets of questions for every topic in your course, as well as giving instant feedback and worked solutions to help improve your mathematical skills.
www.assesson.com.au

Investigation | Rich task

Designing a paved outdoor area



Many paths and entertaining areas are paved with bricks of the same size and shape laid in an interesting design. One such pattern is shown.

To design this pattern, the basic shape of the brick undergoes one or more transformations to different positions.

1. What transformation has occurred to brick 1 to obtain the position of brick 2?
2. What transformation has occurred to brick 2 to obtain the position of brick 3?
3. Describe how the position of brick 3 can be obtained in relation to brick 1.
4. For each of the other numbered bricks in the pattern, describe what transformations to brick 1 are needed to obtain the position of that brick.
5. Explain why this pattern is an example of a tessellation.
6. Design your own paved entertaining area. You may like to use one or more shapes that tessellate to form your pattern. Describe what transformations are involved.

learnON RESOURCES — ONLINE ONLY

Complete this digital doc: Code puzzle: What was Thomas Edison's occupation? (doc-14339)

Answers

Topic 16 Transformations

Exercise 16.2 Axes of symmetry

1. Check with your teacher.

2. a. 3 **b.** 0 **c.** 1 **d.** 1 **e.** 1 **f.** 0 **g.** 1 **h.** 4

3. a. 2 **b.** 2 **c.** 6 **d.** 8 **e.** 5 **f.** 0

4. a. Order 3 **f.** Order 2 **h.** Order 4

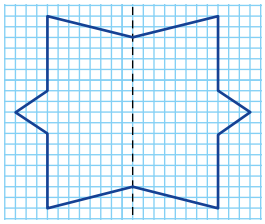
5. a. Order 2 **b.** Order 2 **c.** Order 6 **d.** Order 8 **e.** Order 5 **f.** Order 3

6. a. ABCDEHIKMOTUVWXY **b.** cilovwx **c.** HIOXlox

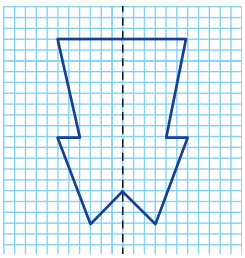
7. C

8. A

9. a.



b.

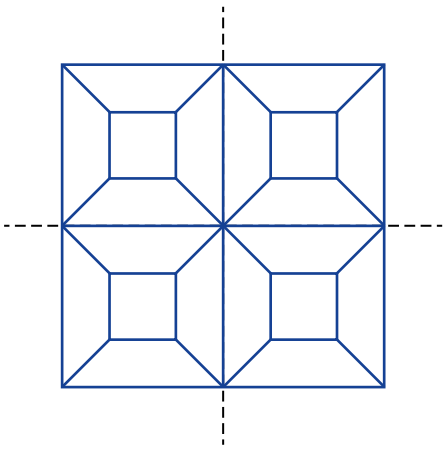


10. Check with your teacher.

11. a. 4, 5, 6, 7

b. Yes, the number of vertices is double the number of axes of symmetry, as each axis of symmetry goes through two vertices.

12. a.



b. Every point on the pattern has to be equally distant from the axis of symmetry on either side of the axis.

13. $\frac{5}{12}$

14. a. 8 axes of symmetry **b.** 7 axes of symmetry

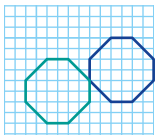
Exercise 16.3 Translations

Key — object — image

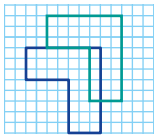
1. a. 1R 2U **b.** 19R 2D **c.** 4L 5D **d.** 10L **e.** 2R 4U **f.** 2R **g.** 2R 9U **h.** 11L 8D
i. 4L 1D **j.** 6R **k.** 7L 5D **l.** 2L 8U

2. a. 1L 6U **b.** 2R 8D **c.** 2R 3U **d.** 12R **e.** 12L 9D **f.** 11L 3D **g.** 8L 1D **h.** 4L 7U

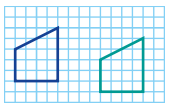
3. a.



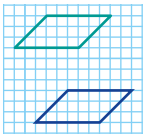
b.



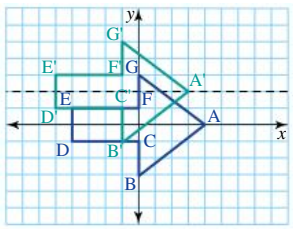
c.



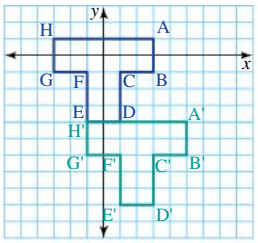
d.



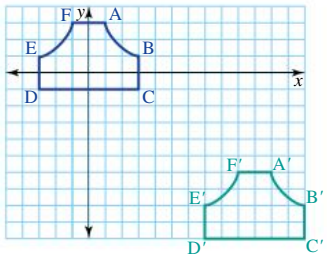
4. i. a.



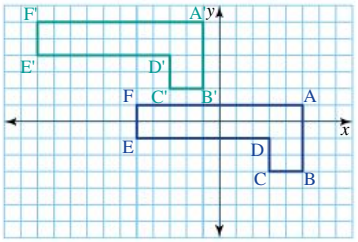
b.



c.



d.



ii. a. $A'(3, 2), B'(-1, -1), C'(-1, 1), D'(-5, 1), E'(-5, 3), F'(-1, 3), G'(-1, 5)$

b. $A'(5, -4), B'(5, -6), C'(3, -6), D'(3, -9), E'(1, -9), F'(1, -6), G'(-1, -6), H'(-1, -4)$

c. $A'(11, -6), B'(13, -8), C'(13, -10), D'(7, -10), E'(7, -8), F'(9, -6)$

d. $A'(-1, 6), B'(-1, 2), C'(-3, 2), D'(-3, 4), E'(-11, 4), F'(-11, 6)$

5. a. 5R 6D **b.** 21R 5D **c.** 2L 1D **d.** 2R 1U **e.** 7R 6U **f.** 19L 6U

6. a. C to D **b.** D to C **c.** E to D **d.** E to B **e.** D to A **f.** E to C

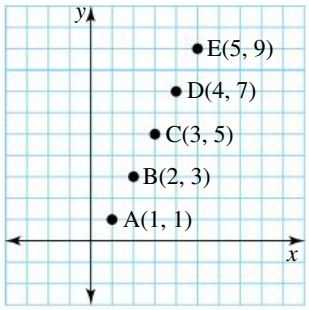
7. D

8. B

9. Check with your teacher.

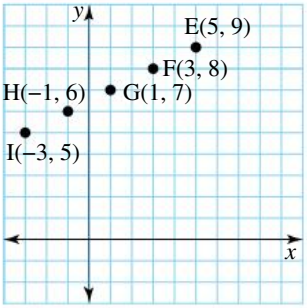
10. Check with your teacher.

11. a.



b. Form a straight line.

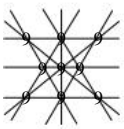
c.



d. Form a straight line.

12. 8 m

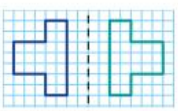
Challenge 16.1



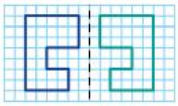
Exercise 16.4 Reflections

Key — object — image

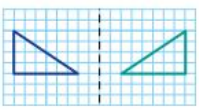
1. a.



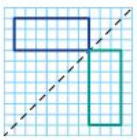
b.



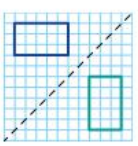
c.



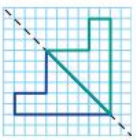
d.



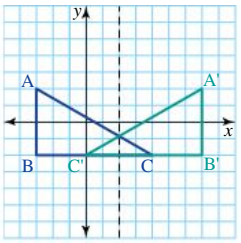
e.



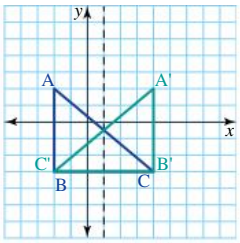
f.



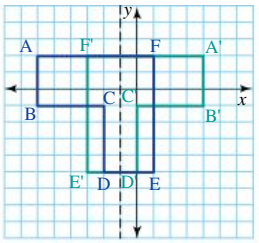
2. i. a.



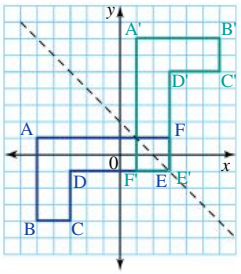
b.



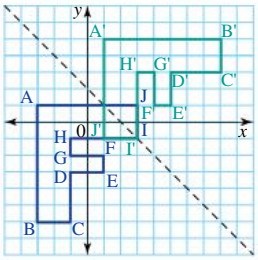
c.



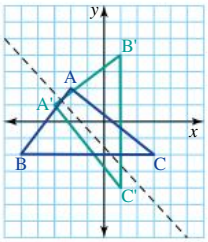
d.



e.



f.



ii. a. A′(6, 2), B′(6, −2), C′(0, −2)

b. A′(4, 2), B′(4, −2), C′(−2, −2)

c. A′(4, 2), B′(4, −1), C′(0, −1), D′(0, −5), E′(−3, −5), F′(−3, 2)

d. A′(1, 7), B′(6, 7), C′(6, 5), D′(3, 5), E′(3, −1), F′(1, −1)

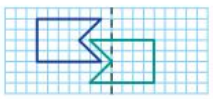
e. A′(1, 5), B′(8, 5), C′(8, 3), D′(5, 3), E′(5, 1), F′(4, 1), G′(4, 3), H′(3, 3), I′(3, −1), J′(1, −1)

f. A′(−4, 1), B′(1, 4), C′(−1, −4)

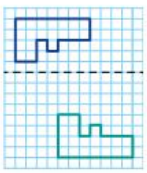
3. Check with your teacher.

4. Check with your teacher.

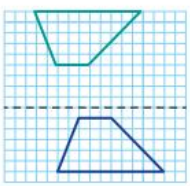
5. a.



b.



c.



6. C

7. When reflected in an axis, the image is the mirror representation of the original object. If the object and its image are next to each other, they touch each other so they could form one object together. As they are both identical, the axis of reflection becomes the axis of symmetry of the newly formed object.

8. A reflection in a horizontal axis requires the object to be reflected in a vertical direction: up or down.

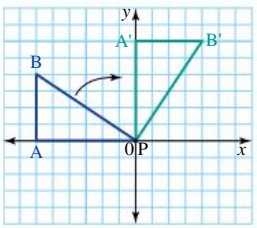
A reflection in a vertical axis requires the object to be reflected in a horizontal direction: left or right.

9. 10:15

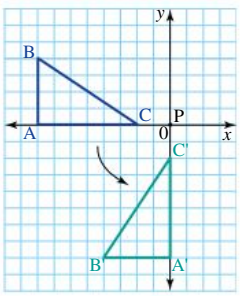
10. Image 2. It is equally distant from the reflection line as the original strawberry, it is the mirror image of the original strawberry and it is placed on the line perpendicular to the reflection line passing through the original strawberry.

Exercise 16.5 Rotations and combined transformations

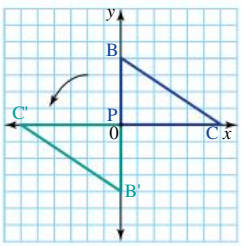
1. i. a.

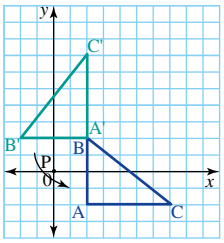


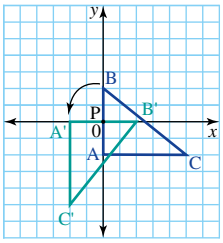
b.

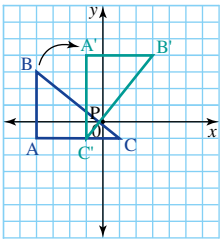


c.



d. 

e. 

f. 

ii. a. $A(-6, 0) \rightarrow A'(0, 6), B(-6, 4) \rightarrow B'(4, 6)$, P is unchanged.

The y-coordinate of the object becomes the x-coordinate of the image, while the x-coordinate of the object changes sign to become the y-coordinate of the image.

b. $A(-8, 0) \rightarrow A'(0, -8), B(-8, 4) \rightarrow B'(-4, -8), C(-2, 0) \rightarrow C'(0, -2)$

The x- and y-coordinates are interchanged from the object to the image, but the change in sign is not consistent.

c. $B(0, 4) \rightarrow B'(0, -4), C(6, 0) \rightarrow C'(-6, 0)$, P is unchanged.

The signs of the coordinates change going from the object to the image.

d. $A(2, -2) \rightarrow A'(2, 2), B(2, 2) \rightarrow B'(-2, 2), C(7, -2) \rightarrow C'(2, 7)$

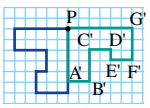
The x- and y-values interchange, with the y-value of the object changing sign as it becomes the x-value of the image.

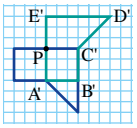
e. $A(0, -2) \rightarrow A'(-2, 0), B(0, 2) \rightarrow B'(2, 0), C(5, -2) \rightarrow C'(-2, -5)$

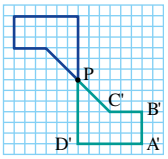
The x- and y-coordinates are interchanged from the object to the image, but the change in sign is not consistent.

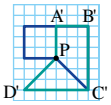
f. $A(-4, -1) \rightarrow A'(-1, 4), B(-4, 3) \rightarrow B'(3, 4), C(1, -1) \rightarrow C'(-1, -1)$

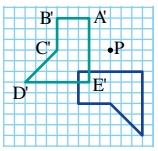
The x- and y-values interchange, with the x-value of the object changing sign as it becomes the y-value of the image.

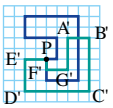
2. a. 

b. 

c. 

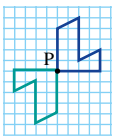
d. 

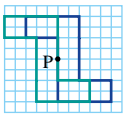
e. 

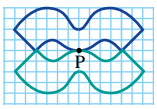
f. 

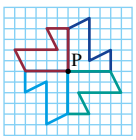
All images are congruent to the original shapes.

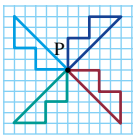
3. a. D **b.** E **c.** F **d.** B **e.** A **f.** C

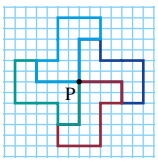
4. a. 

b. 

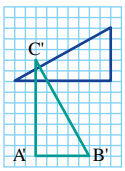
c. 

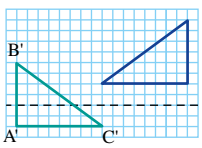
5. a. 

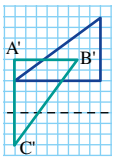
b. 

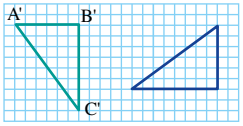
c. 

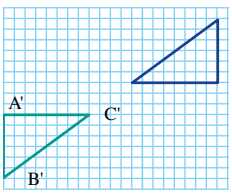
6. Check with your teacher.

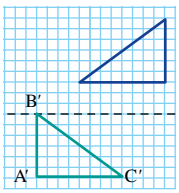
7. a. 

b. 

c. 

d. 

e. 

f. 

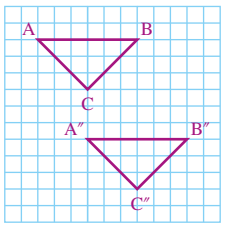
8. a. Congruent **b.** Not congruent **c.** Congruent **d.** Not congruent

9. a. True **b.** True **c.** True **d.** False **e.** False **f.** True

10. D

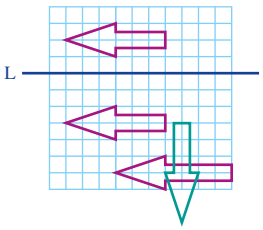
11. E

12. a.



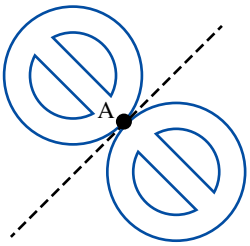
b. Two units right and 4 units down.

13.



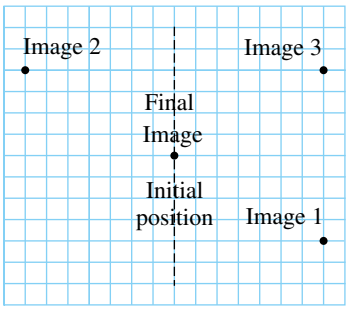
14. Angle ≈ 97°

15. A reflection in dotted line shown.



16. Approximately 1°.

17.



Same position.

Challenge 16.2

a. Clockwise 180° Anticlockwise 180°

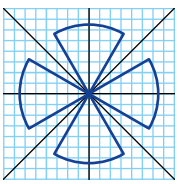
b. Clockwise rotation through 90°, anticlockwise rotation through 270°

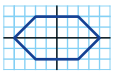
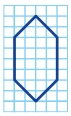
c. Anticlockwise rotation through 90°, clockwise rotation through 270°

16.6 Review

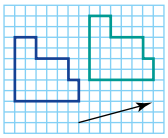
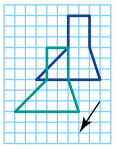
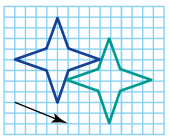
1. D

2.

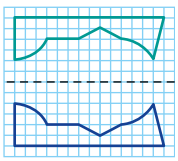
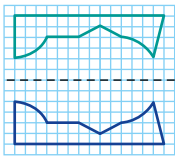
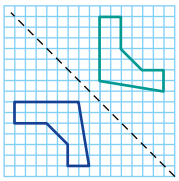


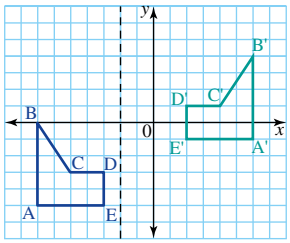
3. a.  **b.** 

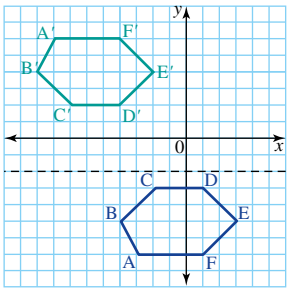
Key — object — image

4. a.  **b.**  **c.** 

5. a. 1U 5R **b.** 3D 2L **c.** 8R 1U **d.** 12L 4U

6. a.  **b.**  **c.** 

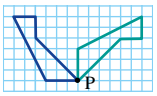
7. i a. 

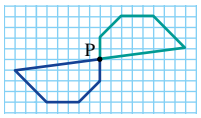
b. 

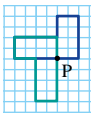
ii. a. $A(-7, -5) \rightarrow A'(6, -1), B(-7, 0) \rightarrow B'(6, 4), C(-5, -3) \rightarrow C'(4, 1), D(-3, -3) \rightarrow D'(2, 1), E(-3, -5) \rightarrow E'(2, -1)$

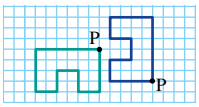
b. $A(-3, -7) \rightarrow A'(-8, 6), B(-4, -5) \rightarrow B'(-9, 4), C(-2, -3) \rightarrow C'(-7, 2), D(1, -3) \rightarrow D'(-4, 2), E(3, -5) \rightarrow E'(-2, 4), F(1, -7) \rightarrow F'(-4, 6)$

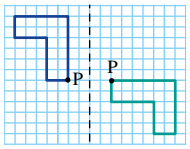
iii. Discuss as a class.

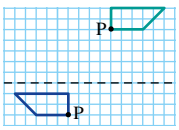
8. a. 

b. 

c. 

9. a. 

b. 

c. 

10. a. Each of the second group of 10 letters has the same pattern as the corresponding letter but with an extra dot underneath.

b. D and F, E and I, H and J, M and U, N and Z, P and V, R and W

c. B, C, E and I

d. Braille is read by touch and translations would not be determined by touch.

e. Answers will vary.

f. Answers will vary.

11. Answers will vary.

Investigation — Rich task

1. 90° clockwise rotation around the bottom right corner of brick 1

2. Translation of one brick-width to the right

3. 90° clockwise rotation around a point halfway between the bases of brick 1 and 3

4. Brick 4: translate one brick-width up;
brick 5: rotate 180° around the point halfway between the closest points connection bricks 1 and 5;
brick 6: rotate 90° clockwise around the bottom right corner of brick 3;
brick 7: rotate 90° clockwise around the bottom right corner of brick 1 and then translate it four brick-widths to the right;
brick 8: rotate 90° clockwise around the point at the bottom left corner of brick 7.

5. This is an example of a tessellation as the pattern repeats itself and there are no gaps.

6. Teacher to check.

TOPIC 17
Views of 3-D shapes

17.1 Overview

Numerous **videos** and **interactivities** are embedded just where you need them, at the point of learning, in your learnON title at www.jacplus.com.au. They will help you to learn the content and concepts covered in this topic.

17.1.1 Why learn this?

Knowledge of geometry and mathematics is essential for understanding how 3-dimensional (3-D) spaces work. Three-dimensional shapes exist all around us — from a cylindrical water bottle to prism shaped boxes, rectangular playing fields, and spherical and oval shaped balls. Drawing a 3-dimensional shape depends on the view you wish to show. Designs and ideas about 3-dimensional shapes can be communicated using 2-dimensional drawings.

17.1.2 What do you know?

assessON

1. **THINK** List what you know about reading scales. Use a thinking tool such as a concept map to show your list.
2. **PAIR** Share what you know with a partner and then with a small group.
3. **SHARE** As a class, create a thinking tool such as a large concept map that shows your class's knowledge of scales.

LEARNING SEQUENCE

17.1 Overview
17.2 Scales and house plans
17.3 Plans and views
17.4 Isometric drawing
17.5 Review

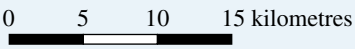
17.2 Scales and house plans

17.2.1 Understanding scales

- A scale on a map or plan describes the ratio between the distance on the map or plan and the actual distance on the Earth's surface.
- A scale may be written several ways.
  - A ratio such as 1 : 100 000 means that 1 cm on a map or plan represents a distance of 100 000 cm on the Earth, or 1 mm on the map or plan represents 100 000 mm on the Earth, etc.
  - A statement such as 1 cm ⇔ 500 km means that 1 cm on the map or plan represents a distance of 500 km on the Earth.
  - A graphical bar scale, such as the one shown in Worked example 1, can be used. In this case, we need to measure the length of the bar to determine the scale ratio.

WORKED EXAMPLE 1

a The scale on a house plan is 1 : 2000. How many metres does a length of 1 cm on the map represent?
b Write the scale 1 cm ⇔ 5000 m as a ratio scale.
c A map shows a scale of 1 cm ⇔ 1500 m. What distance is represented by 18 mm on the map?
d Use the following graphical scale to determine the distance that 1 cm on the map represents.



THINK	WRITE
a 1 Use the ratio to add the same units to both sides. Since the answer asks for cm, use this unit.	**a** 1 : 2000 is the same as: 1 cm ⇔ 2000 cm
2 Bring the length to an appropriate unit. In this case, the answer is required in metres.	2000 cm = (2000 ÷ 100)m = 20 m So, 1 cm ⇔ 20 m.
3 Write the answer.	1 cm on the map represents a length of 20 m in reality.
b 1 Convert the ground units to the same as those of the map.	**b** 5000 m = (5000 × 100) cm = 500 000 cm
2 Write the scale as a ratio.	So, 1 cm ⇔ 5000 m is the same as 1 cm ⇔ 500 000 cm. Expressed as a ratio, this is 1 : 500 000.
c 1 Write a statement explaining this scale.	**c** 1 cm ⇔ 1500 m means that 1 cm on the map represents a distance of 1500 m.
2 Change the question distance to the same units as those for the map.	18 mm = (18 ÷ 10) cm = 1.8 cm
3 Multiply by this scale factor.	1.8 cm represents (1.8 × 1500) m, which is 2700 m or 2.7 km.
d 1 Measure the length of the bars with a ruler.	**d** The bars measure 3.5 cm.

THINK	WRITE
2 Relate this measurement to the length shown in the scale.	This represents a length of 15 km.
3 Write as a scale.	3.5 cm ⇔ 15 km
4 Calculate a 1 cm map distance and write the answer.	So, 1 cm ⇔ $\frac{15}{35}$ km 1 cm on the map represents a distance of 4.3 km.

- Sometimes it is necessary to do the reverse, and convert from a real length to a plan measurement.

WORKED EXAMPLE 2

The length of a wall for a particular house is 12 m. If the scale of a house plan is 1 : 200, what length, on the plan, would represent the actual length of the wall?

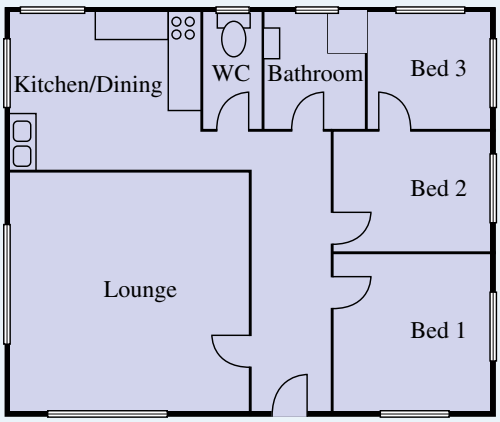
THINK	WRITE
1 Write the scale as a ratio using units of cm.	1 : 200 means that 1 cm on the plan represents an actual length of 200 cm. 1 cm ⇔ 200 cm
2 Convert the actual length from cm to m units.	200 cm = (200 ÷ 100) m = 2 m So 1 cm on the plan represents an actual length of 2 m. 1 cm ⇔ 2 m
3 Divide the actual length by 2 to find how many lots of 2 m there are in 12 m.	$\frac{12\text{ m}}{2\text{ m}} = 6$
4 Multiply both sides of the scale by this scale factor then simplify.	1 cm × 6 ⇔ 2 m × 6 6 cm ⇔ 12 m
5 Answer the question.	The length of the wall on the house plan would be 6 cm.

17.2.2 House plans

- Whenever a house is being constructed, floor plans are supplied to the builder so that the footings, walls, roof etc. are all placed in the correct positions and are the correct length. The scale of the plan is generally given as a ratio. This enables us to calculate all dimensions within the house.

WORKED EXAMPLE 3

At right is a plan for a house.
a Calculate the dimensions of the house.
b Calculate the area of the lounge room.



Scale 1 : 100

THINK	WRITE
a 1 Measure the length and width of the house on the plan.	**a** Length of house on plan = 12 cm Width of house on plan = 10 cm
2 Multiply each of these measurements by 100.	Actual length of house = 12 cm × 100 = 1200 cm = 12 cm Actual width = 10 cm × 100 = 1000 cm = 10 cm
3 Write your answer.	The dimensions of the house are 12 m by 10 m.
b 1 Measure the length and width of the lounge room on the plan.	**b** Length of lounge room on plan = 6 cm Width of lounge room on plan = 6 cm
2 Multiply each of these measurements by 100.	Actual length of lounge room = 6 cm × 100 = 600 cm = 6 m Actual width of lounge room is also 6 m.
3 Calculate the area of the lounge room.	$A = 6^2$ $= 36 \text{ m}^2$
4 Write your answer.	The area of the lounge room is 36 m^2.

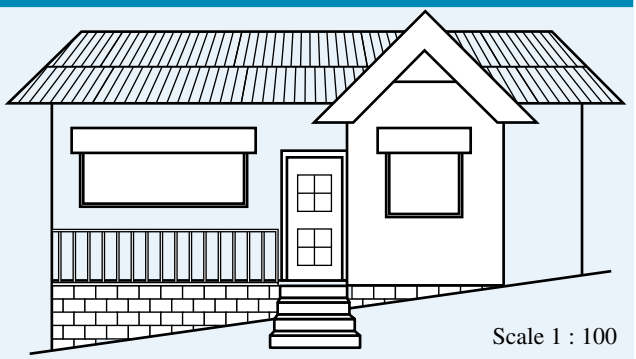
- In addition to a floor plan, a builder needs to know what the house will look like if it is viewed from the front, back and sides. These plans are called *elevations*. These are also drawn to scale.

WORKED EXAMPLE 4

The diagram at right shows the front elevation of a house.

a Calculate the height of the eaves on the lower side of the house.

b Measure the angle of the pitch of the roof.



THINK	WRITE
a 1 Measure the height on the plan for the lower side of the house.	**a** Height on the plan = 3.5 cm
2 Multiply the plan measurement by 100.	Actual height = 3.5 cm × 100 = 350 cm = 3.5 m
3 Write your answer.	The height of the eaves is 3.5 m.
b 1 Measure the angle that the slope of the roof makes with the horizontal.	**b** Angle to horizontal = 45°
2 Write your answer.	The angle of the pitch of the roof is 45°.

- With an understanding of house plans, you should now be able to draw plans of objects familiar to you, such as your home, bedroom or classroom. It is important to include a scale with each plan. Sometimes a legend is also appropriate.

learnON RESOURCES — ONLINE ONLY

Complete this digital doc: SkillSHEET: Metric units of length (doc-6546)

Complete this digital doc: SkillSHEET: Reading scales (how much is each interval worth?) (doc-6547)

Exercise 17.2 Scales and house plans

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1, 2, 3, 4, 5, 6, 7, 8, 14	Questions: 1, 2, 3, 4, 5, 6, 7a, c, e, g, 8, 10a, c, 12, 14, 15	Questions: 1, 2, 3, 4, 5, 6, 7a, d, h, 8, 9, 10, 11, 12, 13, 14, 15

Individual pathway interactivity: int-4391

assessON ONLINE ONLY

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note*: Question numbers may vary slightly.

Fluency

1. WE1 Rewrite the following ratio scales in the form 1 cm ⇔ _____ using the most appropriate units.

a. 1 : 10
b. 1 : 100
c. 1 : 1000
d. 1 : 10 000
e. 1 : 100 000
f. 1 : 5000
g. 1 : 60 000
h. 1 : 400
i. 1 : 750 000
j. 1 : 2 200 000

2. Write the following as ratio scales.

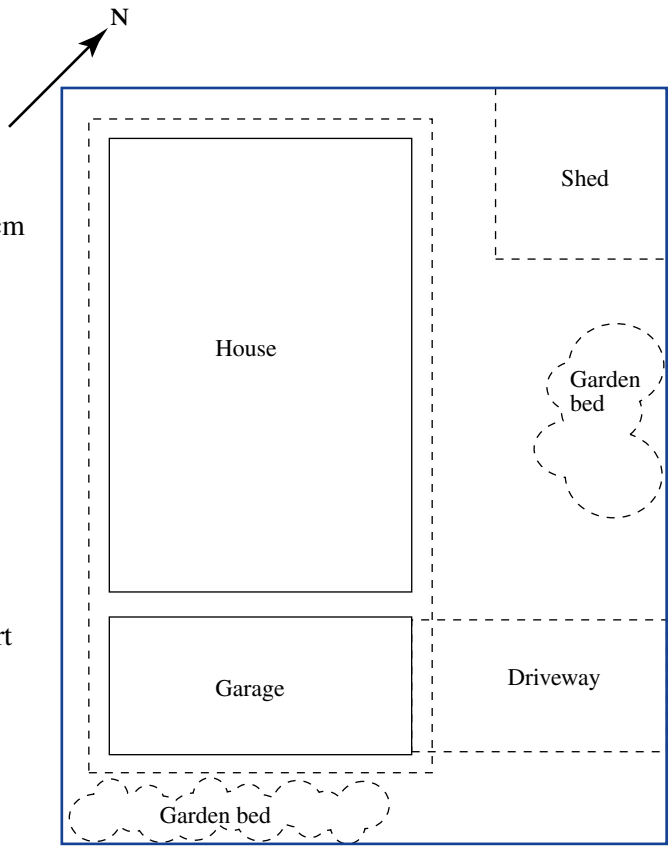
a. 1 cm ⇔ 20 cm
b. 1 cm ⇔ 50 000 cm
c. 1 cm ⇔ 10 m
d. 1 cm ⇔ 200 m
e. 1 cm ⇔ 5 km
f. 1 cm ⇔ 50 km
g. 1 cm ⇔ 5.5 km
h. 2 cm ⇔ 2 km
i. 2 cm ⇔ 25 km
j. 3 cm ⇔ 6 km

3. WE2 The scale of a town plan is given as 1 : 100 000. Use this to find the distances on the plan between:

a. two intersections that are 1 km apart
b. two department stores that are 200 m apart
c. an airport and a motel that are 15 km apart
d. a service station and a bank that are 2.5 km apart
e. the northern and southern extremities, which are 22 km apart
f. two shops that are 500 m apart.

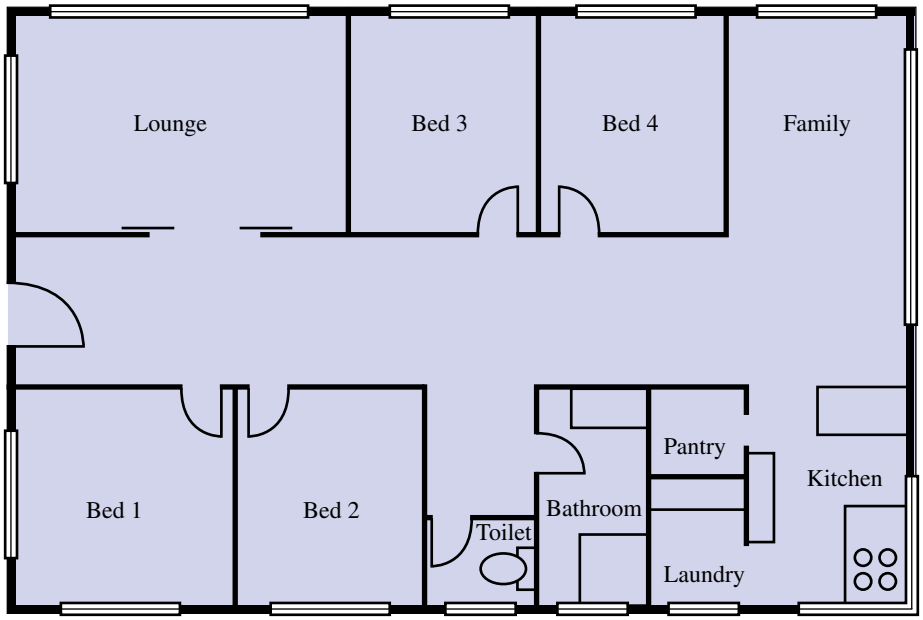
4. WE3 At right is the site plan for a block of land.

a. What are the dimensions of the block of land?
b. What are the dimensions of the house?



Scale 1 : 250

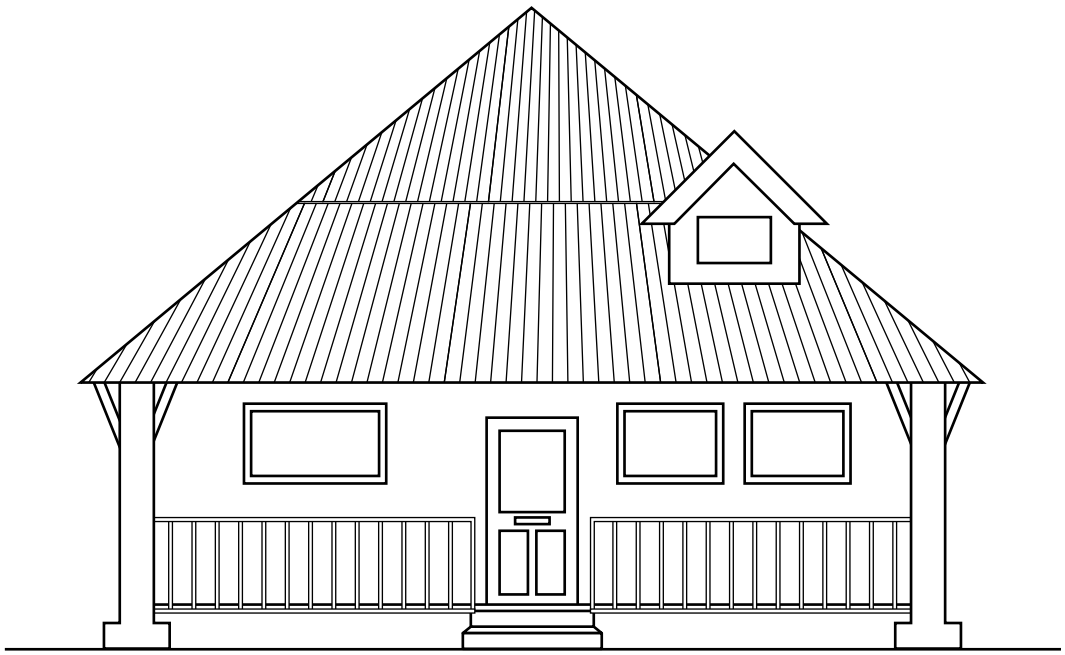
5. A house plan is shown below.



Scale 1 : 150

a. Calculate the dimensions of the house.
b. What are the dimensions of the lounge room?
c. Which bedroom is the largest? What are its dimensions?

6. WE4 Below is the front elevation of a house, drawn to scale.



Scale 1 : 100

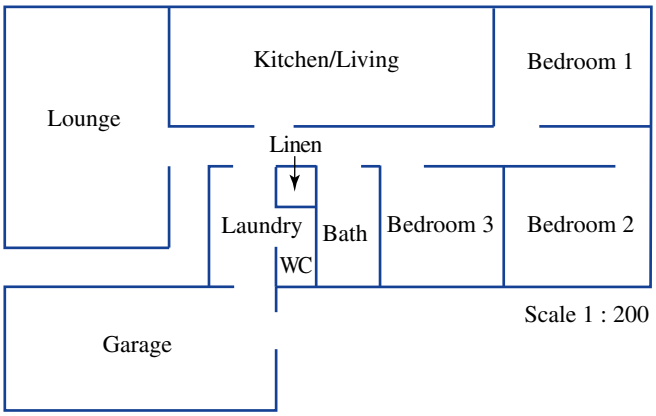
a. Calculate the height of the peak of the roof.
b. Calculate the height of the eaves.
c. Measure the angle of the pitch of the roof.

Understanding

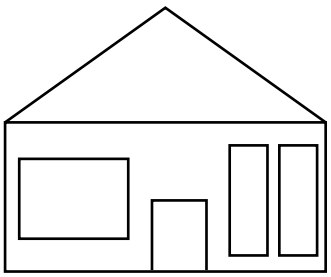
7. If a scale of 1 : 50 is used to make each of the following drawings:
- **a.** what would be the width of the drawing of a house which is 25 m wide
- **b.** what would be the height of the drawing of a man who is 2 m tall
- **c.** what would be the dimensions of the drawing of a rectangular pool 50 m by 20 m
- **d.** what would be the dimensions of the drawing of a table 4 m by 1.5 m
- **e.** what is the actual height of a tree whose height in the drawing is 10 cm
- **f.** what is the actual length of a truck whose length in the drawing is 25 cm
- **g.** what are the dimensions of a television set whose dimensions on the drawing are 10 mm by 8 mm
- **h.** what are the dimensions of a bed whose dimensions on the drawing are 4.5 cm long by 3 cm wide by 1.2 cm high?

8. Referring to the following plan of the house, find:
- **a.** the dimensions of the lounge
- **b.** the width of the hallway
- **c.** the length of the garage
- **d.** the floor area of the kitchen/living room
- **e.** the total floor area of the largest bedroom.

(*Hint:* Use a ruler to find the required lengths on the plan first.)

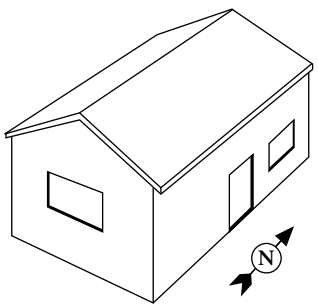


9. Trace the front elevation of the following house into your book. On your diagram write all lengths and angles necessary for the construction of the house.


Scale 1 : 200

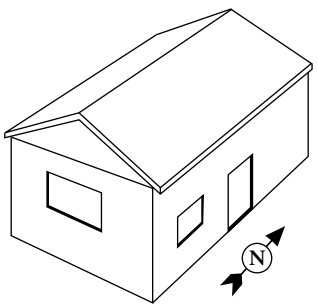
10. The following diagrams are representations of houses with a variety of roof types. Draw a plan of the south and east elevations of these houses. The direction of north is given.

a.



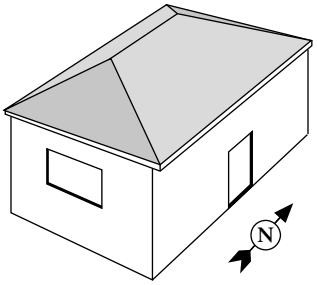
Gable roof

b.



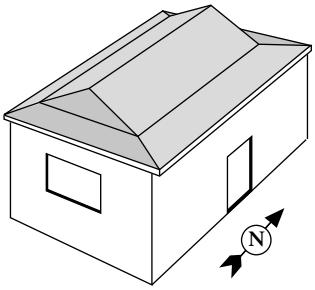
Boxed gable roof

c.



Hip roof

d.



Gambrel
(a roof combining the addition of small gables to a hip roof)

11. Choose an area which is familiar to you, such as your home, bedroom, classroom or school grounds. Draw a plan of your area. Include a scale and a legend. Don't forget to show the northerly direction.

Reasoning

12. A rectangular room has width 3 m and length 5 m.

a. What are the dimensions of the diagram on a scale of 1 : 100?

b. What is the area of the rectangle on the diagram?

c. The diagram has been enlarged so that the dimensions of the new diagram are double the dimensions of the first diagram. What is the area of the rectangle on the new diagram?

d. Calculate the area of a rectangle with dimensions three times bigger than the original diagram.

e. How does enlargement affect the area of a shape?

13. What would happen to the value of an area of a surface if its dimensions were reduced by a factor n?

Problem solving

14. Athena the artist paints portraits. For her present subject, Seth, she uses the following measurement relationships.

- Seth's arm span is the same length as his height.
- The length of Seth's hand is one-tenth of his height.
- The distance across Seth's shoulders is one-quarter his height.

What is the relationship between Seth's arm span, the length of his hand and the distance across his shoulders?

15. Mikaela wants to buy a new fridge that is 800 mm deep and 1 m wide. She is restricted, though, by the kitchen space. The place for the fridge, on the diagram, has dimensions 1.5 cm depth and 2 cm width. If the scale of the drawing is 1:50, will this fridge fit in the space given? State the actual dimensions of the fridge space.

Reflection

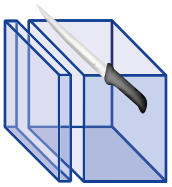
Why is it important to show the position of north on a plan?

CHALLENGE 17.1

A map is photocopied and reduced to 70 per cent of its original size. After the reduction, two towns are 3.5 cm apart and the scale ruler shows that 1.4 cm represents 150 km. Find the actual distance between these two towns and how far apart they were on the original map.

17.3 Plans and views

17.3.1 Prisms



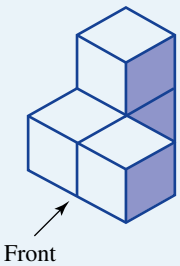
- A prism is a 3-dimensional figure that can be cut into parallel slices or cross-sections that are identical in size and shape.
- Objects or buildings are often constructed by combining prisms.

17.3.2 Plans and views

- An object can be viewed from different angles.
- Architects and draftspersons often draw plans of building sites and various objects when viewed from the front, the side or the top.
- The **front view**, or **front elevation**, is what you see if you are standing directly in front of an object.
- The **side view**, or **side elevation**, is what you see if you are standing directly to one side of the object. You can draw the left view or the right view of an object.
- The **top view**, or **bird's eye view**, is what you see if you are hovering directly over the top of the object looking down on it.

WORKED EXAMPLE 5

The following object is made from 4 cubes.
Draw plans of it showing:
a the front view
b the right view
c the top view.



THINK

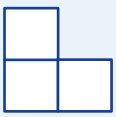
a Make this shape using cubes. Place the shape at a considerable distance and look at it from the front (this way you can see only the front face of each cube). Draw what you see. (Or simply imagine looking at the shape from the front and draw what you see.)

b Look at your model from the right, or imagine that you can see only the right face of each cube and draw what you see.

c Look at your model from the top, or imagine that you can see only the top face of each cube. Draw what you see.

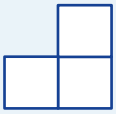
DRAW

a



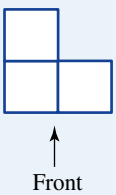
Front view

b



Right view

c



Top view

Front

- Figures such as the one in Worked example 5 can be drawn using isometric dot paper. This will help to give the 3-dimensional perspective of the object.

WORKED EXAMPLE 6

Draw:
i the front view
ii the right view
iii the top view of this solid.

THINK

1 Find an object of similar shape, or visualise the object in your head.

DRAW

Front view

2 Whether viewed from the front, or from the right of the object, the cylindrical shaft will appear as a long thin rectangle. The circular discs will also be seen as a pair of identical rectangles. So the front view and the right view are the same.

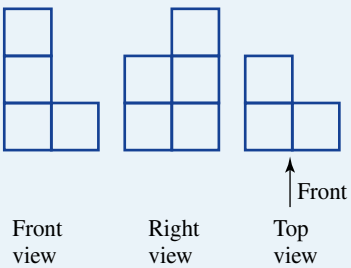
Right view

3 When the object is viewed from above, all we can see is the flat surface of the top disc; that is, a large and a small circle with the same centre. (Note that such circles are called concentric.)

Top view

WORKED EXAMPLE 7

The front, right and top views of a solid are shown. Use cubes to construct the solid.

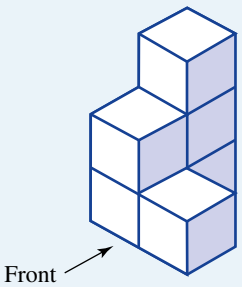


THINK

1 Use cubes to construct the solid.

2 Check carefully that your solid matches each of the 3 views you are given. Make adjustments if necessary.

CONSTRUCT

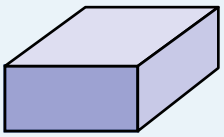


17.3.3 Perspective drawing

- A **2-dimensional representation** of a **3-dimensional object** can be made using a single-point perspective. This shows the object as it appears to the eye in reality. When we look at a picture of a scene, the objects seem to converge or move towards a point in the distance. This point is called a vanishing point.

WORKED EXAMPLE 8

Construct a perspective drawing of the rectangular prism shown.

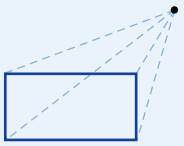


THINK

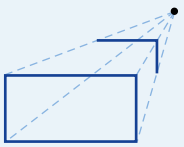
DRAW

1 Draw the front face of the prism as a rectangle.

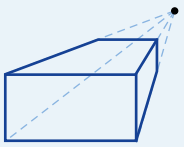
2 Take a vanishing point in the distance behind the object and slightly to the right. Draw dotted lines from this point to the corners of the front face of the prism.



3 On these dotted lines, make the following markings. Draw the top edge of the back face parallel to the top edge of the front face. Also draw the right edge of the back face parallel to the right edge of the front face.



4 Join the top and right-side vertices of the front face to the corresponding vertices of the back face.



learnON RESOURCES — ONLINE ONLY

Try out this interactivity: Plans, elevations and cross-sections (int-0009)

Exercise 17.3 Plans and views

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1, 2, 3, 4, 5, 6, 11, 12	Questions: 1, 2, 3, 4, 5, 6, 9, 11, 12	Questions: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12

Individual pathway interactivity: int-4392

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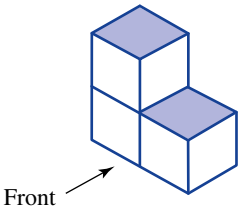
To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note*: Question numbers may vary slightly.

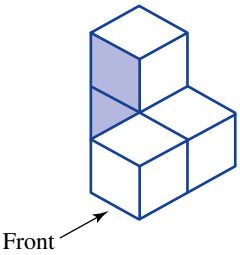
Fluency

1. WE5 The following objects are made from cubes. For every object draw the plans, showing the front view, the right view and the top view. (You may wish to use a set of cubes or building blocks to help you.)

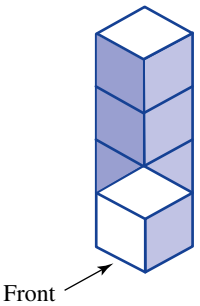
a.



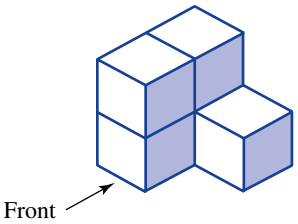
b.



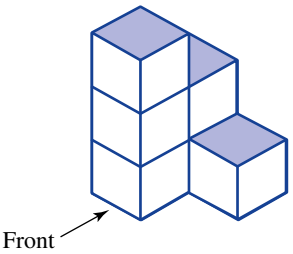
c.



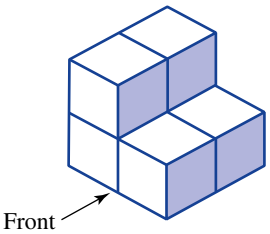
d.



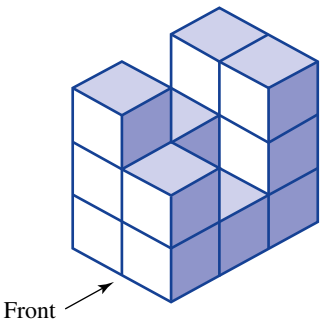
e.



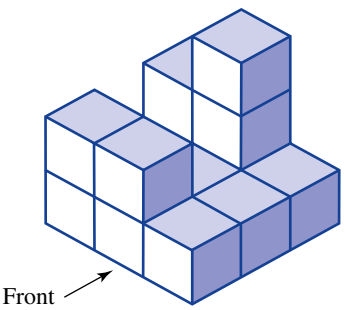
f.



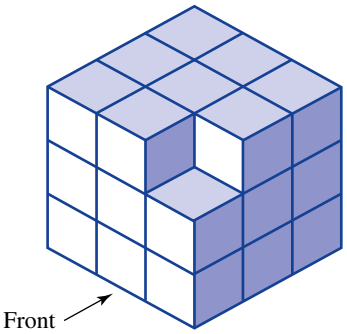
g.



h.



i.



2. WE6 Draw the front, right and top views of each solid shown.

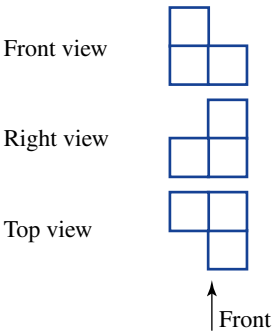
a.

b.

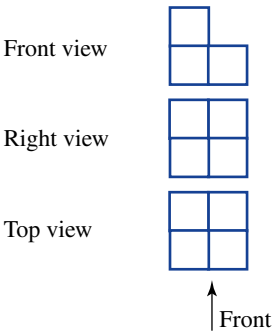
c.

3. WE7 The front, right and top views of a solid are shown. In each case, use cubes to construct the solid.

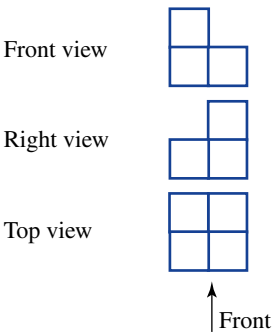
a.



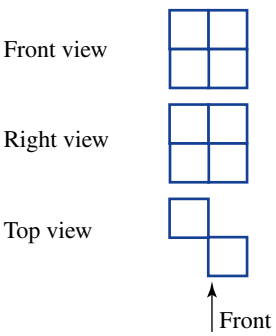
b.



c.



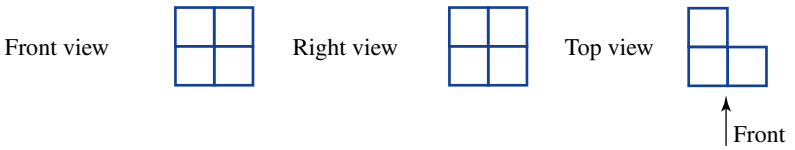
d.



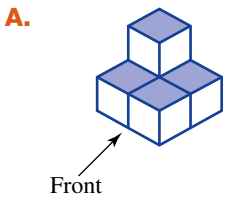
4. WE8 Construct a perspective drawing of a cube.

Understanding

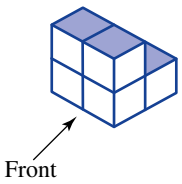
5. MC The front, right and top views of a solid are shown. Which of the given drawings could represent the solid?



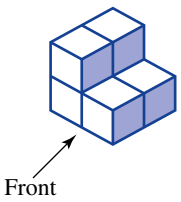
A.



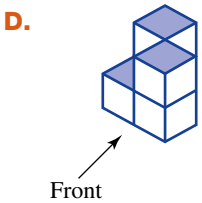
B.



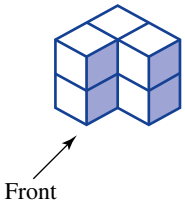
C.



D.



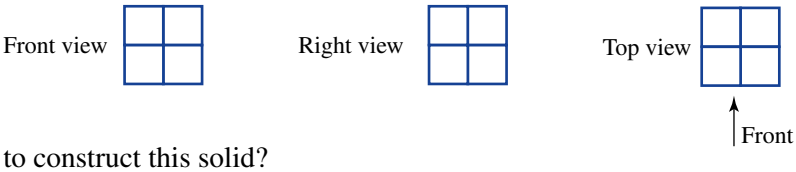
E.



6. a. What shape is the top view of a telephone pole?
b. What shape is the top view of the Melbourne Cricket Ground?
c. What shape is the side view of a bucket?
d. What shape is the top view of a car?

7. a. Draw the side view of a pool table.
b. Draw the front view of your house (seen from the street).
c. Draw the side view of an electric kettle.
d. Draw the top view of your television set.

8. A shape is made using only 4 cubes. Its front view, right view and top view are shown.



a. Is it possible to construct this solid?
b. Describe or draw what this solid would look like.

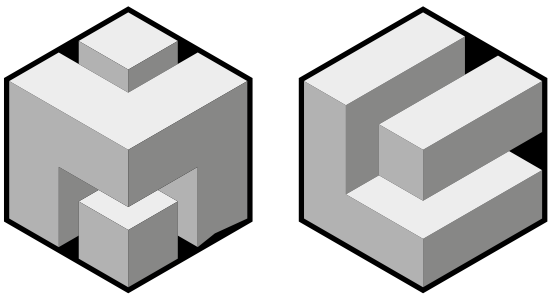
Reasoning

9. Explain why a 3-D object requires three to six diagrams to clearly represent its views.

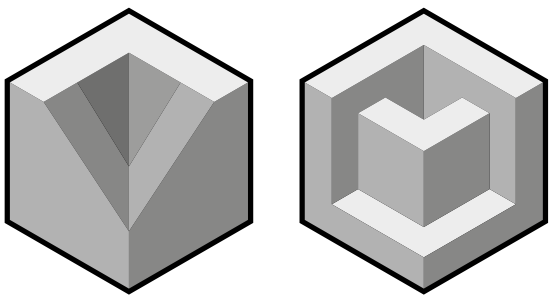
10. How many objects can you construct using the top view shown? Explain your answer with the use of diagrams.

Problem solving

11. Draw top, front and side views for the 3-D objects given.



12. Draw top, front and side views for the 3-D objects given.

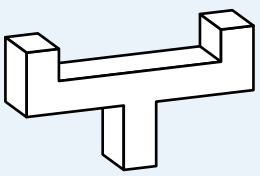


Reflection

Why is the technique of perspective drawing frequently used in artistic paintings?

CHALLENGE 17.2

Using the following diagram, draw its plan, showing the front view, right view and top view.



17.4 Isometric drawing

17.4.1 Isometric drawing

- When working with 3-dimensional models and designs, it is often useful to have the design or model drawn on paper (that is, in 2 dimensions).
- An **isometric drawing** is a 2-dimensional drawing of a 3-dimensional object.
- This picture shows an architect's drawing of a beach hut and environs in isometric view superimposed on the actual hut. Architects and draftspersons often use isometric drawings to give their clients a clear picture of the proposed design.

WORKED EXAMPLE 9

First copy the incomplete figure (Figure 1) onto isometric dot paper. Complete the isometric drawing of the object (Figure 2).

FIGURE 1

FIGURE 2

THINK

Study the object and identify the lines that have already been drawn. Fill in the missing lines on your isometric drawing to match the object.

DRAW

WORKED EXAMPLE 10

Draw the following object on isometric dot paper. (You could construct it first from a set of cubes.)

THINK

DRAW

1 Use cubes to make the object shown (optional). Draw the front face of the object. The vertical edges of the 3-dimensional object are shown with vertical lines on the isometric drawing; the horizontal edges are shown with the lines at an angle (by following the dots on the grid paper).

2 Draw the left face of the object.

3 Add the top face to complete the isometric drawing of the object.

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Complete this digital doc: WorkSHEET: Views of 3-D shapes (doc-6561)

Exercise 17.4 Isometric drawing

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1, 2, 3, 4, 5, 8, 11	Questions: 1, 2, 3a, c, 4, 5, 6, 8, 9, 11	Questions: 1, 2, 3b, d, 4, 5, 6, 7, 8, 9, 10, 11

Individual pathway interactivity: int-4393

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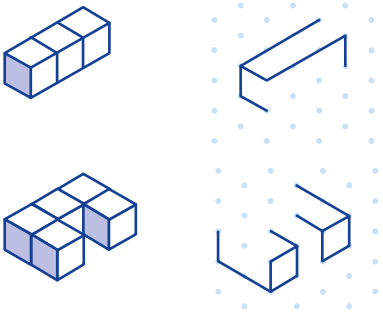
To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note*: Question numbers may vary slightly.

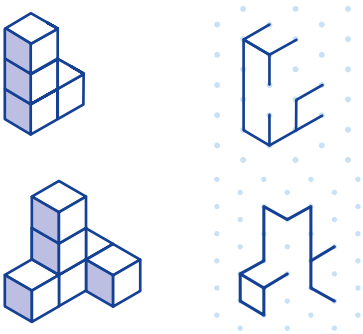
Fluency

1. WE9 Copy the following figures onto isometric dot paper and complete the isometric drawing of the objects shown.

a.



b.



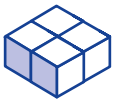
c.

d.

2. WE10 Draw each of the following objects on isometric dot paper. (You might wish to make them first from a set of cubes.)

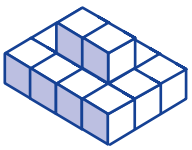
a.

b.



c.

d.



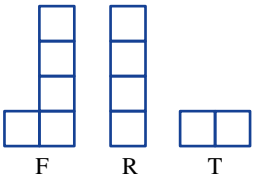
Understanding

3. Construct the following letters using cubes, and then draw the solids on isometric dot paper:

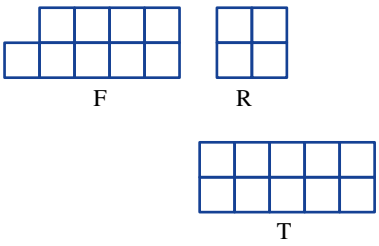
a. the letter T with 5 cubes
b. the letter L with 7 cubes
c. the letter E with 10 cubes
d. the letter H with 7 cubes.

4. Draw these objects, whose front (*F*), right (*R*) and top (*T*) views are given, on isometric dot paper.

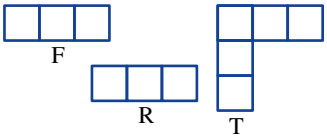
a.



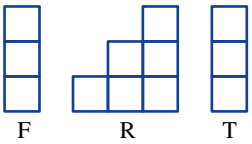
b.



c.

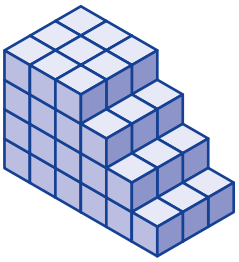


d.

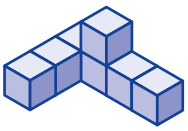


5. Draw the front, right and top views of these objects.

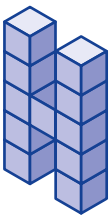
a.



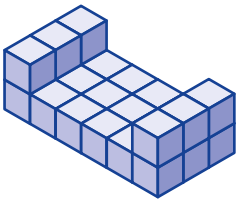
b.



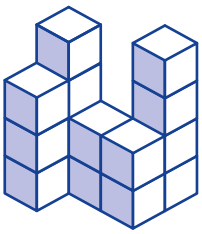
c.



d.



6. Draw the following figure on isometric dot paper.

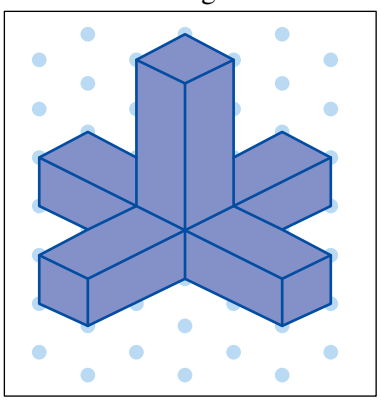


7. Draw a selection of buildings from this photograph of a city skyline on isometric dot paper.

Reasoning

8. On the same isometric dot paper draw the two wooden boxes shown.

9. Draw top, front and side views of the object shown in the following isometric drawing.



Problem solving

10. Sam has a set of wooden cube blocks he uses to build different shapes. His sister Chris challenged him to see how many different ways he could stack 4 cubes. He can only stack them on top of each other or side by side — not one behind the other. The arrangements must be different and not simply a mirror image or rotation of another shape. Draw the different arrangements possible.

11. Jonah is building a brick wall as shown in the following photograph. Draw this part of the wall on isometric dot paper.

Reflection

How does isometric dot paper turn two-dimensional lines into a three-dimensional image?

17.5 Review

17.5.1 Review questions

Fluency

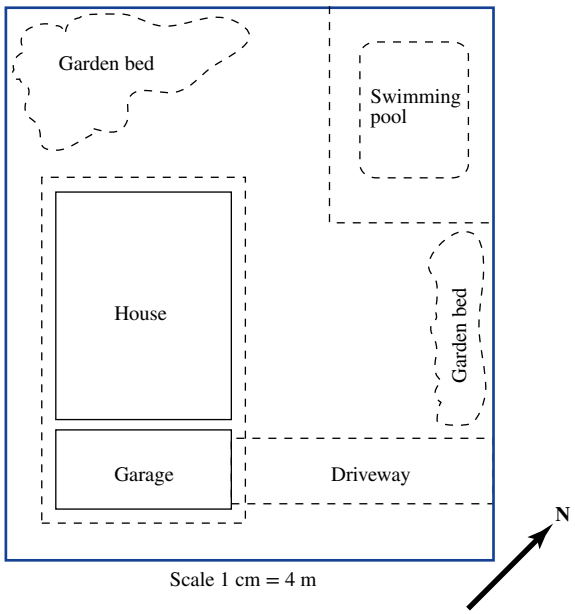
1. The scale on a map is given as 1 : 100 000. Find the actual distances between:
- **a.** two schools that are 5 cm apart on the map
- **b.** two parks that are 2.5 cm apart on the map
- **c.** two farms that are 4 cm apart on the map.

2. A scale of 1 : 200 is used to make models of a tree and a car.
- **a.** What would be the length of the model car if the car is actually 2.5 metres long?
- **b.** What would be the height of the model tree if the tree is actually 6.5 metres tall?

Problem solving

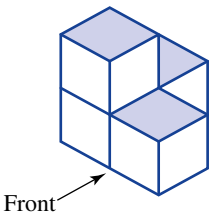
3. On a town plan, the scale is given as 1 : 200 000.
- **a.** Find the actual distances between two schools that are 4.5 cm apart on the plan.
- **b.** Find the distance on the plan between two parks that are 2.5 km apart.

4. The scale diagram below is a site plan for a block of land on which a house is to be built. Use this diagram to answer the following questions.
- **a.** Calculate the dimensions of the block of land.
- **b.** Calculate the dimensions of the house.
- **c.** Calculate the area of the house in square metres.
- **d.** In which direction would you be travelling as you drive into the garage?

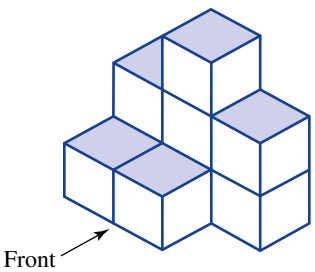


5. Draw the front, side and top views of each of these solids.

a.

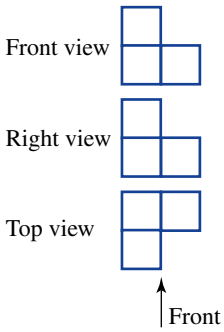


b.

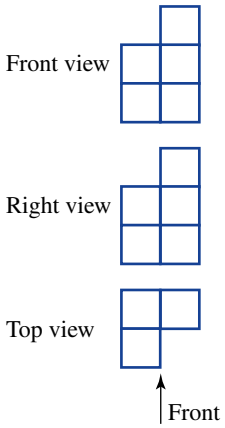


6. The front, side and top view of a solid are shown. Construct this solid, using blocks.

a.

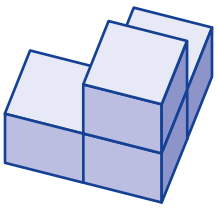


b.

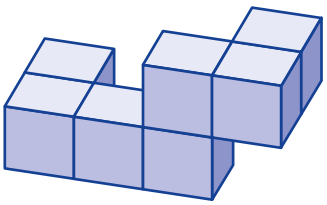


7. Draw the front, right and top views of these objects.

a.

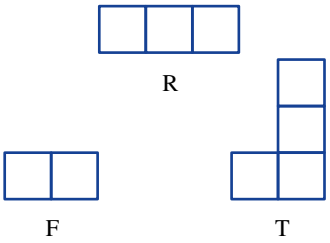


b.

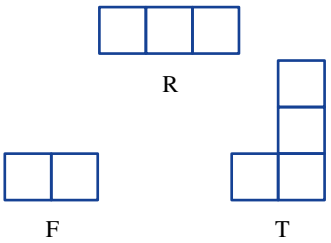


8. Draw isometric views of the objects, whose front, right and top views are given below.

a.



b.



9. A rectangular prism, constructed from the set of cubes, is 3 cubes long, 2 cubes wide and 4 cubes high. Draw an isometric view of the prism.

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- **Try out this interactivity:** Crossword: Topic 17 (int-2743)
- **Try out this interactivity:** Sudoku (int-3177)
- **Complete this digital doc:** Concept map: Topic 17 (doc-10749)

Language

It is important to learn and be able to use correct mathematical language in order to communicate effectively. Create a summary of the topic using the key terms below. You can present your summary in writing or using a concept map, a poster or technology.

2-dimensional representation
3-dimensional object
converge
dimensions
elevations
front view
isometric drawing
legend
map scale
perspective
plan
ratio
scale factor
side view
top view
vanishing point

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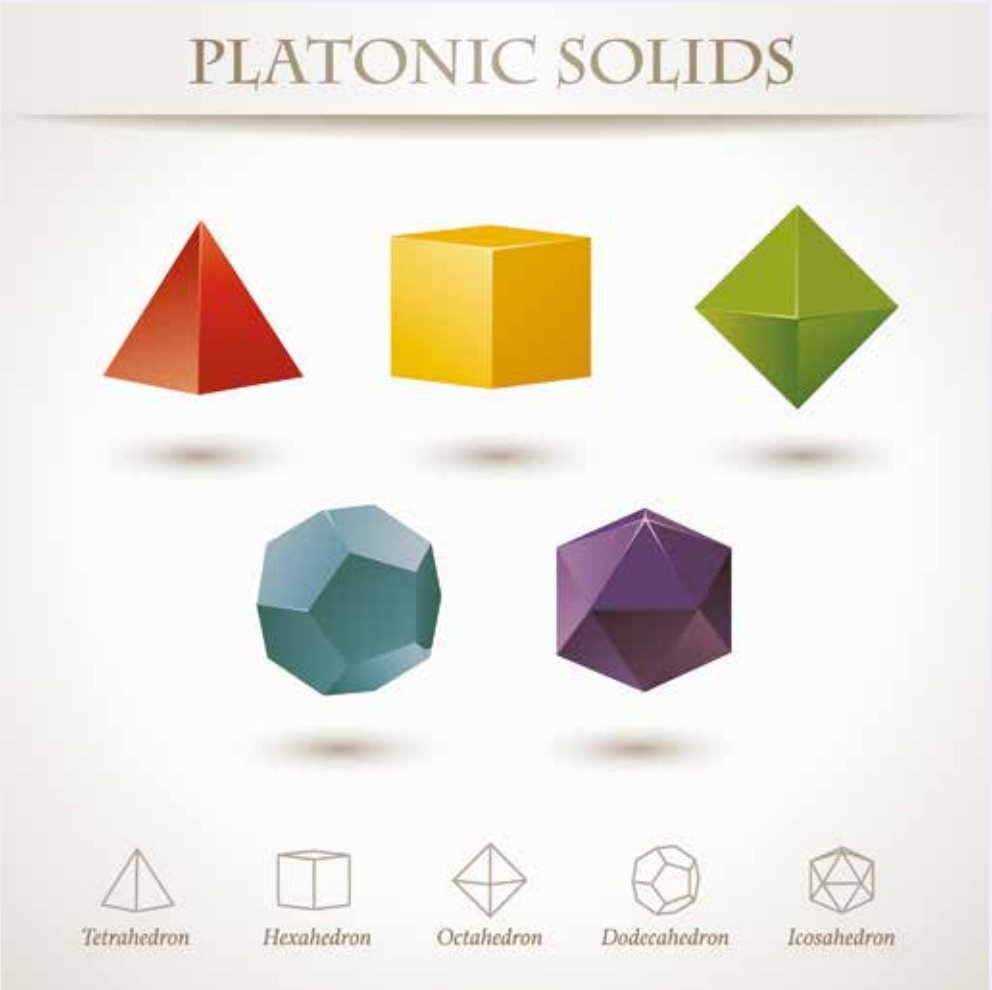
Investigation | Rich task

Making a truncated octahedron



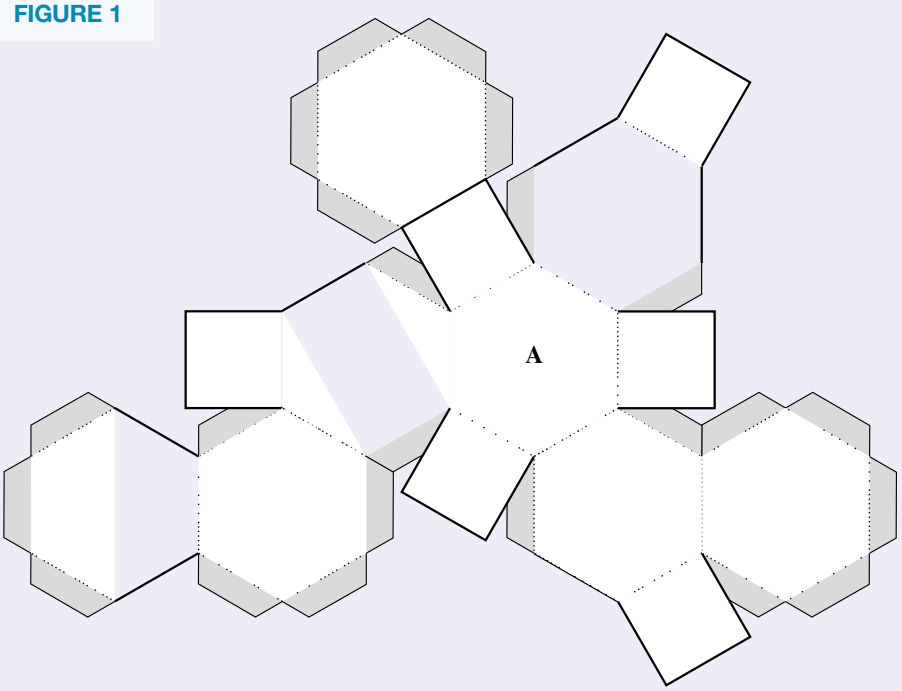
A truncated octahedron is formed when the vertices of an octahedron are cut off. *Truncate* means to cut off. New square faces form when the vertices are cut off, producing the truncated octahedron as shown in Figure 3.

Copy the net that appears in Figure 1 onto stiff cardboard. Cut along the solid outer lines, taking care not to cut the tabs off. Fold upwards along the dotted lines. The best way to fold is to rule a pen along the dotted lines first and then fold upwards. Starting at the central hexagon (labelled *A*), fold each of the surrounding shapes upwards. The 3-dimensional shape will begin to take shape. Glue or sticky-tape the tabs to the inside of each adjoining face.

Carefully examine the net of the truncated octahedron shown in Figure 1.

1. What shapes does it contain?
2. How many of each shape are there?
3. When you have constructed your solid, state the number of faces, vertices and edges associated with the shape.

FIGURE 1



A smaller version of your net is shown in Figure 2, along with the solid form (Figure 3).

FIGURE 2

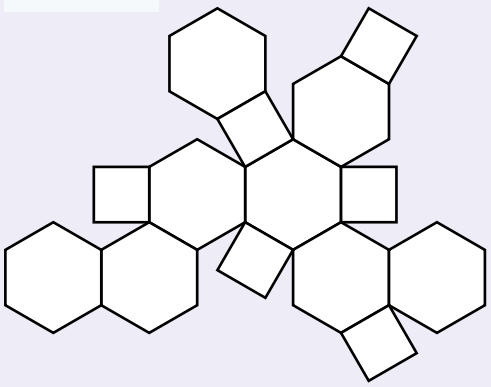
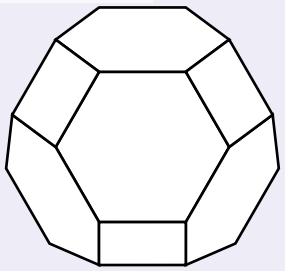


FIGURE 3



4. Choose a colour pattern, using at least three different colours for your net. Show how it will appear on the three-dimensional solid.
5. Investigate other basic three-dimensional shapes and try to construct them using straws. Provide a name and a plan of your shapes' nets.

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Complete this digital doc: Code puzzle: Who invented the 'motion picture' in 1891 and what was it called? (doc-14341)

Answers

Topic 17 Views of 3-D shapes

Exercise 17.2 Scales and house plans

1. a. 1 cm ⇔ 10 cm **b.** 1 cm ⇔ 1 m **c.** 1 cm ⇔ 10 m **d.** 1 cm ⇔ 100 m **e.** 1 cm ⇔ 1 km
f. 1 cm ⇔ 50 km **g.** 1 cm ⇔ 600 m **h.** 1 cm ⇔ 4 km **i.** 1 cm ⇔ 7.5 km **j.** 1 cm ⇔ 22 km

2. a. 1 : 20 **b.** 1 : 50 000 **c.** 1 : 1000 **d.** 1 : 20 000 **e.** 1 : 500 000
f. 1 : 5 000 000 **g.** 1 : 550 000 **h.** 1 : 100 000 **i.** 1 : 125 000 **j.** 1 : 200 000

3. a. 1 cm **b.** 2 mm **c.** 15 cm
d. 2.5 cm **e.** 22 cm **f.** 5 mm

4. a. 20 m × 25 m **b.** 10 m × 15 m

5. a. 18 m × 12 m **b.** 6.75 m × 4.5 m **c.** Bed 1 — 4.5 m × 4.5 m

6. a. 8.5 m **b.** 3.5 m **c.** 40°

7. a. 50 cm **b.** 4 cm **c.** 100 cm × 40 cm **d.** 8 cm × 3 cm
e. 5 m **f.** 12.5 m **g.** 50 cm × 40 cm **h.** 2.25 m × 1.5 m × 0.6 m

8. a. 6 m × 4 m **b.** 1 m **c.** 6.8 m **d.** 24 m^2 **e.** 12 m^2

9. Check with your teacher.

10. Check with your teacher.

11. Check with your teacher.

12. a. 3 cm and 5 cm **b.** 15 m^2 **c.** 60 m^2 **d.** 135 m^2

e. If the dimensions double, the area becomes four times larger. If the dimensions are three times bigger, the area becomes nine times larger. If the dimensions are enlarged by a factor n, the area is enlarged by a factor n^2.

13. If the dimensions were reduced by a factor n, the area would be reduced by a factor n^2.

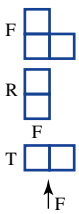
14. Seth's arm span is 10 times the length of his hand and 4 times the distance across his shoulders.

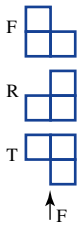
15. No. Depth is 750 cm, width is 1 m.

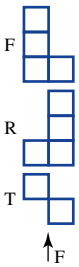
Challenge 17.1

The towns were 5 cm apart on the original map. They are actually 375 km apart.

Exercise 17.3 Plans and views

1. a.


b.


c.


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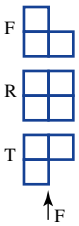
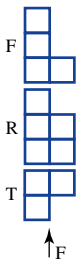
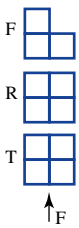
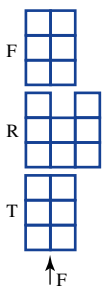
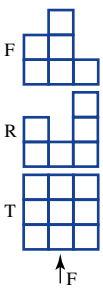
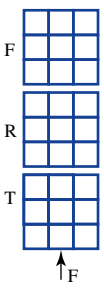


e.


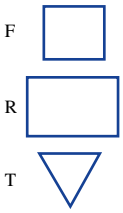
f.


g.


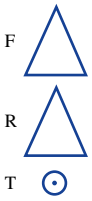
h.


i.


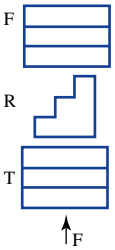
2. a.



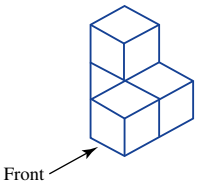
b.



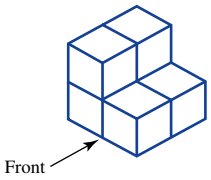
c.



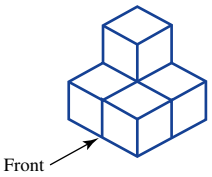
3. a.



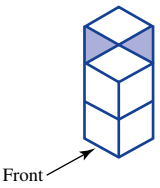
b.



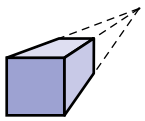
c.



d.



4.



5. D

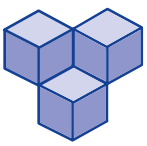
6. a. Circle **b.** Oval **c.** Trapezium **d.** Rectangle

7. a.

b–d Various answers

8. a. It would be difficult to construct this solid.

b.

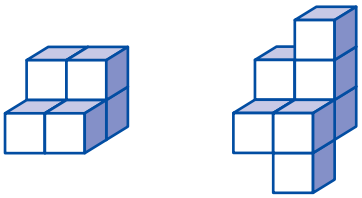


The solid would look like the one shown. The 4th cube is behind the front bottom cube, resting on the surface. The two top cubes have no cubes supporting them.

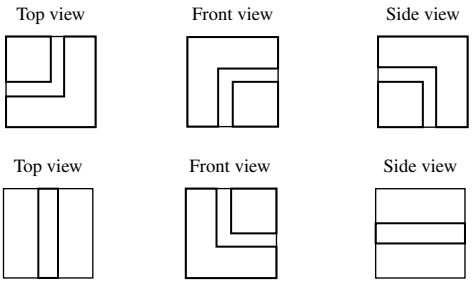
9. Usually a 3-D object will require only 3 views as normally the front and back faces are identical, the top and bottom faces are identical, and the left and right faces are identical.

There are instances when an object has no identical faces. In these situations all views have to be drawn in order to show the details of each face.

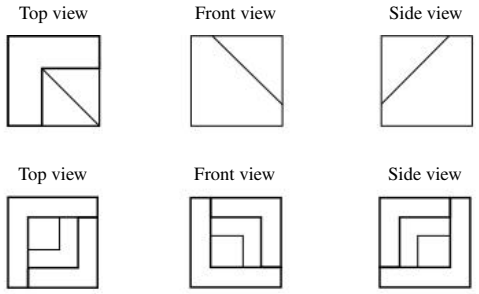
10. An infinity of shapes because there are no restrictions on the other two views.



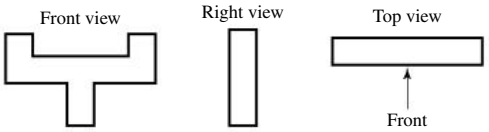
11.



12.



Challenge 17.2



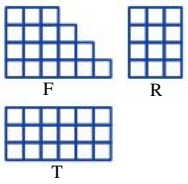
Exercise 17.4 Isometric drawing

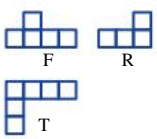
1. a.
b.
c.
d.

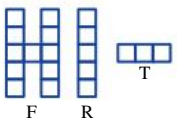
2. a.
b.
c.
d.

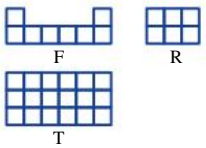
3. a.
b.
c.
d.

4. a.
b.
c.
d.

5. a. 

b. 

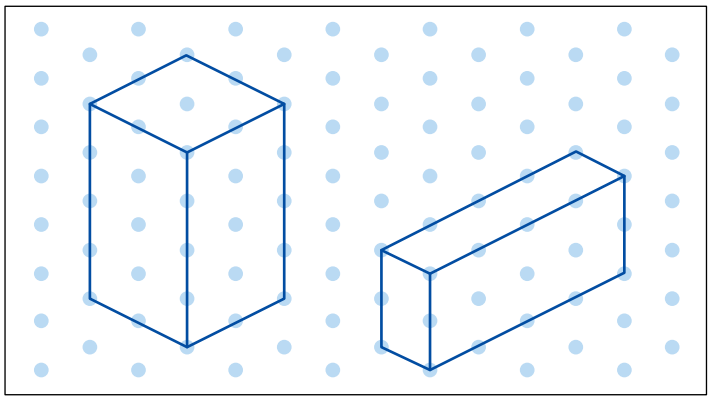
c. 

d. 

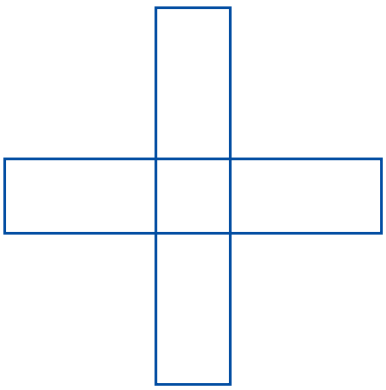
6.

7. Check with your teacher.

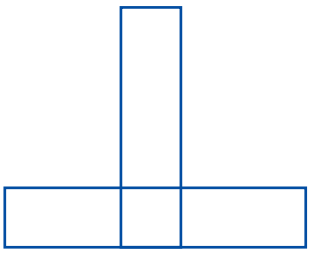
8.



9. Top view

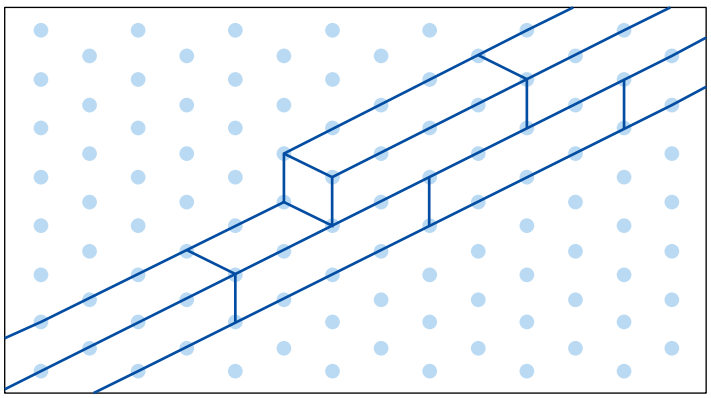


Front and side views



10. 4

11.



17.5 Review

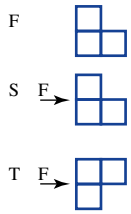
1. a. 5 km **b.** 2.5 km **c.** 4 km

2. a. 1.25 cm **b.** 3.25 cm

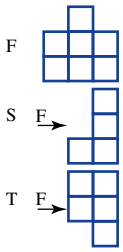
3. a. 9 km **b.** 1.25 cm

4. a. 30 m 3 26 m **b.** 9.6 m 9 12 m **c.** 115.2 m^2 **d.** SW

5. a.



b.



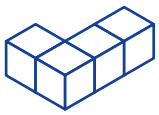
6. a.

b.

7. a.

b.

8. a.



b.

9.

Investigation — Rich task

1. Hexagons and squares

2. 8 hexagons and 6 squares

3. Faces = 14, vertices = 24, edges = 36

4. Teacher to check

5. Teacher to check

TOPIC 18
Money

18.1 Overview

Numerous **videos** and **interactivities** are embedded just where you need them, at the point of learning, in your learnON title at www.jacplus.com.au. They will help you to learn the content and concepts covered in this topic.

18.1.1 Why learn this?

Money is part of our everyday life. We need to understand money and finance because decisions we make concerning money will have significant influence on our future wellbeing.

18.1.2 What do you know?

assessON

1. **THINK** List what you know about money. Use a thinking tool such as a concept map to show your list.
2. **PAIR** Share what you know with a partner and then with a small group.
3. **SHARE** As a class, create a thinking tool such as a large concept map that shows your class's knowledge of money.

LEARNING SEQUENCE

18.1 Overview
18.2 Money
18.3 Money and percentages
18.4 Unitary method
18.5 Review

18.2 Money

18.2.1 Operations with money

In most modern societies, money is used as a medium of exchange for goods and services.

- Australian currency is a decimal system using dollars and cents. There are 100 cents in a dollar.
- To change dollars to cents, you multiply by 100. Multiply, because there will be more cents than dollars.
- To change cents to dollars, you divide by 100. Divide, because there will be fewer dollars than there are cents.
- The Australian government withdrew one-cent and two-cent coins in 1990. Prices are still expressed in one-cent increments. When payment is made by cheque, credit card or electronic means, the exact amount is paid in dollars and cents.
- When cash is being used, amounts are rounded to the nearest 5 cents. For example, 61 cents and 62 cents would be rounded down to 60 cents while 63 cents and 64 cents would be rounded up to 65 cents. Similarly, 66 cents and 67 cents would be rounded down to 65 cents while 68 cents and 69 cents would be rounded up to 70 cents.

WORKED EXAMPLE 1

a Change $2.45 into cents.
b Write 20c in dollars.

THINK	WRITE
a 1 Write the given amount and change dollars to cents by multiplying by 100.	**a** $\$2.45 = (2.45 \times 100)$ cents
2 Write the answer.	$= 245$ cents
b 1 Write the given amount and change to dollars by dividing by 100.	**b** $20c = \$(20 \div 100)$
2 Write the answer.	$= \$0.20$

- To do our shopping, we must be able to use money, to estimate our total shopping bill and calculate the change that we can expect. Most importantly, we need to be able to budget. This means thinking about our available funds before we make a purchase.

WORKED EXAMPLE 2

A $100 note is used to pay for the following:
2 packs of socks @ $16.95 per packet
3 packs of golf balls @ $19.95 per packet.
Estimate the total bill and the change, then find the exact amount for each.

THINK	WRITE
1 Round the cost of the socks to the nearest dollar.	Socks: about $17 per pack
2 Round the cost of the golf balls to the nearest dollar.	Golf balls: about $20 per pack
3 Find the total estimated cost of the socks and golf balls.	Total estimated cost in dollars $= 2 \times 17 + 3 \times 20$ $= 94$
4 Find the change by subtracting the total cost of the purchases from the amount tendered.	Estimated change in dollars $= 100 - 94$ $= 6$
5 Answer the question in a sentence.	The estimated total cost is $94 and the change from $100 is $6.
6 Find the exact cost of the socks and the golf balls.	Total cost $= 2 \times 16.95 + 3 \times 19.95$ $= 33.90 + 59.85$ $= 93.75$
7 Find the change given.	Change $= \$100 - \$93.75$ $= \$6.25$
8 Answer the question in a sentence.	The total cost of the socks and the golf balls is $93.75 and the change from $100 is $6.25.

- Each pair of socks and each pack of golf balls costs 5 cents less than the estimated cost, so the total cost of the goods is (5×5) cents or 25 cents less than the estimated cost.

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Complete this digital doc: SkillSHEET: Rounding money to the nearest 5 cents (doc-6602)
Complete this digital doc: Investigation: How much is one million dollars? (doc-2226)
Complete this digital doc: Investigation: Movie Munchies dollars (doc-2227)

Exercise 18.2 Money

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1, 2, 3, 4, 5, 6, 9, 13, 14, 16, 20	Questions: 1, 2, 3, 4, 5, 7, 8, 10, 12, 13, 14, 16, 18, 20, 21	Questions: 1, 2, 3, 4, 5, 6, 10, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21

Individual pathway interactivity: int-4394

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Fluency

1. WE1a Change each of the following into cents.
a. $7.55 **b.** $3.05 **c.** $2.40 **d.** $9

2. WE1b Change each of the following to dollars.
a. 685 cents **b.** 450 cents **c.** 5 cents **d.** 805 cents

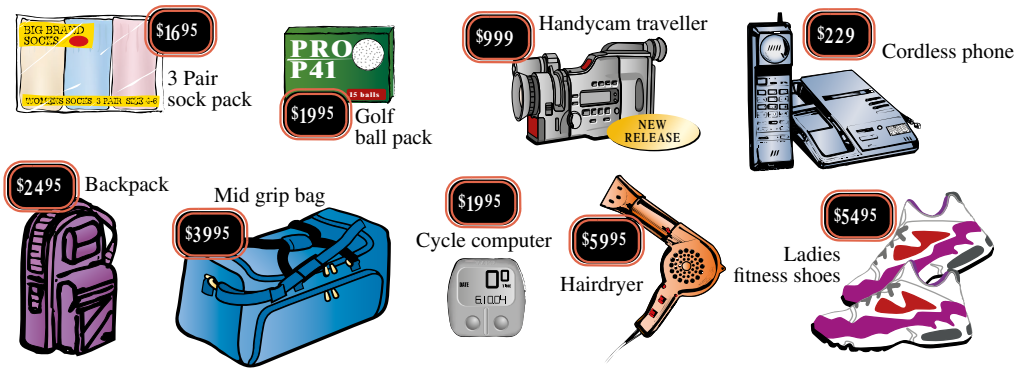
3. Round the following to the nearest 5 cents.
a. $4.76 **b.** $12.61 **c.** $159.28 **d.** $83.44

Understanding

4. The students of Year 9 have raised $245.40 for charity and want to share the money equally among the Children's Hospital, the Guide Dog Appeal and the Red Cross Shield Appeal. How much money will be given to each charity?

5. The cost of 5 packets of chocolate biscuits and a 2-litre bottle of milk was $15.60. If the bottle of milk cost $4.35, how much did a packet of chocolate biscuits cost?

Questions **6–12** refer to these advertised products.



6. How much change would you receive from $50 if you purchased:
a. the socks **b.** the cycle computer **c.** the backpack?

7. Rani can save $15 per week. How many weeks will it take for her to save enough to buy the fitness shoes?

8. John has just selected a new set of golf clubs worth $489.95. He then buys two golf ball packs. What is his total outlay?

9. Janine has just started full-time work. She pays a $100 deposit on the Handycam and the balance in 10 equal monthly instalments. How much is each instalment, assuming she pays no interest charges?

10. Nicole needs a new hairdryer. Her dad lends her $60 to buy the one shown. She agrees to work for him in order to pay him back. They agree on $8 per hour. For how long will she have to work to clear the debt?

11. Veda buys the cordless phone with a $50 deposit and 6 payments of $35 each. How much more than the advertised price does she finally pay? Why would this be?

12. For her birthday, Cristi was given the advertised fitness shoes, the backpack, the socks and the hairdryer by her family. If four people split the expense evenly, how much did each person pay?

For questions **13** and **14**, estimate, then find, the exact value for:

a. the total bill **b.** the change.

13. WE2 A $20 note is used to pay the supermarket bill after purchasing these goods:
3 packets of soup @ $1.89 per packet
2 packs of pasta @ $1.24 per packet
2 boxes of cereal @ $2.95 per box.

14. $30 is used to pay the fruit shop for:
3 kg bananas @ $1.99 per kg
4 kg apples @ $2.95 per kg
2 kg oranges @ $3.25 per kg
$\frac{1}{2}$ kg mushrooms @ $3.90 per kg.

15. A petrol rewards card gives a discount of 4 cents per litre. If I purchase 37.62 litres of petrol at a price of $1.379 per litre then:

a. how much is the cost of the fuel before the discount

b. using the petrol rewards card, what is the discount

c. how much change will I receive if I used $60.00 cash to make the purchase?

Reasoning

16. Sam and Georgia were paid a total of $167.00 for painting a fence. Sam worked for 12 hours and Georgia worked for 8 hours. How much should each person receive?

17. Olivia has 78 fifty-cent coins and 43 two-dollar coins. What percentage of her money is made up of two-dollar coins? Show your working.

18. Fifty people, half of whom were females, were asked if they liked yoghurt. Twenty-seven of the people, including two-fifths of all the males, said they did like yoghurt. How many of the females did not like yoghurt? Show your working.

19. Ingrid offered to pay her brothers $2 for doing her share of the housework each day, but fined them $5 if they forgot to do it. After 4 weeks, Ingrid discovered that she owed her brothers nothing. For how many days did Ingrid's brothers do her share of the housework? Justify your answer.

Problem solving

20. Year 12 students at Minnicom College decided to have a pizza-and-pasta day on the last day of term. Mrs Day decided to offer a limited choice of food and a drink. The restaurant charged her the following: marinara, $6.00; lasagne, $6.50; any small pizza, $5.00. The local supermarket sold drinks for $6.40 per dozen.

a. If 70 students chose pizzas, 15 chose lasagne and 8 chose marinara, and everyone gets a drink, what is the overall cost of the day?

b. If each student is to be charged the same amount, how much will that be? (Remember to round your answer to the nearest 5c.)

c. If there is any change left over, how much is there, and what would you do with it?

21. Christine and Con made a certain number of telephone calls over a 3-month period. There is a charge of 22 cents for each call, and the monthly rental of the phone line is $15.20. The bill for the 3 months comes to $138. How many calls did they make in the 3-month period?

Reflection

In what situations would estimation be useful?

18.3 Money and percentages

18.3.1 Money and percentages

- Signs like those shown above are common when stores are having a sale. To find the percentage of an amount, for example 20% of $50, follow the steps given overleaf.

Step 1:	Write the percentage as an amount out of 100.	$\frac{20}{100}$ of 50
Step 2:	Change the 'of' to a multiplication sign and put the second amount over 1.	$\frac{20}{100} \times \frac{50}{1}$
Step 3:	Simplify the fractions if possible.	$\frac{2}{1} \times \frac{5}{1}$
Step 4:	Multiply the numerators and the denominators.	$\frac{10}{1}$
Step 5:	If left with a fraction, divide the denominator into the numerator. (A calculator may be helpful here.)	$10

WORKED EXAMPLE 3

Find 22% of $40.

THINK	WRITE
1 Write the question.	22% of 40
2 Write the percentage as an amount out of 100, change 'of' to × and write the second amount over 1.	$= \frac{22}{100} \times \frac{40}{1}$
3 Simplify the fractions.	$= \frac{11}{5} \times \frac{4}{1}$
4 Multiply numerators and denominators and write the answer as a mixed number or decimal if appropriate (2 decimal places for money).	$= \frac{44}{5}$ $= 8\frac{4}{5}$ or 8.8
5 Write the answer.	22% of $40 = $8.80

GST

- GST is an abbreviation of *Goods and Services Tax*.
- GST is a tax paid to the government for services and goods bought.
- 10% GST is paid on a bill at a restaurant.
- 10% GST is paid on clothes and other items which we buy.
- The price we see in a shop has already had GST added to it by the seller.

WORKED EXAMPLE 4

Ahmed bought a pair of jeans for $120. What was the price of the jeans before GST was added?

THINK	WRITE
1 The jeans have already had 10% added to the price tag. The jeans are worth 110%.	$110\% = \$120$
2 Need to find 1%. Divide both sides by 110.	$110\% = \$120$ $\frac{110}{110} = \frac{120}{110}$ $1\% = \$1.09$
3 To find 100%, multiply both sides of the equation by 100.	$100\% = 1.09 \times 100$ $100\% = \$109$
4 Answer the question.	The jeans were $109 before GST was added.

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Complete this digital doc: SkillSHEET: Converting a percentage to a decimal fraction (doc-6603)

Complete this digital doc: SkillSHEET: Decreasing a quantity by a percentage (doc-6604)

Complete this digital doc: SkillSHEET: Finding a percentage of a quantity (money) (doc-6605)

Exercise 18.3 Money and percentages

assessON

Individual pathways

PRACTISE
Questions:
1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 13, 14, 17

CONSOLIDATE
Questions:
1 columns 2 and 3, 2, 3, 4a, d, g, 5a, d, g, i, 6a, d, g, 7, 8, 9, 10, 12, 13, 15, 17, 18

MASTER
Questions:
1 column 3, 2, 3, 4c, f, i, 5c, k, 6b, e, h, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18

Individual pathway interactivity: int-4395

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Fluency

1. WE3 Find the following percentages, rounding your answer to 2 decimal places, if appropriate.

a. 10% of $80	**b.** 20% of $50	**c.** 50% of $200
d. 8% of $30	**e.** 12.5% of $200	**f.** 25% of $48
g. 6% of $75	**h.** 48% of $20	**i.** 35% of $15
j. 45% of $18	**k.** 5% of $142	**l.** 55% of $44
m. 7% of $350	**n.** 95% of $28	**o.** 12% of $35

2. Find 10% of the following amounts. (Money answers must be rounded to 2 decimal places.)

a. $32.00	**b.** $12.50	**c.** $75.00	**d.** $167.00	**e.** $1.45
f. $15.98	**g.** $21.09	**h.** $77.77	**i.** $164.20	

3. Look at your answer to each of the problems in question **2**. Copy and complete the following sentence.
To find 10% of an amount, move the position of the decimal point _____ place to the _____.

4. Calculate 10% of the following amounts by moving the position of the decimal point.

a. $23.00	**b.** $57.00	**c.** $130.00	**d.** $64.00	**e.** $81.50
f. $14.80	**g.** $149.60	**h.** $345.90	**i.** $1354.50	

5. Calculate 5% of the following by finding 10% and halving your answer.

a. $12.00	**b.** $36.00	**c.** $68.00
d. $24.00	**e.** $90.00	**f.** $120.00
g. $45.00	**h.** $27.40	**i.** $12.80
j. $33.60	**k.** $508.20	**l.** $235.50

6. Calculate 1% of the following by finding 10% and dividing by 10.

a. $26.00	**b.** $42.00	**c.** $166.00	**d.** $406.00	**e.** $1620.00
f. $14.25	**g.** $7.00	**h.** $5.50	**i.** $16.80	

7. Copy and complete the following sentence.
To find 1% of an amount, move the position of the decimal point _____ places to the _____.

8. Find 15% of the following by finding 10% and 5% then adding them together.

a. $100.00	**b.** $220.00	**c.** $40.00
d. $8.00	**e.** $6.20	**f.** $15.00
g. $19.80	**h.** $568.20	**i.** $150.00

9. MC **a.** 10% of $180.00 is:

A. $1.80	**B.** $9.00	**C.** $18.00	**D.** $90.00

b. 15% of $340.00 is:

A. $34.00	**B.** $17.00	**C.** $68.00	**D.** $51.00

c. 12% of $12.00 is:

A. $1.00	**B.** $1.20	**C.** $1.44	**D.** $2.00

d. 64% of $75 is:

A. $48.00	**B.** $4.80	**C.** $0.48	**D.** $6.40

Understanding

10. Merryn's yearly salary is $46 500. How much does she pay towards her superannuation each year if 5% of her salary goes into superannuation?

11. How much would I save on a pair of shoes that normally costs $45 if there were a 15% discount storewide on all items?

12. A grocery store cut the price of Cadbury's chocolate bars by 20%. The normal price is $3.45. A customer who has a discount coupon is also entitled to a further discount of 7.5%. What would this customer save on the normal price?

13. WE4 A water bill, including GST (10%), is $216.30. What is the cost excluding GST?

14. Barry's electricity bill for 3 months is $870 inclusive of the GST (10% tax).

a. What is the initial cost of electricity? **b.** How much is the GST?

Reasoning

15. Sam operates a fruit shop. He marks all his fruit up by 40% of their cost. If the fruit does not sell readily, he tags it with a sign 'Reduced 40%'. He has such a sign on punnets of strawberries which cost him 75 cents. How much would a customer pay for these strawberries? Justify your answer.

16. Michael arrived home from shopping and realised that he had lost his receipt. He needed to document the GST he paid for tax purposes. Michael remembered that the total amount he paid at the store was $110. He did the following calculation and discovered that he paid $10 GST: $\frac{\$110}{11} = \$10$.

To answer the questions below, consider the following information.

i. When calculating the total price of an item in a store, the vendor adds a 10% Goods and Service Tax (GST) to the cost of the item.

ii. GST = 10% × original cost of the item.

iii. Total cost of item = (Original cost of item) + GST.

iv. Total cost of item = 110% × (Original cost of item).

a. Does Michael's calculation work in this instance? Explain.

b. What does the number 11 represent in this calculation? Explain.

c. If the GST % changes from 10% to 12%, would dividing by 11 give the correct amount of GST? Explain.

Problem solving

17. Stan bought 2 bottles of juice and 6 packets of chips. The total bill was $14.

a. If the juice cost $2.80 per bottle, what did he pay for a packet of chips?

b. What percentage of the total amount of money was the amount he paid for the juice?

c. What percentage of the total amount of money was the amount he paid for one pack of chips?

18. The cost of hiring a taxi is $3.80 plus $1.20 per kilometre travelled.

a. Write an equation, and use it to find the maximum distance that can be travelled in a taxi for $20.

b. If two friends were sharing the cost for the trip, what percentage would each pay?

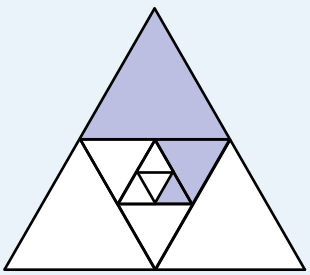
c. If one of the people was going to travel for half of the final distance, what percentages of the fare would each have to pay?

Reflection

Why is it important to have knowledge of percentages when dealing with money?

CHALLENGE 18.1

If each new triangle formed in the figure is an equilateral triangle, express the shaded region of the shape as a common fraction and as a percentage.



18.4 Unitary method

18.4.1 Unitary method

- Many consumers assume that if they buy the larger quantity of a product, they will be getting better value for money. This is not always the case.
- The **unitary method** can be used to work out which item is the best buy by comparing the unit price. Supermarkets in Australia are now obliged to show the unit cost on most products.
- The **unit price** is the price per unit. This may mean the price per 100 mL, the price per 100 g or the price of a single item.
- Remember to represent the same units when you are making comparisons.

WORKED EXAMPLE 5

Three shampoos are sold in the following quantities.
Brand A: 200 mL for $5.38
Brand B: 300 mL for $5.98
Brand C: 400 mL for $8.04
Which shampoo is the best buy?

THINK	WRITE
1 Determine the number of 100-mL units for each shampoo.	Brand A $= \frac{200}{100}$ $= 2$ units Brand B $= \frac{300}{100}$ $= 3$ units Brand C $= \frac{400}{100}$ $= 4$ units
2 Determine the price per unit for each shampoo. Price per unit $= \frac{\text{price}}{\text{100-mL units}}$	Brand A $= \frac{538}{2}$ $=$ \$2.69 per 100 mL Brand B $= \frac{598}{3}$ $=$ \$1.99 per 100 mL Brand C $= \frac{804}{4}$ $=$ \$2.01 per 100 mL
3 Answer the question for the best buy.	Brand B: 300 mL of shampoo for \$5.98

- The smaller the unit cost, the better value of the item.

18.4.2 Unitary method in percentages

- The first step is to find 1% of the amount.
- The next step is to multiply the value of the goods by 100 to get the whole amount (100%).

WORKED EXAMPLE 6

Find the whole amount if 5% is represented by $25.

THINK	WRITE
1 Five per cent of a number equals 25. Find 1% by dividing the number by 5.	5% = $25 1% = 25 ÷ 5 = 5
2 Find 100% by multiplying the amount by 100.	100% = 5 × 100 = $500

WORKED EXAMPLE 7

Ronan operates a sports store at a fixed profit margin of 65%. Ronan sells a tracksuit for $99.95. What would Ronan have paid for the tracksuit?

THINK	WRITE
1 Write the selling price as 165% of the cost price.	165% of cost price = $99.95
2 Find 1% of the cost price by dividing both sides by 165.	1% of cost price = $0.6057576
3 Find 100% of the cost price by multiplying both sides by 100. Give your answer to the nearest cent.	100% of cost price = $60.58
4 Answer in a sentence.	Ronan would have paid $60.58 for the tracksuit.

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Try out this interactivity: Unitary method (int-2365)

Complete this digital doc: WorkSHEET: Money (doc-6617)

Exercise 18.4 Unitary method

assessON

Individual pathways

PRACTISE
Questions:
1, 2, 3, 4, 5, 6, 7, 8, 12, 18, 20

CONSOLIDATE
Questions:
1, 2, 3, 4b, c, e, i, 5, 6, 7, 9, 10, 13, 14, 18, 20

MASTER
Questions:
1, 2, 3, 4c, f, i, 5, 6, 10, 11, 14, 15, 16, 17, 18, 19, 20, 21

Individual pathway interactivity: int-4396

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Fluency

1. How much will you pay for 4.5 kg of apples, given that 2 kg cost $3.80?
2. Compare the cost of 400 g of biscuits for $2.48 and 500 g for $3.10. Which is the better buy?

3. WE5 Which of the following is the best buy?

a.

Chocolate weight	Cost
150 g	$3.25
250 g	$4.75
325 g	$5.50

b.

No. of pages	Cost
80	$1.98
160	$3.38
200	$3.98

4. WE6 Calculate the original amount if:

a. 10% is $18 **b.** 20% is $6 **c.** 25% is $60
d. 40% is $900 **e.** 34% is $60 **f.** 90% is $380
g. 200% is $800 **h.** 120% is $420 **i.** 9% is $54.

5. Calculate:

a. 60%, if 40% is $120 **b.** 38%, if 20% is $6 **c.** 50%, if 25% is $60
d. 150%, if 50% is $900 **e.** 12%, if 75% is $250 **f.** 86%, if 14% is $4200.

Understanding

6. WE7 The profit on an mp3 player is $240. If this is 60% of the cost price, what is:

a. the cost price **b.** the selling price?

7. You spend 40% of your monthly allowance on your mobile phone. What is your monthly allowance if you spend $20 on your mobile phone?

8. Hanh saves 32% of his wages each week. If he saves $220, what is his weekly wage?

9. The retail price of in-line skates is $320. This represents 200% of the cost price. What is the cost price?

10. Alex receives a 15% pay rise. He now receives $97290 a year. How much was his annual income before the pay rise?

11. Which is the best buy?

a. 1.2 kg of apples for $5.20 or 0.75 kg of apples for $3.50?
b. 250 mL of orange juice for $2.95 or 400 mL of orange juice for $4.20?

12. Which is the best buy?
500 g of cheese for $12.80, 200 g of cheese for $5.62 or 80 g of cheese for $2.10?

13. A company made a profit of $238000. This represents a 10% increase in profit on the previous year. What was the profit that year?

14. A camping goods shop operates on a profit margin of 85%. How much would the shop have paid for a sleeping bag that sells for $89.95?

15. After a discount of 15%, an mp3 player was worth $183. What was its value before the discount?

16. During a sale, a retailer allows a discount of 15% off the marked price. His sale price of $60 still gives him a profit of 10%.

a. What did the article cost him? **b.** What was the marked price?

17. A discount of 15% reduced the price of a CD by $3.20.

a. What was the original price of the CD? **b.** What was its selling price?

18. A discount of 22% reduced the price of an outfit by $48.

a. What was its original price? **b.** At what reduced price was it selling?

19. You buy ten pairs of head phones for $15. At what price should you sell a dozen pairs if you wish to make a profit of 25%?

Reasoning

20. An art dealer sold two paintings at an auction. The first painting sold for $7600, making a 22% loss on its cost. The second painting was sold for $5500, making a profit of 44%. Did the art dealer make an overall profit or loss? Show your working.

21. You love to make bracelets. To make them, you need to buy some beads. Brand A costs $7.50 for a box that contains 100 beads; Brand B costs $17.50 for a box that contains 2500 beads. Which brand is the better buy? Explain how you decide.

Problem solving

22. While shopping in a sale, Carly spots a sign that says 'Buy 2 items, get the third half price (the least expensive item will be counted as the third item)'. If she finds a shirt for $25.99, a skirt for $87.99 and a belt for $15.97 and pays cash for the items, how much will Carly end up paying?

23. Some players from a soccer club went to a café to celebrate their win. Each had a burger and a drink. The bill came to $80. When it came time to pay the bill, 3 people had left without paying, so the remaining members had to pay an extra $2 each to cover the bill. If there were originally n people in the group, and the burger and drink deal cost $\$b$, write an equation using these pronumerals to show how the bill was settled. Do not attempt to solve your equation.

Reflection

What are the different situations in which you would use the unitary method?

CHALLENGE 18.2

The price of entry into a theme park has increased by 10% every year since the theme park opened. If the latest price rise increased the tickets to $8.80, what was the price of a ticket 2 years ago?

18.5 Review

18.5.1 Review questions

Fluency

1. Calculate these amounts.

a. $\$2.45 + \$13.20 + \$6.05$ **b.** $\$304.60 - \$126.25$ **c.** $\$9.65 \times 7$

2. What is $\$65.50 + \$11$?

3. Find 10% of each of the following by moving the position of the decimal point. Round your answer to the nearest 5 cents.

a. $63.00 **b.** $42.00 **c.** $105.00 **d.** $216
e. $3.45 **f.** $42.68 **g.** $118.55 **h.** $2125.85

4. Find 5% of the following by finding 10% and halving your answer. Round your answer to the nearest 5 cents.

a. $8.00 **b.** $21.00 **c.** $64.00 **d.** $104.00
e. $35.00 **f.** $52.00 **g.** $205.50 **h.** $77.30

5. Calculate the following. Round your answer to the nearest 5 cents.

a. 1% of $16.00 **b.** 1% of $28.00 **c.** 12% of $42.00 **d.** 30% of $90.00
e. 22% of $220.00 **f.** 43% of $27 **g.** 15% of $19.50 **h.** 8% of $37

6. Mentally calculate 12% of $15.

Problem solving

7. Natalie went shopping and bought a pair of bathers for $38.95, a football for $75.50, four pot plants at $8.75 each and a photograph album for $14.90. How much money did Natalie spend in total?

8. Sally bought a motorbike costing $2785. She paid a deposit of $160 then paid the remainder in 15 equal instalments. How much was each instalment?

9. Jacques' furniture shop had a sale with $\frac{1}{3}$ off the usual price of lounge suites. If the original price of a suite was $5689, what will the sale price be?

10. Heo buys a new television set marked $495. He pays a $100 deposit and 12 payments of $40 each. How much more than the marked price does he finally pay?

11. Estimate the total bill and change, then find the exact amounts when $150 is used to buy:
10 disks at $0.95 each
2 games at $59.95 each
1 file box at $8.95.

12. William owns a hairdressing salon and raises the price of haircuts from $26.50 to $29.95. By what percentage did he increase the price of haircuts?

13. The price of milk increased by 8%. If the original price was $1.84, what is the new price?

14. Find:

a. 20% of $130 **b.** 10% of $25.80 **c.** 15% of $68.60 **d.** 42% of $20

e. 1% of $16.70 **f.** 5% of $186.40 **g.** 27% of $250 **h.** 12% of $36.

15. If you deposit your money in a bank, the bank will give you a small amount of money called interest. The amount of interest they give you depends on how much you have saved and for how long. It will be a certain percentage of the amount deposited. How much interest would you receive if you have $1200 in the bank and the bank calculates interest of 5% on your account balance?

16. Last year Brisbane Roar won 70% of its home and away matches. If there are 20 matches in a season, how many matches did Brisbane Roar win?

17. Sets of 90-minute CDs are sold as in the following packs. Which is the best buy?

Cost ($)	Number of CDs
$3.25	6
$4.99	10
$7.50	15

18. Which is a better buy — 400 g of biscuits costing $2.98, or a pack of biscuits with 400 g + 25% extra, costing $3.28?

19. Antwert buys a pair of jeans for $59.95. The original price tag was covered by a 30% off sticker but the sign on top of the rack said an 'Additional 15% off already reduced prices.'

a. How much did Antwert save on the original price?

b. What percentage of the original cost did he end up saving?

20. Café Noir charges a 1% levy on the bill for trading on Sundays. If the final bill is $55.55, what was the original price, taking into account that the 10% GST has been added and then the levy?

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Try out this interactivity: Word search: Topic 18 (int-2747)

Try out this interactivity: Crossword: Topic 18 (int-2748)

Try out this interactivity: Sudoku: Topic 18 (int-3178)

Complete this digital doc: Concept map: Topic 18 (doc-10751)

Language

It is important to learn and be able to use correct mathematical language in order to communicate effectively. Create a summary of the topic using the key terms below. You can present your summary in writing or using a concept map, a poster or technology.

currency **increments** **profit margin**
estimate **method** **tax**
GST **percentage** **unitary price**

assessON

Link to assessON for questions to test your readiness **FOR** learning, your progress **AS** you learn and your levels **OF** achievement. assessON provides sets of questions for every topic in your course, as well as giving instant feedback and worked solutions to help improve your mathematical skills.

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Investigation | Rich task

Australian currency

Australia's currency is special when we compare it with the currencies of other countries throughout the world. Our coins have distinctive features and our currency notes are unique in terms of their colour and texture. Since decimal currency was introduced in Australia in 1966, our notes and coins have undergone many changes. Only our five, ten and twenty-cent coins are still minted as they were back then. The one and two-cent coins are no longer in circulation, the fifty-cent coin is a different shape, the one and two-dollar notes have been replaced by coins and our notes have changed from paper to a special type of plastic.

Coins have two sides: an *obverse* side and a *reverse* side. The obverse side of all Australian coins depicts our reigning monarch, Queen Elizabeth II, and the year in which the coin was minted. The reverse side depicts a typical Australian feature and sometimes a special commemorative event.

TABLE 1

Coin	Diameter (mm)	Mass (g)	Composition
Five-cent	19.41	2.83	75% Copper, 25% Nickel
Ten-cent	23.60	5.65	75% Copper, 25% Nickel
Twenty-cent	28.52	11.30	75% Copper, 25% Nickel
Fifty-cent	31.51	15.55	75% Copper, 25% Nickel
One-dollar	25.00	9.00	92% Copper, 6% Aluminium, 2% Nickel
Two-dollar	20.50	6.60	92% Copper, 6% Aluminium, 2% Nickel

1. What is depicted on the reverse side of each Australian coin?

Table 1 includes information on Australia's current coins in circulation. Use the table to answer questions 2–4.

2. What are the metal compositions of each of the coins?

3. Which is the heaviest coin and which is the lightest? List the coins in order from lightest to heaviest.

4. Which has the smaller diameter — the five-cent coin or the two-dollar coin? Indicate the difference in size.

TABLE 2

Note	Date of issue	Size (mm)	Average life of notes (months)	
			Plastic	Paper
Five-dollar	07/07/1992 24/04/1995 01/01/2001	130 × 65	40	6
Ten-dollar	01/11/1993	137 × 65	40	8
Twenty-dollar	31/10/1994	144 × 65	50	10
Fifty-dollar	04/10/1995	151 × 65	About 100	24
One-hundred-dollar	15/05/1996	158 × 65	About 450	104

Table 2 displays information on Australia's current notes in circulation. The column on the far right compares the average life of the previously used paper notes with that of the current plastic notes. Use the table to answer questions 5–9.

5. What denomination notes are available in our Australian currency?

6. On what date was Australia's first plastic note issued and what was the denomination of the note?

7. Suggest a reason for the three issue dates for the five-dollar note.

8. Why do you think each note is of a different size?

9. The table clearly shows that the plastic notes last about five times as long as the paper notes we once used. Why do you think the fifty-dollar notes last longer than the five and ten-dollar notes?

The Australian government has gone to great lengths to ensure that it is extremely difficult for the notes to be reproduced, or counterfeited. The Reserve Bank of Australia has identified eight security features. Use the RBA weblink in your Resources tab to answer questions 10–11.

10. Investigate the security features designed to protect our notes from being counterfeited.

11. Select any note and examine its features. Explain how you could check for the genuine nature of the note.

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Complete this digital doc: Code puzzle: What is the meaning of the NZ place name TAUMATAWHAKATANGIHANGAKOAUAUOTAMATEATURIPUKAKAPIKIMAUNGAHORONUKUPOKAIWHENUAKITANATAHU? (doc-14343)

Answers

Topic 18 Money

Exercise 18.2 Money

1. a. 755 cents **b.** 305 cents **c.** 240 cents **d.** 900 cents
2. a. $6.85 **b.** $4.50 **c.** $0.05 **d.** $8.05
3. a. $4.75 **b.** $12.60 **c.** $159.30 **d.** $83.45
4. $81.80
5. $2.25
6. a. $33.05 **b.** $30.05 **c.** $25.05
7. 3.7 = 4 wk
8. $529.85
9. $89.90
10. $7\frac{1}{2}$ hours
11. $31; interest charges
12. $39.20
13. a. $14, $14.05 **b.** $6, $5.95
14. a. $26, $26.22 **b.** $4, $3.80 after rounding to nearest 5 cents
15. a. $51.88 **b.** $1.50 **c.** $9.63
16. Sam receives $100.20, Georgia receives $66.80.
17. 68.8%
18. 8
19. 20 days
20. a. $546.70 **b.** $5.90 **c.** $2
21. 420 calls

Exercise 18.3 Money and percentages

1. a. $8 **b.** $10 **c.** $100 **d.** $2.40
e. $25 **f.** $12 **g.** $4.50 **h.** $9.60
i. $5.25 **j.** $8.10 **k.** $7.10 **l.** $24.20
m. $24.50 **n.** $26.60 **o.** $4.20
2. a. $3.20 **b.** $1.25 **c.** $7.50 **d.** $16.70
e. $0.15 **f.** $1.60 **g.** $2.11 **h.** $7.78
i. $16.42
3. One, left
4. a. $2.30 **b.** $5.70 **c.** $13.00 **d.** $6.40
e. $8.15 **f.** $1.48 **g.** $14.96 **h.** $34.59
i. $135.45
5. a. $0.60 **b.** $1.80 **c.** $3.40 **d.** $1.20
e. $4.50 **f.** $6.00 **g.** $2.25 **h.** $1.37
i. $0.64 **j.** $1.68 **k.** $25.41 **l.** $11.78
6. a. $0.26 **b.** $0.42 **c.** $1.66 **d.** $4.06
e. $16.20 **f.** $0.14 **g.** $0.07 **h.** $0.06
i. $0.17
7. Two, left
8. a. $15 **b.** $33 **c.** $6 **d.** $1.20
e. $0.93 **f.** $2.25 **g.** $2.97 **h.** $85.23
i. $22.5
9. a. C **b.** D **c.** C **d.** A

10. $2325

11. $6.75

12. 90c

13. $196.64

14. a. $790.91 **b.** $79.09

15. 63c

16. a. Yes. If he paid $10 GST then the total cost = $110 − $10 = $100 and 10% of $100 = $10, so the calculation is valid.

b. The total price (110% of original price) is 11 times that of the GST (10% of original price), therefore if you divide the total price by 11, you will get the GST amount.

c. $\frac{112\%}{12\%} \neq 11$

17. a. $1.40 **b.** 40% **c.** 10%

18. a. 13.5 km **b.** 50% and 50% **c.** 25% and 75%

Challenge 18.1

$\frac{21}{64}$, 32.8125%

Exercise 18.4 Unitary method

1. $8.55

2. Neither is better (they are both 62 cents per 100 g).

3. a. The 325-g option is the best buy. **b.** The 200-page option is the best buy.

4. a. $180 **b.** $30 **c.** $240 **d.** $2250

e. $176.47 **f.** $422.22 **g.** $400 **h.** $350

i. $600

5. a. $180 **b.** $11.40 **c.** $120 **d.** $2700

e. $40 **f.** $25 800

6. a. $400 **b.** $640

7. $50

8. $687.50

9. $160

10. $84 600

11. a. 1.2 kg apples is the best buy. **b.** 400 mL of orange juice is the best buy.

12. 500 g of cheese is the best buy.

13. $216 364

14. $48.62

15. $215.29

16. a. $54.55 **b.** $70.59

17. a. $21.33 **b.** $18.13

18. a. $218.18 **b.** $170.18

19. $22.50

20. Loss of $463

21. Brand B; the cost for each bead for Brand B is cheaper (0.7c compared to 7.5c).

22. $121.97

23. $(n - 3)(b + 2) = 80$

Challenge 18.2

$7.27

18.5 Review

1. a. $21.70 **b.** $178.35 **c.** $67.55

2. $76.50

3. a. $6.30 **b.** $4.20 **c.** $10.50 **d.** $21.60

e. $0.35 **f.** $4.25 **g.** $11.85 **h.** $212.60

4. a. \$0.40 **b.** \$1.05 **c.** \$3.20 **d.** \$5.20
e. \$1.75 **f.** \$2.60 **g.** \$10.30 **h.** \$3.85

5. a. \$0.15 **b.** \$0.30 **c.** \$5.05 **d.** \$27.00
e. \$48.40 **f.** \$11.60 **g.** \$2.95 **h.** \$2.95

6. \$1.80

7. \$164.35

8. \$175

9. \$3792.67

10. \$85

11. Estimates are total, \$140; change, \$10; exact amounts are \$138.35, \$11.65.

12. 13%

13. \$1.99

14. a. \$26 **b.** \$2.58 **c.** \$10.29 **d.** \$8.40
e. \$0.17 **f.** \$9.32 **g.** \$67.50 **h.** \$4.32

15. \$60

16. 14

17. Options are 50c per CD, 49.9c per CD, 54.17c per CD.
Therefore the \$4.99 pack is best buy.

18. First pack costs 76c per 100 g; second pack costs 66c per 100 g; so the second pack is the best buy.

19. a. Saved \$40.81 **b.** 40.5% saved

20. \$50

Investigation — Rich task

1. Five-cent, echidna; ten-cent, lyrebird; twenty-cent, platypus; fifty-cent, Coat of Arms; one-dollar, kangaroo; two-dollar, Aboriginal tribal elder

2. The five-, ten-, twenty and fifty-cent coins all contain 75% copper and 25% nickel. The one and two-dollar coins contain 92% copper, 6% aluminium and 2% nickel.

3. The fifty-cent coin is the heaviest and the five-cent coin is the lightest. The order from lightest to heaviest is: five-cent, ten-cent, two-dollar, one-dollar, twenty-cent and fifty-cent coin.

4. The five-cent diameter is the smallest diameter with a difference of 1.09 mm.

5. Five-, ten-, twenty-, fifty and one-hundred-dollar notes are available.

6. The first plastic note was issued on 7 July 1992 and was a five-dollar note. (The table does not include the commemorative ten-dollar note released as a part of Australia's bicentennial celebrations in 1988.)

7. Three different notes produced. Original 1992, new colour 1995, Federation note 2001.

8. To assist blind people in determining the denomination of each note.

9. They are not handled as often as the notes with the smaller denomination.

10–11. Teacher to check.

TOPIC 19
Numeracy 3

19.1 Overview

Numerous **videos** and **interactivities** are embedded just where you need them, at the point of learning, in your learnON title at www.jacplus.com.au. They will help you to learn the content and concepts covered in this topic.

19.1.1 Why learn this?

Understanding numeracy helps us make sense of mathematical ideas and how they relate to everyday life.

Numeracy includes counting, addition, subtraction, multiplication and division, fractions, decimals, measuring, understanding shapes and graphs.

19.1.2 What do you know?

assessON

1. **THINK** List what you know about numeracy. Use a thinking tool such as a concept map to show your list.
2. **PAIR** Share what you know with a partner and then with a small group.
3. **SHARE** As a class, create a thinking tool such as a large concept map that shows your class's knowledge of numeracy.

LEARNING SEQUENCE

19.1 Overview
19.2 Set E
19.3 Set F

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Watch this eLesson: NAPLAN: strategies and tips (eles-1688)

19.2 Set E

19.2.1 Calculator allowed

1. In May 2013 the number of students at a school was 1200. Given that the number of pupils in May 2014 was 1320, find the percentage increase in the number of students.

2. A carpenter is looking for the smallest nail diameter in order to construct a cabinet. Choose the smallest diameter from the following sizes:

A. 0.22 mm **B.** 0.109 mm **C.** 0.072 mm **D.** 0.284 mm

3. Jamie earns $26 from mowing lawns and $27 from delivering pamphlets each week. Jamie also receives $20 per week in pocket money from his parents. Each week he keeps $27 in spending money and banks the rest. Calculate the amount that Jamie banks each week.

4. Twenty students were surveyed on the type of music they liked. The results were:

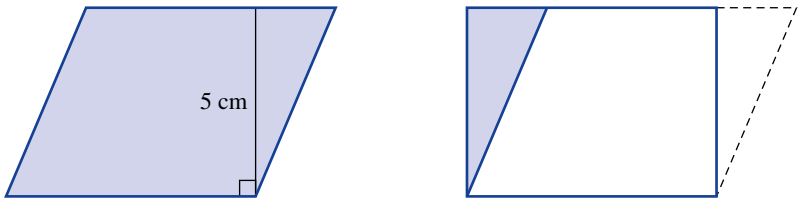
- $\frac{1}{5}$ liked pop
- $\frac{1}{10}$ liked rock
- $\frac{1}{4}$ liked hip hop
- $\frac{1}{20}$ liked classical
- the rest of the students liked more than one type.

How many students liked more than one type of music?

5. A rectangular luggage label has an area of 96 cm^2 and a width of 8 cm. What is the perimeter of the label?

A. 40 cm **B.** 20 cm **C.** 12 cm **D.** 32 cm

6. The area of this parallelogram is 40 cm^2. What will be the perimeter of the rectangle created by moving the right-angled triangle as shown?



7. A computer costs $1200. It is reduced by 5% in a sale. What is the sale price of the computer?

A. $1000 **B.** $1195
C. $1140 **D.** $60

8. Gordon is given a new PIN for his debit card. He calculates that if he divides his PIN by his age, which is 14, and then adds his house number, which is 105, the answer will be 225. What is the PIN?

9. A skip that holds 3.6 cubic metres has been delivered. You can use a wheelie bin with a capacity of 240 L to transport your rubbish to the skip. How many trips will you make to fill the skip? (1 m^3 = 1000 litres.)

A. 15 **B.** 14 **C.** 150 **D.** 140

10. I buy 12 CDs at a cost of $65.00. If I return 25% of them to the shop, how much money will I get back?

A. $3.00 **B.** $59.58 **C.** $16.25 **D.** $21.67

11. How many of these cards show values the same as $\frac{2}{5}$?

20%	$\frac{20}{50}$	$\frac{6}{30}$	40%	0.40

A. 4 **B.** 3 **C.** 2 **D.** 1

12. When 2^8 is divided by 2^4, the answer is:

A. 2^2 **B.** 2^4 **C.** 2^{12} **D.** 4^4

13. Which of the following has the largest value?

A. $3^2 \times 3^2$ **B.** $3^3 \times 3^2$ **C.** $\frac{3^8}{3^2}$ **D.** $\frac{3^2}{3^8}$

14. The table on the right shows the time that Angela spent running this week. What was the average time spent running?

A. 210 minutes **B.** 50 minutes
C. 35 minutes **D.** 30 minutes

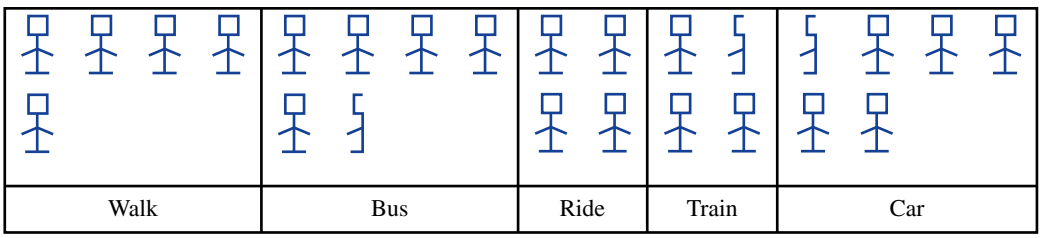
Day	Time
Monday	35 minutes
Tuesday	25 minutes
Wednesday	30 minutes
Thursday	42 minutes
Friday	50 minutes
Saturday	28 minutes

15. Justin buys some footy cards for $8.95 and soft drink for $3.80. He pays with a $20 note. How much change does he receive?

A. $6.75 **B.** $7.80
C. $7.25 **D.** $7.76

16. The students of Year 7 did a survey on how they travel to school, and recorded their data in a pictograph. How many students are in Year 7?

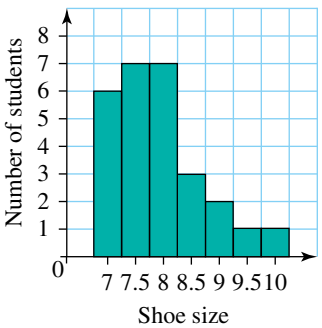
= 6 students = 3 students



A. 150 **B.** 147 **C.** 141 **D.** 132

17. The column graph represents the shoe sizes of 28 Year 7 students in a mathematics class. What percentage of students has a shoe size greater than 7.5?

A. 50% **B.** 30%
C. 60% **D.** 70%



18. Ann, Meg and Sally shared a pizza. Ann ate $\frac{1}{4}$ and Meg ate $\frac{1}{6}$ of the pizza. What was left for Sally?

A. $\frac{1}{5}$ **B.** $\frac{4}{5}$ **C.** $\frac{5}{12}$ **D.** $\frac{7}{12}$

19. You buy 3.5 kg of chicken at $15.30 per kg and 2.5 kg of potatoes at 90 cents per kg. You have only $40. You try to work out by how much you are short. Which number sentence best describes your problem?

A. $(3.5 \times 15.30 + 2.5 \times 90) - 40$ **B.** $40 - 3.5 \times 15.30 + 2.5 \times 0.90$

C. $3.5 + 2.5 \times 15.30 \times 0.90 - 40$ **D.** $40 - (3.5 \times 15.30 + 2.5 \times 0.90)$

20. The planet Mercury has a diameter that is 38% less than the planet Earth's diameter, which is 12 756 km. What is the diameter of Mercury? (Round to the nearest kilometre.)

21. The table shows the prices for admission to the museum.

Admission	
Adults	$19.80
Children	$12.25
Family (2 adults and 2 children)	$49.75

How much do you save by buying a family ticket instead of buying 2 adults and 2 children?

A. $14.35 **B.** $17.75 **C.** $10.75 **D.** $7.55

22. The maximum temperatures in Perth and at Mt Buller were recorded for one week.

	Mon.	**Tues.**	**Wed.**	**Thurs.**	**Fri.**	**Sat.**	**Sun.**
Perth	16	12	14	10	8	9	11
Mt Buller	−3	−1	−6	−2	−3	−1	−4

On which day is the temperature difference the greatest?

A. Monday **B.** Wednesday **C.** Friday **D.** Sunday

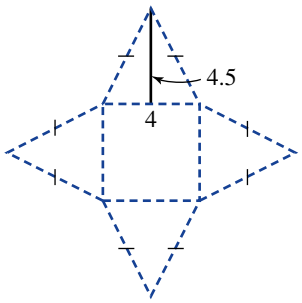
23. What is the area of this star-shaped diagram?

A. 56 unit^2

B. 52 unit^2

C. 61 unit^2

D. 96 unit^2



24. The following table shows the ages of people on a bus.

	0–19	**20–29**	**30–39**	**40–49**	**50–59**	**60–69**	**70–79**	**>79**
Male	3	1	5	2	0	4	1	0
Female	2	2	7	1	3	4	0	1

Which is the closest percentage of people on the bus who were less than 30 years old?

A. 20% **B.** 22%

C. 24% **D.** 26%

25. Fill in the missing terms in the following number pattern.

45, ___, 37, 33, ___, ___ ….

26. I have a dripping tap that increases my water consumption by 3%. I used 64 kilolitres over a 3-month period and my tap was constantly dripping over that period of time. How many litres could I have saved by fixing my tap?

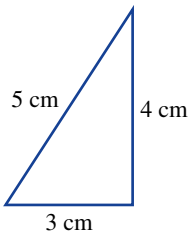
27. The average bath uses two-and-a-half times as much water as a three-minute shower. If the average shower uses 15 litres of water per minute, how many more litres are used by having a bath?

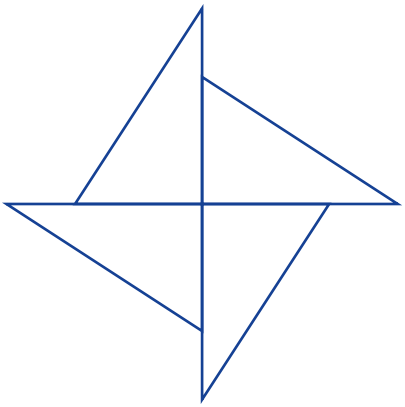
A. 80 L **B.** 67.5 L **C.** 77 L **D.** 60.5 L

28. At 10 drips per minute, a leaking tap wastes 3000 litres of water in a year. At this rate, how many millilitres of water are wasted per drip? (Round to 2 decimal places.)

A. 1.92 mL **B.** 13 mL **C.** 0.57 mL **D.** 14 mL

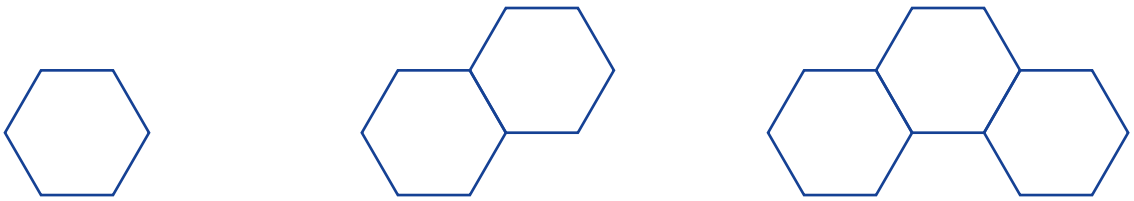
29. A star shape is made by combining four copies of the single triangle shown. What is the star's perimeter?





A. 32 cm **B.** 26 cm **C.** 24 cm **D.** 20 cm

30. You have a number of regular hexagon tiles with sides of 6 cm. If 6 tiles are joined in the pattern presented, what will be the perimeter?

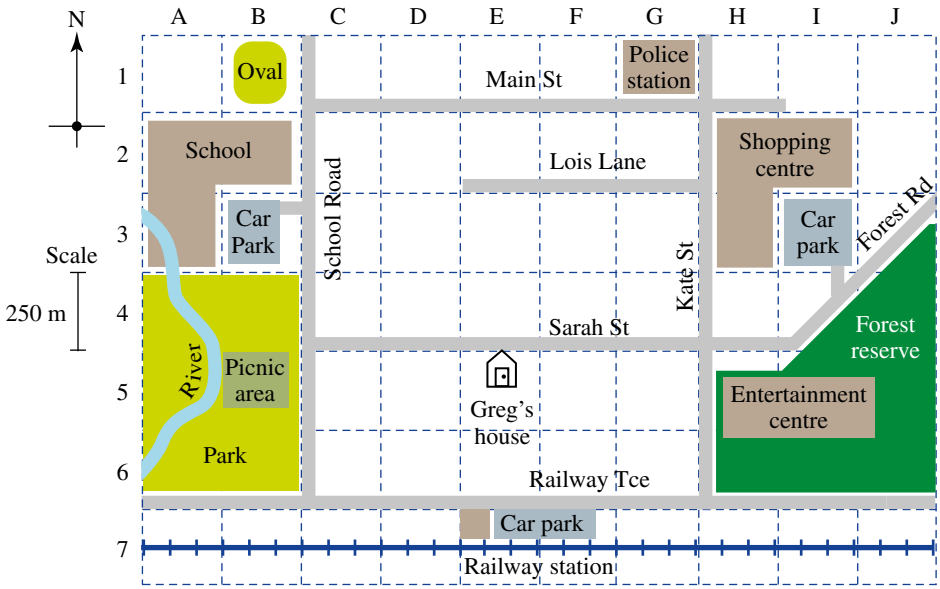


A. 172 cm **B.** 156 cm **C.** 164 cm **D.** 160 cm

19.3 Set F

19.3.1 Non-calculator

Use the following map to answer questions **1**, **2** and **3**.



1. What is the direct (straight line) distance from the front of Greg's house to the car park entrance near the school?

A. 1000 m **B.** 750 m **C.** 625 m **D.** 500 m

2. I meet a friend at E6. We walk west 0.75 km, then 1.25 km north and finally 1 km east. Where are we on the map?

A. D2 **B.** E2 **C.** F2 **D.** F1

3. Which of the following directions will I need to follow to get to the railway station from the house?

A. Turn left and walk 625 m, turn left and walk 500 m, turn left and walk 250 m.

B. Turn right and walk 625 m, turn left and walk 500 m, turn left and walk 500 m.

C. Turn left and walk 625 m, turn left and walk 500 m, turn left and walk 500 m.

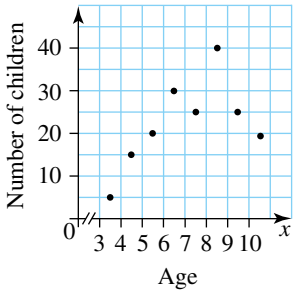
D. Turn left and walk 625 m, turn right and walk 500 m, turn left and walk 500 m.

4. How many of the shapes below are the same as X?

A. 0 **B.** 1 **C.** 2 **D.** 3

5. From the graph, how many of the children are less than 7 years of age?

A. 30 **B.** 25 **C.** 70 **D.** 27



6. Students were asked to state how many SMSs they sent in one day. From the tally chart, what are the values of x, y and z?

Number of SMS	Tally	Frequency
0	|||	3
1	𝍸 ||	x
2	𝍸 ||||	y
3	𝍸 𝍸 𝍸 |	z
4	||||	4
5	𝍸 |	6

A. 7, 9 and 16 **B.** 6, 8 and 15 **C.** 8, 10 and 17 **D.** 7, 18 and 48

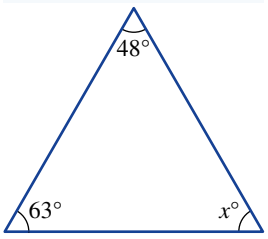
7. What is the value in degrees of the angle x in the triangle shown in Figure 1?

8. Jacinta is painting her bedroom. The area of the walls is 59 m^2, and each can of paint will cover an area of about 11 m^2. Estimate how many cans of paint Jacinta will need to buy.

A. 4 cans **B.** 1 can

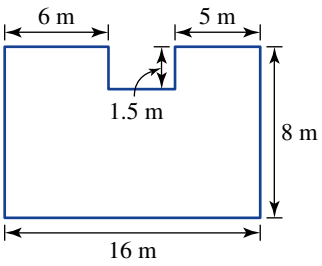
C. 5 cans **D.** 6 cans

FIGURE 1



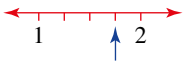
9. Figure 2 shows the plan of a swimming pool. What is its perimeter?

FIGURE 2



A. 36.5 m **B.** 51 m
C. 46 m **D.** 62 m

10. What is the reading on this scale?



A. 1.50 **B.** 1.55
C. 1.75 **D.** 1.8

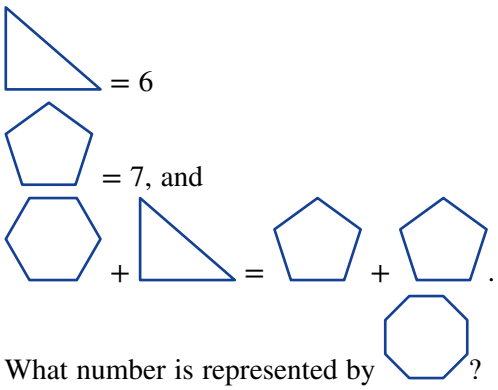
11. Which of the following is most likely to be the diameter of a dinner plate?

A. 12 cm **B.** 25 cm
C. 50 cm **D.** 75 cm

12. You arrive at a backpackers on 29 March and leave on 3 April. If you pay $45 per night, what is your total bill?

A. $225 **B.** $270
C. $315 **D.** $45

13. Let



What number is represented by ?

A. 6 **B.** 7 **C.** 8 **D.** 9

14. In a classroom, $\frac{6}{10}$ of the students are female. What percentage are boys?

A. 60% **B.** 50% **C.** 35% **D.** 40%

15. A map has a scale of 1 : 35 000. How much does 1 cm on the map represent in real life?

A. 350 m **B.** 3 m **C.** 3500 m **D.** 35 km

16. You divided 60 sweets between your two brothers in the ratio of their ages. If Nyg is 7 and Tyng is 5, how many does Tyng receive?

A. 25 **B.** 35 **C.** 5 **D.** 12

17. One angle of an isosceles triangle measures 40°. If the other two angles are the equal angles, find the measure of each of those angles.

A. 50° **B.** 140° **C.** 70° **D.** 40°

18. If X represents the number of stamps that Alice had, and Alice then gave 15 stamps to her friend, which expression shows how many stamps Alice has now?

A. 15 + X **B.** 15 − X
C. X − 15 **D.** X ÷ 15

19. What are the whole numbers that make 9 − __ > 4 true?

A. 0, 1, 2, 3, 4, 5 **B.** 0, 1, 2, 3, 4
C. 0, 1, 2 **D.** 5

20. A calculator display shows 4.33333. Which problem could have given that answer on the calculator?

A. 4 + .3 **B.** 4 × .3
C. 33 333 + .4 **D.** $\frac{13}{3}$

21. You are given 4 cards, each with a fraction on it. Can you arrange them from the smallest to the largest?

A. $\frac{1}{12}, \frac{1}{3}, \frac{3}{6}, \frac{3}{4}$ **B.** $\frac{1}{3}, \frac{3}{4}, \frac{3}{6}, \frac{1}{12}$ **C.** $\frac{1}{12}, \frac{1}{3}, \frac{3}{4}, \frac{3}{6}$ **D.** $\frac{1}{3}, \frac{1}{12}, \frac{3}{6}, \frac{3}{4}$

22. 120 people attended the Year 7 production. Tickets cost $16.70. Half the people bought programs at $4.00. How much money was collected?

23. The table below shows information about a Year 7 class and mobile phones.

	Own mobile	Do not own mobile
Boys	8	10
Girls	10	4

What is the probability of choosing a girl who owns a mobile phone?

A. $\frac{7}{16}$ **B.** $\frac{5}{16}$ **C.** $\frac{9}{16}$ **D.** $\frac{5}{9}$

24. Which one of the following does not represent 12×6?

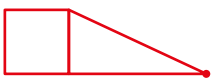
A. $10 \times 6 + 2 \times 6$ **B.** $7 \times 6 + 5 \times 6$

C. $12 \times 4 + 12 \times 2$ **D.** $12 \times 3 + 12 \times 2$

25. $5 is taken off the price of all items for one day only. How much would you pay for items worth $32, $48.85, $57.95, $114, $151.75, $201.65?

A. $601.20 **B.** $575.20 **C.** $600.35 **D.** $576.20

26. When you rotate the following diagram anticlockwise 90° about the point, which of the following will be the result?



A.

B.

C.

D.

27. Which of these has the least value?

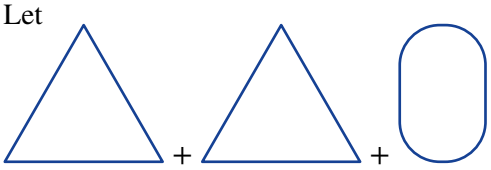
A. 20% **B.** $\frac{3}{5}$ **C.** $\frac{1}{10}$ **D.** 0.3

28. Hector is sixty-nine years old. He was born on 29 February. How many times has he had a birthday?

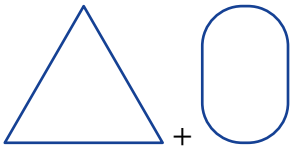
29. A rope measures $3\frac{2}{3}$ m. Which of the following has the same measurement?

A. $\frac{7}{3}$ m **B.** $\frac{8}{3}$ m **C.** $\frac{10}{3}$ m **D.** $\frac{11}{3}$ m

30. Let

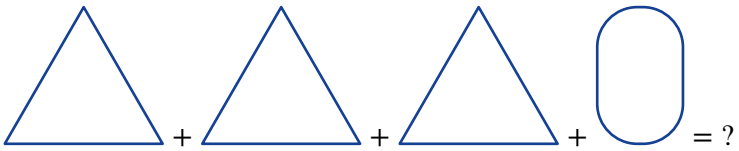


= 600, and



= 350.

What do these shapes total when they are combined?



= ?

A. 750 **B.** 850 **C.** 950 **D.** 1050

Answers

Topic 19 Numeracy 3

19.2 Set E

19.2.1 Calculator allowed

1. 10%	**2.** C	**3.** $46	**4.** 8
5. A	**6.** 26 cm	**7.** C	**8.** 1680
9. A	**10.** C	**11.** B	**12.** B
13. C	**14.** C	**15.** C	**16.** C
17. A	**18.** D	**19.** D	**20.** 7909 km
21. A	**22.** B	**23.** B	**24.** B
25. 41, 29 and 25	**26.** 1920 L	**27.** B	**28.** C
29. C	**30.** B		

19.3 Set F

19.3.1 Non-calculator

1. B	**2.** D	**3.** C	**4.** C
5. C	**6.** A	**7.** 69°	**8.** D
9. B	**10.** C	**11.** B	**12.** A
13. C	**14.** D	**15.** A	**16.** A
17. C	**18.** C	**19.** B	**20.** D
21. A	**22.** $2244	**23.** B	**24.** D
25. D	**26.** A	**27.** C	**28.** 17 times
29. D	**30.** B		

TOPIC 20 Programming

20.1 Overview

Numerous **videos** and **interactivities** are embedded just where you need them, at the point of learning, in your learnON title at www.jacplus.com.au. They will help you to learn the content and concepts covered in this topic.

20.1.1 Why learn this?

Computer programming is the process of creating instructions for a computer to follow to perform tasks. Many different languages can be used to program computers. These languages have specific purposes. For example, JavaScript is used in web pages to create interactive experiences like games; C is a computer language often embedded in programs in electronic devices like TVs and digital cameras.

This topic applies the computer language JavaScript in most of the examples and questions. JavaScript was chosen for its popularity and availability on nearly all web browsers.

According to Steve Jobs, the co-founder of Apple and Pixar, "everybody in this country [the USA] should learn how to program a computer because it teaches you how to think".

20.1.2 What do you know?

assessON

1. **THINK** Write what you know about computer programming languages. Use a thinking tool such as a concept map to display your ideas.
2. **PAIR** Share what you know with a partner and then with a small group.
3. **SHARE** As a class, combine all the ideas into one large concept map or poster.

LEARNING SEQUENCE

online only

20.1 Overview
20.2 Variables
20.3 Expressions
20.4 Sequences
20.5 Algorithms
20.6 Review

20.2 Variables

20.2.1 Values

- Variables in computer programs are similar to variables used in algebra. Variables are used to store values.
- In algebra, the value of a variable is fixed once it's defined. For example, the two equations $x = 2$ and $x = 3$ cannot be simultaneously true. Once x has been defined to be 2 ($x = 2$) it cannot be changed to be 3 ($x \neq 3$).
- In a computer program, the value of a variable can be changed at any point in the sequence. For example, the variable `x` is first assigned the value `2`, and then changed to the value `3`.
- Three of the most common types of values used in computer programming are:
  - **number** (a numerical value): `1`, `2`, `-3`, `1.5`, `4.15359`
  - **string** (any text in quotation marks `"..."`): `"Some text"`, `"123"`, `"A string"`
  - **Boolean** (used for logic; this type only has two possible values): `true, false`

WORKED EXAMPLE 1

Classify the following values as number, string or Boolean.

a `"Aidan"` **b** `1.2` **c** `"This is text"`
d `true` **e** `"23"` **f** `"true"`
g `""` **h** `-101`

THINK	WRITE
a `"Aidan"` is in quotation marks, so this value is a string.	**a** String
b `1.2` is a numerical value not in quotation marks, so this value is a number.	**b** Number
c `"This is text"` is in quotation marks, so this value is a string.	**c** String
d `true` is a Boolean value.	**d** Boolean
e `"23"` is in quotation marks, so it is a string.	**e** String
f `"true"` is in quotation marks, so it is a string.	**f** String
g `""` is an empty pair of quotation marks, so it is a string.	**g** String
h `-101` is a numerical value, so it is a number.	**h** Number

20.2.2 Variable names

- A powerful feature of computers is their ability to store vast amounts of information accurately for any required amount of time. Computer languages use memory locations to store values. These named storage containers are called **variables**.
- Variables can have long or short names. Often longer names are used to remind the programmer the meaning of the information to be stored. The following are all valid variable names:
  `x, y, i, age, name, first_name, number_of_people, bankBalance`
- There are complex rules that determine what can be used for a valid variable name. For simplicity, this topic restricts the variable names to three simple rules.
- Variable names:
  1. must not start with a number
  2. can only contain upper-case and lower-case letters, numbers and the underscore character `(_)`, and cannot contain spaces
  3. cannot be JavaScript keywords.

  Note: The following are JavaScript keywords, which should not be used as variable names.

```
abstract, arguments, Array, boolean, break, byte, case,
catch, char, class, const, continue, Date, debugger,
default, delete, do, double, else, enum, eval, export,
extends, final, finally, float, for, function, goto,
hasOwnProperty, if, implements, import, in, Infinity,
instanceof, int, interface, isFinite, isNaN, isPrototypeOf,
length, let, long, Math, name, NaN, native, new, null,
Number, Object, package, private, protected, prototype,
public, return, short, static, String, super, switch,
synchronized, this, throw, throws, toString, transient,
true, try, typeof, undefined, valueOf, var, void, volatile,
while, with, yield, false
```

WORKED EXAMPLE 2

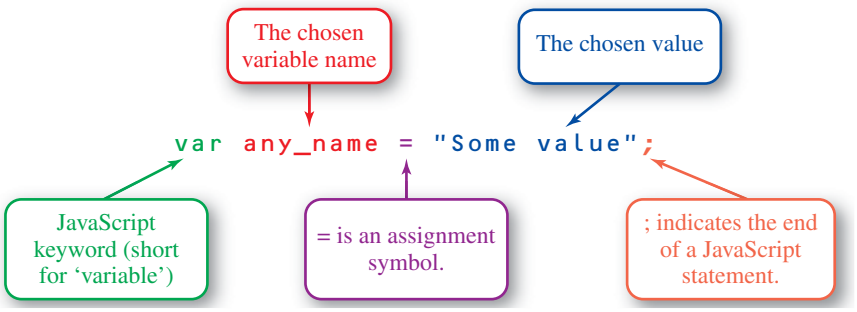
Is each of the following a valid variable name? (Answer either true or false for each name.)

a `a_variable` **b** `1_first` **c** `a%percentage`
d `camelCase_12` **e** `this` **f** `function`

THINK	WRITE
a `a_variable` is valid because it only uses letters and the underscore character.	**a** True
b `1_first` is invalid because it uses a number as the first character.	**b** False
c `a%percentage` is invalid because it uses a `%` character.	**c** False
d `camelCase_12` is valid as it only uses letters, numbers and the underscore character. It also does not start with a number.	**d** True
e `this` is invalid because `this` is a JavaScript keyword.	**e** False
f `function` is invalid because `function` is a JavaScript keyword.	**f** False

20.2.3 Assigning values to variables

- Values are assigned to variables differently depending on the computer language used. When using JavaScript, the first time a value is assigned to a variable the instruction should begin with the JavaScript keyword `var`. JavaScript uses the following structure to assign a value to a variable.



WORKED EXAMPLE 3

Write a JavaScript statement to make the following variable assignments.

a Assign the value `10` to the variable `ten`.
b Assign `true` to the Boolean variable `I_can_write_code`.
c Assign `"JavaScript"` to the variable `language`.

THINK	WRITE
a 1 Identify the variable name, which in this case is `ten`.	**a**
2 Identify the value, which in this case is `10`.	
3 Apply the JavaScript structure to write the statement.	`var ten = 10;`
b 1 Identify the variable name, which in this case is `I_can_write_code`.	**b**
2 Identify the value, which in this case is `true`.	
3 Apply the JavaScript structure to write the statement.	`var I_can_write_code = true;`
c 1 Identify the variable name, which in this case is `language`.	**c**
2 Identify the value, which in this case is `"JavaScript"`.	
3 Apply the JavaScript structure to write the statement.	`var language = "JavaScript";`

learnON RESOURCES — ONLINE ONLY

Try out this interactivity: CodeBlocks Activity 1 (int-6573)

Exercise 20.2 Variables

assessON

Individual pathways

PRACTISE
Questions:
1–9, 11, 13

CONSOLIDATE
Questions:
1a–j, 2a–g, 3a–h, 4a–g, 5a–m, 6a–f, 8–13, 15

MASTER
Questions:
1k–q, 2h–m, 3i–o, 4h–k, 5n–w, 6g–j, 9, 10, 11d–g, 12g–k, 13–16

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note*: Question numbers may vary slightly.

Fluency

1. WE1 Classify the following values as number, string or Boolean.

a. `123`
b. `-12`
c. `3.6`
d. `"A string"`
e. `"Is this a string?"`
f. `"10"`
g. `true`
h. `"animals"`
i. `false`
j. `"-10"`
k. `917`
l. `-10000000`
m. `"false"`
n. `"9999"`
o. `""`
p. `0`
q. `"Yes"`

2. Is each of the following a number value? (Answer either true or false for each value.)

a. `"12"`	**b.** `"-13.22342"`	**c.** `3.14159265359`
d. `1237123`	**e.** `"1237123"`	**f.** `true`
g. `false`	**h.** `Three`	**i.** `"0"`
j. `Nine`	**k.** `Number12`	**l.** `-80`
m. `4`	**n.** `0.0000001`	

3. Is each of the following a string value? (Answer either true or false.)

a. `"A string value"`	**b.** `Another string value`
c. `12`	**d.** `"12"`
e. `Four`	**f.** `"Four"`
g. `"true"`	**h.** `true`
i. `"false"`	**j.** `This is a string`
k. `"This is not a string"`	**l.** `false`
m. `Five`	**n.** `"Five"`
o. `18276`	

4. Is each of the following a Boolean value? (Answer either true or false.)

a. `true`	**b.** `false`	**c.** `"true"`
d. `12`	**e.** `Yes`	**f.** `No`
g. `"Yes"`	**h.** `"No"`	**i.** `21344`

5. WE2 Variable names:

1. must not start with a number
2. can only contain upper-case and lower-case letters, numbers and the underscore character `(_)`, and cannot contain spaces
3. cannot be JavaScript keywords. The following are some JavaScript keywords, which should not be used as variable names: `true, var, false.`

Given these rules, is each of the following a valid variable name? (Answer either true or false.)

a. `variable`	**b.** `value`
c. `x1`	**d.** `x2`
e. `1x`	**f.** `2x`
g. `ten`	**h.** `10`
i. `percentage%variable`	**j.** `percentage_variable`
k. `fourteen`	**l.** `14variable`
m. `ThisIsAVariableName`	**n.** `this_is_also_a_variable_name`
o. `100_variable_names_sitting_in_a_row`	
p. `Variable name`	**q.** `a`
r. `i`	**s.** `j`
t. `_`	**u.** `_private`
v. `true`	**w.** `false`

Understanding

6. Fill in the variable, value and type for the following JavaScript assignment statements.

	Variable	Value	Type	JavaScript statement
e.g.	`a_variable`	`"text"`	String	`var a_variable = "text";`
a				`var x = 10;`
b				`var nine = 9;`
c				`var aBoolean = true;`
d				`var _ = "A string";`

	Variable	Value	Type	JavaScript statement
e				`var _long_var = true;`
f				`var areYouTall = false;`
g				`var about_e = 2.718281828459;`
h				`var sin_90 = 1;`
i				`var isThis = "false";`
j				`var a_Boolean = true;`

7. What are the two possible JavaScript Boolean values?

8. Can a value assigned to a variable in a computer program be changed?

9. WE3 Write a JavaScript statement to make the following variable assignments.

a. The value `10` assigned to the variable named `count`.

b. The value `"Data"` assigned to the variable called `information`.

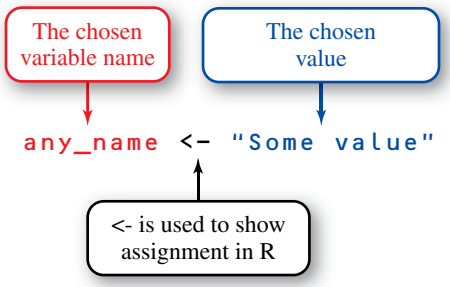
c. The value `9` assigned to the variable `threeSquared`.

d. `true` assigned to the variable `isThisJavaScript`.

e. The variable `meaningOfLife` storing the value `42`.

f. `classSize` holding the value `25`.

10. The computer language R has a different structure to assign values to variables, which is shown to the right.



Rewrite the following JavaScript assignment statements in the computer language R.

a. `var timeInSeconds = 120;`

b. `var distanceInMiles = 40;`

c. `var firstName = "Ben";`

d. `var middleName = "Violet";`

e. `var numberLeft = 19;`

f. `var littleOverThree = 3.1;`

11. Is each of the following a valid JavaScript assignment statement? (Answer either true or false for each.)

a. `var someVariable = 10;`

b. `var thisIsBoolean = "A string";`

c. `var time <- 10;`

d. `time <- 100`

e. `var aString = 10;`

f. `var thisLooksValid == 10;`

g. `var isThisBoolean = false;`

12. The Python computer language uses slightly different Boolean values: `True` and `False`.
Note that JavaScript's Boolean values are `true` and `false` (the case of the first letter is different).
Python uses the assignment structure:

```
variableName = "some value"
```

Rewrite the JavaScript assignment statements shown in the table below as Python assignment statements.

	JavaScript assignment statement	Python assignment statement
a	`var x = 10;`	
b	`var nine = 9;`	

	JavaScript assignment statement	Python assignment statement
c	`var aBoolean = true;`	
d	`var _ = "A string";`	
e	`var _longer_variable = true;`	
f	`var areYouTall = false;`	
g	`var nearly4 = 3.99;`	
h	`var about_e = 2.718281828459;`	
i	`var sin_90 = 1;`	
j	`var isThisABoolean = "false";`	
k	`var is_this_a_Boolean = true;`	

Reasoning

13. Why is it important to have rules for the structure of variable names?

14. Variable names:

1. must not start with a number

2. can only contain upper-case and lower-case letters, numbers and the character `_`

3. cannot be JavaScript keywords like `var`, `true` and `false`.

i. Which of the rules listed do the following variable names violate?

ii. In each case explain why the rule is important.

a. `true` **b.** `false` **c.** `1w` **d.** `2`

e. `-value` **f.** `variable name` **g.** `"variableName"`

Problem solving

15. Using only the characters `"a"`, `"B"`, `"_"` and `"1"`, list all the valid variable names:

a. of length 1

b. of length 2 without repeating characters

c. of length 2, where characters can be repeated.

16. Variable names:

1. must not start with a number

2. can only contain upper and lower case letters, numbers and the underscore character `(_)`, and cannot contain spaces

3. cannot be JavaScript keywords. The following are JavaScript keywords which should not be used as variable names:

```
do, if, in, for, int, let,
NaN, new, try, var, byte,
case, char, Date, else, enum, eval, goto, long,
Math, name, null, this, true, void, with, false
```

Given these rules, how many possible valid variables names are there:

a. of length 1? **b.** of length 2? **c.** of length 3? **d.** of length 4?

Reflection

What is the difference between a number value, a string value and a Boolean value?

20.3 Expressions

20.3.1 Numerical expressions

- Expressions are used to perform calculations on existing values to generate new values.
- Computer programs use similar symbols and rules as standard mathematics to create numerical expressions.
- Numerical expressions involving numbers, brackets, plus and minus in JavaScript evaluate as you would expect in any mathematical expression.

WORKED EXAMPLE 4

Evaluate the JavaScript for the following expressions.

a `10 + 2` **b** `5 - 3`

c `1.2 + 3.4` **d** `3 + 6 - (5 - 4)`

THINK	WRITE
a Add `10` and `2`.	**a** `10 + 2 = 12`
b Subtract `3` from `5`.	**b** `5 - 3 = 2`
c Add `1.2` and `3.4`.	**c** `1.2 + 3.4 = 4.6`
d Write out the expression. Calculate the sub-expression `(5 - 4)`. Calculate the final expression.	**d** `3 + 6 - (5 - 4) = 3 + 6 - (1)` `= 8`

- In JavaScript, for multiplication, use the character `*`; for division, use the character `/`.
- Fractions can be evaluated using the division character, but the numerator and denominator expressions need to be put into brackets.

$$\frac{2+3+1}{1+1} = \texttt{(2+3+1)/(1+1)}$$

Note: The characters `*` and `/` are only acceptable for use in programs and are not acceptable conventions for writing mathematics generally.

WORKED EXAMPLE 5

Write the following mathematical expressions as JavaScript expressions.

a 8×6 **b** $2 \times (5 - 3)$

c $12 \div (2 \times 2)$ **d** $\dfrac{12 + 3}{6 - 1}$

THINK	WRITE
a Replace the × with a `*`.	**a** `8 * 6`
b Replace the × with a `*`.	**b** `2 * (5 - 3)`
c Replace the × with a `*` and the ÷ with a `/`.	**c** `12/(2 * 2)`
d The fraction is a division. The numerator and denominator require brackets.	**d** `(12 + 3)/(6 - 1)`

20.3.2 String expressions

- Expressions are not restricted to numerical expressions. Strings can also belong to expressions.
- Two or more strings can be concatenated (joined together) using the `+` symbol. For example, `"Hello" + " " + "World"` evaluates to `"Hello World"`.

WORKED EXAMPLE 6

a Write a JavaScript expression to concatenate the strings `"Strings"`, `" "`, `"can"`, `"have express"`, `"ions"`.
b Write the expression.
c Write the string `"I can write expressions"` as an expression of single words and spaces.

THINK	WRITE
a Replace the commas with `+`.	**a** `"Strings" + " " + "can" + "have express" + "ions"`
b Write the expression.	**b** `"Strings can have expressions"`
c Replace the spaces with `" " + " " + "`.	**c** `"I" + " " + "can" + " " + "write" + " " + "expressions"`

- A **character** is a string of length `1`. A string is a list of characters. Each character position has an incrementing index starting at `0`.

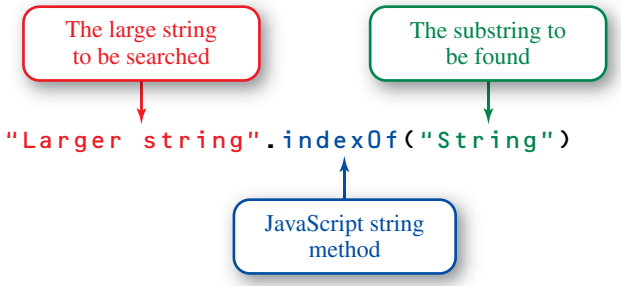
The string `"a fox"`	
Character	**Index**
`"a"`	0
`" "`	1
`"f"`	2
`"o"`	3
`"x"`	4

20.3.3 String methods

- JavaScript has several **methods** reserved for string expressions. These methods are used to manipulate a string.

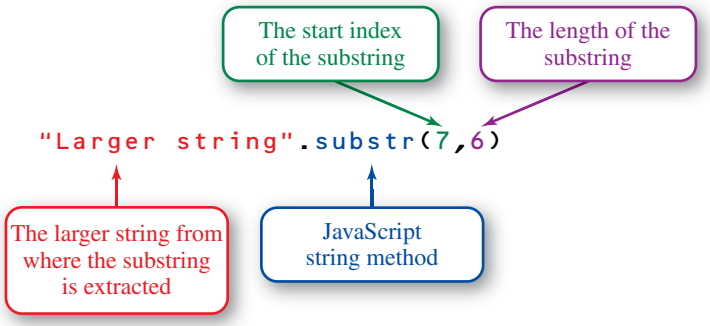
String index of method

- The JavaScript method `indexOf` is used to find the first occurrence of a substring inside a larger string. The value returned is an index (starting at `0`) of the first occurrence. The `indexOf` method uses the structure shown in the following figure to define an expression.



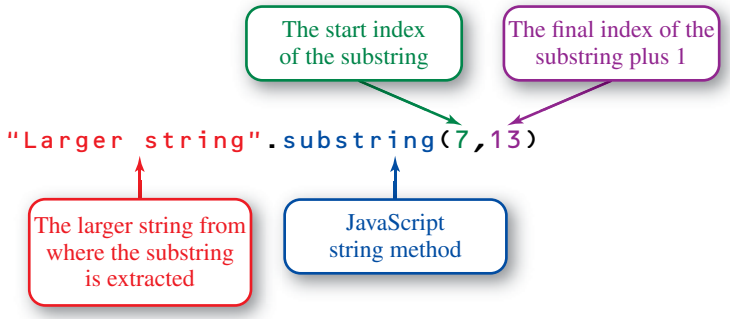
String substr method

- The JavaScript method `substr` extracts a substring from a larger string. The method requires the substring's starting index and length. The `substr` method uses the following structure to define an expression.



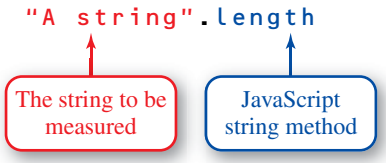
String substring method

- The JavaScript method `substring` also extracts a substring from a larger string. The method also requires the substring's starting index, but is different from the `substr` method because it requires the final index plus 1. The `substring` method uses the following structure to define an expression.



String length method

- The JavaScript method `length` measures the length of a string. The `length` method uses the following structure to define an expression.



WORKED EXAMPLE 7

Evaluate the following JavaScript expressions manually.

a `"Some text".indexOf("ex")`
b `"Some other".substr(6,3)`
c `"Some more".substring(6,8)`
d `"Final text".length`

THINK **WRITE**

a Think about the index of each character. Remember that indexing starts from zero.

S	o	m	e		t	e	x	t
0	1	2	3	4	5	6	7	8

In `"Some text"` the first occurrence of `"ex"`starts at index `6`.

S	o	m	e		t	e	x	t
0	1	2	3	4	5	6	7	8

a 6

b Think about the index of each character.

S	o	m	e		o	t	h	e	r
0	1	2	3	4	5	6	7	8	9

Look at the first three characters starting at index `6`.

S	o	m	e		o	t	h	e	r
0	1	2	3	4	5	6	7	8	9

b `"the"`

c Think about the index of each character.

S	o	m	e		m	o	r	e
0	1	2	3	4	5	6	7	8

Look at the characters starting at index `6` and ending at index `8 - 1 = 7`.

S	o	m	e		m	o	r	e
0	1	2	3	4	5	6	7	8

c `"or"`

d The number of characters in the string `"Final text"` is `10`.

	F	i	n	a	l		t	e	x	t
Index	0	1	2	3	4	5	6	7	8	9
Length	1	2	3	4	5	6	7	8	9	10

d 10

20.3.4 Assigning expressions

- Results of expressions can be stored as a variable.

WORKED EXAMPLE 8

Write a JavaScript statement to assign the following JavaScript expressions to the variable `store`.

a `27 / (12 - 3)` **b** `(6 + 4) * (1 + 12)`

c `"Store lat" + "er"`

THINK **WRITE**

Think about the assigning structure

```
var store = "value";
```

required for JavaScript.

a Replace `"value"` with the expression `27 / (12 - 3)`.	**a** `var store = 27 / (12 - 3);`
b Replace `"value"` with the expression `(6 + 4) * (1 + 12)`.	**b** `var store = (6 + 4) * (1 + 12);`
c Replace `"value"` with the expression `"Store lat" + "er"`.	**c** `var store = "Store lat" + "er";`

learnON RESOURCES — ONLINE ONLY

Try out this interactivity: CodeBlocks Activity 2 (int-6574)

Exercise 20.3 Expressions

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–5, 6a–h, 7, 8, 11	Questions: 1a–e, 2a–e, 3a–e, 4a–g, 5, 6a–h, 7, 8, 10a–c, 11a–c, 12, 14	Questions: 1f–h, 2f–j, 3f–h, 4h–m, 5a–c, 6i–l, 7d–f, 8, 9, 10d–f, 11c–e, 12–15

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Fluency

1. WE4 Evaluate the following JavaScript expressions manually.

a. `2 + 13`
b. `52 - 3`
c. `-21 + 34`
d. `-12 - 13`
e. `12 + (3 - 2)`
f. `57 - (6 + 3)`
g. `57 - (6 - 3)`
h. `43 - 16 - (12 - (13 - 6)) + 2`

2. Evaluate the following JavaScript expressions manually.

a. `20 / 5`
b. `5 * 3`
c. `8 * 2`
d. `(2 + 13) / 5`
e. `(7 + 12) / (2 + 3)`
f. `100 / 25 - 9 / 3`
g. `5 * 3 + 16 / 4`
h. `8 + 6 * 2 / 3`
i. `((12 - 3) / (5 - 2) + 16 / 8 - 4) / 2`
j. `(-48 / 8 - 4) / 2 + (22 - 13) / (15 / 5)`

3. Evaluate the following JavaScript expressions manually.

a. `"Quotation marks" + " at " + "the start" + " and end"`

b. `"This " + "is a" + " string"`
c. `"Hello " + "World" + "!"`
d. `"Express" + "ions ma" + "ke strings f" + "un"`
e. `"Happy :" + ")"`
f. `"Java" + "Script"`
g. `("String " + "expressions") + (" using" + " brackets")`
h. `"Remember" + " the two " + " spaces"`

4. WE7 Evaluate the following JavaScript expressions manually.
a. `"Four".length`
b. `("Java" + "Script").length`
c. `"Java".length + "Script".length`
d. `"The quick brown fox jumps over the lazy dog".length`
e. `"Moving forward".indexOf("i")`
f. `"The quick brown fox".length`
g. `"Find the e".indexOf("e")`
h. `"Own this gown".indexOf("own")`
i. `"The quick brown fox".substr(4,5)`
j. `"The quick brown fox".substr(12,3)`
k. `"The quick brown fox".substring(4,5)`
l. `("The sub" + "string is the answer").substring(4,13)`
m. `"The quick brown fox".substr(4,8).length`

5. Evaluate the following JavaScript expressions manually.
a. `"Find the four".substr(4 + 1 + 3 + 1,4)`
b.
```
"Find five wait find four".substr(4 * 5,
"four".length)
```
c.
```
"Take apart put together".substr(0,22 / 2) +
"put together"
```
d. `"Stop here".indexOf("e")`
e. `("The sub" + "string is the answer").substring(4,39 / 3)`

Understanding

6. WE5 Write the following mathematical expressions as JavaScript expressions.

a. 6×11
b. $54 \div 3$
c. $\frac{12}{5}$
d. $6(2 + 3)$
e. $(2 + 4)(-3 + 5)$
f. $\frac{7 + 2}{4 - 1}$
g. $3 \times 7 + 3$
h. $100 \div 25 + 4 + 4 \times 3$
i. $\frac{12 + 2(4 - 2)}{5}$
j. $6(2 + 5)(4 - 1) - 3$
k. $\frac{(2 + 4)(-2 + 6)}{3}$
l. $\frac{2 + 2}{3 - 2} \times \frac{1 + 2}{4 - 1}$

7. Write the expression to extract the string `"and"` from the string below with the method:
i. `substr`
ii. `substring`.
a. `"This fox and sheep"`
b. `"Find and back there"`
c. `"Use hands"`
d. `"Search the land"`
e. `"ksanbjhandasdkjh"`
f. `"Earth and Mars"`

8. `charAt(index)` is the same as `substr(index,1)`.
`charAt` returns one character at the specified index. For example,
`"Find the letter h".charAt(6)` evaluates to `"h"`.
Evaluate the following JavaScript expressions.
a. `"The letter x is at index 23".charAt(23)`
b. `"Where is z hiding?".charAt(9)`
c. `"Pick 5 today".charAt(5)`
d. `"Find the last character.".charAt(`
`"Find the last character.".length-1)`

9. There are many more string methods that can be used in expressions.
`toUpperCase` converts all lower-case letters to upper-case letters.
`toLowerCase` converts all the upper-case letters to lower-case letters.
`"To upper case".toUpperCase()` evaluates to `"TO UPPER CASE"`.
`"To Lower Case".toLowerCase()` evaluates to `"to lower case"`.
Evaluate the following JavaScript expressions manually.
a. `"Make all upper case".toUpperCase()`
b. `"Make ALL lower case".toLowerCase()`
c. `"A title".toUpperCase()`
d. `"STOP SHOUTING".toLowerCase()`

10. The `%` symbol is used to find the remainder after an integer division. `7%3` evaluates to `1`, as `3` divides into `7` twice with a `1` remainder.
Evaluate the following JavaScript expressions manually
a. `20%3` **b.** `19%5` **c.** `200%8` **d.** `12%5` **e.** `6%5` **f.** `3%120`

11. WE8 Write a JavaScript statement to assign the following JavaScript expressions to the variable `finalResult`.
a. `27 - (16 - 3)`
b. `(3 + 23) / (10 + 3)`
c. `"Assign" + "ed to finalResult"`
d. `"Len".length + "gth".length`
e. `18%5`

Reasoning

12. Write a JavaScript expression and assign to the stated variable for the following situations.

a. Tom has 12 toy cars and 2 friends. Write a numerical expression to divide the cars between the three of them and assign it to the variable `numberOfCarsEach`.
b. Kate has saved $20 and earned $15 today. Write an expression to evaluate how much money she has now in dollars and assign it to the variable `totalCash`.
c. Vanessa wrote the text "My Trip" and she would like to change all the letters to upper case using the string method `toUpperCase`. Write an expression to convert the letters to upper case and assign it to the variable `myTitle`.

13. Write a JavaScript expression and assign it to the variable `questionResult` for the following questions.
a. Calculate $10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$.
b. Add the even numbers between 1 to 19.
c. Join the strings `"Th","e"," f","in","al re","sult is 2","2"` and then calculate the length.

Problem solving

14. Rearrange all the characters `123456789*` to form a valid expression:

a. that evaluates to the smallest possible number.

b. that evaluates to the largest possible number.

15. Given an English word, write a JavaScript program to translate the word into Pig Latin.
Output the translated Pig Latin to the console.
To translate a word into Pig Latin, move the first letter to the end of the word and then add `"ay"`. For example, `"javascript"` becomes `"avascriptjay"`.
The program has been started below.

```
var englishWord = "program";
```

Reflection

How will you remember the different string methods?

20.4 Sequences

20.4.1 Comments

- Computer programs are usually made up of more than one statement. This list of statements is run in order as a sequence.
- Knowing a computer language is often not enough for people to understand the code of a program easily.
- Comments are added to the code to give hints about the function of the program. The comments are completely ignored by the computer, so are only for the benefit of the reader.
- JavaScript comments start with `//` and the rest of the text on that line is ignored by the computer. For example

```
var x = 2; // The smallest prime number.
```

  - The first part of the line '`var x = 2;`' is the actual JavaScript code to be run.
  - The second part '`// The smallest prime number`' is ignored by the computer and is only there for the reader's reference. The text could be changed and it would make no difference to the running of the program.

WORKED EXAMPLE 9

Add a comment to each statement of the form `// variable = value` where `variable` is the variable name and `value` is the evaluated value.

a `var a = 2 + 1 / 10;` **b** `var time = 30 + 15;`

c `var F5 = 1 * 2 * 3 * 4 * 5;`

THINK	WRITE
a 1 The variable name is `a`.	**a**
2 The calculated value is 2 + 1/10 = 2.1.	
3 Add the comment to the statement.	`var a = 2 + 1 / 10; // a = 2.1`
b 1 The variable name is `time`.	**b**
2 The calculated value is 30 + 15 = 45.	
3 Add the comment to the statement.	`var time = 30 + 15; // time = 45`

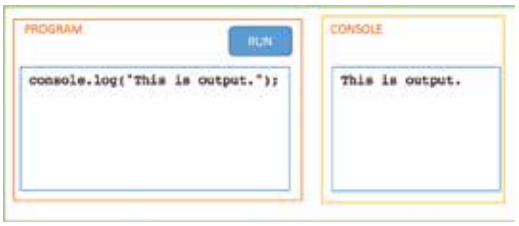
c 1 The variable name is `F5`. c

2 The calculated value is
$1 \times 2 \times 3 \times 4 \times 5 = 120$.

3 Add the comment to the statement. `var F5 = 1 * 2 * 3 * 4 * 5; // F5 = 120`

20.4.2 Output

- The **console** is a special region in a web browser for monitoring the running of JavaScript programs. Most web browsers, including Chrome, Firefox, Safari, Internet Explorer, Microsoft Edge and Opera, allow you activate the console through the menu options.
- The JavaScript console.log function can be used to see the result of an expression or value. This function outputs results to the console. A diagram of this using the given statement and output is shown on the next page.



  - Statement: `console.log("This is output in the console");`
  - Output: `This is the output in the console`
    Note: The quotation marks in strings are not shown in the output.

WORKED EXAMPLE 10

What is the output to the console after running the following statements?

a `console.log((16 + 4)/(11 - 1));`
b `console.log(("Test" + " " + "this").length);`
c `console.log("Con" + "sole out" + "put");`

THINK	WRITE
a 1 The expression `(16 + 4) / (11 - 1)` equals 2.	a
2 The statement simplifies to `console.log(2)`.	2
b 1 The expression `"Test" + " " + "this"` equals `"Test this"`. `"Test this"` has a length of 9.	b
2 The statement simplifies to `console.log(9)`.	9
c 1 The expression `"Con" + "sole out" + "put"` equals `"Console output"`.	c
2 The statement simplifies to `console.log("Console output")`.	
3 The `console.log` output does not show the quotation marks.	Console output

WORKED EXAMPLE 11

i Write a JavaScript statement to output to the console the following expressions.
ii Write the output.

a `(2 + 12) / (9 - 2)` **b** `"putter".substr(0,3)`

THINK		WRITE	
Think about the structure of the `console.log();` statement.			
a i	Put the expression inside the brackets.	**a i**	`console.log((2 + 12) / (9 - 2));`
ii	Write the output.	**ii**	`2`
b i	Put the expression inside the brackets.	**b i**	`console.log("putter".substr(0,3));`
ii	Write the output.	**ii**	`put`

20.4.3 Store and use a result

- A common sequence of statements is to:
  1. calculate an expression and store the result in a variable
  2. use the stored result in a new expression.

```
var storedForLater = "An" + "expression";
var newString = storedForLater + "with more";
// Output "An expression with more" console.log(newString);
```

WORKED EXAMPLE 12

For each statement in the following programs:
- **find the `variable` name between `var` and `=`**
- **calculate the `value` assigned to this `variable`.**

Append a comment to each statement indicating the progress. The comment should be of the form `// variable = value`.

a
```
var r = 10 / 2;
var area = r * r;
```
b
```
var a = 2 * 5;
var b = 20 + a;
var c = 30 + b;
var d = 4 * a * c - b * b;
```
c
```
var y = 2 + 8;
var y = 3 + 4;
var x = 3 * y;
```

THINK	WRITE
a 1 The variable is `r`. `10 / 2` is `5`.	**a** `var r = 10 / 2; // r = 5`
2 The variable is `area`. `5 * 5` is `25`.	`var area = r * r; // area = 25`
b 1 The variable is `a`. `2 * 5` is `10`.	**b** `var a = 2 * 5; // a = 10`
2 The variable is `b`. `20 + 10` is `30`.	`var b = 20 + a; // b = 30`
3 The variable is `c`. `30 + 30` is `60`.	`var c = 30 + b; // c = 60`
4 The variable is `d`. `4 * 10 * 60 - 30 * 30` is `1500`.	`var d = 4 * a * c - b * b; // d = 1500`
c 1 The variable is `y`. `2 + 8` is `10`.	**c** `var y = 2 + 8; // y = 10`
2 The variable is `y`. `3 + 4` is `7`. `y` now has a new value and the old value is lost from memory.	`var y = 3 + 4; // y = 7`
3 The variable is `x`. `3 * 7` is `21`.	`var x = 3 * y; // x = 21`

WORKED EXAMPLE 13

Manually simulate running the programs shown below. What is the final value stored in the variable `x`?

a
```
var a = 10 + 2;
var x = a * a;
```
b
```
var x1 = "one";
var x2 = "two";
var s = " ";
var x = x1 + s + x2;
```
c
```
var r = 4;
var r = 6;
var x = r + r;
```

THINK	WRITE
a 1 Write the equation.	**a** `a = 10 + 2`
2 Perform the addition.	`a = 12`
3 Write the equation (use `*` not ×).	`x = a * a`
4 Substitute `12` for `a`.	`x = 12 * 12`
5 Perform the multiplication.	`x = 144`
b 1 Write the first assignment.	**b** `x1 = "one"`
2 Write the second assignment.	`x2 = "two"`
3 Write the third assignment.	`s = " "`
4 Write the fourth equation.	`x = x1 + s + x2`
5 Substitute values into `x1`, `x2` and `s`.	`x = "one" + " " + "two"`
6 Evaluate the expression.	`x = "one two"`

c 1	Write the assignment.	`c r = 4`
2	Write the reassignment.	`r = 6`
3	Write the equation.	`x = r + r`
4	Substitute `6` for `r`.	`x = 6 + 6`
5	Evaluate the expression.	`x = 12`

20.4.4 Sequence

- The sequence of statements in a program is very important. Rearranging statements in a program can change the end result and/or cause an error.

WORKED EXAMPLE 14

a Find all the different programs that can be created from rearranging the three statements in the following table.

Reference	Statement
A	`var a = 2;`
B	`a = a + 3;`
C	`a = a * a;`

b Add `console.log(a);` to the end of each program.

c Find the end result for each of the programs.

THINK

a There six possible ways to arrange three statements.

ABC	ACB
BAC	BCA
CAB	CBA

b Add the `console.log(a)` statement to the end of each program.

WRITE

a

A
B
C
	`var a = 2;`
	`a = a + 3;`
	`a = a * a;`
		A
		C
		B
			`var a = 2;`
			`a = a * a;`
			`a = a + 3;`

B
A
C
	`a = a + 3;`
	`var a = 2;`
	`a = a * a;`
		B
		C
		A
			`a = a + 3;`
			`a = a * a;`
			`var a = 2;`

C
A
B
	`a = a * a;`
	`var a = 2;`
	`a = a + 3;`
		C
		B
		A
			`a = a * a;`
			`a = a + 3;`
			`var a = 2;`

b

A
B
C
 	
	`var a = 2;`
	`a = a + 3;`
	`a = a * a;`
	`console.log(a);`
		A
		C
		B
		
			`var a = 2;`
			`a = a * a;`
			`a = a + 3;`
			`console.log(a);`

B
A
C
 	
	`a = a + 3;`
	`var a = 2;`
	`a = a * a;`
	`console.log(a);`
		B
		C
		A
		
			`a = a + 3;`
			`a = a * a;`
			`var a = 2;`
			`console.log(a);`

C
A
B
 	
	`a = a * a;`
	`var a = 2;`
	`a = a + 3;`
	`console.log(a);`
		C
		B
		A
		
			`a = a * a;`
			`a = a + 3;`
			`var a = 2;`
			`console.log(a);`

c For each program, take the value of `a` through each step.

c

A
B
C
	`var a = 2;`
	`a = 2 + 3;`
	`a = 5 * 5;`
	The final result is `25`.
		A
		C
		B
			`var a = 2;`
			`a = 2 * 2;`
			`a = 4 + 3;`
			The final result is `7`.

B
A
C
	`a = a + 3;`
	`var a = 2;`
	`a = 2 * 2;`
	The final result is `4`.
		B
		C
		A
			`a = a + 3;`
			`a = a * a;`
			`var a = 2;`
			The final result is `2`.

C
A
B
	`a = a * a;`
	`var a = 2;`
	`a = 2 + 3;`
	The final result is `5`.
		C
		B
		A
			`a = a * a;`
			`a = a + 3;`
			`var a = 2;`
			The final result is `2`.

20.4.5 Computer languages

- Computers follow instructions called computer programs, which are written in a computer language.
- Each computer language has its own clear set of rules and structure called a syntax.
- There are many different computer languages. Some examples include JavaScript, TypeScript, Python, Ruby, C#, C, C++, R and PHP.
- Below are sections of programs using different computer languages. All these programs perform the same key tasks:
  - Assign the value `10` to the variable `x`.
  - Calculate `2 + x` (`x` is currently storing the value `10`).
  - Evaluate this to get the value `12`.
  - Assign the evaluated value `12` to the variable `y`.

JavaScript
`var x = 10;`
`var y = 2 + x;`
	TypeScript
	`var x:number = 10;`
	`var y:number = 2 + x;`
		Python and Ruby
		`x = 10`
		`y = 2 + x`

C#, C and C++
`int x = 10;`
`int y = 2 + x;`
	R
	`x <- 10`
	`y <- 2 + x`
		PHP
		`$x = 10;`
		`$y = 2 + $x;`

WORKED EXAMPLE 15

Consider the following R program.

```
p <- 4
r <- p + 3
```

Rewrite the program in JavaScript.

THINK

Compare the two example programs.

R	JavaScript
`x <- 10` `y <- 2 + x`	`var x = 10;` `var y = 2 + x;`

WRITE

R	JavaScript
`p <- 4` `r <- p + 3`	`var p = 4;` `var y = p + 3;`

Note the changes required to go from R to JavaScript statements.

- Add `"var"` to the start.
- Change `"<-"` to `"="`.
- Append `";"` to each statement.

learnON RESOURCES — ONLINE ONLY

Try out this interactivity: CodeBlocks Activity 3 (int-6575)

Exercise 20.4 Sequences

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–8, 12, 14	Questions: 1, 2a–c, 3, 4a–d, 5, 6, 7a–c, 8, 9, 12, 14, 15	Questions: 2d–f, 3, 4d–f, 6b, c, 7c–e, 8–16

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note*: Question numbers may vary slightly.

Fluency

1. The program statements in the table below relate to a rectangle. Match the following JavaScript comments with the appropriate statements.

```
// set the height to 15
// set the width to 10
// calculate half the perimeter
// calculate the area
// calculate the perimeter
```

Program	Comment
`var width = 10;`	
`var height = 15;`	
`var halfAround = width + height;`	
`var allAround = 2 * halfAround;`	
`var area = width*height;`	

2. WE10 What is the output to the console after running the following statements?

a.
```
console.log((18 + 2)/(6 - 1));
```

b.
```
console.log(("How long is" + " " + "this").length);
```

c.
```
console.log("No" +
  "quotation marks in console output");
```

d.
```
console.log(("This is d").substring(8,9));
```

e.
```
console.log(("Measure the length").length);
```

f.
```
console.log(("The length is 16").substr(14,2));
```

3. WE13 Manually simulate running the following programs. What is the final value stored in the variable `z`?

a.
```
var y = 14 + 16;
var z = y / 3;
```
b.
```
var x1 = "cat";
var x2 = "jumped";
var x3 = "slept";
var s = " ";
var z = x1 + s + x3;
```
c.
```
var radius = 1;
var radius = 22;
var z = radius + radius;
```

4. Manually simulate running the following programs. What will be the output to the console?

a.
```
var a = 10;
console.log(a * a + 5);
```
b.
```
var aString = "Ten";
console.log(aString.length * 3 + 1);
```
c.
```
var a = 3;
var b = 4;
var c = 5;
console.log(a * a + b * b - c * c);
```
d.
```
var single = "One";
var double = "two";
var three = single + " plus " + double;
console.log(three);
```
e.
```
var x = "10";
var y = "20";
console.log(x + y);
```
f.
```
var x = 10;
var y = 20;
var z = x + y;
console.log(y);
```

5. Match the JavaScript statements below with the equivalent C++ statements in the table.

```
var aVariable = 6 * 8 * 7 * 9;
var x = 10 + 3;
var y = 3 + 10;
var x = 10 + 4;
var x = 10 * 4;
```

C++ statements	JavaScript statements
`int y = 10 + 3;`	
`int x = 10 + 4;`	
`int x = 3 + 10;`	
`int x = 10 * 4;`	
`int aVariable = 9 * 8 * 7 * 6;`	

Understanding

6. WE9 To each statement, add a comment of the form `// variable = value`, where `variable` is the variable name and `value` is the evaluated value.

a.
```
var nearly_e = 2 + 7 / 10 + 1 / 100 + 8 / 1000;
```
b.
```
var marathonKm = 26 * 1.6;
```
c.
```
var secondsPerDay = 60 * 60 * 24;
```

7. WE11 Write a JavaScript statement to output the following expressions to the console.

a. `(3 + 41) / (9 + 2)`
b. `"putter".substr(1,5)`
c. `2 + 3 - 5`
d. `(2 * 12)/(8 - 2)`
e. `"life".substring(1,3)`

8. WE12 For each statement in the programs below:

- find the `variable` name between `var` and =
- calculate the `value` assigned to this `variable`.

Append a comment of the form `// variable=value` to each statement indicating the progress.

a.
```
var side1 = 3 * 4;
var side2 = side1 / 2;
var area = side1 * side2;
```
b.
```
var a = 5 + 2;
var b = a + a;
var c = b + b;
var delta = 4 * a * c - b * b;
```
c.
```
var reassign = 12 + 18;
var reassign = 1 + 41;
var result = 3 * reassign;
```

9. WE15 Rewrite the following R program in JavaScript.

```
r <- 8
r_less_5 <- r - 5
```

10. a. Rewrite the following JavaScript program in C.

```
var minutes = 34;
var seconds = minutes * 60;
```

b. Write a JavaScript program to convert 3 hours into minutes.

11. What is the most likely programing language of the following statements?

a.
```
var thisIsTyped:number = 101;
```
b.
```
storeMeHere <- 19
```
c.
```
$dollars = 10 + 13;
```
d.
```
var webLanguage = "JavaSc" + "ript";
```
e.
```
var newType:string = "Another type of " + "script";
```
f.
```
console.log("Nothing new here");
```

12. WE14 **a.** Find all the different programs that can be created from rearranging the following three statements.

Reference	Statement
A	`var e = 5;`
B	`e = (e + 7) * 2;`
C	`e = e / e;`

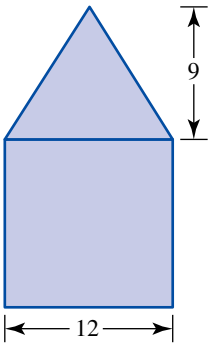
b. Add `console.log(e);` to the end of each program.

c. Find the end result for each of the programs.

Reasoning

13. Write a JavaScript program to calculate the `perimeter` of a rectangle with a `width` of `30` units and `length` of `20` units.

14. Fill in the gaps for the following JavaScript program to calculate the area of a composite shape. The shape is made up of a square of `width 12` units and triangle of `height 9` units.




```
var ______ = 12;
var _______ = 9;
var triangleArea = height * ______ / 2;
var squareArea = ______ * width;
var totalArea = squareArea + _____________;
```

Problem solving

15. Rearrange the following program so that it starts with `x = 3` and finishes with `x = 100`.

Hint: Assume `x` is always a positive whole number through the entire program sequence.

```
var x = 3;
x = x / 24;
x = x - 96;
x = x + 9;
x = 10 * x; (Hint: This step is in the correct position.)
x = 110 - x;
x = x - 10;
x = x * x;
x = 10
```

16. Fill in the single character gaps below using the nine digits `123456789` so the program finishes with `x = 1`. The `8` has already been filled in.

Hint: Assume `x` is always a positive whole number through the entire program sequence.

```
var x = _;
x = x / _;
x = x + _;
x = x / _;
x = x + 8;
x = x / _;
x = x + _;
x = x - _;
x = _ - x;
x = 1
```

Reflection

Why is the sequence of steps so important in a program?

20.5 Algorithms

20.5.1 Define a function

- In JavaScript a function is used to give a `name` to a section of code. The code below shows an example of the structure used to do this.

```
function name() {
  statement 1
  statement 2
  statement n
}
```

Note: All functions start and end with curly braces `{}` as shown in the example above.

WORKED EXAMPLE 16

i What is the name of each of the following functions?
ii What is the output to the console if the code inside the function was run?

a
```
function triangleArea() {
    var base = 10;
    var height = 3;
    console.log(base * height / 2);
}
```

b
```
function outputVolume() {
    var width = 3;
    var height = 4;
    var depth = 5;
    console.log(width * height * depth);
}
```

c
```
function complexFormula() {
    var w = 6;
    var h = 7;
    var d = 8;
    console.log(2 * w * h + 2 * w * d + 2 * h * d);
}
```

THINK

a i Identify the `name` after the function keyword.

ii 1 Write the first assignment.

2 Write the second assignment.

3 Write the output expression.

4 Substitute `10` for `base` and `3` for `height`.

5 Evaluate the output expression.

WRITE

a i `triangleArea`

ii `base = 10`

`height = 3`

`base * height / 2`

`= 10 * 3 / 2`

`= 15`

b i Identify the `name` after the function keyword.	**b i** `outputVolume`
ii 1 Write the first assignment.	**ii** `width = 3`
2 Write the second assignment.	`height = 4`
3 Write the third assignment.	`depth = 5`
4 Write the output expression.	`width * height * depth`
5 Substitute `3` for `width`, `4` for `height` and `5` for `depth`.	`= 3 * 4 * 5`
6 Evaluate the output expression.	`= 60`
c i Identify the `name` after the function keyword.	**c i** `complexFormula`
ii 1 Write the first assignment.	**ii** `w = 6`
2 Write the second assignment.	`h = 7`
3 Write the third assignment.	`d = 8`
4 Write the output expression.	`2 * w * h + 2 * w * d + 2 * h * d`
5 Substitute `6` for `w`, `7` for `h` and `8` for `d`.	`= 2 * 6 * 7 + 2 * 6 * 8 + 2 * 7 * 8`
6 Evaluate the output expression.	`= 292`

20.5.2 Calling a function

- Defining a function only tells the computer what to do if the function is **called**. In order for a function to run, it needs to be called with a separate statement.
- Once a function with the `name` has been defined, it can be called with the statement `name();`.

WORKED EXAMPLE 17

Given the following function names, what is the statement to call the function?

a `outputTime` **b** `updateResults` **c** `createNew`

THINK	WRITE
a Identify the function `name` and join with `();`.	**a** `outputTime();`
b Identify the function `name` and join with `();`.	**b** `updateResults();`
c Identify the function `name` and join with `();`.	**c** `createNew();`

- To be a valid program, a function must be defined before it can be called. A function can be called many times once defined, as shown in the following example code.

```
function name() {
  statement 1
  statement 2
  statement n
}
name();
name();
name();
```

WORKED EXAMPLE 18

Are the following valid programs? (Answer either true or false.)

a
```
function something() {
  console.log("Output");
}
something();
something();
```

b
```
function doesNothing() {
}
doesNothing();
```

c
```
wrongOrder();
function wrongOrder() {
  console.log("Does not work");
}
```

d
```
function function1() {
  console.log("Function 1");
}
console.log("In the middle");
function1();
```

THINK	WRITE
a This program is valid as `something` is defined before being called.	**a** True
b This program is valid as `doesNothing` is defined before being called.	**b** True
c This program is invalid as `wrongOrder` is called before being defined.	**c** False
d This program is valid as `function1` is defined before being called. A program is still valid if it has a statement between the definition and calling statement.	**d** True

WORKED EXAMPLE 19

a Write a function named `outputResults` to output `"Some results"` to the console.
b Write a statement to call the function.
c Combine the function definition and statement into one program.
d Write the output.

THINK	WRITE
a Identify the `name` of the function.	**a** `function outputResults() {`
Write the output statement.	`  console.log("Some results");`
Finalise the function definition.	`}`
b Write the calling statement by adding `();` to the end of the function.	**b** `outputResults();`

c Write the function definition.	**c** `function outputResults() {`
	`  console.log("Some results");`
	`}`

Write the calling statement.	`outputResults();`
d Write the output.	**d** `Some results`

20.5.3 Input

- In JavaScript a function can be given an `input` to vary the function's behaviour as shown below.

```
function name(input) {
  statement 1
  statement 2
  statement n
}
```

WORKED EXAMPLE 20

i What is the name of each of the following functions?
ii What is the name of the input?
iii What is the output to the console if the functions were called with the input value 10?

a
```
function outputDouble(inputValue) {
  console.log(2 * inputValue);
}
```
b
```
function outputVolume(side) {
  console.log(side * side * side);
}
```
c
```
function sixTimesSquared(x) {
  var squared = x * x;
  console.log(6 * squared);
}
```

THINK	WRITE
a i Identify the `name` after the `function` keyword.	**a i** `outputDouble`
ii Identify the `input` inside the function brackets ().	**ii** `inputValue`

iii	1	Write the output statement.	iii `console.log(2 * inputValue)`
	2	Substitute `10` for `inputValue`.	`console.log(2 * 10)`
	3	Evaluate the expression.	`console.log(20)`
	4	Write the output to console.	`20`
b i		Identify the `name` after the `function` keyword.	b i `outputVolume`
ii		Identify the `input` inside the function brackets ().	ii `side`
iii	1	Write the output statement.	iii `console.log(side * side * side)`
	2	Substitute `10` for `side`.	`console.log(10 * 10 * 10)`
	3	Evaluate the expression.	`console.log(1000)`
	4	Write the output to console.	`1000`
c i		Identify the `name` after the `function` keyword.	c i `sixTimesSquared`
ii		Identify the `input` inside the function brackets ().	ii `x`
iii	1	Write the assignment expression	iii `squared = x * x`
	2	Substitute `10` for `x`.	`squared = 10 * 10`
	3	Evaluate the assignment.	`squared = 100`
	4	Write the output statement.	`console.log(6 * squared)`
	5	Substitute `100` for `squared`.	`console.log(6 * 100)`
	6	Evaluate the expression.	`console.log(600)`
	7	Write the output to console.	`600`

- In JavaScript a function can be given zero, one or more inputs; for example, the inputs could be `x1,x2,…` as shown in the following code. The inputs are given as a list, separated by commas.

```
function name(x1,x2,…) {
  statement 1
  statement 2
  statement n
}
```

WORKED EXAMPLE 21

i How many inputs do each of the following functions have?
ii What are the names of the inputs? (Write these in order, with no spaces and separated with commas.)

a
```
function outputSquared(ignore,x) {
  console.log(x * x);
}
```

b
```
function outputVolume(w,h,d) {
  console.log(w * h * d);
}
```

c
```
function requiresNothing() {
  console.log("Always the same");
}
```

THINK	WRITE
a i Count the inputs inside the function brackets ().	**a i** 2
ii Identify the inputs inside the function brackets ().	**ii** `ignore, x`
b i Count the inputs inside the function brackets ().	**b i** 3
ii Identify the inputs inside the function brackets ().	**ii** `w, h, d`
c i Count the inputs inside the function brackets ().	**c i** 0
ii Identify that there are no inputs inside the function brackets ().	**ii** There are no inputs.

- A function with inputs `x1,x2,...` can be called with the corresponding values `v1,v2,....` The value `v1` is assigned to the input `x1`, the value `v2` is assigned to the input `x2`, etc.

```
function name(x1,x2,…) {
  statement 1
  statement 2
  statement n
}
name(v1,v2,…);
```

WORKED EXAMPLE 22

What is the output to the console of each of the following programs?

a
```
function outputCubed(x) {
  console.log(x * x * x);
}
outputCubed(3);
```

```
b function rectangleArea(w,h) {
    console.log(w * h);
  }
  rectangleArea(11,2);
c function noInputs() {
    console.log("Never changes");
  }
  noInputs();
```

THINK	WRITE
a 1 The input `x` equals `3`.	**a** `x = 3`
2 Write the output statement.	`console.log(x * x * x)`
3 Substitute `3` for `x`.	`console.log(3 * 3 * 3)`
4 Evaluate the expression	`console.log(27)`
5 Write the output to console.	`27`
b 1 The input `w` equals `11`.	**b** `w = 11`
2 The input `h` equals `2`.	`h = 2`
3 Write the output statement.	`console.log(w * h)`
4 Substitute `11` for `w` and `2` for `h`.	`console.log(11 * 2)`
5 Evaluate the expression.	`console.log(22)`
6 Write the output to console.	`22`
c 1 The function has no inputs.	**c**
2 Write the output statement.	`console.log("Never changes")`
3 Write the output to console.	`Never changes`

20.5.4 Return value

- A JavaScript function will execute until a return statement is reached, at which point the function stops running the section of code and output is returned to the 'caller'. The output can then be stored in a variable `stored`. The structure of the function and storing the result is shown below.

```
function name(x1,x2,...) {
  statement 1
  statement 2
  statement n
  return output;
}
var stored = name(v1,v2,...);
```

WORKED EXAMPLE 23

Manually simulate running each of the following programs. What is the final value stored in the variable x?

a
```
function volumePrism(height,area) {
    return height * area;
}
var x = volumePrism(10,40);
```

b
```
function mean(x1,x2,x3,x4) {
    total = x1 + x2 + x3 + x4;
    return total / 4;
}
var x = mean(2,4,6,8);
```

THINK		WRITE	
a	**1** Write the assignment.	**a**	`x = volumePrism(10, 40)`
	2 The input `h` equals `10`.		`height = 10`
	3 The input `area` equals `40`.		`area = 40`
	4 Write the return expression.		`volumePrism(10, 40) = height * area`
	5 Substitute `10` for `h` and `40` for `area`.		`volumePrism(10, 40) = 10 * 40`
	6 Evaluate the return expression.		`volumePrism(10, 40) = 400`
	7 Write the assignment.		`x = volumePrism(10, 40)`
	8 Substitute `400` for `volumePrism(10,40)`.		`x = 400`
b	**1** Write the assignment.	**b**	`x = mean(2, 4, 6, 8)`
	2 The input `x1` equals `2`.		`x1 = 2`
	3 The input `x2` equals `4`.		`x2 = 4`
	4 The input `x3` equals `6`.		`x3 = 6`
	5 The input `x4` equals `8`.		`x4 = 8`
	6 Write the first assignment inside the function.		`total = x1 + x2 + x3 + x4`
	7 Substitute `2` for `x1`, `4` for `x2`, `6` for `x3` and `8` for `x4`.		`total = 2 + 4 + 6 + 8`
	8 Evaluate the expression.		`total = 20`
	9 Write the return expression.		`mean(2, 4, 6, 8) = total / 4`
	10 Substitute `20` for `total`.		`mean(2, 4, 6, 8) = 20 / 4`
	11 Evaluate the return expression.		`mean(2, 4, 6, 8) = 5`
	12 Write the assignment.		`x = mean(2, 4, 6, 8)`
	13 Substitute `5` for `mean(2,4,6,8)`.		`x = 5`

20.5.5 Design an algorithm

- An **algorithm** is a step-by-step set of tasks to solve a particular problem. A program is the implementation of the algorithm.
- Designing an algorithm to solve a problem involves:
  - determining the inputs
  - determining the output required
  - breaking the problem into simple steps.

  These steps are not code, but an explanation of what the program is required to do at this point.

WORKED EXAMPLE 24

Design an algorithm to solve each of the problems below.

i Determine the inputs.
ii Determine the output required.
iii Break the problem into simple steps.
a Convert from a decimal to a percentage.
b Calculate the total area of a number of squares with the same side length.
c Calculate the probability of a picking a red marble from a bag of red marbles and blue marbles.

THINK			WRITE		
a	**i**	Write the input.	**a**	**i**	`decimal`
	ii	Write the output required.		**ii**	`percentage`
	iii	Write step 1.		**iii**	Find the percentage from a decimal.
b	**i**	Write the inputs.	**b**	**i**	`squares,length`
	ii	Write the output required.		**ii**	`area`
	iii	**1** Write step 1.		**iii**	Calculate area of each square.
		2 Write step 2.			Calculate total area.
c	**i**	Write the inputs.	**c**	**i**	`red,blue`
	ii	Write the output required.		**ii**	`probability`
	iii	**1** Write step 1.		**iii**	Find the total number of marbles.
		2 Write step 2.			Find the probability of a red marble.

20.5.6 Implementing an algorithm

- Implementing an algorithm in JavaScript involves:
  - designing an algorithm
  - writing function inputs
  - writing JavaScript statements for each step
  - returning the required result (output).

WORKED EXAMPLE 25

Implement an algorithm in JavaScript to solve each of the problems below.
a Convert from a decimal to a percentage.
b Calculate the total area of a number of squares with the same side length.
c Calculate the probability of a picking a red marble from a bag of red marbles and blue marbles.

THINK	WRITE
a 1 Design an algorithm.	**a** See Worked example 24a.
2 Write the function input.	`function toPercentage(decimal) {`
3 Write step 1.	`// Find percentage from decimal` `var percentage = 100 * decimal;`
4 Return the required result.	`return percentage;` `}`
b 1 Design an algorithm.	**b** See Worked example 24b.
2 Write the function inputs.	`function area(number,length) {`
3 Write step 1.	`// Calculate area of each square` `var squareArea = length * length;`
4 Write step 2.	`// Calculate total area` `var totalArea = number * squareArea;`
5 Return the required result.	`return totalArea;` `}`
c 1 Design an algorithm.	**c** See Worked example 24c.
2 Write the function inputs.	`function prob(red,blue) {`
3 Write step 1.	`// Find total number of marbles` `var marbles = red + blue;`
4 Write step 2.	`// Probability of a red marble` `var probability = red / marbles;`
5 Return the required result.	`return probability;` `}`

Exercise 20.5 Algorithms

assessON

Individual pathways

PRACTISE	CONSOLIDATE	MASTER
Questions: 1–9	Questions: 1, 2a–d, 3, 4a–c, 5a–c, 6–9, 11, 12	Questions: 2d–f, 3, 4c, d, 5b–d, 6, 7b, c, 8–14

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au. *Note*: Question numbers may vary slightly.

Fluency

1. WE16 **i.** What is the name of each of the following functions?

ii. What is the output to the console if each function was called?

a.
```
function rectangleArea() {
    var width = 30;
    var height = 56;
    console.log(width * height);
}
```

b.
```
function outputResults() {
    var result1 = 1;
    var result2 = 5;
    var result3 = 15;
    var sum = result1 + result2 + result3;
    console.log(sum / 3);
}
```

c.
```
function totalTimeMinutes() {
    var timeHours1 = 16;
    var timeHours2 = 71;
    console.log(60 * (timeHours1 + timeHours2) / 2);
}
```

2. WE17 Given the following function names, what is the statement to call each function?

a. `callThis`
b. `divideThenMultiply`
c. `outputMeanValue`
d. `outputTotal`
e. `outputCircleArea`
f. `calculateMode`

3. WE18 Are each of the following valid programs? (Answer either true or false.)

a.
```
function defineFirst() {
    console.log("Output");
}
defineFirst();
```

b.
```
function whyCallMe() {
}
whyCallMe();
```

c.
```
callLater();
function callLater() {
    console.log("Too late now");
}
```

d.
```
function definedFunction() {
    console.log("Output second and third");
}
console.log("Output first");
definedFunction();
definedFunction();
```

4. WE20 For each function below, answer the following questions.

i. What is the name of the function?

ii. What is the name of the input?

iii. What is the output to the console if the function was called with the input value 6?

a.
```
function outputTriple(input) {
    console.log(input + input + input);
}
```

b.
```
function outputArea(s) {
    console.log(s * s);
}
```
c.
```
function cubeVolume(length) {
    var area = length * length;
    console.log(length * area);
}
```
d.
```
function twoTimesSquare(a) {
    console.log(2 * a * a);
}
```

5. WE21 **i.** How many inputs do the each of following functions have?

ii. What are the names of the inputs for each function? (Write the inputs in order with no spaces and separated with commas.)

a.
```
function threeInputs(x,y,z) {
    console.log(x * x + y * y + z * z);
}
```
b.
```
function ignoreInputs(ignore1,ignore2) {
    console.log(10);
}
```
c.
```
function constantOuput(input) {
    console.log("Never changing");
}
```
d.
```
function sumMany(a0,a1,a2,a3,a4,a5,a6,a7) {
    console.log(a0 + a2 + a3 + a4 + a5 + a6 + a7);
}
```

Understanding

6. WE19 **a.** Write a function named `outputBedTime` to output to the console `"9:00PM"`.

b. Write a statement to call the function.

c. Combine the function definition and statement into one program.

7. WE22 What is the output to the console of each of the following programs?

a.
```
function outputPower5(x) {
    console.log(x * x * x * x * x);
}
outputPower5(3);
```
b.
```
function squareArea(side) {
    console.log(side * side);
}
squareArea(40);
```
c.
```
function oneInput(butIgnored) {
    console.log("No input required");
}
oneInput(23847324);
```

8. WE23 Manually simulate running the following program. What is the final value stored in the variable `x`?
```
function findArea(width,height) {
  return width * height;
}
var x = findArea(12,5);
```

9. Manually simulate running the following program. What is the final value stored in the variable `x`?

```
function multiply(x1,x2) {
  return x1 * x2;
}
function multiplyAddCombinations(a,b,c) {
  var combination1 = multiply(a,b);
  var combination2 = multiply(b,c);
  var combination3 = multiply(c,a);
  return combination1 + combination2 + combination3;
}
var x = multiplyAddCombinations(2,3,4);
```

10. Manually simulate running the following program. What is the final value stored in the variable `x`?

```
function f1() { return 1; }
function f2() { return 1; }
function f3() { return f1() + f2(); }
function f4() { return f2() + f3(); }
function f5() { return f3() + f4(); }
function f6() { return f4() + f5(); }
function f7() { return f5() + f6(); }
function f8() { return f6() + f7(); }
var x = f8();
```

Reasoning

11. WE24 Design an algorithm to solve each of the problems below.

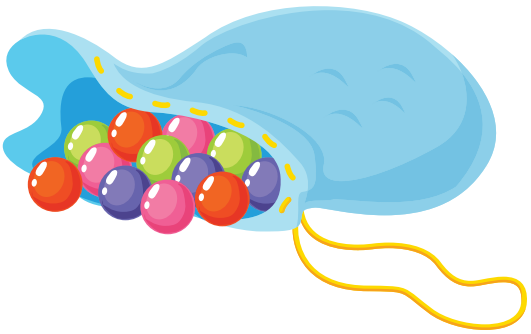
i. Determine the inputs.

ii. Determine the output required.

iii. Break the problem into simple steps.

a. Convert from a percentage to a decimal.

b. Calculate the probability of picking a red marble from a bag of red, purple, green and pink marbles.



12. WE25 Implement an algorithm in JavaScript to solve each of the problems below.

a. Convert from a percentage to a decimal.

b. Calculate the probability of picking a red marble from a bag of red, purple, green and pink marbles.

Problem solving

13. Write a function named `someFunction` that returns the following results when called.

Statement	y
`var y = someFunction(1);`	2
`var y = someFunction(12);`	156
`var y = someFunction(6);`	42
`var y = someFunction(7);`	56

14. Extension

Design and implement a program that accepts the inputs a, b and c. The program returns the value x, which solves the mathematical equation. $ax + b = c$.

Reflection

Where do you see applications of algorithms in everyday life?

20.6 Review

20.6.1 Review questions

Fluency

1. Classify the following values as number, string or Boolean.

a. `64`
b. `-2`
c. `2.34`
d. `"-22"`
e. `true`
f. `1222.3`
g. `"Some random text"`
h. `86`
i. `"true"`
j. `"999"`
k. `false`

2. Is each of the following a string value? (Answer either true or false.)

a. `"A string value"`
b. `This is a string`
c. `true`
d. `374`
e. `" 1234"`
f. `"1234"`
g. `false`
h. `value1 + value2`
i. `1.234`
j. `1.1.1.1.1`
k. `"3464812.9879879"`

3. Is each of the following a number value? (Answer either true or false.)

a. `"Forty"`
b. `"40"`
c. `2653`
d. `"0"`
e. `0`
f. `true`
g. `Eighteen`
h. `false`
i. `"238476"`
j. `-6.22`

4. Is each of the following a Boolean value? (Answer either true or false.)

a. `false`
b. `true`
c. `"true"`
d. `"false"`
e. `False`
f. `True`
g. `true`
h. `1`

5. Variable names:

1. must not start with a number
2. can only contain upper-case and lower-case letters, numbers and the underscore character _
3. cannot be JavaScript keywords. The following are some JavaScript keywords that should not be used as variable names: `true, var, false`.

Is each of the following a valid variable name? (Answer either true or false.)

a. `thisIsOk`
b. `output_results`
c. `output results`
d. `outputResults`
e. `1stResult`
f. `secondResult`
g. `3rdResult`
h. `four%two`
i. `no good`

6. Evaluate each of the following JavaScript expressions manually.

a. `2 + 31`
b. `84 - 34`
c. `-14 + 101`
d. `"No" + "t a number"`
e. `"Length ten".length`
f. `(17 + 3 * (14 - 6) / 2 - 2) / 9`
g. `11 * 7`
h. `18 * 2`
i. `(2 + 31) * 4 / 12`
j. `16 + (6 - 5) * 12`
k. `"Remember quot" + "ation marks"`
l. `(7 - 5) / 2 + (12 + 16) / 14`
m. `("Join" + "This").length`
n. `(12 + 13) / 5 + (4 + 16) / 4`

o. `200 / 40`

p. `"Join".length + "This".length`

7. Match the JavaScript comments below with the appropriate statements in the programs in the table.

```
// calculate the area of triangle 1
// calculate the area of triangle 2
// triangle 2 has half the height of triangle 1
// set the base of triangle 1 to 16
// triangle 2 has double the base of triangle 1
// set the height of triangle 1 to 10
```

Program	Comment
`var height1 = 10;`	
`var base1 = 16;`	
`var height2 = height1 / 2;`	
`var base2 = 2 * base1;`	
`var area1 = height1 * base1 / 2;`	
`var area2 = height2 * base2 / 2;`	

8. What is output to the console after running each of the following statements?

a. `console.log((38 + 12) / (16 - 6));`

b. `console.log(("Measure the " + " " + "length").length);`

c. `console.log("Console output has no " + "quotation marks");`

d. `console.log(("This is g").substring(8,9));`

e. `console.log(("Length is 12").substr(14,2));`

9. Manually simulate running the following programs. What is the final value stored in the variable `p`?

a.
```
var v = 4 + 96;
var p = v / 4;
```

b.
```
var y1 = "heals";
var y2 = "all";
var y3 = "time";
var s = " ";
var p = y3 + s + y1 + s + y2;
```

c.
```
var changing = 12;
var changing = 11;
var changing = 1;
var p = changing + changing + changing;
var changing = 2;
var x = changing;
```

10. Manually simulate running the following programs. What will be output to the console?

a.
```
var b = 11;
console.log(b * b + b);
```

b.
```
var fourteen = "14";
console.log(fourteen.length * 5 + 4);
```

c.
```
var x = 9;
var y = 12;
var z = 15;
console.log(x * x + y * y - z * z);
```

d.
```
var single = "One";
var double = "two";
var triple = "three";
var three = single + " plus " + double + " equals " + triple;
console.log(three);
```
e.
```
var x = "Java";
var y = "Script";
console.log(x + y);
```

11. Match the JavaScript statements below with the equivalent R statements in the table.

```
var y = 15;
var t = 10;
var y = 4 + 10;
var t = "10";
```

R statements	JavaScript statements
`t <- 10`	
`t <- "10"`	
`y <- 10 + 4`	
`y <- 4 + 11`	

12. **i.** What is the name of each of the following functions?
ii. What is the output to the console if each function was called?

a.
```
function averageWidthScaled() {
    var width1 = 30;
    var width2 = 50;
    var averageWidth = (width1 + width2) / 2;
    var scale = 40;
    console.log(averageWidth * scale);
}
```
b.
```
function averageResults() {
    var x1 = 31;
    var x2 = 335;
    var x3 = 1335;
    var x4 = 13;
    var Sum = x1 + x2 + x3 + x4;
    var Average = Sum / 4;
    console.log("Average");
}
```

13. Given the following function names, what is the statement to call each function?

a. `interestingName`
b. `outputTime`
c. `meanValue`

14. Are the following valid programs? (Answer either true or false.)

a.
```
function correctOrder() {
    console.log("Working");
}
correctOrder();
```
b.
```
function I_do_nothing() {
}
I_do_nothing();
```

c.
```
jumpTheGun();
function jumpTheGun() {
   console.log("False start");
}
```
d.
```
function callMeAgain() {
   console.log("Do it again");
}
console.log("First time");
callMeAgain();
callMeAgain();
callMeAgain();
```

15. **i.** What is the name of each of the following functions?
ii. What is the name of the input in each case?
iii. What is the output to the console if each function was called with the input value `7`?

a.
```
function doubleIt(doubleThis) {
   console.log(doubleThis + 2 * doubleThis - doubleThis);
}
```
b.
```
function outputCubeVolume(side) {
   console.log(side * side * side);
}
```

16. **i.** How many inputs do each of the following functions have?
ii. What are the names of the inputs in each case? (Write the inputs in order with no spaces and separated with commas.)

a.
```
function countThem(x,y) {
   console.log(x / x + y / y);
}
```
b.
```
function useInputs(use1,use2,use3) {
   var total = use1 + use2 + use3;
   console.log("Maybe ignore them");
}
```

17. What is the output to the console of the following program?
```
function power3(y) {
   console.log(y * y * y);
}
function power6(x) {
   console.log(power3(x) * power3(x));
}
power6(2);
```

18. Manually simulate running the program below. What is the final value stored in the variable `volume`?
```
function findVolume(width,height,depth) {
   return width * height * depth;
}
var volume = findVolume(3,6,8);
```

Problem solving

19. Manually simulate running the following program. What is the final value stored in the variable `x`?
```
function dice1(a,b,c,d,e,f) {
   return d;
}
```

```
function dice2(y1,y2,y3,y4,y5,y6){
   return y5;
}
function twoDice(z1,z2,z3,z4,z5,z6) {
   var role1 = dice1(z1,z2,z3,z4,z5,z6);
   var role2 = dice2(z1,z2,z3,z4,z5,z6);
   var total = role1 + role2;
   return total;
}
var x = twoDice(6,5,4,3,2,1);
```

20. Manually simulate running the following program. What is the final value stored in the variable `x`?

```
function f1() { return 1; }
function f2() { return 1; }
function f3() { return 1; }
function f4() { return f1() + f2() + f3(); }
function f5() { return f2() + f3() + f4(); }
function f6() { return f3() + f4() + f5(); }
function f7() { return f4() + f5() + f6(); }
var x = f7();
```

21. Design an algorithm to solve each of the problems below.

i. Determine the inputs.
ii. Determine the output required.
iii. Break the problem into simple steps.

a. Take the base to the power of 4 and return the value.
b. Find the mean of five numbers.
c. Calculate the missing angle of a triangle given the other two angles.

22. Implement an algorithm in JavaScript to solve each of the problems below.

a. Take the base to the power of 4 and return the value.
b. Find the mean of 5 numbers.
c. Calculate the missing angle of a triangle given the other two angles.

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Language

It is important to learn and be able to use correct mathematical language in order to communicate effectively. Create a summary of the topic using the key terms below. You can present your summary in writing or using a concept map, a poster or technology.

algorithm **console** **number**
Boolean **called** **string**
character **method** **variable**

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Answers

Exercise 20.2 Variables

1. **a.** Number **b.** Number **c.** Number **d.** String **e.** String **f.** String
g. Boolean **h.** String **i.** Boolean **j.** String **k.** Number **l.** Number
m. String **n.** Boolean **o.** String **p.** String **q.** Number **r.** String

2. **a.** False **b.** False **c.** True **d.** True **e.** False **f.** False **g.** False **h.** False **i.** False
j. False **k.** False **l.** True **m.** True **n.** True

3. **a.** True **b.** False **c.** False **d.** True **e.** False **f.** True **g.** True **h.** False **i.** True
j. False **k.** True **l.** False **m.** False **n.** True **o.** False

4. **a.** True **b.** True **c.** False **d.** False **e.** False **f.** False **g.** False **h.** False **i.** False

5. **a.** True **b.** True **c.** True **d.** True **e.** False **f.** False **g.** True **h.** False **i.** False
j. True **k.** True **l.** False **m.** True **n.** True **o.** False **p.** False **q.** True **r.** True
s. True **t.** True **u.** True **v.** False **w.** False

6.

	Variable	Value	Type	JavaScript statement
e.g.	`a_variable`	`"text"`	String	`var a_variable = "text";`
a	`x`	`10`	Number	`var x = 10;`
b	`nine`	`9`	Number	`var nine = 9;`
c	`aBoolean`	`true`	Boolean	`var aBoolean = true;`
d	`_`	`"A string"`	String	`var _ = "A string";`
e	`_long_var`	`true`	Boolean	`var _long_var = true;`
f	`areYouTall`	`false`	Boolean	`var areYouTall = false;`
g	`about_e`	`2.718281828459`	Number	`var about_e = 2.718281828459;`
h	`sin_90`	`1`	Number	`var sin_90 = 1;`
i	`isThis`	`"false"`	String	`var isThis = "false";`
j	`a_Boolean`	`true`	Boolean	`var a_Boolean = true;`

7. true, false

8. Yes

9. **a.** `var count = 10;`
b. `var information = "Data";`
c. `var threeSquared = 9;`
d. `var isThisJavaScript = true;`
e. `var meaningOfLife = 42;`
f. `var classSize = 25;`

10. **a.** `timeInSeconds <- 120`
b. `distanceInMiles <- 40`
c. `firstName <- "Ben"`
d. `middleName <- "Violet"`
e. `numberLeft <- 19`
f. `littleOverThree <- 3.1`

11. **a.** True **b.** True **c.** False **d.** False **e.** True **f.** False **g.** True

12.

	JavaScript assignment statement	Python assignment statement
a	`var x = 10;`	`x = 10`
b	`var nine = 9;`	`nine = 9`
c	`var aBoolean = true;`	`aBoolean = True`
d	`var _ = "A string";`	`_ = "A string"`
e	`var _longer_variable = true;`	`_longer_variable = True`
f	`var areYouTall = false;`	`areYouTall = False`
g	`var nearly4 = 3.99;`	`nearly4 = 3.99`
h	`var about_e = 2.718281828459;`	`about_e = 2.718281828459`
i	`var sin_90 = 1;`	`sin_90 = 1`
j	`var isThisABoolean = "false";`	`isThisABoolean = "false"`
k	`var is_this_a_Boolean = true;`	`is_this_a_Boolean = True`

13. • Avoid confusion with values and keywords.
- Keep the variable names readable.
- Avoid confusion with mathematical and other symbols used by the computer language.

14. a. i. violates rule 3
ii. Avoid confusion with the Boolean value `true`.
b. i. violates rule 3
ii. Avoid confusion with the Boolean value `false`.
c. i. violates rule 1
ii. Avoid confusion with the numerical value 1.
d. i. violates rule 1
ii. Avoid confusion with the numerical value 2.
e. i. violates rule 2
ii. Avoid confusion with the mathematical symbol negative.
f. i. violates rule 2
ii. The space indicates two different variable names.
g. i. violates rule 2
ii. The quotation marks indicate that is a string value.

15. a. `a`, `B`, `_` **b.** `aB`, `a_`, `a1`, `Ba`, `B_`, `B1`, `_a`, `_B`, `_1`
c. `aa`, `aB`, `a_`, `a1`, `Ba`, `BB`, `B_`, `B1`, `_a`, `_B`, `___`, `_1`

16. a. $26 + 26 + 1 = 53$ (26 upper-case letters and 26 lower-case letters; plus 1 symbol _)
b. $53 \times 63 - 3 = 3336$ (53 characters in the first position; 63 characters in the second position as the 10 digits can be included in this position; minus the 3 JavaScript keywords of length 2)
c. $53 \times 63 \times 63 - 7 = 210\,350$ (53 characters in the first position; 63 characters in the second and third position; minus the 7 JavaScript keywords of length 3)
d. $53 \times 63 \times 63 \times 63 - 16 = 13\,252\,475$ (53 characters in the first position; 63 characters in the second, third and fourth position; minus the 16 JavaScript keywords of length 4)

Exercise 20.3 Expressions

1. a. `15` **b.** `49` **c.** `13` **d.** `-25` **e.** `13` **f.** `48` **g.** `54` **h.** `24`

2. a. `4` **b.** `15` **c.** `16` **d.** `3` **e.** `3.8` **f.** `1` **g.** `19` **h.** `12` **i.** `0.5` **j.** `-2`

3. a. `"Quotation marks at the start and end"` **b.** `"This is a string"`
c. `"Hello World!"` **d.** `"Expressions make strings fun"`
e. `"Happy :)"` **f.** `"JavaScript"`
g. `"String expressions using brackets"` **h.** `"Remember the two spaces"`

4. a. `4` **b.** `10` **c.** `10` **d.** `43` **e.** `3`
f. `19` **g.** `7` **h.** `10` **i.** `"quick"` **j.** `"own"`
k. `"q"` **l.** `"substring"` **m.** `8`

5. a. `"four"` **b.** `"four"`
c. `"Take apart put together"` **d.** `6`
e. `"substring"`

6. a. `6 * 11` **b.** `54 / 3`
c. `12 / 5` **d.** `6 * (2 + 3)`
e. `(2 + 4) * (-3 + 5)` **f.** `(7 + 2) / (4 - 1)`
g. `3 * 7 + 3` **h.** `100 / 25 + 4 + 4 * 3`
i. `(12 + 2 * (4 - 2)) / 5` **j.** `6 * (2 + 5) * (4 - 1) - 3`
k. `(2 + 4) * (-2 + 6) / 3` **l.** `(2 + 2) / (3 - 2) * (1 + 2)/(4 - 1)`

7. a. i. `"This fox and sheep".substr(9,3)`
ii. `"This fox and sheep".substring(9,12)`
b. i. `"Find and back there".substr(5,3)`
ii. `"Find and back there".substring(5,8)`

c. i. `"Use hands".substr(5,3)`

ii. `"Use hands".substring(5,8)`

d. i. `"Search the land".substr(12,3)`

ii. `"Search the land".substring(12,15)`

e. i. `"ksanbjhandasdkjh".substr(7,3)`

ii. `"ksanbjhandasdkjh".substring(7,10)`

f. i. `"Earth and Mars".substr(6,3)`

ii. `"Earth and Mars".substring(6,9)`

8. a. `"x"` b. `"z"` c. `"5"` d. `"."`

9. a. `"MAKE ALL UPPER CASE"` b. `"make all lower case"`

c. `"A TITLE"` d. `"stop shouting"`

10. a. 2 b. 4 c. 0 d. 2 e. 1 f. 3

11. a.
```
var finalResult = 27 - (16 - 3);
```
b.
```
var finalResult = (3 + 23) / (10 + 3);
```
c.
```
var finalResult = "Assign" + "ed to finalResult";
```
d.
```
var finalResult = "Len".length + "gth".length;
```
e.
```
var finalResult = 18%5;
```

12. a.
```
var numberOfCarsEach = 12 / 3;
```
b.
```
var totalCash = 20 + 15;
```
c.
```
var myTitle = "My Trip".toUpperCase();
```

13. a.
```
var questionResult = 10 * 9 * 8 * 7 * 6 * 5 * 4 * 3 * 2 * 1;
```
b.
```
var questionResult = 2 + 4 + 6 + 8 + 10 + 12 + 14 + 16 + 18;
```
c.
```
var questionResult = ("Th" + "e" + " f" + "in" + "al re" + "sult is 2" +
  "2").length;
```

14. a. `1*23456789 or 23456789*1` b. `9642*87531 or 87531*9642`

15. Many answers are possible. A sample answer is shown.
```
var englishWord = "program";
var wordLength = englishWord.length;
var firstLetter = englishWord.
substr(0, 1);
var restOfWord = englishWord.substring
(1, wordLength);
var pigLatin = restOfWord + firstLetter + "ay";
console.log(pigLatin);
```

Exercise 20.4 Sequences

1.

Program	Comment
`var width =  10;`	`// set the width to 10`
`var height = 15;`	`// set the height to 15`
`var halfAround = width + height;`	`// calculate half the perimeter`
`var allAround = 2 * halfAround;`	`// calculate the perimeter`
`var area = width * height;`	`// calculate the area`

2. a. 4 b. 16 c. No quotation marks in console output

d. d e. 18 f. 16

3. a. 10 b. `"cat slept"` c. 44

4. a. 105 b. 10 c. 0

d. One plus two e. 1020 f. 20

5.

C++ statements	JavaScript statements
`int y = 10 + 3;`	`var y = 3 + 10;`
`int x = 10 + 4;`	`var x = 10 + 4;`
`int x = 3 + 10;`	`var x = 10 + 3;`
`int x = 10 * 4;`	`var x = 10 * 4;`
`int aVariable = 9 * 8 * 7 * 6;`	`var aVariable = 6 * 8 * 7 * 9;`

6. a.
```
var nearly_e = 2 + 7 / 10 + 1 / 100 + 8 / 1000; // nearly_e = 2.718
```
b.
```
var marathonKm = 26 * 1.6; // marathonKm = 41.6;
```
c.
```
var secondsPerDay = 60 * 60 * 24; // secondsPerDay = 86400
```

7. a.
```
console.log((3 + 41) / (9 + 2));
```
b.
```
console.log("putter".substr(1,5));
```
c.
```
console.log(2 + 3 - 5);
```
d.
```
console.log((2 * 12)/(8 - 2));
```
e.
```
console.log("life".substring(1,3));
```

8. a.
```
var side1 = 3 * 4; // side1 = 12
var side2 = side1 / 2; // side2 = 6
var area = side1 * side2; // area = 72
```
b.
```
var a = 5 + 2; // a = 7
var b = a + a; // b = 14
var c = b + b; // c = 28
var delta = 4 * a* c - b * b;
// delta = 588
```
c.
```
var reassign = 12 + 18; // reassign = 30
var reassign = 1 + 41; // reassign = 42
var result = 3 * reassign; // result = 126
```

9.
```
var r = 8;
var r_less_5 = r - 5;
```

10. a.
```
int minutes = 34;
int seconds = minutes * 60;
```
b.
```
var hours = 3;
var minutes = hours * 60;
```

11. a. TypeScript **b.** R **c.** PHP **d.** JavaScript **e.** TypeScript **f.** JavaScript

12. a and **b**

A
B
C
	`var e = 5;`
	`e = (e + 7) * 2;`
	`e = e / e; console.log(e);`
		A
		C
		B
			`var e = 5;`
			`e = e / e;`
			`e = (e + 7) * 2; console.log(e);`

B
A
C
	`e = (e + 7) * 2;`
	`var e = 5;`
	`e = e / e; console.log(e);`
		B
		C
		A
			`e = (e + 7) * 2;`
			`e = e / e;`
			`var e = 5; console.log(e);`

C
A
B
	`e = e / e;`
	`var e = 5;`
	`e = (e + 7) * 2; console.log(e);`
		C
		B
		A
			`e = e / e;`
			`e = (e + 7) * 2;`
			`var e = 5; console.log(e);`

c.

A B C	1	A C B	16
B A C	1	B C A	5
C A B	24	C B A	5

13.
```
var width = 30;
var length = 20;
var perimeter = 2 * width + 2 * length;
```
14.
```
var width = 12;
var height = 9;
var triangleArea = height * width / 2;
var squareArea = width * width;
var totalArea = squareArea + triangleArea;
```
15.
```
var x = 3;
x = x + 9;
x = x * x;
x = x - 96;
x = 10 * x;
x = x / 24;
x = x - 10;
x = 110 - x;
```
16.
```
var x = 9;
x = x / 3;
x = x + 1;
x = x / 2;
x = x + 8;
x = x / 5;
x = x + 7;
x = x - 6;
x = 4 - x;
```

Exercise 20.5 Algorithms

1. **a. i.** `rectangleArea` **ii.** 1680
   b. i. `outputResults` **ii.** 7
   c. i. `totalTimeMinutes` **ii.** 2610
2. **a.** `callThis();` **b.** `divideThenMultiply();` **c.** `outputMeanValue();`
   d. `outputTotal();` **e.** `outputCircleArea();` **f.** `calculateMode();`
3. **a.** True **b.** True **c.** False **d.** True
4. **a. i.** `outputTriple` **ii.** `input` **iii.** 18
   b. i. `outputArea` **ii.** `s` **iii.** 36
   c. i. `cubeVolume` **ii.** `length` **iii.** 216
   d. i. `twoTimesSquare` **ii.** `a` **iii.** 72
5. **a. i.** 3 **ii.** `x,y,z`
   b. i. 2 **ii.** `ignore1,ignore2`
   c. i. 1 **ii.** `input`
   d. i. 8 **ii.** `a0,a1,a2,a3,a4,a5,a6,a7`
6. **a.**
```
function outputBedTime() {
    console.log("9:00PM");
}
```
   b.
```
outputBedTime();
```
   c.
```
function outputBedTime() {
    console.log("9:00PM");
}
outputBedTime();
```

7. a. `243` **b.** `1600` **c.** `No input required`

8. `60`

9. `26`

10. `21`

11. a. i. `percentage` **ii.** `decimal` **iii.** Find percentage from decimal.

b. i. `red,purple,green,pink`

ii. `probability`

1. Find total number of marbles.

2. Find probability of picking a red marble.

12. a.
```
function toDecimal(percentage) {
  var decimal = percentage / 100;
  return decimal;
}
```

b.
```
function calculateProbability(red,purple,green,pink) {
  // Find total number of marbles
  var marbles = red+purple+green+pink;
  // Probability of a red marble
  var probability = red / marbles;
  return probability;
}
```

13.
```
function someFunction(x) {
  return x * x + x;
}
```

14.
```
function xSolve(a,b,c) {
  // Algebraically solve for x
  // a * x + b = c
  // a * x + b - b = c - b
  // a * x = c - b
  // a * x / a = (c - b) / a
  // x = (c - b) / a
  // Calculate x given a, b and c
  return (c - b) / a;
}
```

20.6 Review

1. a. Number **b.** Number **c.** Number **d.** String **e.** Boolean **f.** Number
g. String **h.** Number **i.** String **j.** String **k.** Boolean

2. a. True **b.** False **c.** False **d.** False **e.** True **f.** True
g. False **h.** False **i.** False **j.** False **k.** True

3. a. False **b.** False **c.** True **d.** False **e.** True **f.** False
g. False **h.** False **i.** False **j.** True

4. a. True **b.** True **c.** False **d.** False **e.** False **f.** False
g. True **h.** False

5. a. True **b.** True **c.** False **d.** True **e.** False **f.** True
g. False **h.** False **i.** False

6. a. 33 **b.** `50`
c. 87 **d.** `"Not a number"`
e. 10 **f.** 3
g. 77 **h.** 36

i. 11 **j.** 28

k. `"Remember quotation marks"` **l.** 3

m. 8 **n.** 10

o. 5 **p.** 8

7.

Program	Comment
`var height1 = 10;`	`// set the height of triangle 1 to 10`
`var base1 = 16;`	`// set the base of triangle 1 to 16`
`var height2 = height1 / 2;`	`// triangle 2 has half the height`
`var base2 = 2 * base1;`	`// triangle 2 has double the base`
`var area1 = height1 * base1 / 2;`	`// calculate the area of triangle 1`
`var area2 = height2 * base2 / 2;`	`// calculate the area of triangle 2`

8. a. `5`

b. `18`

c. `Console output has no quotation marks`

d. `g`

e. `12`

9. a. `25` **b.** `"time heals all"` **c.** `3`

10. a. `132` **b.** `14` **c.** `0`

d. `One plus two equals three` **e.** `JavaScript`

11.

R statements	JavaScript statements
`t <- 10`	`var t = 10;`
`t <- "10"`	`var t = "10";`
`y <- 10 + 4`	`var y = 4 + 10;`
`y <- 4 + 11`	`var y = 15;`

12. a. i. `averageWidthScaled` **ii.** `1600`

b. i. `averageResults` **ii.** `Average`

13. a. `interestingName();` **b.** `outputTime();` **c.** `meanValue();`

14. a. True **b.** True **c.** False **d.** True

15. a. i. `doubleIt` **ii.** `doubleThis` **iii.** `14`

b. i. `outputCubeVolume` **ii.** `side` **iii.** `343`

16. a. i. `2` **ii.** `x,y`

b. i. `3` **ii.** `use1,use2,use3`

17. `64`

18. `144`

19. `5`

20. `17`

21. a. i. `base` **ii.** `value`

iii. 1) Multiple base by base by base by base.

b. i. `x1,x2,x3,x4,x5` **ii.** `mean`

iii. 1) Find the total of `x1,x2,x3,x4,x5.` **iv.** 2) Divide the total by 5.

c. i. `angle1,angle2` **ii.** `angle3`

iii. 1) Find the total of known angles. **iv.** 2) Subtract the above total from 180.

22. a.
```
function toPower4(base) {
    // Multiple base by base by base by base
    var value = base * base * base * base;
    return value;
}
```

b.
```
function findMean(x1,x2,x3,x4,x5) {
    // Find the total of x1,x2,x3,x4,x5
    var total = x1 + x2 + x3 + x4 + x5;
    // Divide the total by 5
    var mean = total / 5;
    return mean;
}
```

c.
```
function findMissingAngle(angle1,angle2) {
    // Find the total of known angles
    var knownTotal = angle1 + angle2;
    // Subtract the above total from 180
    var angle3 = 180 - knownTotal;
    return angle3;
}
```

GLOSSARY

acute angle: angle greater than 0° but less than 90°
acute-angled triangle: triangle where each angle is acute; that is, less than 90°
adjacent angles: angles lying next to another angle
algebra: the branch of mathematics that deals with solving mathematical problems involving an unknown value
algorithm: a step-by-step set of tasks to solve a particular problem. A program is an implementation of an algorithm.
allied angles: see *co-interior angles*
alternate angles: angles on opposite sides of the transversal positioned between the parallel lines. These angles are equal in size.
angle: rotation between a pair of lines that meet at a point
area: amount of surface enclosed by the shape; measured in square units, such as square metres, m^2, and square kilometres, km^2
area references: grid references on a map that refer to a whole square and not a single point
arithmagon: triangular figure in which the two numbers at the ends of each line add to the number along the line
arms: lines that meet at a point to form an angle
associative law for addition: a number law that refers to the order in which three numbers may be added or subtracted
associative law for multiplication: a number law that refers to the order in which three numbers may be multiplied or divided
axes: horizontal and vertical lines with scales that form the Cartesian plane
axis of symmetry: line that divides a shape into two identical parts
backtracking: the process of working backwards through a flowchart. Each step is the inverse operation of the corresponding step in the flowchart. It can be used to solve equations.
bar graph: a graph that has horizontal bars instead of vertical columns. Categories are graphed on the vertical axis and the frequencies (numbers) on the horizontal axis; compare *column graph*
base: the number being repeatedly multiplied in a power term. The power is the number of times the base is written.
BIDMAS: the order in which calculations are performed. The order is brackets; index (or power); division, multiplication from left to right; addition and subtraction from left to right.
bird's eye view: view observed from the top of the object looking down on it; also *top view*
Boolean: a JavaScript data type with two possible values: `true` or `false`. JavaScript Booleans are used to make logical decisions.
called: in programming, a defined function is called from another part of the program. Calling a function will run the statements defined inside the function. If a function has a name it is called with the statement `name();`.
capacity: maximum amount of fluid that can be contained in an object. The units may be the same as for volume or litres and millilitres may be used.
Cartesian plane: area formed by a horizontal line with a scale (x-axis) joined to a vertical line with a scale (y-axis). The point of intersection of the lines is called the origin.
centre of rotation: a specific point around which an object is rotated
certain: in probability, an outcome that will happen
character: in programming, a string of length `1`. A JavaScript character is used to represent a letter, digit or symbol.
coefficient: the number written in front of a term

co-interior angles: on the same side of the transversal positioned between the parallel lines. These angles are supplementary; also *allied angles*

column graph: graph in which equal width columns are used to represent the frequencies (numbers) of different categories

common factor: a factor common to more than one number

commutative law for addition: a number law that refers to the order in which two numbers may be added or subtracted

commutative law for multiplication: a number law that refers to the order in which two numbers may be multiplied or divided

complementary angles: sum (add) to 90°

composite number: number with more than two factors

composite shape: figure made up of more than one basic shape

congruent figures: figures that have exactly the same size and shape

console: a special region in a web browser for monitoring the running of JavaScript programs

constant: a value that is not a variable

conversion factors: in measurement, they are used to convert one unit of measurement to another

coordinates: on a number place, two numbers that give the position of a point on the Cartesian plane. The first number is the x-coordinate and the second number is the y-coordinate.

corresponding angles: angles on the same side of the transversal, and which are both either above or below the parallel lines. The angles are equal in size.

cube numbers: numbers with three equal factors; for example, $8 = 2 \times 2 \times 2$ and $27 = 3 \times 3 \times 3$

cube root ($\sqrt[3]{\ }$): number that multiplies by itself three times to equal the original number. Finding a cube root is the opposite of cubing a number.

decomposition method: a method of subtraction. The larger number is decomposed by taking 10 from the tens column and adding it to the units column. It may also involve taking 100 from the hundreds column and adding it to the tens column.

denominator: the bottom term of a fraction. It shows the total number of parts into which the whole has been divided.

difference: the answer obtained when two numbers are subtracted

digits: first nine counting numbers and zero

directed number: a number that has both size and direction, for example, $+3$ and -7

distributive law: a rule that states that each term inside a pair of brackets is to be multiplied by the term outside the brackets

dividend: the quantity in a division that is to be divided

divisor: the quantity in a division by which the dividend is divided

dot plot: consists of a horizontal axis labelled and evenly scaled, with each data value represented by a dot

equation: mathematical statement containing a left-hand side, a right-hand side and an equals sign between them

equilateral: describes a triangle with all sides equal in length. All angles will be equal too.

even chance: in probability, an outcome that has a 50% chance of happening

event: in probability, a result that may be obtained

experiment: the process in probability of performing repeated trials of an activity for obtaining data in order to be able to predict the chances of certain things happening

experimental probability: probability determined by conducting an experiment and gathering data

expression: mathematical statement made up of pronumerals (letters) and numbers. Expressions do not contain an equals sign.

exterior angle: angle formed when any side of a closed shape is extended outwards. The exterior angle and the interior angle adjacent to it are supplementary (add up to 180°).

factor: a whole number that divides exactly into another whole number, without any remainder
factor tree: a diagram that displays the prime factors of a composite number. Each branch shows a factor of all numbers above it. The last numbers at the bottom of the tree are the prime factors of the original number.
favourable outcome: in probability, the result that we are looking for, the desired outcome
finite decimal: a decimal with a fixed number of decimal places
flowcharts: diagrams that illustrate the process by which an input number becomes an output number in an equation or formula
formula: rule that is written using pronumerals
fraction: used to describe parts of a whole number
frequency: number of times a result or piece of data occurs
frequency distribution table: table used to organise data by recording the number of times each data value occurs.
front view: view observed from directly in front of an object
highest common factor (HCF): largest factor that is common to all the numbers given
impossible: in probability, an outcome that can never happen
improper fraction: fraction whose numerator is larger than its denominator, for example, $\frac{3}{4}$
index (indices): indicates the number of times a base is to be multiplied when written in index form; also *power*
input number: the first number in a flowchart
integers: positive whole numbers, negative whole numbers and zero
interior angle: angle inside a shape
inverse operation: operation that 'undoes' a previous operation. Addition and subtraction are inverse operations; and multiplication and division are inverse operations of each other.
irregular quadrilateral: quadrilateral that does not have any special properties
isometric: a transformation that does not alter the size or shape of the original object
isometric drawing: a 2-dimensional drawing of a 3-dimensional object
isosceles triangle: describes a triangle that has two sides of equal length. The two angles opposite the equal sides will also be equal.
kite: quadrilateral in which two pairs of adjacent sides are equal in length and one pair of opposite angles (those between the sides of unequal length) are equal
legends: definitions of different symbols or colours used when drawing graphs
like terms: terms that contain exactly the same pronumeral (letter) part; for example, $5a$ and $33a$ are like terms but $4ab$ is not.
long-term trend: in probability, the trend observed for results from a very large number of trials. This tends to match the results of theoretical probability.
lowest common multiple (LCM): lowest multiple that two or more numbers have in common
mean: in summary statistics, the sum of all the values (scores) divided by the number of values — also called the average
median: in summary statistics, the middle value if the number of data is odd; or the average of the two middle values if the number of data is even. Data must first be in numerical order.
method: in JavaScript, a defined function applied to an object. For example, the method indexOf can be applied to a string, `"string".indexOf("in")`, to find the index of the substring `"in"`.
mixed number: number that is made up of a whole number and a fraction, for example $2\frac{3}{4}$
mode: in summary statistics, the most common value in a set of data
multiple: an answer obtained when a number is multiplied by another whole number; for example, 4, 8, 12, 16 are multiples of 4
negative integer: whole numbers less than zero
number (programming): a JavaScript data type that represents a numerical value

numerator: top term of a fraction. It shows how many parts there are.
obtuse angle: angle greater than 90° but less than 180°
obtuse-angled triangle: triangle that has one angle greater than 90°
opposite integer: numbers equidistant from zero, but one is positive and one is negative
ordered pair: in a number plane, see *coordinates*
ordered pair: in probability, the pair of items written in brackets separated by a comma; for example, when tossing a coin and rolling a die, an outcome might be a 'Head' and a '6'. This would be written as (H, 6) in ordered pair notation.
origin: point where the x-axis and the y-axis intersect on the Cartesian plane; that is, the point with coordinates $(0, 0)$
outcome: in probability, the particular result of an experiment
outlier: extreme value in a set of data
output number: the last number in a flowchart
pan balance scale: a scale with two pans that can be used to show whether things have equal mass. It can be used in Mathematics to show whether two expressions are equal in value.
parallelogram: quadrilateral with both pairs of opposite sides parallel to each other. Rectangles, squares and rhombuses are parallelograms.
per cent: the amount out of 100, or *per hundred*; for example, 50% means 50 out of 100 or $\frac{50}{100}$
perigon: angle that equals 360° (a full circle); also *revolution*
perimeter: distance around the outside (border) of a shape
perpendicular: at right angles (90°)
place value: the position of a digit in a number
positive integers: whole numbers greater than zero
power: see indices
prime factors: factors that are only prime numbers
prime number: counting number that has exactly two factors, itself and 1
probability: likelihood or chance of a particular event (result) occurring
product: result of a multiplication
pronumeral: letter used in place of a number
proper fraction: fraction whose numerator is smaller than its denominator, for example $\frac{3}{4}$
protractor: instrument used to measure angles in degrees
quadrant: one of four regions of the Cartesian plane produced by the intersection of the x- and y-axes
quadrilateral: 2-dimensional, closed shape formed by four straight lines
quotient: result of a division
random numbers: a selection of numbers that follow no particular order or pattern
range: in summary statistics, the difference between the highest and lowest values (scores)
rational numbers: numbers that can be written as fractions, where the denominator is not zero
ratio: a comparison of two or more quantities of the same kind
rational numbers: numbers that can be written as fractions, where the denominator is not zero
reciprocal: the reciprocal of a number is obtained by first expressing it as a fraction then tipping the fraction upside down; for example, $\frac{3}{4}$ is the reciprocal of $\frac{3}{4}$. A mixed number must be converted to an improper fraction first and then tipped.
rectangle: parallelogram in which the opposite sides are equal in length and all angles are equal to 90°
recurring decimals: decimals that have one or more digits repeated continuously; for example, 0.999… They can be expressed exactly by placing a dot or horizontal line over the repeating digits as in this example. $8.343\,434 = 8.\overline{34}$ or $8.\overline{34}$
reflection: exact image of an object, as seen in a mirror; as far behind the mirror as the object is in front of it. Reflections always have reversed orientations; right appears left and left appears right.
reflex angle: angle greater than 180° but less than 360°

revolution: see *perigon*
rhombus: a parallelogram in which all sides are equal and opposite angles are equal; plural *rhombi*
right angle: angle that equals 90°
right-angled triangle: triangle that has one of its angles equal to 90° (a right angle). A small square is placed in that angle to mark the right angle.
rotation: the turning of an object about a certain point — the centre of rotation
rounding: technique used to give an approximate answer to a question. If the second digit is 0, 1, 2, 3 or 4, the first digit stays the same. If the second is 5, 6, 7, 8 or 9, the first digit is rounded up to the next value. This also applies to decimals. For example, 2.456 rounded to 2 decimal places is 2.46.
rule: pattern that links a set of input numbers to a set of output numbers in algebra; for example, 'adding 5' is a rule
sample space: in probability, the complete set of outcomes or results obtained from an experiment. It is shown as a list enclosed in a pair of braces (curled brackets) {}.
scale: a series of marks used to measure length, mass, temperature or any other quantity
scalene: describes a triangle that has no equal sides. The angles will all be unequal too.
sequence: a number pattern
side view: view observed when standing directly to one side of an object. The left view or the right view of the object can be drawn
simulation (probability): experiment that uses simple devices such as coins, dice, a pack of cards and spinners to model or mimic a real life situation
square: parallelogram in which all sides are equal and all angles are equal to 90°
square numbers: numbers that have two equal factors; for example, $9 = 3 \times 3$ and $36 = 6 \times 6$
square root ($\sqrt{}$): number that multiplies by itself to equal the original number. Finding a square root is the opposite of squaring a number; for example, $\sqrt{47} = 7$.
straight angle: angle that equals 180°
string: a JavaScript data type that represents text
substitution: process by which a number replaces a pronumeral in a formula
successful trial: a trial that results is the outcome you wanted
sum: answer obtained when numbers are added together
supplementary angles: sum to 180°
term: part of an equation or expression. Terms can be expressed as one or more than one pronumeral, or a number only. Examples include $5x$, $3ab$, xyz, 8 and so on.
theoretical probability: probability determined by predicting outcomes rather than by gathering data
3-dimensional object: shape that has depth, as well as length and width
top view: see *bird's eye view*
transformation: ordered set of directions that enables a shape to be moved from one place to another. At the start the shape is called the original object and after transformation it is called the image. Examples of transformations are translations, reflections and rotations.
translation: horizontal (left/right) or vertical (up/down) movement
transversal: a line that intersects a pair (a set) of parallel lines
trapezium: quadrilateral in which one pair of opposite sides is parallel
trial: number of times a probability experiment is conducted
triangle: 2-dimensional, closed shape formed by three straight lines
2-dimensional representation: plan or net that can be folded to form a polyhedron
unitary method: a method comparing the unit cost of varying quantities of items to help determine the best buy
unit price: the price for a single unit of the item in question. This can be the price of a specified quantity (e.g. 1 kg or 100 mL) or of a single unit (e.g. 1 banana).

variable: a symbol in an equation or expression that may take many different values
variable (programming): a named container or memory location that holds a value
vertex: the point where two lines meet; plural *vertices*
vertically opposite angles: special angles formed when two straight lines intersect. The two non-adjacent angles are called vertically opposite angles. These angles are equal in size.
vinculum: horizontal bar separating the numerator from the denominator
volume: amount of space a 3-dimensional object occupies. The units used are cubic units, such as cubic metres, m^3, and cubic centimetres, cm^3. A volume of $1\,m^3$ is the space occupied by a cube of side length 1 m.
whole number: a counting number: 0, 1, 2, 3, 4, …
***x*-axis:** horizontal line with a scale that is part of the Cartesian plane
***y*-axis:** vertical line with a scale that is part of the Cartesian plane

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